Design Exercise 02 - Design of a Gear-Driven Speed Reducer Shaft

Team Member Name	Student ID
Christopher Jin	2018141521058
Ian Li	2018141521045

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1 System Descriptions

A conveyor system as shown in Figure 1 is a fast and efficient mechanical handling apparatus for automatically transporting loads and materials within an area. This system minimizes human error, lowers workplace risks and reduces labor costs — among other benefits. They are useful in helping to move bulky or heavy items from one point to another. A conveyor system may use a belt, wheels, rollers, or a chain to transport objects (Berg, 2016).

Figure 1: A classic conveyor system.

In this project, we need to design a pair of spur gears that are used to not only transmit the power but also change the speed of shaft into desired values in the conveyor system so that the motor can drive the conveyor belt in a desired speed.

2 Design Objectives

There are three main goals in our design of a gearset as shown below:

1. Determine the preliminary design of the gearset according to the layout.
2. Final design needs to survive both pitting and bending fatigue.
3. Final design needs to exhibit adequate contact ratio.

3 Design Limitations

3.1 Design Constraints

1. The center distance is to be as small as reasonably possible.
2. A life of 5 years of 2000 hours/year operation is desired, but full power will be transmitted only about 10% of the time, with half power the other 90%
3. Likelihood of failure during the 5 years should not exceed 5%.
4. My design is based on full power operation time.
5. Choose hardened-steel for the spur gears. Steel gear material will be selected to provide relatively high strength at relatively low cost.
6. The pinion and gear will be machined and then ground. In accordance with good practice, a case-hardening procedure will be specified that will leave compressive residual stresses in the gear-tooth surfaces.
7. Specify high surface hardness of 660 Bhn for pinion to obtain the minimum center distance. Keep pinion-tooth hardness about 10 percent higher than the gear-tooth hardness.
8. Choose the common 20° full-depth involute tooth form.
9. Safety factor of 1.25 for failure by surface fatigue.
10. Final design needs to survive both pitting and bending fatigue.
11. Final design needs to exhibit adequate contact ratio.

3.2 Design Assumptions

To simplify our model and eliminate the complexity, we make the following main assumptions in this literature. All assumptions will be re-emphasized once they are used in the construction of our model.

1. Shock loading from the motor and driven machine is negligible.
2. Surface pitting is the primary failure mode and pinion should fail before gear.
3. The full load is applied to the tip of a single tooth.
4. The radial component, W_r, is negligible.

5. The load is distributed uniformly across the full face width, which is a nonconservative assumption.
6. Forces which are due to tooth sliding friction are negligible.
7. Stress concentration in the tooth fillet is negligible.
8. The pinion-tooth hardness is about 10 percent higher than the gear-tooth hardness in Brinell scale.
9. The gearset we design is the commercial enclosed gear units.
10. The gearset is put in the middle of two bearings.
11. In this design, we only consider the uncrowned teeth.
12. Because the standard surface conditions for gear teeth have not yet been established, the surface condition factor is assumed to be 1.
13. The backup ratio m_B is larger than 1.2.

4 Notations and Definitions

In this work, we use the nomenclature in Table 1 in the model construction. Other none-frequent-used symbols will be introduced once they are used.

Table 1a: Notations used in this literature

Symbol	Meaning	unit
C_e	Mesh alignment correction factor	–
C_f	Surface condition factor	–
C_H	Hardness-ratio factor	–
C_{ma}	Mesh alignment factor	–
C_{mc}	Load correction factor	–
C_{mf}	Face load-distribution factor	–
C_p	Elastic coefficient	–
C_{pf}	Pinion proportion factor	–
C_{pm}	Pinion proportion modifier	–
d	Pitch diameter	in
d_P	Pitch diameter, pinion	in
d_G	Pitch diameter, gear	in
F	Net face width of narrowest member	in
H	Power	hp
H_B	Brinell hardness	–
H_{BG}	Brinell hardness of gear	–
H_{BP}	Brinell hardness of pinion	–
I	Geometry factor of pitting resistance	–
J	Geometry factor for bending strength	–

Table 1b: Notations used in this literature (Continued)

Symbol	Meaning	unit
K_B	Rim-thickness factor	–
K_f	Fatigue stress-concentration factor	–
K_m	Load-distribution factor	–
K_o	Overload factor	–
K_R	Reliability factor	–
K_s	Size factor	–
K_T	Temperature factor	–
K_v	Dynamic factor	–
m	Module	–
m_B	Backup ratio	–
m_C	Contact ratio	–
m_F	Face-contact ratio	–
m_G	Gear ratio (never less than 1)	–
m_N	Load-sharing ratio	–
m_t	Transverse module	–
N	Number of stress cycles	–
N_G	Number of teeth on gear	–
N_P	Number of teeth on pinion	–
n	Speed	rev/min
n_P	Pinion speed	rev/min
P	Diametral pitch	teeth/in
P_d	Transverse diametral pitch	teeth/in
p_b	Base pitch	in
Q_v	Quality number	–
R	Reliability	–
r_{ap}	Addendum radius of the mating pinion	in
r_{ag}	Addendum radius of the mating gear	in
r_{bp}	Base circle radius of the mating pinion	in
r_{bg}	Base circle radius of the mating gear	in
S_c	AGMA surface endurance strength	ksi
S_t	AGMA bending strength	ksi
S	Bearing span	in
S_1	Pinion offset from center span	in
S_F	Safety factor–bending	–
S_H	Safety factor–pitting	–
W^t	Transmitted load	lbf
Y_N	Stress-cycle factor for bending strength	–
Z_N	Stress-cycle factor for pitting resistance	–
σ	Bending stress, AGMA	ksi
σ_c	Contact stress from AGMA relationships	ksi
σ_{all}	Allowable bending stress, AGMA	ksi
$\sigma_{c,all}$	Allowable contact stress, AGMA	ksi
ϕ	Pressure angle	$^\circ$
ϕ_n	Normal pressure angle	$^\circ$

5 Potential Failure Assessment

In this design, there are many factors that can lead to failure. We list the factors that may cause the failure of the system we designed in Table 2:

Table 2: Potential Failure Assessment.

Failure Scenarios	Critical Parameters	Design Acceptance Criteria	Risk Priorities (High/Medium/Low)
Failure due to fatigue of bending stress at the root of gear teeth	Allowable stress numbers	Safety factor of fatigue of bending stress is larger than 1.25	High
Failure due to fatigue of contact stress at the root of gear teeth	Allowable stress numbers	Safety factor of fatigue of contact stress is larger than 1.25	High
Failure due to fatigue on the shaft	Transmitted load & Shaft properties	Safety factor of fatigue of shaft is larger than 2	Low

6 Design Thought Process

6.1 Iteration 01 - Initial Trial

6.1.1 Iteration Setup and Basic Theories

First, we make the priori decisions as follows:

- Function: 100 hp, 3600 rpm for motor and 900 rpm for load shaft, $R = 0.95$, $N = 2.16 \times 10^8$ cycles for motor and $N = 5.4 \times 10^7$ cycles for load shaft, $K_o = 1$
- Material: Nitralloy 135M, grade 1
- Design factor for unquantifiable exingencies: $n_d = 2$
- Tooth system: $\phi_n = 20°$
- Tooth count: $N_P = 20$ teeth, $N_G = 80$ teeth (no interference)
- Quality number: $Q_v = 8.5$, use grade 1 material
- Assume $m_B \geq 1.2$ so that $K_B = 1$

Note that according to Constraint 7, the only material we can choose is Nitralloy, whose Rockwell C Scale Hardness of the case is between 62 and 65 (near 660 Bhn). Therefore, we choose Nitralloy 135M, grade 1 for trial. We also notice that the Rockwell C Scale Hardness for the core is between 32 and 36 (302–335 Brinell). Hence, in order to satisfy the second requirement in Constraint 7, which is that we need to keep pinion-tooth hardness about 10 percent higher than the gear-tooth hardness, we can only choose two sides of hardness in this range as attainable because 10 percent higher of 302 Brinell is 332.2 Brinell based on Assumption 8, which is near the maximum hardness that is attainable–335 Brinell. Finally, we make the decision that the

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

Figure 2: Values of the Lewis Form Factor Y (Shigley & Mischke, 2005).

pinion and gear are made from Nitralloy 135M, grade 1 with core hardnesses of 335 Brinell on the pinion and core hardnesses of 302 Brinell on the gear.

Next, we select a trial diametral pitch of $P_d = 4$ teeth/in. Thus, $d_P = 20/4 = 5$ in and $d_G = 80/4 = 20$ in. From Figure 2, we can know that $Y_P = 0.3220$ and $Y_G = 0.4374$ (interpolated). From Figure 3, we can know that $J_P = 0.3360$ and $J_G = 0.4240$.

And from the question statement, pitch-line velocity can be derived by Equation 1.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi\,(5)\,3600}{12} = 4712.4 \text{ fpm} \tag{1}$$

Hence, the transmitted force between the pinion and gear is shown in Equation 2.

$$W^t = \frac{33000H}{V} = \frac{33000\,(100)}{4712.4} = 700.2817 \text{ lbf} \tag{2}$$

Then, the bending stress and pitting resistance (contact stress) will be calculated based on two fundamental stress equations used in the AGMA methodology as shown in Equation 3 and 4.

The fundamental equation for bending stress is

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \tag{3}$$

where W^t is the tangential transmitted load, lbf
K_o is the overload factor
K_v is the dynamic factor
K_s is the size factor
P_d is the transverse diametral pitch
F is the face width of the narrower member, in
K_m is the load-distribution factor
K_B is the rim-thickness factor
J is the geometry factor for bending strength

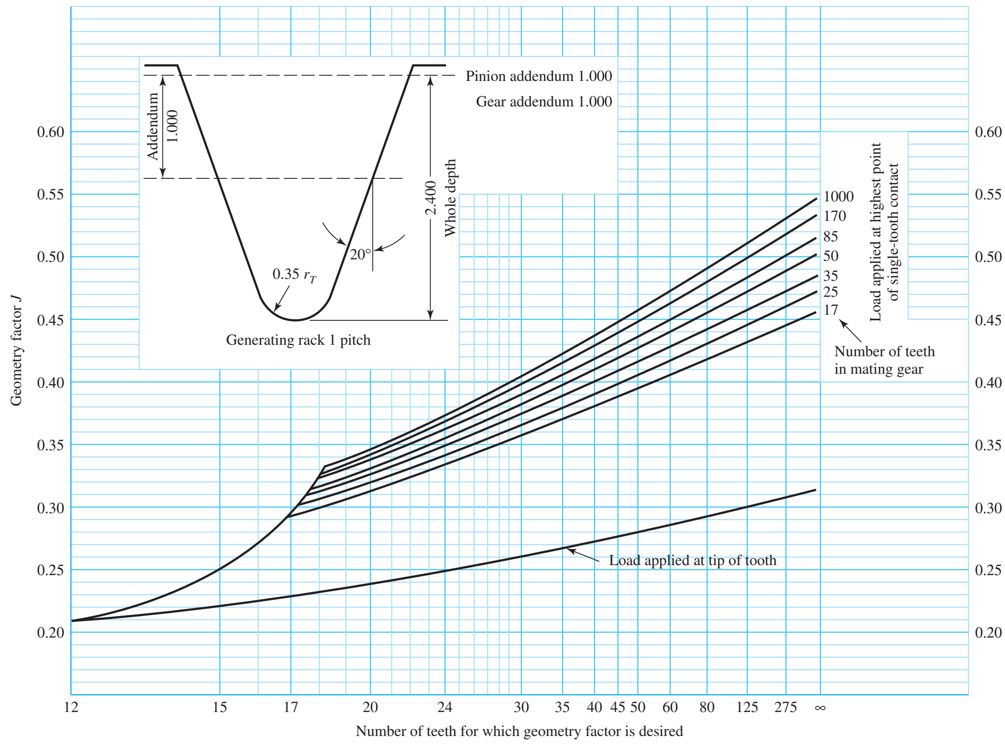

Figure 3: Spur-gear geometry factors J (Shigley & Mischke, 2005).

The fundamental equation for pitting resistance (contact stress) is

$$\sigma_c = C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I}} \tag{4}$$

where W^t, K_o, K_v, K_s, K_m, F, and b are the same terms as defined for Equation 3
C_p is an elastic coefficient, $\sqrt{\mathrm{lbf/in^2}}$
C_f is the surface condition factor
d_P is the pitch diameter of the pinion, in
I is the geometry factor for pitting resistance

In addition, instead of using the term strength, AGMA uses data termed allowable stress numbers and designates these by the symbols σ_{all} and $\sigma_{c,all}$. In this approach the strengths are modified by various factors that produce limiting values of the bending stress and the contact stress as shown in Equation 5 and 6.

The equation for the allowable bending stress is

$$\sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R} \tag{5}$$

where S_t is the allowable bending stress, lbf/in^2
Y_N is the stress-cycle factor for bending stress
K_T is the temperature factor
K_R is the reliability factor
S_F is the AGMA factor of safety, a stress ratio

The equation for the allowable contact stress $\sigma_{c,all}$ is

$$\sigma_{c,all} = \frac{S_c}{S_H} \frac{Z_N C_H}{K_T K_R} \tag{6}$$

where S_c is the allowable contact stress, lbf/in^2
Z_N is the stress-cycle factor
C_H is the hardness ratio factors for pitting resistance
K_T is the temperature factor
K_R is the reliability factor
S_H is the AGMA factor of safety, a stress ratio

In Equation 5 and 6, all other symbols are factors except S_t and S_c. The derivation of S_t can be seen in Figure 4, whose equation is shown in Equation 7.

$$S_t = 86.2 H_B + 12730 \text{ psi} \tag{7}$$

The results come out that the allowable bending stress for pinion and gear are equal to $(S_t)_P = 41607$ psi and $(S_t)_G = 38762$ psi, respectively. Also, from Shigley & Mischke (2005)'s book, we can find that the allowable contact stress for Nitralloy 135M, grade 1 is $S_c = 17000$ psi for any core hardness.

The evaluation of all these factors in Equation 3, 4, 5, and 6 is explained in the following of this subsection.

6.1.2 Determination of Factors in Equation 3, 4, 5, and 6

Dynamic Factor Dynamic factors are used to account for inaccuracies in the manufacture and meshing of gear teeth in action.

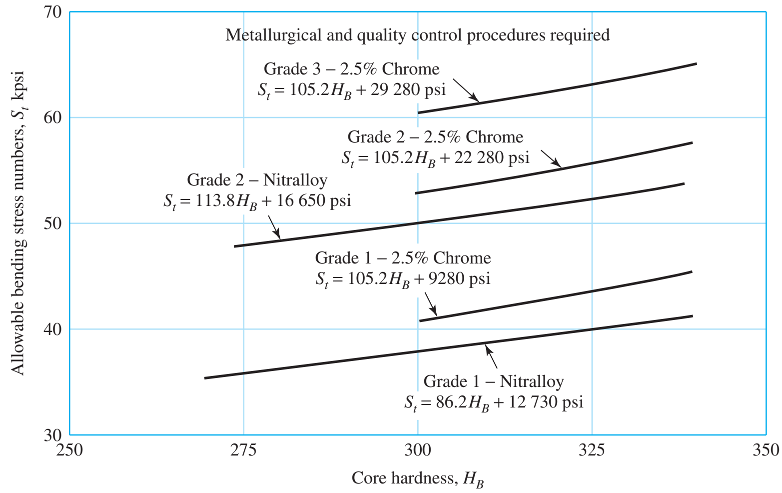

Figure 4: Allowable bending stress numbers for nitriding steel gears, S_t, in Equation 5 (Shigley & Mischke, 2005).

In an attempt to account for effects including inaccuracies produced in the generation of the tooth profile, vibration of the tooth during meshing due to the tooth stiffness, magnitude of the pitch-line velocity, and so on, AGMA has defined a set of quality numbers, Q_v. These numbers define the tolerances for gears of various sizes manufactured to a specified accuracy. Equation 8 for the dynamic factor is based on these Q_v numbers.

$$K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B \tag{8}$$

where

$$\begin{aligned} A &= 50 + 56\,(1 - B) \\ B &= 0.25\,(12 - Q_v)^{2/3} \end{aligned} \tag{9}$$

Because in the scenario of first iteration, we try quality number: $Q_v = 8.5$, the dynamic factor can be calculated, which is equal to $K_v = 1.4612$.

Reliability Factor The reliability factor accounts for the effect of the statistical distributions of material fatigue failures.

The functional relationship between K_R and reliability is highly nonlinear. When interpolation is required, linear interpolation is too crude. A log transformation to each quantity produces a linear string. A least-squares regression fit is shown in Equation 10.

$$K_R = \begin{cases} 0.658 - 0.0759 \ln (1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln (1 - R) & 0.99 \leq R \leq 0.9999 \end{cases} \tag{10}$$

Because in the design scenario, $R = 0.95$, the reliability factor can be calculated, which is equal to $K_R = 0.8854$.

Temperature Factor For oil or gear-blank temperatures up to 250°F (120°C), we use $K_T = 1$.

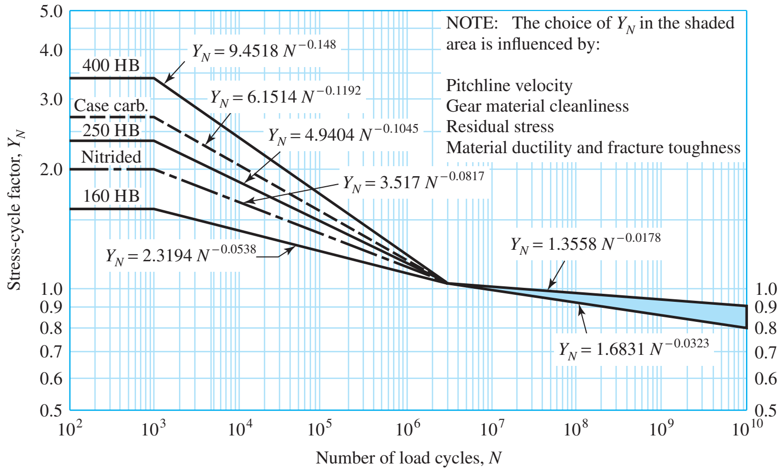

Figure 5: Repeatedly applied bending strength stress-cycle factor Y_N (Shigley & Mischke, 2005).

Hardness-Ratio Factor The pinion generally has a smaller number of teeth than the gear and consequently is subjected to more cycles of contact stress. If both the pinion and the gear are through hardened, then a uniform surface strength can be obtained by making the pinion harder than the gear. A similar effect can be obtained when a surface-hardened pinion is mated with a through-hardened gear. The hardness-ratio factor C_H is used only for the gear. Its purpose is to adjust the surface strengths for this effect. For the pinion, $C_H = 1$. For the gear, C_H is obtained from Equation 11.

$$C_H = 1.0 + A'\left(m_G - 1.0\right) \tag{11}$$

where

$$A' = \begin{cases} 0 & \frac{H_{BP}}{H_{BG}} < 1.2 \\ 8.98\left(10^{-3}\right)\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29\left(10^{-3}\right) & 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7 \\ 0.00698 & \frac{H_{BP}}{H_{BG}} > 1.7 \end{cases} \tag{12}$$

In the scenario of problem setup, we can know that the hardness ratio of pinion and gear is near 1.1. Therefore, the hardness-ratio factor will always be $C_H = 1$ in our design.

Stress-Cycle Factors The purpose of the stress-cycle factors Y_N and Z_n is to modify the gear strength for lives other than 10^7 cycles. Values for these factors are given in Figure 5 and 6.

Because in this design scenario, both pinion cycles and gear cycles are larger than 10^7, the stress-cycle factors Y_N and Z_n can be calculated as shown in Equation 13 and 14.

$$Y_N = 1.3558N^{-0.0178} \tag{13}$$

$$Z_N = 1.4488N^{-0.023} \tag{14}$$

From Equation 13 and 14, we can calculate the stress-cycle factors for pinion and gear, which are equal to $(Y_N)_P = 0.9635$ & $(Z_N)_P = 0.9318$ and $(Y_N)_G = 0.9875$ & $(Z_N)_G = 0.9620$, respectively.

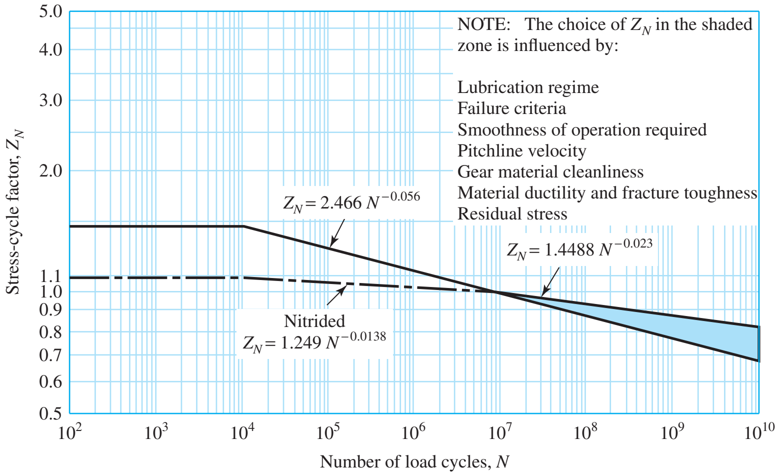

Figure 6: Pitting resistance stress-cycle factor Z_N (Shigley & Mischke, 2005).

Size Factor The size factor reflects nonuniformity of material properties due to size. It depends upon many factors including tooth size, diameter of part, ratio of tooth size, face width, etc. Standard size factors for gear teeth have not yet been established for cases where there is a detrimental size effect. The calculation of size factor–Lewis's geometry incorporated into the Marin size factor in fatigue is shown in Equation 15.

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} \tag{15}$$

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p (Shigley & Mischke, 2005). But in the question statement, we need to tentatively choose width F at the maximum of the normal range, $14/P$, for minimum center distance. Therefore, we tentatively choose a midrange face width as attainable, $F = 4p = 4\pi/P = 4\pi/4 = 3.14$ in.

Substituting the results from Figure 2 and also the trial of $F = 3.14$ in and $P = 4$ teeth/in into Equation 15, we can obtain the size factors for both pinion and gear, which are equal to $(K_s)_P = 1.1416$ and $(K_s)_G = 1.1510$, respectively.

Load-Distribution Factor The load-distribution factor modified the stress equations to reflect nonuniform distribution of load across the line of contact. The ideal is to locate the gear "midspan" between two bearings at the zero slope place when the load is applied. And according to our Assumption 10, our gearset is put in the ideal place.

The load-distribution factor under these conditions is currently given by the face load distribution factor, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc} \left(C_{pf} C_{pm} + C_{ma} C_e \right) \tag{16}$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \tag{17}$$

$$C_{pf} = \begin{cases} \frac{F}{10d_p} - 0.025 & F \le 1 \text{ in} \\ \frac{F}{10d_p} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \frac{F}{10d_p} - 0.1109 + 0.0207F - 0.000228F^2 & 17 < F \le 40 \text{ in} \end{cases} \tag{18}$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle mounted pinion with } S_1/S \ge 0.175 \end{cases} \tag{19}$$

$$C_{ma} = A + BF + CF^2 \tag{20}$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \tag{21}$$

The definition of distances S and S_1 used in evaluating C_{pm} in Equation 19 can be known by Figure 7 and the values of A, B, C for Equation 20 can be found in Figure 8.

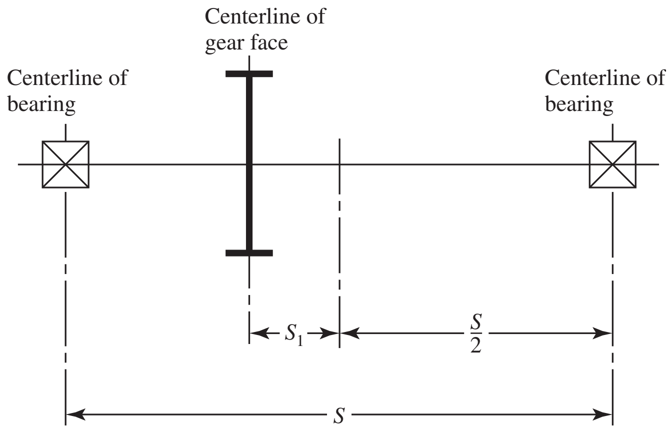

Figure 7: Definition of distances S and S_1 used in evaluating C_{pm} in Equation 19 (Shigley & Mischke, 2005).

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

Figure 8: Empirical Constants A, B, and C for Equation 20 (Shigley & Mischke, 2005).

According to Assumption 11, we can know from Equation 17 that $C_{mc} = 1$. And because the face width $F = 3.14$ in, we need to use second condition in Equation 18 to calculate C_{pf}, which turns out that $C_{pf} = 0.0646$. In addition, according to our Assumption 10, we can know that $S_1 = 0$ in Figure 7. Hence, combined with Equation 19, we can obtain $C_{pm} = 1$. C_{ma} can be derived based on Assumption 9. We can know the values of A, B, C for Equation 20, which is $A = 0.127$, $B = 0.0158$, and $C = -0.930\left(10^{-4}\right)$. Substituting these values and $F = 3.14$ in into Equation 20, we can get $C_{ma} = 0.1757$. Moreover, because our gearset is not adjusted at assembly and compatibility is not improved by lapping, $C_e = 1$ in Equation 21. From what has been mentioned above, we can eventually calculate the load-distribution factor, which is $K_m = 1.2403$.

Elastic Coefficient From Shigley & Mischke (2005)'s book, we can find that if the material of pinion and gear in the gearset are both steel, the elastic coefficient is equal to $C_p = 2300\ \sqrt{\text{psi}}$.

Surface-Strength Geometry Factor The factor I is also called the pitting-resistance geometry factor by AGMA. For external gears, the surface-strength geometry factor can be calculated using Equation 22.

$$I = \frac{\cos\phi\sin\phi}{2m_N}\frac{m_G}{m_G+1} \tag{22}$$

where $m_N = 1$ for spur gears and m_G is the speed ratio defined as Equation 23.

$$m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P} \tag{23}$$

Based on Constraint 8, the surface-strength geometry factor can be obtained, which is $I = 0.1286$.

6.1.3 Feasibility Analysis and Safety Factor Calculation

With the above estimates of K_s and K_m from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Equation 3 and 5, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue, we obtain Equation 24.

$$(F)_{bend} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J}\frac{K_T K_R}{S_t Y_N} \tag{24}$$

From Equation 24, we can derive the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue for both pinion and gear, which are $(F)_{bend,P} = 0.7619$ in and $(F)_{bend,G} = 0.6375$ in, respectively. The results are much smaller than our trial face width, indicating that our scheme is feasible.

Equating Equation 4 and 6, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue, we obtain Equation 25.

$$(F)_{wear} = \left(\frac{C_p K_T K_R}{S_c Z_n}\right) n_d W^t K_o K_v K_s \frac{K_m C_f}{d_p I} \tag{25}$$

From Equation 25, we can derive the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue for both pinion and gear, which are $(F)_{wear,P} = 0.7450$ in and $(F)_{wear,G} = 0.7047$ in, respectively. The results are much smaller than our trial face width, indicating that our scheme is feasible.

Finally, we will calculate the AGMA factor of safety for both pinion and gear.

Pinion tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for pinion is equal to $(\sigma)_P = 5490.2$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 26.

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} \tag{26}$$

The result turns out that $(S_F)_P = 8.2470$, which is much larger than our required safety factor 1.25, indicating that our scheme is feasible.

Gear tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for gear is equal to $(\sigma)_G = 4386.5$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 26.

The result turns out that $(S_F)_G = 9.8565$, which is much larger than our required safety factor 1.25, indicating that our scheme is feasible.

Pinion tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for pinion is equal to $(\sigma_c)_P = 61607$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of pinion is shown in Equation 27.

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c} \tag{27}$$

The result turns out that $(S_H)_P^2 = 8.4337$, which is much larger than our required safety factor 1.25, indicating that our scheme is feasible.

Gear tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for gear is equal to $(\sigma_c)_G = 61859$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of gear is shown in Equation 27.

The result turns out that $(S_H)_G^2 = 8.9157$, which is much larger than our required safety factor 1.25, indicating that our scheme is feasible.

6.1.4 Conclusion and Ideas for Improvement in the Next Iteration

In this iteration, we can observe that all safety factors for tooth bending and wear are pretty huge. The reason which causes the large safety factors is that the diametral pitch we select is much small, which leads to the bulky diameter of pinion and gear. That is not the outcome we expect. This also violates Constraint 1. Wherefore, in next iteration, we will increase the diametral pitch so that the center distance of the gearset will be reduced.

6.2 Iteration 02 - Increase Diametral Pitch

6.2.1 Iteration Setup and Basic Theories

In the first iteration in Section 6.1, we find that the diametral pitch we select is too small. Therefore, in this iteration, we will increase our diametral pitch so that the center distance of the gearset will be reduced. We make the same priori decisions as Section 6.1.1.

In this iteration, we select a trial diametral pitch of $P_d = 40$ teeth/in. Thus, $d_P = 20/40 = 0.5$ in and $d_G = 80/40 = 2$ in.

And from the question statement, pitch-line velocity can be derived by Equation 28.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi (0.5) 3600}{12} = 471.24 \text{ fpm} \tag{28}$$

Hence, the transmitted force between the pinion and gear is shown in Equation 29.

$$W^t = \frac{33000H}{V} = \frac{33000 (100)}{471.24} = 7002.817 \text{ lbf} \tag{29}$$

Other parameters in this section are the same as Section 6.1.1. In the following subsection, we will explore the change of factors due to the change of diametral pitch.

6.2.2 Determination of Factors in Equation 3, 4, 5, and 6 that Change

In this subsection, we will discuss the factors that will change due to the change of diametral pitch.

Size Factor The calculation of size factor is shown in Equation 30.

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} \tag{30}$$

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p (Shigley & Mischke, 2005). But in the question statement, we need to tentatively choose width F at the maximum of the normal range, $14/P$, for minimum center distance. Therefore, we tentatively choose a midrange face width as attainable, $F = 4p = 4\pi/P = 4\pi/40 = 0.314$ in.

Substituting the results from Figure 2 and also the trial of $F = 0.314$ in and $P = 40$ teeth/in into Equation 30, we can obtain the size factors for both pinion and gear, which are equal to $(K_s)_P = 0.8923$ and $(K_s)_G = 0.8996$, respectively.

Load-Distribution Factor The load-distribution factor under these conditions is currently given by the face load distribution factor, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc} (C_{pf} C_{pm} + C_{ma} C_e) \tag{31}$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \tag{32}$$

$$C_{pf} = \begin{cases} \frac{F}{10d_p} - 0.025 & F \leq 1 \text{ in} \\ \frac{F}{10d_p} - 0.0375 + 0.0125F & 1 < F \leq 17 \text{ in} \\ \frac{F}{10d_p} - 0.1109 + 0.0207F - 0.000228F^2 & 17 < F \leq 40 \text{ in} \end{cases} \tag{33}$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle mounted pinion with } S_1/S \geq 0.175 \end{cases} \tag{34}$$

$$C_{ma} = A + BF + CF^2 \tag{35}$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \quad (36)$$

The definition of distances S and S_1 used in evaluating C_{pm} in Equation 34 can be known by Figure 7 and the values of A, B, C for Equation 35 can be found in Figure 8.

According to Assumption 11, we can know from Equation 32 that $C_{mc} = 1$. And because the face width $F = 0.314$ in, we need to use first condition in Equation 33 to calculate C_{pf}, which turns out that $C_{pf} = 0.0378$. In addition, according to our Assumption 10, we can know that $S_1 = 0$ in Figure 7. Hence, combined with Equation 34, we can obtain $C_{pm} = 1$. C_{ma} can be derived based on Assumption 9. We can know the values of A, B, C for Equation 35, which is $A = 0.127$, $B = 0.0158$, and $C = -0.930\left(10^{-4}\right)$. Substituting these values and $F = 0.314$ in into Equation 35, we can get $C_{ma} = 0.1320$. Moreover, because our gearset is not adjusted at assembly and compatibility is not improved by lapping, $C_e = 1$ in Equation 36. From what has been mentioned above, we can eventually calculate the load-distribution factor, which is $K_m = 1.1698$.

6.2.3 Feasibility Analysis and Safety Factor Calculation

With the above estimates of factors from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Equation 3 and 5, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue, we obtain Equation 37.

$$(F)_{bend} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J} \frac{K_T K_R}{S_t Y_N} \quad (37)$$

From Equation 37, we can derive the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue for both pinion and gear, which are $(F)_{bend,P} = 44.6006$ in and $(F)_{bend,G} = 37.3174$ in, respectively. The results are much larger than our trial face width, indicating that our scheme is not feasible.

6.2.4 Conclusion and Ideas for Improvement in the Next Iteration

In this iteration, we can observe that the diametral pitch we select is too large, which leads to the failure of the whole gearset. That is not the outcome we expect. Wherefore, in next iteration, we will set the diametral pitch to an appropriate value so that the safety factor is within the normal range.

6.3 Iteration 03 - Set Appropriate Diametral Pitch

6.3.1 Iteration Setup and Basic Theories

In the previous iterations in Section 6.1 and 6.2, we find that we need to set the diametral pitch we select to an appropriate value. Therefore, in this iteration, we will change our diametral pitch so that the center distance of the gearset will be as small as possible along with the reasonable factor of safety. We make the same priori decisions as Section 6.1.1.

In this iteration, we select a trial diametral pitch of $P_d = 6$ teeth/in. Thus, $d_P = 20/6 = 3.3333$ in and $d_G = 80/6 = 13.3333$ in.

And from the question statement, pitch-line velocity can be derived by Equation 38.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi\,(3.3333)\,3600}{12} = 3141.6 \text{ fpm} \quad (38)$$

Hence, the transmitted force between the pinion and gear is shown in Equation 39.

$$W^t = \frac{33000H}{V} = \frac{33000\,(100)}{3141.6} = 1050.4 \text{ lbf} \tag{39}$$

Other parameters in this section are the same as Section 6.1.1. In the following subsection, we will explore the change of factors due to the change of diametral pitch.

6.3.2 Determination of Factors in Equation 3, 4, 5, and 6 that Change

In this subsection, we will discuss the factors that will change due to the change of diametral pitch. The factors which we don't discuss in this subsection means that those factors will not change if we alter the diametral pitch.

Size Factor The calculation of size factor is shown in Equation 40.

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \tag{40}$$

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p (Shigley & Mischke, 2005). But in the question statement, we need to tentatively choose width F at the maximum of the normal range, $14/P$, for minimum center distance. Therefore, we tentatively choose a midrange face width as attainable, $F = 4p = 4\pi/P = 4\pi/6 = 2.0944$ in.

Substituting the results from Figure 2 and also the trial of $F = 2.0944$ in and $P = 6$ teeth/in into Equation 40, we can obtain the size factors for both pinion and gear, which are equal to $(K_s)_P = 1.0931$ and $(K_s)_G = 1.1021$, respectively.

Load-Distribution Factor The load-distribution factor under these conditions is currently given by the face load distribution factor, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) \tag{41}$$

where

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \tag{42}$$

$$C_{pf} = \begin{cases} \frac{F}{10d_p} - 0.025 & F \le 1 \text{ in} \\ \frac{F}{10d_p} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \frac{F}{10d_p} - 0.1109 + 0.0207F - 0.000228F^2 & 17 < F \le 40 \text{ in} \end{cases} \tag{43}$$

$$C_{pm} = \begin{cases} 1 & \text{for straddle mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle mounted pinion with } S_1/S \ge 0.175 \end{cases} \tag{44}$$

$$C_{ma} = A + BF + CF^2 \tag{45}$$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \tag{46}$$

The definition of distances S and S_1 used in evaluating C_{pm} in Equation 44 can be known by Figure 7 and the values of A, B, C for Equation 45 can be found in Figure 8.

According to Assumption 11, we can know from Equation 42 that $C_{mc} = 1$. And because the face width $F = 2.0944$ in, we need to use second condition in Equation 43 to calculate C_{pf}, which turns out that $C_{pf} = 0.0515$. In addition, according to our Assumption 10, we can know that $S_1 = 0$ in Figure 7. Hence, combined with Equation 44, we can obtain $C_{pm} = 1$. C_{ma} can be derived based on Assumption 9. We can know the values of A, B, C for Equation 45, which is $A = 0.127$, $B = 0.0158$, and $C = -0.930\left(10^{-4}\right)$. Substituting these values and $F = 2.0944$ in into Equation 45, we can get $C_{ma} = 0.1597$. Moreover, because our gearset is not adjusted at assembly and compatibility is not improved by lapping, $C_e = 1$ in Equation 46. From what has been mentioned above, we can eventually calculate the load-distribution factor, which is $K_m = 1.2112$.

6.3.3 Feasibility Analysis and Safety Factor Calculation

With the above estimates of factors from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Equation 3 and 5, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue, we obtain Equation 47.

$$(F)_{bend} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J} \frac{K_T K_R}{S_t Y_N} \tag{47}$$

From Equation 47, we can derive the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue for both pinion and gear, which are $(F)_{bend,P} = 0.7619$ in and $(F)_{bend,G} = 0.6375$ in, respectively. The results are much smaller than our trial face width, indicating that our scheme is feasible.

Equating Equation 4 and 6, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue, we obtain Equation 48.

$$(F)_{wear} = \left(\frac{C_p K_T K_R}{S_c Z_n}\right) n_d W^t K_o K_v K_s \frac{K_m C_f}{d_p I} \tag{48}$$

From Equation 48, we can derive the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue for both pinion and gear, which are $(F)_{wear,P} = 1.4859$ in and $(F)_{wear,G} = 1.4056$ in, respectively. The results are smaller than our trial face width, indicating that our scheme is feasible.

Finally, we will calculate the AGMA factor of safety for both pinion and gear.

Pinion tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for pinion is equal to $(\sigma)_P = 16425$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 49.

$$S_F = \frac{S_t Y_N / \left(K_T K_R\right)}{\sigma} \tag{49}$$

The result turns out that $(S_F)_P = 2.7566$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Gear tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for gear is equal to $(\sigma)_G = 13123$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 49.

The result turns out that $(S_F)_G = 3.2946$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Pinion tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for pinion is equal to $(\sigma_c)_P = 1.0656 \times 10^5$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of pinion is shown in Equation 50.

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c} \tag{50}$$

The result turns out that $(S_H)_P^2 = 2.8190$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Gear tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for gear is equal to $(\sigma_c)_G = 1.0700 \times 10^5$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of gear is shown in Equation 50.

The result turns out that $(S_H)_G^2 = 2.9801$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

6.3.4 Conclusion and Ideas for Improvement in the Next Iteration

In this iteration, we can observe that all safety factors for tooth bending and wear are slightly larger than our required safety factor 1.25, which is already in an ideal situation. Later, we discover a strange phenomenon. In China, the price of nitralloy is actually similar to that of carburized steel. Sometimes, the price of nitralloy is even lower than the price of carburized steel, that's the reason why we use nitralloy for the first iteration in Section 6.1, 6.2, and 6.3. However, in the US, nitralloy is at least twice the cost of caburized steel. Moreover, we notice the fact that the surface hardness of some highly strengthened carburized steel can reach 660 Bhn. It's not that only nitralloy can reach 660 Bhn as mentioned in Section 6.1.1. In addition, the allowable bending stress and the allowable wear stress of caburized steel are larger than nitralloy, so that its mechanical performance is better. In this way, we can make the center distance smaller. Therefore, in all subsequent iterations, we use caburized and hardened steel as the material of our gearset. The purpose of this is to save the manufacturing cost of the gearset as much as possible and reduce the center distance.

6.4 Iteration 04 - Change Material

6.4.1 Iteration Setup and Basic Theories

In the iterations above in Section 6.1, 6.2, and 6.3, we analyzed the results of using Nitralloy 135M as gear material. Carburized and hardened steel has similar properties to Nitralloy 135M, but it has the lower cost. Therefore, in this iteration, we will calculate the minimum diameter using carburized and hardened steel as the basic material of the gears. Likewise, we make the same priori decisions as Section 6.1.1 except the material we use.

Based on the information given in Association et al. (2004), the carburized and hardened steel is also available under the Constraint 7, whose Rockwell C Scale Hardness of the case is between 55 and 64 (near 660 Bhn). Therefore, the grade 1 of this kind of steel is chosen for trial. To satisfy the second requirement in Constraint 7, we choose two sides of hardness to be 600 in gear side and 660 Bhn in pinion side, which is within limiting range.

Next, we select a trial diametral pitch of $P_d = 10$ teeth/in. Thus, $d_P = 20/10 = 2$ in and $d_G = 80/10 = 8$ in. From Figure 2, we can know that $Y_P = 0.3220$ and $Y_G = 0.4374$ (interpolated). From Figure 3, we can know that $J_P = 0.3360$ and $J_G = 0.4240$.

And from the question statement, pitch-line velocity can be derived by Equation 51.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi\,(2)\,3600}{12} = 1885.0 \text{ fpm} \tag{51}$$

Hence, the transmitted force between the pinion and gear is shown in Equation 52.

$$W^t = \frac{33000H}{V} = \frac{33000\,(100)}{1885.0} = 1750.7 \text{ lbf} \tag{52}$$

The value of S_t and S_c are given from Figure 9 and Figure 10, which are 55000 psi and 180000 psi, respectively.

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame or induction hardened with type A pattern	See Table 8	45 000	55 000	—
	Flame or induction hardened with type B pattern	See Table 8	22 000	22 000	—
	Carburized and hardened	See Table 9	55 000	65 000 or 70 000	75 000
	Nitrided (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Figure 9: Repeatedly Applied Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears (Shigley & Mischke, 2005).

6.4.2 Determination of Factors in Equation 3, 4, 5, and 6

In this subsection, we will discuss the factors that will change due to the change of diametral pitch.

Size Factor The calculation of size factor is shown in Equation 53.

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} \tag{53}$$

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p (Shigley & Mischke, 2005). But in the question statement, we need to tentatively choose width

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi Grade 1	Grade 2	Grade 3
Steel	Through hardened	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame or induction hardened	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened	See Table 9*	180 000	225 000	275 000
	Nitrided (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided	90.0 HR15N	176 000	196 000	216 000

Figure 10: Repeatedly Applied Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears (Shigley & Mischke, 2005).

F at the maximum of the normal range, $14/P$, for minimum center distance. Therefore, we tentatively choose a midrange face width as attainable, $F = 4p = 4\pi/P = 4\pi/40 = 1.257$ in.

Substituting the results from Figure 2 and also the trial of $F = 1.257$ in and $P = 10$ teeth/in into Equation 53, we can obtain the size factors for both pinion and gear, which are equal to $(K_s)_P = 1.0349$ and $(K_s)_G = 1.04346$, respectively.

Load-Distribution Factor The load-distribution factor under these conditions is currently given by the face load distribution factor, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) \tag{54}$$

where $F = 1.257$ in shares the same condition of Iteration 01. Based on the same calculation process, the load-distribution factor is $K_m = 1.1877$ when $C_{pf} = 0.04104$.

6.4.3 Feasibility Analysis and Safety Factor Calculation

From Equation 24, we can derive the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue for both pinion and gear, which are $(F)_{bend,P} = 2.7948$ in and $(F)_{bend,G} = 2.1786$ in, respectively. The results are much larger than our trial face width, indicating that our scheme is not feasible. When we don't consider the effect of the unquantifiable elements, the safety factors for pinion and gear's bending stress of this system are 0.8993 and 1.1536. The safety factors of contact stress are 0.8831 and 0.9080 respectively.

6.4.4 Conclusion and Ideas for Improvement in the Next Iteration

In this iteration, we can observe that to prevent the effect of unquantifiable elements makes the huge difference of the width of the gear system. Therefore, we need to find a material and diametral pitch to hold the unquantifiable forces.

6.5 Iteration 05 - Set Appropriate Diametral Pitch

6.5.1 Iteration Setup and Basic Theories

In the iteration 04 in Section 6.4, we find that we need to set the diametral pitch we select to an appropriate value. Therefore, in this iteration, we will change our diametral pitch so that the center distance of the gearset will be as small as possible along with the reasonable factor of safety. We make the same priori decisions as Section 6.4.1.

Next, we select a trail diametral pitch of $P_d = 7$ teeth/in. Thus, $d_P = 20/7 = 2.8571$ in and $d_G = 80/7 = 11.4286$ in.

And from the question statement, pitch-line velocity can be derived by Equation 55.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi\,(2.8571)\,3600}{12} = 2692.8 \text{ fpm} \tag{55}$$

Hence, the transmitted force between the pinion and gear is shown in Equation 56.

$$W^t = \frac{33000H}{V} = \frac{33000\,(100)}{2692.8} = 1225.5 \text{ lbf} \tag{56}$$

6.5.2 Determination of Factors in Equation 3, 4, 5, and 6

In this subsection, we will discuss the factors that will change due to the change of diametral pitch. The factors which we don't discuss in this subsection means that those factors will not change if we alter the diametral pitch.

Size Factor The calculation of size factor is shown in Equation 57.

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} \tag{57}$$

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p (Shigley & Mischke, 2005). But in the question statement, we need to tentatively choose width F at the maximum of the normal range, $14/P$, for minimum center distance. Therefore, we tentatively choose a midrange face width as attainable, $F = 4p = 4\pi/P = 4\pi/7 = 1.7952$ in.

Substituting the results from Figure 2 and also the trial of $F = 1.7952$ in and $P = 7$ teeth/in into Equation 57, we can obtain the size factors for both pinion and gear, which are equal to $(K_s)_P = 1.0752$ and $(K_s)_G = 1.0841$, respectively.

Load-Distribution Factor The load-distribution factor under these conditions is currently given by the face load distribution factor, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) \tag{58}$$

where $F = 1.7952$ in shares the same condition of Iteration 1. Based on the same calculation process, the load-distribution factor is $K_m = 1.2028$ when $C_{pf} = 0.0478$.

6.5.3 Feasibility Analysis and Safety Factor Calculation

With the above estimates of factors from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Equation 3 and 5, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue, we obtain Equation 59.

$$(F)_{bend} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J} \frac{K_T K_R}{S_t Y_N} \tag{59}$$

From Equation 59, we can derive the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue for both pinion and gear, which are $(F)_{bend,P} = 1.5000$ in and $(F)_{bend,G} = 1.1693$ in, respectively. The results are smaller than our trial face width, indicating that our scheme is feasible.

Equating Equation 4 and 6, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue, we obtain Equation 60.

$$(F)_{wear} = \left(\frac{C_p K_T K_R}{S_c Z_n}\right) n_d W^t K_o K_v K_s \frac{K_m C_f}{d_p I} \tag{60}$$

From Equation 60, we can derive the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue for both pinion and gear, which are $(F)_{wear,P} = 1.7295$ in and $(F)_{wear,G} = 1.6360$ in, respectively. The results are smaller than our trial face width, indicating that our scheme is feasible.

Finally, we will calculate the AGMA factor of safety for both pinion and gear.

Pinion tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for pinion is equal to $(\sigma)_P = 25005$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 61.

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} \tag{61}$$

The result turns out that $(S_F)_P = 2.3936$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Gear tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for gear is equal to $(\sigma)_G = 19979$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 61.

The result turns out that $(S_F)_G = 3.0707$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Pinion tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for pinion is equal to $(\sigma_c)_P = 1.3148 \times 10^5$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of pinion is shown in Equation 62.

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c} \tag{62}$$

The result turns out that $(S_H)_P^2 = 2.0759$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Diametral Pitch P(teeth/in)	
Coarse	2, $2\frac{1}{4}$, $2\frac{1}{2}$, 3, 4, 6, 8, 10, 12, 16
Fine	20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200

Module m(mm/tooth)	
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45

Figure 11: Tooth Sizes in General Uses (Shigley & Mischke, 2005).

Gear tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for gear is equal to $(\sigma_c)_G = 1.3202 \times 10^5$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of gear is shown in Equation 62.

The result turns out that $(S_H)_G^2 = 2.1945$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

6.5.4 Conclusion and Ideas for Improvement in the Next Iteration

In this iteration, we find that the center distance is not significantly reduced compared to Iteration 03 in Section 6.3. In order to be able to better reduce our center distance, we must choose higher-strength materials. It can be seen from Figure 9 and Figure 10 that the same material will greater strength with higher grade, and the price difference of different grades is not very big. Therefore, in the next iteration, we will choose grade 3 materials to reduce our center distance.

6.6 Iteration 06 - Change Grade of Material & Set Appropriate Diametral Pitch

6.6.1 Iteration Setup and Basic Theories

In the iteration 05 in Section 6.5,we find that we need to further increase the strength of materials to minimize the center distance. Therefore, in this iteration, we will choose carburized and hardened steel Grade 3 from Figure 9 and Figure 10 as our material. The value of S_t and S_c are given in Figure 9 and Figure 10, which are 75000 psi and 275000 psi, respectively.

We also notice that there are some tooth sizes used generally as shown in Figure 11.

Hence, we select a trial diametral pitch of $P_d = 8$ teeth/in from Figure 11. Thus, $d_P = 20/8 = 2.5000$ in and $d_G = 80/8 = 10.0000$ in.

And from the question statement, pitch-line velocity can be derived by Equation 63.

$$V = \frac{\pi d_P n_P}{12} = \frac{\pi\,(2.5000)\,3600}{12} = 2356.2 \text{ fpm} \tag{63}$$

Hence, the transmitted force between the pinion and gear is shown in Equation 64.

$$W^t = \frac{33000H}{V} = \frac{33000\,(100)}{2356.2} = 1400.6 \text{ lbf} \tag{64}$$

6.6.2 Determination of Factors in Equation 3, 4, 5, and 6

In this subsection, we will discuss the factors that will change due to the change of diametral pitch and material grade. The factors which we don't discuss in this subsection means that those factors will not change if we alter the diametral pitch.

Size Factor The calculation of size factor is shown in Equation 65.

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} \tag{65}$$

As a general rule, spur gears should have a face width F from 3 to 5 times the circular pitch p (Shigley & Mischke, 2005). But in the question statement, we need to tentatively choose width F at the maximum of the normal range, $14/P$, for minimum center distance. Therefore, we tentatively choose a midrange face width as attainable, $F = 4p = 4\pi/P = 4\pi/8 = 1.5708$ in.

Substituting the results from Figure 2 and also the trial of $F = 1.5708$ in and $P = 8$ teeth/in into Equation 65, we can obtain the size factors for both pinion and gear, which are equal to $(K_s)_P = 1.0600$ and $(K_s)_G = 1.0687$, respectively.

Load-Distribution Factor The load-distribution factor under these conditions is currently given by the face load distribution factor, C_{mf}, where

$$K_m = C_{mf} = 1 + C_{mc} \left(C_{pf} C_{pm} + C_{ma} C_e \right) \tag{66}$$

where $F = 1.5708$ in shares the same condition of Iteration 1. Based on the same calculation process, the load-distribution factor is $K_m = 1.1966$ when $C_{pf} = 0.0450$.

6.6.3 Feasibility Analysis and Safety Factor Calculation

With the above estimates of factors from the trial diametral pitch, we check to see if the mesh width F is controlled by bending or wear considerations. Equating Equation 3 and 5, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue, we obtain Equation 67.

$$(F)_{bend} = n_d W^t K_o K_v K_s P_d \frac{K_m K_B}{J} \frac{K_T K_R}{S_t Y_N} \tag{67}$$

From Equation 67, we can derive the face width $(F)_{bend,P}$ and $(F)_{bend,G}$ necessary to resist bending fatigue for both pinion and gear, which are $(F)_{bend,P} = 1.3872$ in and $(F)_{bend,G} = 1.0813$ in, respectively. The results are smaller than our trial face width, indicating that our scheme is feasible.

Equating Equation 4 and 6, substituting $n_d W^t$ for W^t, and solving for the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue, we obtain Equation 68.

$$(F)_{wear} = \left(\frac{C_p K_T K_R}{S_c Z_n} \right) n_d W^t K_o K_v K_s \frac{K_m C_f}{d_p I} \tag{68}$$

From Equation 68, we can derive the face width $(F)_{wear,P}$ and $(F)_{wear,G}$ necessary to resist wear fatigue for both pinion and gear, which are $(F)_{wear,P} = 0.9344$ in and $(F)_{wear,G} = 0.8839$ in, respectively. The results are smaller than our trial face width, indicating that our scheme is feasible.

Finally, we will calculate the AGMA factor of safety for both pinion and gear.

Pinion tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for pinion is equal to $(\sigma)_P = 36038$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 69.

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma} \tag{69}$$

The result turns out that $(S_F)_P = 2.2647$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Gear tooth bending We know that the bending stress induced by W^t in bending is shown in Equation 3, from which we can calculate that the bending stress induced by W^t in bending for gear is equal to $(\sigma)_G = 28793$ psi.

AGMA factor of safety for bending stress induced by W^t in bending of pinion is shown in Equation 69.

The result turns out that $(S_F)_G = 2.9054$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Pinion tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for pinion is equal to $(\sigma_c)_P = 1.5784 \times 10^5$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of pinion is shown in Equation 70.

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c} \tag{70}$$

The result turns out that $(S_H)_P^2 = 3.3621$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

Gear tooth wear We know that the bending stress induced by W^t in wear is shown in Equation 4, from which we can calculate that the wear stress induced by W^t in wear for gear is equal to $(\sigma_c)_G = 1.5849 \times 10^5$ psi.

AGMA factor of safety for wear stress induced by W^t in wear of gear is shown in Equation 70.

The result turns out that $(S_H)_G^2 = 3.5543$, which is larger than our required safety factor 1.25, indicating that our scheme is feasible.

6.6.4 Conclusion

This is our last iteration. In this iteration, we used the strongest material to minimize the center distance. In addition, we also analyze that the diametral pitch we choose is feasible and the entire gearset can operate safely.

6.6.5 Determination of Various Relevant Parameters

In this subsection, we will determine the parameters relevant to the gearset we design in this iteration.

Addendum The addendum a is the radial distance between the top land and the pitch circle (Shigley & Mischke, 2005). According to AGMA gear design, the addendum of the pinion and gear is equal to $a = 0.1250$ in.

Dedendum The dedendum b is the radial distance from the bottom land to the pitch circle (Shigley & Mischke, 2005). According to AGMA gear design, the dedendum of the pinion and gear is equal to $b = 0.1563$ in.

Whole Depth The whole depth H_t is the sum of the addendum and the dedendum. Therefore, $h = a + b = 0.2813$ in.

Clearance The clearance c is the amount by which the dedendum in a given gear exceeds the addendum of its mating gear. Hence, $c = b - a = 0.0313$ in.

Contact Ratio The contact ratio m_C is a measure of overlapping tooth action which is necessary to assure smooth, continuous action. For example, as one pair of teeth passes out of action, a succeeding pair of teeth must have already started action. The hunting ratio is the ratio of the number of gear and pinion teeth. It is a means of ensuring that each tooth in the pinion contacts every tooth in the gear before it contacts any gear tooth a second time. The calculation of contact ratio is shown in Equation 71.

$$m_C = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - C \sin\phi}{p_b} \tag{71}$$

where r_{ap} (1.3750 in) & r_{ag} (5.1250 in) are addendum radii of the mating pinion and gear
r_{bp} (1.0950 in) & r_{bg} (4.8450 in) are base circle radii of the mating pinion and gear
p_b is the base pitch and $p_b = p \cos\phi$

Calculating Equation 71 yields that the contact ratio of the gearset we design is equal to $m_C = 1.205$, which satisfies Constraint 11.

7 Final Design Summary

7.1 Designed System Concept

The diagram of the gearset we design is shown in Figure 12 and the details of the gear and pinion are shown in Figure 13 and 14, respectively. Because our picture is a vector diagram, you can zoom in infinitely to view its specific details.

The description of the whole system is as follows: the motor drives the pinion, and then the pinion transfers the torque to the gear, and the gear drives the conveyor belt so that the conveyor belt can move smoothly to transport the goods. The gearset we design has two functions. The first function is to transmit torque, and the second function is to reduce the speed of the motor.

7.2 Material Selection

The material we select is carburized and hardened steel Grade 3 for both pinion and gear, whose mechanical properties are $S_t = 75000$ psi and $S_c = 275000$ psi.

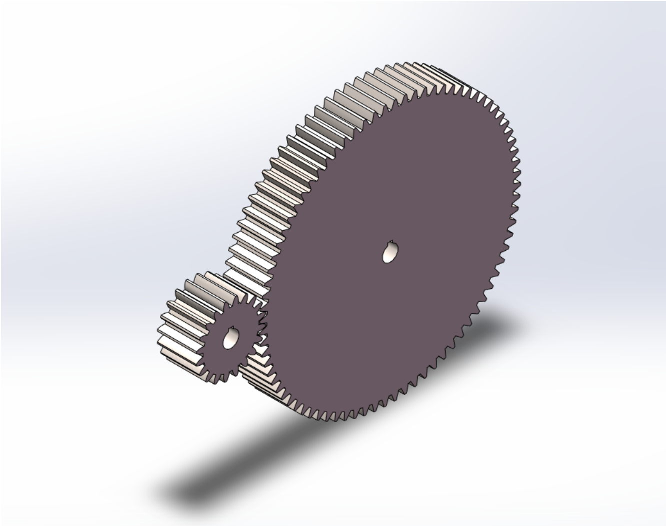

Figure 12: The gearset we design.

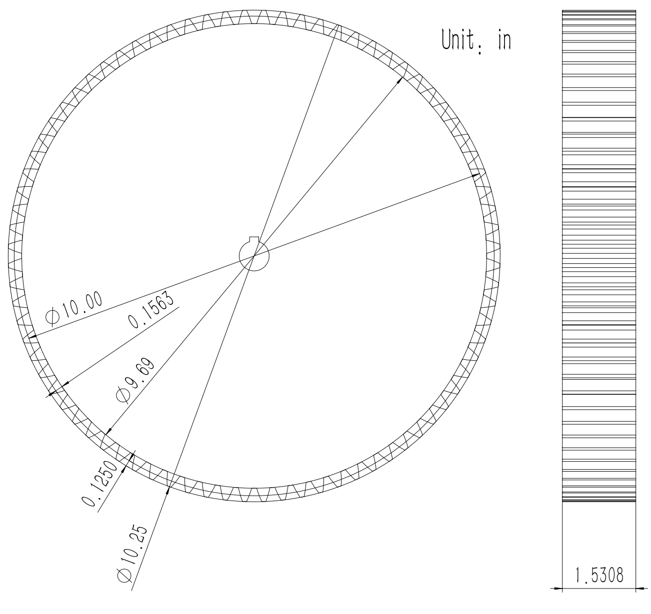

Figure 13: The details of the gear we design.

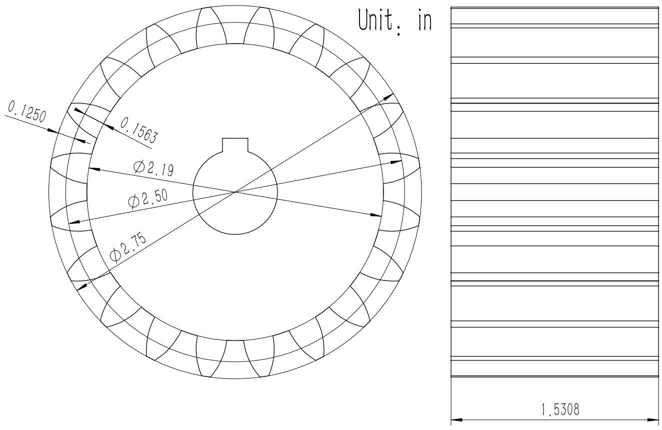

Figure 14: The details of the pinion we design.

7.3 Design Validation

All design validations are shown in Section 6.6.3. Overall, the system we designed is very stable.

8 Future Design Improvement

In this design exercise, we still have many parameters that have not been optimized. The future design improvement is shown following.

1. When calculating the minimum width, safety factor n=2 is introduced, which is a complete estimate value, in order to ensure that failure will not occur when unquantifiable impulse appears. In future experiments, we can detect the emergence of the largest impulse and set an accurate value.

2. Rim-thickness has been neglected in the design of the gear, and it is necessary to determine Thickness and its parameters in the subsequent design.

3. The radial component, W^r is neglected in our calculation, further analysis such as (Computer Aided Engineering) CAE can be used to take that into consideration.

4. The temperature with this gear system is assumed to be below 120°C, which should be test in the future.

References

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