Design Exercise 02 - Design of a Gear-Driven Speed Reducer Shaft

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1 System Descriptions

Gear drives as shown in Figure 1, sometimes referred to as gear trains and gearboxes, are mechanisms composed of an assembly of gears, shafts, and other machine components used to mount rotating parts. They form a mechanical system used to transfer shaft power from a driver such as engines, turbines, or motors to the driven piece of machinery. Gear drives can alter the transmitted power by using different configurations of gears (Li et al., 2017).

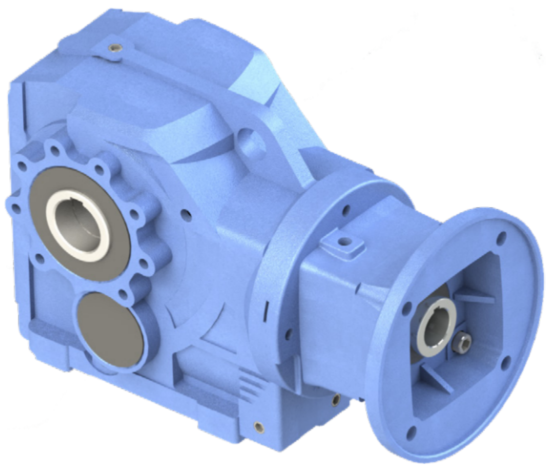

Figure 1: Typical gear drives.

Gear drives can increase or decrease the rotational speed of the output shaft. A common use of gear drives is for reducing speeds of motors and engines that typically run at thousands of revolutions per minute (rpm). These are known as speed reducers. By reducing the speed, torque is increased. This force amplification characteristic is one of the main functions of speed reducers (Deng et al., 2020).

Gears are the main components of gear drives. Gears are toothed rolling elements which mesh with one another by engaging their teeth. Due to the large dynamic forces involved, gears are made with alloyed steel. The properties of these metals are also modified by heat treatment to reach the right strength and stiffness required for its application.

Other components of gear drives are the shafts, keys, couplings, bearings, housing, and flanges (Xiangkai et al., 1999). The shafts are the components that connect the gear drive with input and output systems. Keys and couplings are used to lock the shafts of the driver and driven equipment onto the gear drive. Bearings are the machine elements that support the shafts while reducing friction. The housing and flanges are usually made monolithic. The housing encloses and supports the whole assembly while the flanges are used for mounting.

The schematic diagram of the speed reducer we need to design is shown in Figure 2.

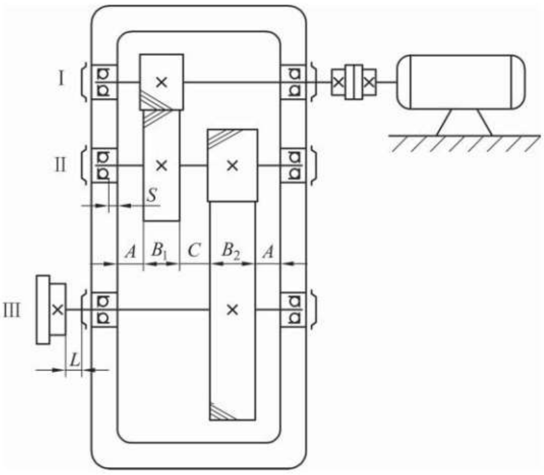

Figure 2: Schematic diagram for the speed reducer we need to design.

The power is transmitted from the motor on the input side to shaft I and to the speed reducer. Then, shaft I and shaft II are connected by helical gear so that torque can be effectively transmitted. After torque is transferred to shaft II, power is transferred to shaft III through the helical gear connecting shaft II and shaft III. In this way, the reduced torque is transferred from shaft III to the output side. What we want to design is this shaft III. The simplified schematic diagram for the shaft III is shown in Figure 3.

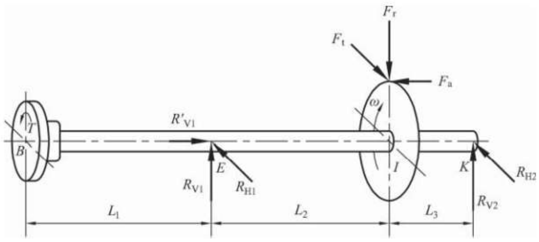

Figure 3: Simplified schematic diagram for the shaft III we need to design.

2 Design Objectives

There are three main goals in our design of a gear-driven speed reducer shaft as shown below:

1. Design the shaft so that it can transmit the torque from the input side to the output side.
2. Design the connection and also the power transmission between the gear and the shaft.
3. Design the two bearing to support the rotation and also the axial load of the shaft.

3 Design Limitations

3.1 Design Constraints

1. Helical gear I is key-mounted to the output shaft III.
2. The power to be transmitted by the output shaft III is 9.5 kW.
3. The output shaft rotates at a rotational speed of 120 rpm.
4. Operating temperature of the gearbox is near 60°C.
5. Expected service life of the gearbox is 1500 hours.
6. The reliability factor of gearbox is equal to 90%.
7. Acting forces on helical gear I are: tangential force $F_t = 4558$ N, radial force $F_r = 1699$ N, and axial force $F_a = 1002$ N. Previous structural design also decided the spans of $L_1 = 118.4$ mm, $L_2 = 113.6$ mm, and $L_3 = 53.6$ mm.
8. Pitch diameter of helical gear I on the output shaft is 324 mm.
9. Must use a temper-treated, medium-carbon steel as the shaft material to retain the production cost.
10. Material will be delivered in round bar and machined down to the final shaft dimensions.
11. Selected material must have strength to provide a safety factor of 1.5 resisting shaft fatigue failure.

3.2 Design Assumptions

To simplify our model and eliminate the complexity, we make the following main assumptions in this literature. All assumptions will be re-emphasized once they are used in the construction of our model.

1. In the force analysis, the force of all face-to-face contact is simplified to the force at one point, which means the bearing reactions is simple-supports, which are also shown in Figure 4.
2. The shaft diameter is less than 51 mm.
3. The fillet on the shaft is all shoulder fillet—sharp ($D/d = 1.5$ and $r/d = 0.02$).
4. The contribution of fatigue damaging from axial load is negligible.
5. Key and keyway design will not affect the fatigue failure of the shaft.
6. Failure occurs whenever the maximum shear stress τ_{max} in a design part exceeds the max shear stress at yield point in a tensile test specimen of the same material.
7. Failure is to occur whenever the octahedral shear stress (τ_{oct}) for any stress state equals or exceeds the octahedral shear stress for the simple tension-test specimen at yield point.
8. Axial loads are commonly ignored during feasibility study because the contribution from axial load is usually very small at critical locations.
9. The application factor of 02-Series Angular-Contact Ball Bearings is equal to 1.5.

4 Notations and Definitions

In this work, we use the nomenclature in Table 1 in the model construction. Other none-frequent-used symbols will be introduced once they are used.

Table 1: Notations used in this literature

Symbol	Meaning	unit
S_y	Yield strength of material	MPa
S_{ut}	Tensile strength of material	MPa
S'_e	Endurance limit	MPa
S_e	Modified endurance limit	MPa
σ'_{max}	von Mises maximum stress	MPa
σ_m	Midrange normal stress	MPa
σ_a	Alternating normal stress	MPa
τ_m	Midrange shear stress	MPa
τ_a	Alternating shear stress	MPa
$F_{i(\cdot)}$	Induced loads	N
$F_{e(\cdot)}$	Dynamic equivalent loads	N
C_0	Static load ratings	N
C_{10}	Dynamic load ratings	N
D	Basic size	mm
d	Diameter of shaft	mm
r	Fillet radius	mm
δ_F	Fundamental deviation	mm
ΔD	International tolerance grade	mm
Δd	International tolerance grade	mm
M_m	Midrange bending moments	$\mathrm{N \cdot m}$
M_a	Alternating bending moments	$\mathrm{N \cdot m}$
T_m	Midrange bending torques	$\mathrm{N \cdot m}$
T_a	Alternating bending torques	$\mathrm{N \cdot m}$
R	Reliability	–
K_t	Static stress-concentration factors for bending	–
K_{ts}	Static stress-concentration factors for torsion	–
K_f	Fatigue stress-concentration factors for bending	–
K_{fs}	Fatigue stress-concentration factors for torsion	–
a	Neuber constant	–
FS	Safety factor	–
x_D	Dimensionless design life	–
T_F	Degree Fahrenheit	°F
n	Rotational speed	rpm
L_D	Desired bearing life	rev
$\mathcal{L}$	Service life	h

5 Potential Failure Assessment

In this design, there are many factors that can lead to failure. We list the factors that may cause the failure of the system we designed in Table 2:

Table 2: Simply supported beam data table for result calculations.

Failure Scenarios	Critical Parameters	Design Acceptance Criteria	Risk Priorities (High/Medium/Low)
Static yielding failure along shaft	Material and diameter of shaft	Safety factor of static yield is larger than 1.5	Medium
Fatigue failure along shaft	Material and diameter of shaft	Safety factor of fatigue is larger than 1.5	High
Failure due to torque transmission along shaft	Material and diameter of shaft	Safety factor of shearing stress is larger than 1.5	Medium
Key and keyway failure	Material and dimension of key and keyway	Safety factor of key design is larger than 1.5	High
Spalling at the inner ring of the ball bearing	Equivalent force acting on the bearing	Application factor of the bearing is around 1.5	Medium

6 Design Thought Process

6.1 Iteration 01

6.1.1 Material Selection

According to Constraint 9, we must use a temper-treated, medium-carbon steel as the shaft material to retain the production cost. In *ASM Metals Reference Book*, we select AISI No. 1040, quenched and tempered with temperature 205°C, whose physical constants and mechanical properties are listed in Equation 1.

$$\begin{cases} S_{ut} = 779 \text{ MPa} = 113 \text{ ksi} \\ S_y = 593 \text{ MPa} = 86 \text{ ksi} \end{cases} \tag{1}$$

6.1.2 Force Analysis

The reactions in the xz plane from Figure 4b are shown in Equation 2 and 3.

$$R_{H1} = \frac{4558\,(53.6)}{167.2} = 1461.2 \text{ N} \tag{2}$$

$$R_{H2} = \frac{4558\,(113.6)}{167.2} = 3096.8 \text{ N} \tag{3}$$

The reactions in the xy plane from Figure 4c are shown in Equation 4 and 5.

$$R_{V1} = \frac{1699\,(53.6)}{167.2} + \frac{162324}{167.2} = 1515.5 \text{ N} \tag{4}$$

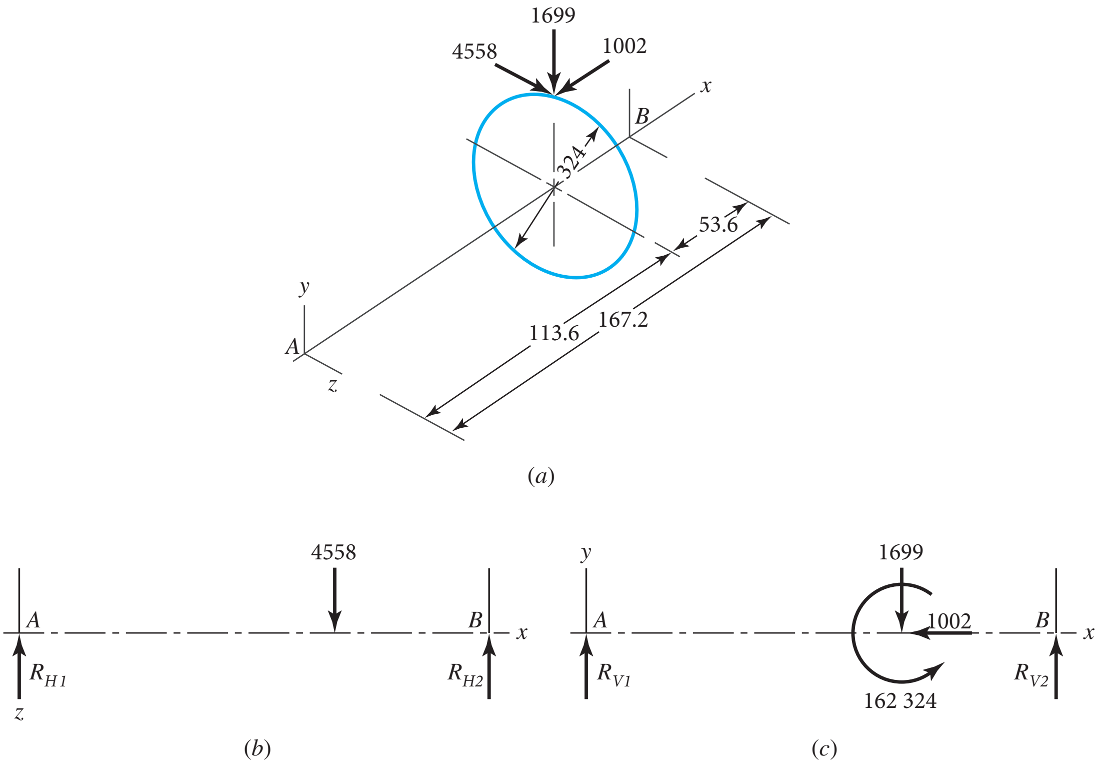

Figure 4: Essential geometry of helical gear and shaft. Length dimensions in mm, loads in N, couple in N · mm. (a) Sketch (not to scale) showing thrust, radial, and tangential forces. (b) Forces in xz plane. (c) Forces in xy plane.

$$R_{V2} = \frac{1699\,(113.6)}{167.2} - \frac{162324}{167.2} = 183.5072 \text{ N} \tag{5}$$

The radial loads F_{rA} and F_{rB} are the vector additions of R_{H1} and R_{V1}, and R_{H2} and R_{V2} as shown in Equation 6 and 7, respectively.

$$F_{rA} = \sqrt{R_{H1}^2 + R_{V1}^2} = \sqrt{1461.2^2 + 1515.5^2} = 2105.2 \text{ N} \tag{6}$$

$$F_{rB} = \sqrt{R_{H2}^2 + R_{V2}^2} = \sqrt{3096.8^2 + 183.5072^2} = 3102.3 \text{ N} \tag{7}$$

6.1.3 Static Yield Analysis

From Section 6.1.2, we can know that the maximum bending moment acting along the shaft is located near the gear, which is shown in Equation 8.

$$M_{max} = \max\{F_{rA} \cdot L_2, F_{rB} \cdot L_3\} = 239.1477 \text{ N} \cdot \text{m} \tag{8}$$

From what has been calculated above, we can know that for a rotating shaft, the constant bending moment will create a completely reversed bending stress as shown in Equation 9 and

10.

$$\begin{cases} M_a = M_{max} = 239.1477 \text{ N} \cdot \text{m} \\ M_m = 0 \text{ N} \cdot \text{m} \end{cases} \tag{9}$$

$$\begin{cases} T_a = 0 \text{ N} \cdot \text{m} \\ T_m = F_a \cdot r = 738.3960 \text{ N} \cdot \text{m} \end{cases} \tag{10}$$

From Figure 5, we estimate $K_t = 2.7$ and $K_{ts} = 2.2$ for sharp fillet.

	Bending	Torsional	Axial
Shoulder fillet—sharp ($r/d = 0.02$)	2.7	2.2	3.0
Shoulder fillet—well rounded ($r/d = 0.1$)	1.7	1.5	1.9
End-mill keyseat ($r/d = 0.02$)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Missing values in the table are not readily available.

Figure 5: First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts} (Shigley & Mischke, 2005).

In order to find the reduced value of K_t and K_{ts}, we need to make use of *Neuber equation*, which is given by Equation 11 and 12.

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \tag{11}$$

$$K_{fs} = 1 + \frac{K_{ts} - 1}{1 + \sqrt{a/r}} \tag{12}$$

where $\sqrt{a}$ is defined as the *Neuber constant* and is a material constant given by Equation 13 and 14.

For bending or axial:

$$\sqrt{a} = 0.246 - 3.08\left(10^{-3}\right) S_{ut} + 1.51\left(10^{-5}\right) S_{ut}^2 - 2.67\left(10^{-8}\right) S_{ut}^3 \tag{13}$$

For torsion:

$$\sqrt{a} = 0.190 - 2.51\left(10^{-3}\right) S_{ut} + 1.35\left(10^{-5}\right) S_{ut}^2 - 2.67\left(10^{-8}\right) S_{ut}^3 \tag{14}$$

It is always necessary to consider the possibility of static failure in the first load cycle. The Soderberg criteria inherently guards against yielding, as can be seen by noting that its failure curve is conservatively within the yield (Langer) line on Figure 6. The ASME Elliptic also takes yielding into account, but is not entirely conservative throughout its entire range. This is evident by noting that it crosses the yield line in Figure 6. The Gerber and modified Goodman criteria do not guard against yielding, requiring a separate check for yielding. A von Mises maximum stress is calculated for this purpose as shown in Equation 15.

$$\sigma'_{max} = \left[\left(\sigma_m + \sigma_a\right)^2 + 3\left(\tau_m + \tau_a\right)^2\right]^{1/2} = \left[\left(\frac{32K_f\left(M_m + M_a\right)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}\left(T_m + T_a\right)}{\pi d^3}\right)^2\right]^{1/2} \tag{15}$$

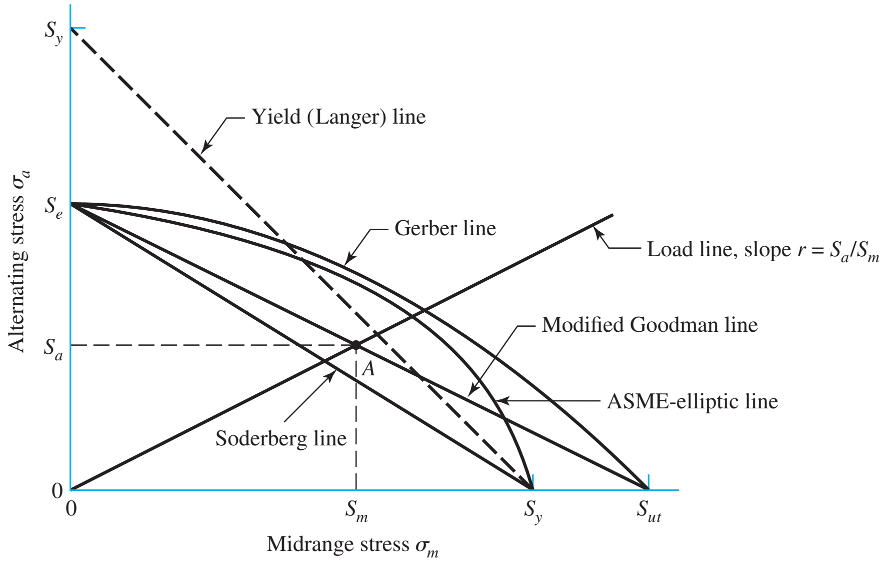

Figure 6: Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to the strength S_a, which, paired with σ_m, is the limiting value of σ_a (Shigley et al., 1972).

$$FS = \frac{S_y}{\sigma'_{max}} \tag{16}$$

In Equation 16, solving for diameter d for the shaft yields Equation 17.

$$d = 0.03278 \text{ m} = 32.78 \text{ mm} \tag{17}$$

6.1.4 Fatigue-Failure Analysis

For steels, we will estimate the endurance limit as Equation 18.

$$S'_e = 0.5 S_{ut} \tag{18}$$

where S_{ut} is the minimum tensile strength. The prime mark on S'_e in Equation 18 refers to the rotating-beam specimen itself. We wish to reserve the unprimed symbol S_e for the endurance limit of an actual machine element subjected to any kind of loading.

Marin identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. A Marin equation is therefore written as Equation 19.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{19}$$

where k_a = surface condition modification factor
k_b = size modification factor
k_c = load modification factor
k_d = temperature modification factor
k_e = reliability factor
k_f = miscellaneous-effects modification factor
S'_e = rotary-beam test specimen endurance limit
S_e = endurance limit at the critical location of a machine part in the geometry and condition of use

In the next, we will calculate each factor in Equation 19 in turn.

Surface condition modification factor Here, we consider that the cost of the shaft is a bit high, so we try our best to use better surface processing methods to make the surface condition modification factor k_a as large as possible. Based on this, we polish the round bar, whose surface condition modification factor is shown in Equation 20.

$$k_a = 1 \tag{20}$$

Size modification factor Based on the result calculated in Section 6.1.3, we can temporarily assume that the diameter of the shaft is less than 51 mm. The size factor has been evaluated using 133 sets of data points (Mischke, 1987). The results for bending and torsion may be expressed as Equation.

$$k_b = (d/7.62)^{-0.107} \tag{21}$$

Load modification factor According to the force analysis in Section 6.1.2, we can know that the shaft we need to design is acted by torsion, axial loading, and bending. When torsion is combined with other loading, such as bending, we need to set the load modification factor as shown in Equation .

$$k_c = 1 \tag{22}$$

Temperature modification factor According to Constraint 4, we can know that the operating temperature of the gearbox is near 60°C, for which we can calculate the temperature modification factor as shown in Equation 23.

$$k_d = 0.975 + 0.432\left(10^{-3}\right)T_F - 0.115\left(10^{-5}\right)T_F^2 + 0.104\left(10^{-8}\right)T_F^3 - 0.595\left(10^{-12}\right)T_F^4 \tag{23}$$

Reliability factor According to Constraint 6, we can know that the reliability factor of gearbox is equal to 90%. We have verified that the reliability factor does not have much impact on the determination of the final shaft diameter, and the impact is about 3% if we change the reliability from 90% to 99.9%. For the convenience of subsequent bearing design, we set the reliability of the shaft to 99.9%. Therefore, we can calculate the reliability factor as shown in Equation 24.

$$k_e = 0.753 \tag{24}$$

Miscellaneous-effects modification factor There is no miscellaneous-effects mentioned in the problem statement, so we set the miscellaneous-effects modification factor to be 1.

For fatigue-failure analysis, we implement DE-Goodman criteria as shown in Equation 25.

$$\frac{1}{FS} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs}T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs}T_m\right)^2\right]^{1/2}\right\} \tag{25}$$

In Equation 25, solving for diameter d for the shaft yields Equation 26.

$$d = 0.03882 \text{ m} = 38.82 \text{ mm} \tag{26}$$

Combining the calculations in Section 6.1.3 and 6.1.4, we can get the minimum allowable diameter of shaft equal to 38.82 mm. In order to facilitate the later processing and calculation, we use commonly used dimensions to design our shaft – the diameter of shaft is designed to be 40 mm.

6.1.5 Clearance Fit Design between Shaft and Gear

Up to now, we have determined that the diameter of the shaft we use for shaft III, which is equal to 40 mm. This diameter is also the basic size of the clearance fit we are going to design. Here, we decide to use close running fit, which is designed for running on accurate machines and for accurate location at moderate speeds and journal pressures (Cao et al., 2019).

According to Table 7-20, the symbol for the close running fit is H8/f6. And the basic size of the system is equal to 40 mm. According to Table A-11, the tolerance grade of H8 is $\Delta D = 0.039$ mm and the tolerance grade of f6 is $\Delta d = 0.016$ mm. Also, according to Table A-12, the fundamental deviation for f6 is $\delta_F = -0.025$ mm. Therefore, the minimum shaft outer diameter is equal to

$$d_{min} = D - \Delta d = 40 \text{ mm} - 0.016 \text{ mm} = 39.984 \text{ mm} \tag{27}$$

And the maximum shaft outer diameter is equal to

$$d_{max} = D = 40 \text{ mm} \tag{28}$$

Therefore, the range of the shaft outer diameter is $[39.984, 40]$ mm.

Then, the minimum gear inner diameter is equal to

$$D_{min} = D - \delta F = 40 \text{ mm} - (-0.025 \text{ mm}) = 40.025 \text{ mm} \tag{29}$$

And the maximum gear inner diameter is equal to

$$D_{max} = D - \delta F + \Delta D = 40 \text{ mm} - (-0.025 \text{ mm}) + 0.039 \text{ mm} = 40.064 \text{ mm} \tag{30}$$

According to Equation 29 and 30, the range of the gear inner diameter is $[40.025, 40.064]$ mm.

6.1.6 Key & Keyway Design between Shaft and Gear

We choose a cold-drawn low-carbon mild steel which is generally available for key stock, such as UNS G10180, with a yield strength of 370 MPa. Because the basic size of the shaft we design is equal to 40 mm, we set the dimension of key and keyway as following (Shigley et al., 2004). The schematic diagram of key and keyway is shown in Figure 7.

Keyway	Width $W = 12$ mm	Key	Width $W = 12$ mm
	Depth $h = 3.3$ mm		Depth $T = 8$ mm

The minimum length of key and keyway can be determined by shearing failure and bearing stress failure as follows.

Shearing Failure:
$$L = \frac{4FST_m}{WDS_y} = 0.0249 \text{ m} = 24.9 \text{ mm} \tag{31}$$

Bearing Stress Failure:
$$L = \frac{4FSKT_m}{TDS_y} = 0.0486 \text{ m} = 48.6 \text{ mm} \tag{32}$$

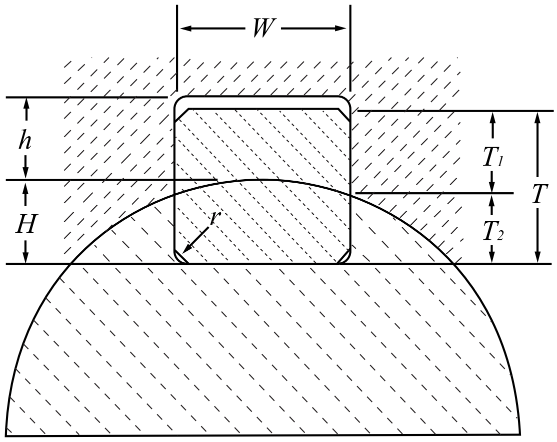

Figure 7: Key and keyway dimensions.

In addition, the maximum length of a key is limited by the hub length of the attached element, and should generally not exceed 1.5 times the shaft diameter, which is equal to 60 mm. The minimum key length should be $\geq 25\%$ larger than the shaft diameter, which is equal to 50 mm.

Combining shearing failure, bearing stress failure, and also the minimum key length requirement corresponding to shaft diameter, we choose 50 mm as the length of key and keyway, which is the same as the width of gear.

The problem has arisen - the distance between the end of a keyseat and the start of the shoulder fillet is greater than one-tenth of the shaft diameter, which will result in a great stress concentration near the shoulder fillet. Therefore, in Iteration 02 in Section 6.2, we will try to change our material selection of the shaft to reduce the shaft diameter so that the length of the key and keyway will be reduced.

6.1.7 Design of Gear Axial Fixation on Shaft

There are two ways to fix the helical gear axially to the shaft.

- Use keyless bushings to fix it (without flat keys).
- While adding flat keys with set screws, the shaft end is threaded and locked with precision nuts.

6.1.8 Bearing Design

For the first try for bearing design, we use Tapered Roller Bearing because it has greater load-carrying capacity and can carry both radial and thrust (axial) loads, or any combination of the two, at the same time.

Using K of 1.5 as the initial guess for each bearing, the induced loads from the bearings are shown in Equation 33 and 34.

$$F_{iA} = \frac{0.47F_{rA}}{K_A} = 659.629 \text{ N} \tag{33}$$

$$F_{iB} = \frac{0.47F_{rB}}{K_B} = 972.054 \text{ N} \tag{34}$$

Under direct mounting case, since F_{iA} is clearly less than $F_{iB} + F_{ae}$, bearing A carries the net thrust load. Therefore, the dynamic equivalent loads are shown in Equation 35 and 36.

$$F_{eA} = 0.4F_{rA} + K_A\left(F_{iB} + F_{ae}\right) = 3803.161 \text{ N} \tag{35}$$

$$F_{eB} = F_{rB} = 3102.3 \text{ N} \tag{36}$$

Under indirect mounting case, since F_{iA} is larger than $F_{iB} - F_{ae}$, bearing B carries the net thrust load. Therefore, the dynamic equivalent loads are shown in Equation 37 and 38.

$$F_{eA} = F_{rA} = 2105.2 \text{ N} \tag{37}$$

$$F_{eB} = 0.4F_{rB} + K_B\left(F_{iA} + F_{ae}\right) = 3733.3635 \text{ N} \tag{38}$$

If we select tapered roller bearing from the table in the textbook, we find that the diameter of tapered roller bearing is smaller than 15 mm, which usually is an indication not needing a tapered roller bearing. Therefore, in the following iteration, we will try the angular contact ball bearing.

6.2 Iteration 02

6.2.1 Material Selection

According to Constraint 9, we must use a temper-treated, medium-carbon steel as the shaft material to retain the production cost. In *ASM Metals Reference Book*, we select AISI No. 1050, quenched and tempered with temperature 205°C this time, whose physical constants and mechanical properties are listed in Equation 39.

$$\begin{cases} S_{ut} = 1120 \text{ MPa} = 163 \text{ ksi} \\ S_y = 807 \text{ MPa} = 117 \text{ ksi} \end{cases} \tag{39}$$

6.2.2 Static Yield Analysis

From Section 6.2.2, we can know that for a rotating shaft, the constant bending moment will create a completely reversed bending stress as shown in Equation 40 and 41.

$$\begin{cases} M_a = M_{max} = 239.1477 \text{ N} \cdot \text{m} \\ M_m = 0 \text{ N} \cdot \text{m} \end{cases} \tag{40}$$

$$\begin{cases} T_a = 0 \text{ N} \cdot \text{m} \\ T_m = F_a \cdot r = 738.3960 \text{ N} \cdot \text{m} \end{cases} \tag{41}$$

From Figure 5, we estimate $K_t = 2.7$ and $K_{ts} = 2.2$ for sharp fillet.

In order to find the reduced value of K_t and K_{ts}, we need to make use of *Neuber equation*, which is given by Equation 42 and 43.

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} \tag{42}$$

$$K_{fs} = 1 + \frac{K_{ts} - 1}{1 + \sqrt{a/r}} \tag{43}$$

It is always necessary to consider the possibility of static failure in the first load cycle. The Soderberg criteria inherently guards against yielding, as can be seen by noting that its failure curve is conservatively within the yield (Langer) line on Figure 6. The ASME Elliptic also takes yielding into account, but is not entirely conservative throughout its entire range. This is evident by noting that it crosses the yield line in Figure 6. The Gerber and modified Goodman criteria do not guard against yielding, requiring a separate check for yielding. A von Mises maximum stress is calculated for this purpose as shown in Equation 44.

$$\sigma'_{max} = \left[(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2\right]^{1/2} = \left[\left(\frac{32K_f (M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \tag{44}$$

$$FS = \frac{S_y}{\sigma'_{max}} \tag{45}$$

In Equation 45, solving for diameter d for the shaft yields Equation 46.

$$d = 0.03000 \text{ m} = 30.00 \text{ mm} \tag{46}$$

6.2.3 Fatigue-Failure Analysis

For steels, we will estimate the endurance limit as Equation 47.

$$S'_e = 0.5 S_{ut} \tag{47}$$

where S_{ut} is the minimum tensile strength. The prime mark on S'_e in Equation 47 refers to the rotating-beam specimen itself. We wish to reserve the unprimed symbol S_e for the endurance limit of an actual machine element subjected to any kind of loading.

Marin identified factors that quantified the effects of surface condition, size, loading, temperature, and miscellaneous items. A Marin equation is therefore written as Equation 48.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e \tag{48}$$

In the next, we will calculate each factor in Equation 48 in turn.

Surface condition modification factor Here, we consider that the cost of the shaft is a bit high, so we try our best to use better surface processing methods to make the surface condition modification factor k_a as large as possible. Based on this, we polish the round bar, whose surface condition modification factor is shown in Equation 49.

$$k_a = 1 \tag{49}$$

Size modification factor Based on the result calculated in Section 6.2.2, we can temporarily assume that the diameter of the shaft is less than 51 mm. The size factor has been evaluated using 133 sets of data points (Mischke, 1987). The results for bending and torsion may be expressed as Equation.

$$k_b = (d/7.62)^{-0.107} \tag{50}$$

Load modification factor According to the force analysis in Section 6.1.2, we can know that the shaft we need to design is acted by torsion, axial loading, and bending. When torsion is combined with other loading, such as bending, we need to set the load modification factor as shown in Equation .

$$k_c = 1 \tag{51}$$

Temperature modification factor According to Constraint 4, we can know that the operating temperature of the gearbox is near 60°C, for which we can calculate the temperature modification factor as shown in Equation 52.

$$k_d = 0.975 + 0.432\left(10^{-3}\right)T_F - 0.115\left(10^{-5}\right)T_F^2 + 0.104\left(10^{-8}\right)T_F^3 - 0.595\left(10^{-12}\right)T_F^4 \tag{52}$$

Reliability factor According to Constraint 6, we can know that the reliability factor of gearbox is equal to 90%. Here, we change our reliability factor to the commonly used one, which is 95% Therefore, we can calculate the reliability factor as shown in Equation 53.

$$k_e = 0.868 \tag{53}$$

Miscellaneous-effects modification factor There is no miscellaneous-effects mentioned in the problem statement, so we set the miscellaneous-effects modification factor to be 1.

For fatigue-failure analysis, we implement DE-Goodman criteria as shown in Equation 54.

$$\frac{1}{FS} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2}\right\} \tag{54}$$

In Equation 54, solving for diameter d for the shaft yields Equation 55.

$$d = 0.03391 \text{ m} = 33.91 \text{ mm} \tag{55}$$

Combining the calculations in Section 6.2.2 and 6.2.3, we can get the minimum allowable diameter of shaft equal to 33.91 mm. In order to facilitate the later processing and calculation, we use commonly used dimensions to design our shaft – the diameter of shaft is designed to be 34 mm.

6.2.4 Clearance Fit Design between Shaft and Gear

Up to now, we have determined that the diameter of the shaft we use for shaft III, which is equal to 34 mm. This diameter is also the basic size of the clearance fit we are going to design. Here, we decide to use close running fit, which is designed for running on accurate machines and for accurate location at moderate speeds and journal pressures (Cao et al., 2019).

According to Table 7-20, the symbol for the close running fit is H8/f6. And the basic size of the system is equal to 34 mm. According to Table A-11, the tolerance grade of H8 is $\Delta D = 0.039$ mm and the tolerance grade of f6 is $\Delta d = 0.016$ mm. Also, according to Table A-12, the fundamental deviation for f6 is $\delta_F = -0.025$ mm. Therefore, the minimum shaft outer diameter is equal to

$$d_{min} = D - \Delta d = 34 \text{ mm} - 0.016 \text{ mm} = 33.984 \text{ mm} \tag{56}$$

And the maximum shaft outer diameter is equal to

$$d_{max} = D = 34 \text{ mm} \tag{57}$$

Therefore, the range of the shaft outer diameter is $[33.984, 34]$ mm.

Then, the minimum gear inner diameter is equal to

$$D_{min} = D - \delta F = 34 \text{ mm} - (-0.025 \text{ mm}) = 34.025 \text{ mm} \tag{58}$$

And the maximum gear inner diameter is equal to

$$D_{max} = D - \delta F + \Delta D = 34 \text{ mm} - (-0.025 \text{ mm}) + 0.039 \text{ mm} = 34.064 \text{ mm} \tag{59}$$

According to Equation 58 and 59, the range of the gear inner diameter is $[34.025, 34.064]$ mm.

6.2.5 Key & Keyway Design between Shaft and Gear

We choose a cold-drawn low-carbon mild steel which is generally available for key stock, such as UNS G10180, with a yield strength of 370 MPa. Because the basic size of the shaft we design is equal to 34 mm, we set the dimension of key and keyway as following (Shigley et al., 2004). The schematic diagram of key and keyway is shown in Figure 7.

Keyway	Width $W = 10$ mm	Key	Width $W = 10$ mm
	Depth $h = 3.3$ mm		Depth $T = 8$ mm

The minimum length of key and keyway can be determined by shearing failure and bearing stress failure as follows.

$$\text{Shearing Failure:} \quad L = \frac{4FST_m}{WDS_y} = 0.0352 \text{ m} = 35.2 \text{ mm} \tag{60}$$

$$\text{Bearing Stress Failure:} \quad L = \frac{4FSKT_m}{TDS_y} = 0.0572 \text{ m} = 57.2 \text{ mm} \tag{61}$$

The problem has arisen again, the distance between the end of a keyseat and the start of the shoulder fillet is greater than one-tenth of the shaft diameter, which will result in a great stress concentration near the shoulder fillet. Therefore, in Iteration 03 in Section 6.3, we will try to change our material selection of the key to length of key to handle the torque transmission.

6.3 Iteration 03

6.3.1 Key & Keyway Design between Shaft and Gear

We choose a cold-drawn low-carbon mild steel which is generally available for key stock, such as UNS G10400, with a yield strength of 490 MPa. Because the basic size of the shaft we design is equal to 34 mm, we set the dimension of key and keyway as following (Shigley et al., 2004). The schematic diagram of key and keyway is shown in Figure 7.

Keyway	Width $W = 10$ mm	Key	Width $W = 10$ mm
	Depth $h = 3.3$ mm		Depth $T = 8$ mm

The minimum length of key and keyway can be determined by shearing failure and bearing stress failure as follows.

$$\text{Shearing Failure:} \quad L = \frac{4FST_m}{WDS_y} = 0.0266 \text{ m} = 26.6 \text{ mm} \tag{62}$$

$$\text{Bearing Stress Failure:} \quad L = \frac{4FSKT_m}{TDS_y} = 0.0432 \text{ m} = 43.2 \text{ mm} \tag{63}$$

In addition, the maximum length of a key is limited by the hub length of the attached element, and should generally not exceed 1.5 times the shaft diameter, which is equal to 51 mm. The minimum key length should be $\geq 25\%$ larger than the shaft diameter, which is equal to 42.5000 mm.

Combining shearing failure, bearing stress failure, and also the minimum key length requirement corresponding to shaft diameter, we choose 43.2 mm as the length of key and keyway.

6.3.2 Bearing Design

The total number of bearing on shaft III, NB is 2. And according to Constraint 6, we can know that the individual bearing reliabilities, if equal, must be shown in Equation 64.

$$R_A = R_B = R_T^{\frac{1}{NB}} = 0.94868 \tag{64}$$

The desired bearing life is shown in Equation 65.

$$L_D = \mathcal{L}n = (1500\ \text{h}) \times (120\ \text{rpm}) \times \frac{60\ \text{min}}{1\ \text{h}} = 1.08 \times 10^7\ \text{rev} \tag{65}$$

From Equation 6 and 7, we can know the radial reaction force on the bearing A and B. Combined with the given external loads on the gear train, we can assume that because the force acting on the radical direction is not very big and axial force exists here, the 02-Series Angular-Contact Ball Bearings are used.

The dimensionless design life for both bearings is shown in Equation 66.

$$x_D = \frac{L_D}{L_{10}} = 10.8 \tag{66}$$

For Angular ball Bearing, the parameters when calculating C_{10} is $a' = 3$ and $R = 94.868\%$. Since the inner shaft is rotating while the outer house keeps static, $V = 1$. The maximum fillet radius of the bearings is $r/d = 0.02$. Let's first calculate the parameter for bearing A.

1st Iteration We select 20 mm bore bearing.

First, we verify whether the sharp fillet can satisfy the fillet radius requirement of 20 mm bore bearing as shown in Equation 67.

$$r = 0.02d = 0.4\ \text{mm} < 1.0\ \text{mm} \tag{67}$$

Also, we notice that the Static Load Ratings, C_0, of 02-Series Angular-Contact Ball Bearings - 20 mm is equal to 6.55 kN, we can get that

$$\frac{F_a}{C_0} = 0.153 \tag{68}$$

which makes e from Figure 8 approximately 0.329. Now $F_a/(VF_r) = 0.4760$, which is greater than 0.329, so we find Y_2 by interpolation as shown in Figure 9.

Therefore, the equivalent force action on bearing A is shown in Equation 69.

$$F_e = X_2 V F_r + Y_2 F_a = 1986.5\ \text{N} \tag{69}$$

We know that the Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$, an application factor of 1.5, and $a = 3$ for the ball bearing at A, the catalog rating should be equal to or greater than C_{10} in Equation 70.

$$C_{10} = a_f F_e \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R)^{1/b}} \right]^{1/a} = 9.749\ \text{kN} < 13.3\ \text{kN} \tag{70}$$

Hence, in next iteration, we can make bore diameter smaller to save the cost.

		$F_a/(VF_r) \le e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

Figure 8: Equivalent radial load factors for ball bearings (Shigley et al., 1972).

F_a/C_0	Y_2	
0.110		
0.153	Y_2	from which $Y_2 = 1.35$
0.170		

Figure 9: Interpolation to get Y.

2[rd] Iteration We select 17 mm bore bearing.

First, we verify whether the sharp fillet can satisfy the fillet radius requirement of 17 mm bore bearing as shown in Equation 71.

$$r = 0.02d = 0.34 \text{ mm} < 0.6 \text{ mm} \tag{71}$$

Also, we notice that the Static Load Ratings, C_0, of 02-Series Angular-Contact Ball Bearings - 17 mm is equal to 4.75 kN, we can get that

$$\frac{F_a}{C_0} = 0.2109 \tag{72}$$

which makes e from Figure 8 approximately 0.355. Now $F_a/(VF_r) = 0.4760$, which is greater than 0.355, so we find Y_2 by interpolation as shown in Figure 10.

Therefore, the equivalent force action on bearing A is shown in Equation 73.

$$F_e = X_2 V F_r + Y_2 F_a = 2431.4 \text{ N} \tag{73}$$

We know that the Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$, an application factor of 1.5, and $a = 3$ for the ball bearing at A, the catalog rating should be equal

F_a/C_0	Y_2	
0.170		
0.211	Y_2	from which $Y_2 = 1.25$
0.280		

Figure 10: Interpolation to get Y.

to or greater than C_{10} in Equation 74.

$$C_{10} = a_f F_e \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R)^{1/b}} \right]^{1/a} = 9364.0 \text{ N} < 9.95 \text{ kN} \tag{74}$$

Therefore, 17 mm bore and 40mm outside diameter should be used for bearing A.

However, according to Equation 75, 76, and 77, the shaft in the bearing must bear the torque passed by the gear, from which we can know that the minimum diameter for the outer part of the Shaft is 24.09 mm as shows in Equation 78. Based on this, 25 mm bore and 52 mm outside diameter angular-contact ball bearing is used for A.

$$FS = \frac{S_y/2}{\tau_{max}} \tag{75}$$

$$\tau_{max} = \frac{T_m c}{J} \tag{76}$$

$$J = \frac{\pi d^4}{32} \tag{77}$$

$$d = 0.02409 \text{ m} = 24.09 \text{ mm} \tag{78}$$

Then, we calculate the parameter for bearing B.

The radical load is the only force acting on the bearing. However, although it doesn't hold the axial load, angular ball bearing is stilling using to prevent impact and vibration on this system.

We verify whether the sharp fillet can satisfy the fillet radius requirement of 20 mm bore bearing as shown in Equation 79.

$$r = 0.02d = 0.4 \text{ mm} < 1.0 \text{ mm} \tag{79}$$

We know that the Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$, an application factor of 1.5, and $a = 3$ for the ball bearing at A, the catalog rating should be equal to or greater than C_{10} in Equation 80.

$$C_{10} = a_f F_r \left[\frac{x_D}{x_0 + (\theta - x_0)(1 - R)^{1/b}} \right]^{1/a} = 11.95 \text{ kN} < 13.3 \text{ kN} \tag{80}$$

The angular ball bearing with 20 mm bore and 47 mm outer diameter is used for bearing B.

Finally, we will verify the reliability improvement. For bearing A, $C_{10} = 14.8$ kN and $C_0 = 7.65$ kN. We can get Equation 81.

$$\frac{F_a}{C_0} = 0.131 \tag{81}$$

F_a/C_0	Y_2	
0.110		
0.131	Y_2	from which $Y_2 = 1.40$
0.170		

Figure 11: Interpolation to get Y.

which makes e from Figure 8 approximately 0.314. Now $F_a/(VF_r) = 0.4760$, which is greater than 0.314, so we find Y_2 by interpolation as shown in Figure 11.

Therefore, the equivalent force action on bearing A is shown in Equation 82.

$$F_e = X_2 V F_r + Y_2 F_a = 2.583 \text{ kN} \tag{82}$$

Based on this, we can know that the reliability of bearing A is shown in Equation 83.

$$R_A = \exp\left\{ -\left[\frac{x_D \left(\frac{a_f F_e}{C_{10}}\right)^a - x_0}{\theta - x_0} \right]^b \right\} = 0.9921 \tag{83}$$

For bearing B, $C_{10} = 13.3$ kN and $C_0 = 6.55$ kN. We can get Equation 84.

$$R_B = \exp\left\{ -\left[\frac{x_D \left(\frac{a_f F_r}{C_{10}}\right)^a - x_0}{\theta - x_0} \right]^b \right\} = 0.9683 \tag{84}$$

Combining Equation 83 and 84, we can know that the total reliability of this two bearings system is shown in Equation.

$$R = R_A R_B = 0.9418 > 0.9 \tag{85}$$

which is acceptable.

To keep the bearings stabilize in the same places, the retaining rings are supposed to be used at the outer sides of the bearings as shown in Figure 12. With the shoulder on the shaft and the retaining rings on the other side, the movement of the bearings on the shaft direction would be limited.

7 Final Design Summary

7.1 Designed System Concept

So far, the shaft III of the speed reducer has been designed. The following is a description of the entire system. The power is transferred from other shafts to the gear. The gear transfers these powers to the shaft III through the key and keyway. The entire shaft is fixed to the housing of the speed reducer through two bearings at both ends, so as to transfer the reduced power to the output side.

The panoramic diagram of the shaft we designed is shown in Figure 12. Because our picture is a vector diagram, you can zoom in infinitely to view its specific details. To assemble the output shaft III, the first thing that needs to be done is to place the key used to prevent the gear from turning. Then, the gear should be put in and ensure a stable assembly with the key. After that, the two bearings need to be placed at both ends of the shaft and placed with the house. Finally, a retaining ring is used to locate the bearings.

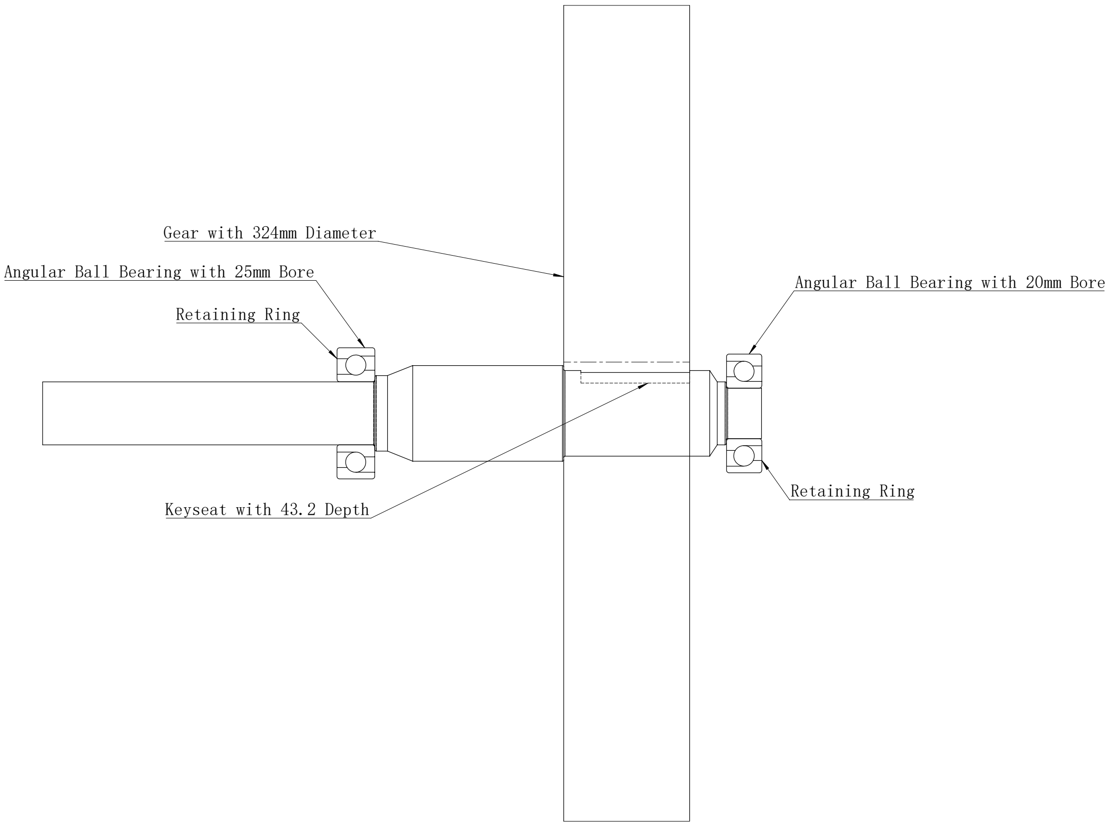

Figure 12: Panoramic diagram of the designed shaft.

The specific dimensions of the shaft III of the speed reducer we designed are marked in the figure 13. Also, you can zoom in infinitely to view its specific details.

7.2 Material Selections

7.2.1 Material Selection for Shaft

According to Constraint 9, we must use a temper-treated, medium-carbon steel as the shaft material to retain the production cost. In *ASM Metals Reference Book*, we select AISI No. 1050, quenched and tempered with temperature 205°C this time, whose physical constants and mechanical properties are listed in Equation 86.

$$\begin{cases} S_{ut} = 1120 \text{ MPa} = 163 \text{ ksi} \\ S_y = 807 \text{ MPa} = 117 \text{ ksi} \end{cases} \tag{86}$$

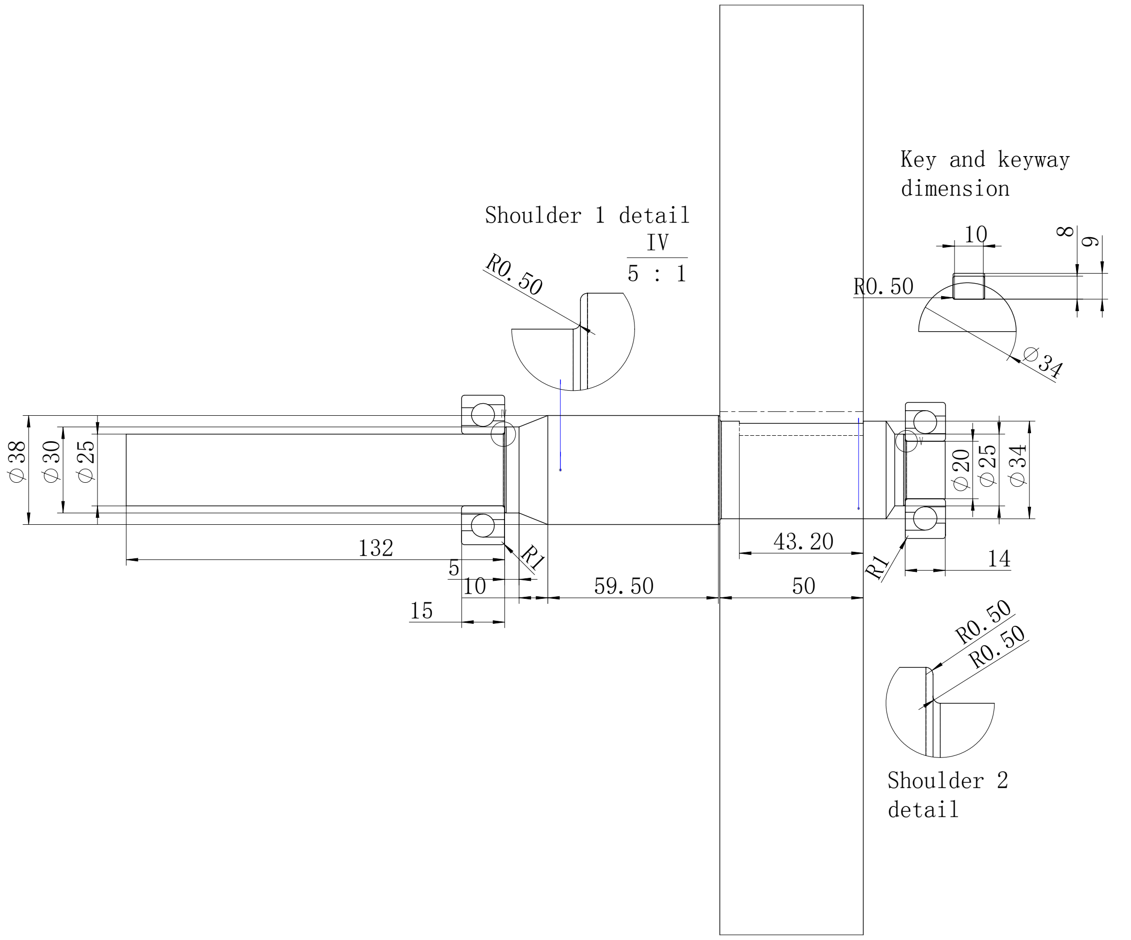

Figure 13: Dimension of the designed shaft.

7.2.2 Material Selection for Key

We choose a cold-drawn low-carbon mild steel which is generally available for key stock, such as UNS G10400, with a yield strength of 490 MPa.

7.2.3 Bearing Type

For bearing A, we select 25 mm bore and 52 mm outside diameter angular-contact ball bearing. For bearing B, we select 20 mm bore and 47 mm outer diameter angular-contact ball bearing.

7.3 Design Validation

All design validations are shown in Section 6.3.1 and 6.3.2. Overall, the system we designed is very stable.

8 Future Design Improvement

Of course, in this design exercise, we still have many parameters that have not been optimized. For example, in actual situations, a certain gap should exist between the upper end of the key and the gear to help the key to transmit the torque as shown in Figure 7, but in our design In, we did not consider. In addition, we just mentioned the connection method of bearing and shaft, bearing and shell, shaft and gear vaguely, but did not expand their specific design methods in detail. Therefore, in future designs, we need to more clearly determine the parameters and physical models of these connection.

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