Design Exercise 01 - Power Transmission Hub Design

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1.1 System Descriptions

When buying a car, many consumers are the first to be attracted by the beautiful appearance, and then observe the interior decoration, understand the engine displacement related data and so on. However, all those who choose to buy a car must not ignore the driving of the car, because it is directly related to the road performance of the car.

The driving of most cars is undertaken by the driving shaft. A drive shaft is a rod-like component that transmits the torque of the engine to the wheels together with the gearbox and drive axle, so that the automobile generates driving force. It's essentially the shaft that drives your vehicle (Kim et al., 2004). In this design exercise, we will design a hub for a motor with a known rated torque so that the motor can transfer the power it generates to the wheel, so that the car gets energy to move forward.

Figure 1 shows the exploded view of typical vehicle. As shown in the figure, most cars are driven by front-wheel drive. The car's engine is placed at the front end of the car and drives the front wheels of the car to rotate.

Figure 2 shows an exploded view of a typical motor/hub/wheel assembly. In this illustration the hub is pressed onto the output shaft of the drive motor and is secured using an interference fit. The wheel slides onto the other end of the hub via a very close clearance fit. A locking bracket is used to bolt the motor onto the vehicle chassis. In this picture, we have omitted the

Figure 1: Exploded view of typical vehicle.

suspension system used to support the weight of the entire car. The body is directly connected to the wheels through the suspension system.

The reminder of sections are arranged as follows: In Section 1.2, we will discuss our design goals so that we can determine the direction of our design. In Section 1.3, we will come up with the constraints and assumptions of the design. In Section 1.4, we will list the notations and definitions that will be used in this design exercise. In Section 1.5, we will assess the potential failure In Section 1.6, we will conduct design iterations in accordance with our ideas. Here, we will encounter many failures and problems, but in the end, we still design a good solution to transmit torque. In Section 1.7, we will summarize the final results of our design iteration.

1.2 Design Objectives

There are three main goals in our design of drive shaft as shown below:

1. Attaching the hub to the motor.
2. Attaching the wheel or other power transmission device to the hub.
3. Transmitting the power through the hub.

1.3 Design Limitations

1.3.1 Design Constraints

1. Space between driveshaft end and wheel center hub is 33 mm.
2. Power specs of the motor is rated to deliver 25 hp at 30 rpm.
3. The output shaft of the motor is 45 mm and the shaft is made of AISI 4340 material, Q&T at 800°F.

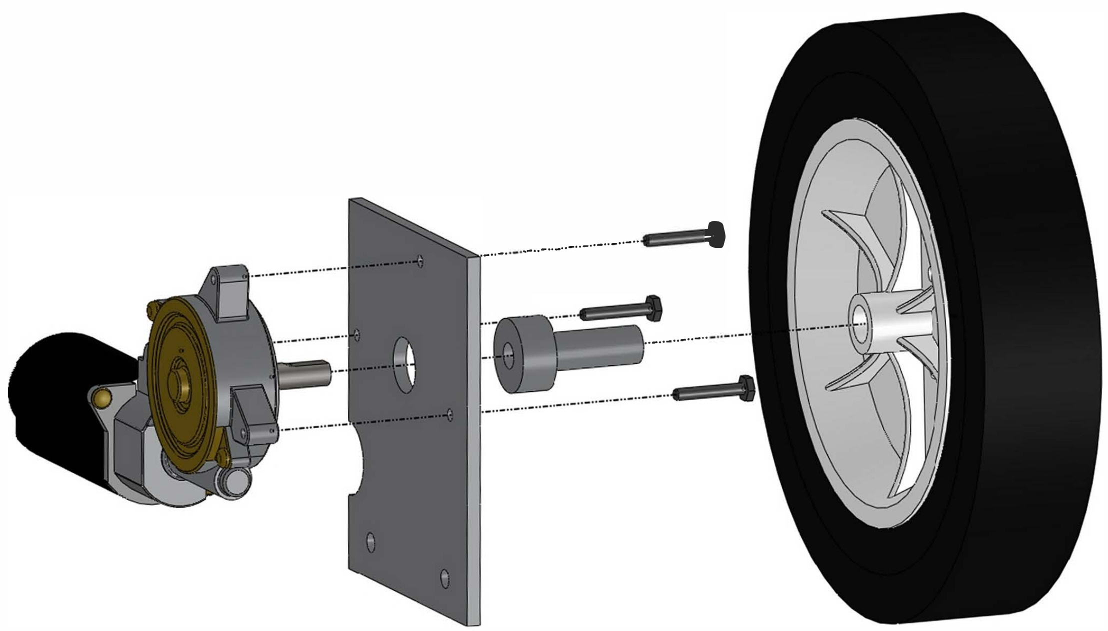

Figure 2: Exploded view of drive motor/hub/wheel assembly.

4. Motor should not fail before the hub in all circumstances.
5. For motor supplier, nominal diameter of the driveshaft can't be changed.
6. Length-to-diameter (L/D) ratio for interference-fit prefers to be kept between 1.0 to 2.5.
7. The width of the wheel center hub is equal to 10 cm.

1.3.2 Design Assumptions

To simplify our model and eliminate the complexity, we make the following main assumptions in this literature. All assumptions will be re-emphasized once they are used in the construction of our model.

Assumption 1. The friction coefficient for the surface between AISI 4340 material, Q&T at 800°F and Cast 333.0 T6 Aluminum Alloys is equal to $\mu = 0.3$ (Rusin et al., 2016).

Assumption 2. The driveshaft is only responsible for torque transmission, not the gravity transmission. Instead, it is through connecting the car's suspension system to the car wheels that the wheel can withstand the car's gravity (Grega et al., 2017).

Assumption 3. Ignore the gravity of the hub we designed.

Assumption 4. The interference between the shaft and the hub is evenly distributed, and the forces received in different places are the same (Abdul-Kadir et al., 2008).

Assumption 5. The friction coefficient for the surface between AISI 4340 material, Q&T at 800°F and AISI 4340 material, Q&T at 800°F is equal to $\mu = 0.3$ (Blau, 2008).

Assumption 6. The speed of a car changes slowly during operation, and there will be no rapid acceleration or deceleration.

1.4 Notations and Definitions

In this work, we use the nomenclature in Table 1 in the model construction. Other none-frequent-used symbols will be introduced once they are used.

Table 1: Notations used in this literature

Symbol	Meaning	unit
E_i	Modulus of Elasticity of inner shaft	GPa
v_i	Poisson's Ratio of inner shaft	–
E_o	Modulus of Elasticity of outer hub	GPa
v_o	Poisson's Ratio of outer hub	–
δ	Radial interference	mm
R	Approximate radius at interface	mm
d	Approximate diameter at interface	mm
L	Approximate length at interface	mm
c	Hub outer radius	mm
a	Shaft inner radius	mm
b_o	Hub inner radius	mm
b_i	Shaft outer radius	mm
D	Basic size	mm
δ_F	Fundamental deviation	mm
ΔD	International tolerance grade	mm
Δd	International tolerance grade	mm
P	Contact pressure at interference	MPa
$(\sigma_r)_{b_o}$	Radial stress of hub inner surface	MPa
$(\sigma_r)_{b_i}$	Radial stress of shaft outer surface	MPa
$(\sigma_\theta)_{b_o}$	Hoop stress of hub inner surface	MPa
$(\sigma_\theta)_{b_i}$	Hoop stress of shaft outer surface	MPa
T	Torque transmit through shaft and hub	$\mathrm{N \cdot m}$
J	Second moment of inertia of the area	m^4
τ	Torsion transmit through shaft and hub	MPa
μ	Coefficient of friction	–
n	Safety factor	–
S_y	Yield strength of material	MPa
S_{ut}	Tensile strength of material	MPa

1.5 Potential Failure Assessment

In this design, there are many factors that can lead to failure. We list the factors that may cause the failure of the system we designed in Table 2:

1.6 Design Thought Process

1.6.1 Iteration 01

Choose material Under normal circumstances, a relatively soft metal like aluminum will be used in a typical interference fit. So, we look for the aluminum that we seem to be suitable for in the appendix of the textbook.

Table 2: Simply supported beam data table for result calculations.

Failure Scenarios	Critical Parameters	Design Acceptance Criteria	Risk Priorities (High/Medium/Low)
Failure occurs during interference fit	Outer diameter of hub and the modulus of elasticity of hub	Safety factor of interference fit is larger than 2	High
Failure caused by torque	Outer diameter of hub	Safety factor of torsion is larger than 2	High
Slip due to insufficient friction	Inner diameter of hub	Safety factor of friction is larger than 2	Low
Failure at shaft when transmitting torque	Outer diameter of shaft	Safety factor of shaft is larger than 2	Low

From Table A-5, we can know the physical constants of aluminum-Modulus of Elasticity and Poisson's Ratio as shown in Equation 1 and 2, respectively.

$$E_o = 71.7 \text{ GPa} \tag{1}$$

$$v_o = 0.333 \tag{2}$$

And in appendix, the mechanical properties of some aluminum alloys is listed in Table A-24. Because under normal case, the cost of casting will be cheaper than the cost of wrought, so in Table A-24, we choose cast aluminum alloys. In addition, we think that the material we use is for the transmission shaft of the car, so its strength should be relatively high. After browsing all the materials in this table, we choose Cast 333.0 T6 Aluminum Alloys, whose mechanical properties is shown in Equation 3.

$$\begin{cases} S_{ut} = 289 \text{ MPa} = 42 \text{ ksi} \\ S_y = 207 \text{ MPa} = 30 \text{ ksi} \end{cases} \tag{3}$$

Sort out known parameters There are many known parameters in the problem file, we list them in this paragraph.

1. The output shaft of the motor is 45 mm.
2. The shaft is made of AISI 4340 material, Q&T at 800°F, whose physical constants and mechanical properties are listed in Equation 4, 5, and 6 according to Table A-5, A-21, and A-22 in the appendix.

$$E_i = 207.0 \text{ GPa} \tag{4}$$

$$v_i = 0.292 \tag{5}$$

$$\begin{cases} S_{ut} = 1470 \text{ MPa} = 213 \text{ ksi} \\ S_y = 1360 \text{ MPa} = 198 \text{ ksi} \end{cases} \tag{6}$$

3. Power specs of the motor is rated to deliver 25 hp at 30 rpm, from which we can know that the torque transmitted through shaft is shown in Equation 7.

$$T = \frac{H}{n} = \frac{25 \text{ hp}}{30 \text{ rpm}} = 5.934 \times 10^3 \text{ N} \cdot \text{m} \tag{7}$$

4. The width of the center hub in the wheel is 10 mm.

Determine the type of interference fit There are three kinds of fit methods in the categories of interference fit-locational interference fits, medium drive fits, and force fits. Therefore, we need to determine which type of fit we use when assemble the motor shaft with the hub we design. Let's discuss the functions and advantages of each type of fit.

- Locational interference fits are used where accuracy of location is prime importance and are used for parts requiring rigidity and alignment with no special requirements for bore pressure. The parts can be assembled or disassembled using cold pressing and greater forces or hot pressing. Such fits are not for parts designated to transmit frictional loads from one part to another by virtue of the tightness of fit. It is the standard press fit for steel, cast iron, or brass to steel assemblies. The amount of interference is too small for satisfactory press fit to be obtained in materials of low modulus of elasticity such as light alloys (Kanber, 2007).

- Medium drive fits are suitable for ordinary steel parts, or shrink fits on light sections. Medium Drive Fits are about the tightest fits that can be used with high-grade cast-iron external members (Croccolo et al., 2010).

- Force fits are designed for transfers of torsional moments using friction forces between Holes and Shafts. Force and Shrink Fits are a type of interference fit which seek to maintain constant hole pressure for all sizes. The interferece varies almost directly the diameter of the parts. The interference between the min and the max value is kept small to ensure that resulting pressure are reasonable. The value of interference/loading capacity of the fit increases with increasing class of the fit (Paul et al., 2015).

According to above analysis, we can find that the **force fits** are most suitable for our design because we use the relative soft material-aluminum and we want the hub we design to transmit the torque from the drive motor to the wheel.

Determine the inner and outer diameter of the hub we design Because we now know the outer diameter of the shaft, and then we decided to use force fit, we can determine the inner diameter of the hub we design.

According to Table 7-20, the symbol for force-fit is H7/u6. And the basic size of the system is equal to 45 mm. According to Table A-11, the tolerance grade of H7 is $\Delta D = 0.025$ mm and the tolerance grade of u6 is $\Delta d = 0.016$ mm. Also, according to Table A-12, the fundamental deviation for u6 is $\delta_F = +0.070$ mm. Therefore, the maximum shaft outer diameter is equal to

$$d_{max} = D = 45 \text{ mm} \tag{8}$$

And the minimum shaft outer diameter is equal to

$$d_{min} = D - \Delta d = 45 \text{ mm} - 0.016 \text{ mm} = 44.984 \text{ mm} \tag{9}$$

Therefore, the range of the shaft outer diameter is $[44.984, 45]$ mm. But according to Design Constraints 5, the nominal diameter of the driveshaft can't be changed for motor supplier. Hence, the outer diameter of shaft is fixed to 45 mm. The only thing we can change is the inner diameter of the hub we design.

The maximum hub inner diameter is equal to

$$D_{max} = D - \delta F = 45 \text{ mm} - 0.070 \text{ mm} = 44.930 \text{ mm} \tag{10}$$

And the minimum hub inner diameter is equal to

$$D_{min} = D - \delta F - \Delta D = 45 \text{ mm} - 0.070 \text{ mm} - 0.025 \text{ mm} = 44.905 \text{ mm} \tag{11}$$

According to Equation 10 and 11, the range of the hub inner diameter is $[44.905, 44.930]$ mm. Hence, we can get that the maximum interference is equal to

$$2\delta_{max} = d_{max} - D_{min} = 45 \text{ mm} - 44.905 \text{ mm} = 0.095 \text{ mm} \tag{12}$$

And the minimum interference is equal to

$$2\delta_{min} = d_{max} - D_{max} = 45 \text{ mm} - 44.930 \text{ mm} = 0.070 \text{ mm} \tag{13}$$

For the outer diameter of the hub, ASME BPVC suggests $d_o \geq 1.25 d_i$ to qualify for thick-walled cylinder. In our case, the outer diameter of hub, $2c$, should be larger than $1.25 \times 45 \text{ mm} = 56.25$ mm.

Experiment with outer diameter of hub with $2c = 60$ mm We first try $2c = 60$ mm.

The contact pressure of the interference fit is equal to

$$\begin{aligned} P &= \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{c^2+R^2}{c^2-R^2}+v_o\right)+\frac{1}{E_i}\left(\frac{R^2+a^2}{R^2-a^2}-v_i\right)\right]} \\ &= \frac{\frac{(0.095 \text{ mm})}{2}}{\frac{(45.000 \text{ mm})}{2} \times \left[\frac{1}{71.7\times10^9 \text{ Pa}}\left(\frac{\left(\frac{60 \text{ mm}}{2}\right)^2+\left(\frac{45.000 \text{ mm}}{2}\right)^2}{\left(\frac{60 \text{ mm}}{2}\right)^2-\left(\frac{45.000 \text{ mm}}{2}\right)^2}+0.333\right)+\frac{1}{207\times10^9 \text{ Pa}}\left(\frac{\left(\frac{45.000 \text{ mm}}{2}\right)^2+0^2}{\left(\frac{45.000 \text{ mm}}{2}\right)^2-0^2}-0.292\right)\right]} \\ &= 36.477 \text{ MPa} \end{aligned} \tag{14}$$

And we consider the maximum torque that the designed hub can transmit to get the minimum length that the motor shaft is forced into the hub we design, which is shown in Equation 15 and 16.

$$T = \mu\left(\pi d L\right) P R \tag{15}$$

$$\begin{aligned} \Longrightarrow L = \frac{T}{\mu\left(\pi d\right) P R} &= \frac{\left(5.934 \times 10^3 \text{ N}\cdot\text{m}\right)}{\pi \times (45.000 \text{ mm}) \times 0.3 \times (36.477 \text{ MPa}) \times \left(\frac{45.000 \text{ mm}}{2}\right)} \\ &= 0.1705\ m = 170.5 \text{ mm} \end{aligned} \tag{16}$$

The length-to-diameter (L/D) ratio for interference-fit we design is equal to

$$\frac{L}{D} = \frac{170.5 \text{ mm}}{45.000 \text{ mm}} = 3.7889 \tag{17}$$

which cannot meet our Design Constraints 6.

Next, we examine the torsion we need to withstand. In order to ensure our safety factor of torsion to be near 2, we should let the torsion in our hub be one fourth of the yielding strength of the material, which is near 50 MPa.

$$\tau = \frac{Tc}{J} = \frac{16T}{\pi d^3} = \frac{16 \times \left(5.934 \times 10^3 \text{ N}\cdot\text{m}\right)}{\pi \text{d}^3} = 50 \text{ MPa} \tag{18}$$

$$\Longrightarrow d = 0.0845 \text{ m} = 84.5 \text{ mm} \tag{19}$$

Therefore, it seems our original design of c is not suitable. Next, we will try $2c = 90$ mm to guarantee our hub can withstand the torsion in the hub. If we are lucky enough, the length-to-diameter (L/D) ratio for interference-fit we design will also meet our requirements.

1.6.2 Iteration 02

In this iteration, we will try $2c = 90$ mm and $2b_o = 44.905$ mm with Cast 333.0 T6 Aluminum Alloys as material.

Experiment with outer diameter of hub with $2c = 90$ mm We try $2c = 90$ mm.

The contact pressure of the interference fit is equal to

$$\begin{aligned}P &= \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{c^2+R^2}{c^2-R^2}+v_o\right)+\frac{1}{E_i}\left(\frac{R^2+a^2}{R^2-a^2}-v_i\right)\right]} \\ &= \frac{\frac{(0.095\ \text{mm})}{2}}{\frac{(45.000\ \text{mm})}{2}\times\left[\frac{1}{71.7\times10^9\ \text{Pa}}\left(\frac{\left(\frac{90\ \text{mm}}{2}\right)^2+\left(\frac{45.000\ \text{mm}}{2}\right)^2}{\left(\frac{90\ \text{mm}}{2}\right)^2-\left(\frac{45.000\ \text{mm}}{2}\right)^2}+0.333\right)+\frac{1}{207\times10^9\ \text{Pa}}\left(\frac{\left(\frac{45.000\ \text{mm}}{2}\right)^2+0^2}{\left(\frac{45.000\ \text{mm}}{2}\right)^2-0^2}-0.292\right)\right]} \\ &= 67.427\ \text{MPa}\end{aligned} \tag{20}$$

And we consider the maximum torque that the designed hub can transmit to get the minimum length that the motor shaft is forced into the hub we design, which is shown in Equation 21 and 22.

$$T = \mu\left(\pi d L\right) P R \tag{21}$$

$$\begin{aligned}\Longrightarrow L &= \frac{T}{\mu\left(\pi d\right) P R} = \frac{\left(5.934\times10^3\ \text{N}\cdot\text{m}\right)}{\pi\times(45.000\ \text{mm})\times0.3\times(67.427\ \text{MPa})\times\left(\frac{45.000\ \text{mm}}{2}\right)} \\ &= 0.0922\ m = 92.2\ \text{mm}\end{aligned} \tag{22}$$

The length-to-diameter (L/D) ratio for interference-fit we design is equal to

$$\frac{L}{D} = \frac{92.2\ \text{mm}}{45.000\ \text{mm}} = 2.04889 \tag{23}$$

which can meet our Design Constraints 6.

And then, we check the safety factor for interference fit.

The radial stresses at the interface of shaft and hub are shown in Equation 24.

$$\left(\sigma_r\right)_{b_i} = \left(\sigma_r\right)_{b_o} = -P = -67.427\ \text{MPa} \tag{24}$$

The hoop stresses at the interface of shaft and hub are shown in Equation 25 and 26.

$$\left(\sigma_\theta\right)_{b_i} = -P\frac{R^2+a^2}{R^2-a^2} = -\left(67.427\ \text{MPa}\right)\frac{\left(\frac{45.000\ \text{mm}}{2}\right)^2+0^2}{\left(\frac{45.000\ \text{mm}}{2}\right)^2-0^2} = -67.427\ \text{MPa} \tag{25}$$

$$\left(\sigma_\theta\right)_{b_0} = P\frac{c^2+R^2}{c^2-R^2} = \left(67.427\ \text{MPa}\right)\frac{\left(\frac{90\ \text{mm}}{2}\right)^2+\left(\frac{45.000\ \text{mm}}{2}\right)^2}{\left(\frac{90\ \text{mm}}{2}\right)^2-\left(\frac{45.000\ \text{mm}}{2}\right)^2} = 114.38\ \text{MPa} \tag{26}$$

We apply Maximum-Shear-Stress Theory to find our safety factor as shown in Equation 27 and 28.

$$n\cdot\frac{2c^2}{c^2-R^2}P = S_y \tag{27}$$

$$\Longrightarrow n = \frac{S_y}{\frac{2c^2}{c^2-R^2}P} = \frac{207\ \text{MPa}}{\frac{2\times\left(\frac{90\ \text{mm}}{2}\right)^2}{\left(\frac{90\ \text{mm}}{2}\right)^2-\left(\frac{45.000\ \text{mm}}{2}\right)^2}\times\left(67.427\ \text{MPa}\right)} = 1.1512 \tag{28}$$

Obviously, this safety factor cannot meet our needs at all, and it is likely to fail during interference fit. This may be because the 2c we designed was too large, so that the radial stress and hoop stress became relatively large during the interference fit. So in the next iteration, we go to the number of $2c$ at the margin of the torsion safety factor, which is $2c = 85$ mm.

1.6.3 Iteration 03

In this iteration, we will try $2c = 85$ mm and $2b_o = 44.905$ mm with Cast 333.0 T6 Aluminum Alloys as material.

Experiment with outer diameter of hub with $2c = 85$ mm We try $2c = 85$ mm.

The contact pressure of the interference fit is equal to

$$\begin{aligned} P &= \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{c^2+R^2}{c^2-R^2}+v_o\right)+\frac{1}{E_i}\left(\frac{R^2+a^2}{R^2-a^2}-v_i\right)\right]} \\ &= \frac{\frac{(0.095\ \text{mm})}{2}}{\frac{(45.000\ \text{mm})}{2}\times\left[\frac{1}{71.7\times10^9\ \text{Pa}}\left(\frac{\left(\frac{85\ \text{mm}}{2}\right)^2+\left(\frac{45.000\ \text{mm}}{2}\right)^2}{\left(\frac{85\ \text{mm}}{2}\right)^2-\left(\frac{45.000\ \text{mm}}{2}\right)^2}+0.333\right)+\frac{1}{207\times10^9\ \text{Pa}}\left(\frac{\left(\frac{45.000\ \text{mm}}{2}\right)^2+0^2}{\left(\frac{45.000\ \text{mm}}{2}\right)^2-0^2}-0.292\right)\right]} \\ &= 64.218\ \text{MPa} \end{aligned} \tag{29}$$

And we consider the maximum torque that the designed hub can transmit to get the minimum length that the motor shaft is forced into the hub we design, which is shown in Equation 30 and 31.

$$T = \mu\left(\pi d L\right)PR \tag{30}$$

$$\begin{aligned} \Longrightarrow L = \frac{T}{\mu\left(\pi d\right)PR} &= \frac{\left(5.934\times10^3\ \text{N}\cdot\text{m}\right)}{\pi\times(45.000\ \text{mm})\times0.3\times(64.218\ \text{MPa})\times\left(\frac{45.000\ \text{mm}}{2}\right)} \\ &= 0.0968\ m = 96.8\ \text{mm} \end{aligned} \tag{31}$$

The length-to-diameter (L/D) ratio for interference-fit we design is equal to

$$\frac{L}{D} = \frac{96.8\ \text{mm}}{45.000\ \text{mm}} = 2.15111 \tag{32}$$

which can meet our Design Constraints 6.

And then, we check the safety factor for interference fit.

The radial stresses at the interface of shaft and hub are shown in Equation 33.

$$\left(\sigma_r\right)_{b_i} = \left(\sigma_r\right)_{b_o} = -P = -64.218\ \text{MPa} \tag{33}$$

The hoop stresses at the interface of shaft and hub are shown in Equation 34 and 35.

$$\left(\sigma_\theta\right)_{b_i} = -P\frac{R^2+a^2}{R^2-a^2} = -\left(64.218\ \text{MPa}\right)\frac{\left(\frac{45.000\ \text{mm}}{2}\right)^2+0^2}{\left(\frac{45.000\ \text{mm}}{2}\right)^2-0^2} = -64.218\ \text{MPa} \tag{34}$$

$$\left(\sigma_\theta\right)_{b_0} = P\frac{c^2+R^2}{c^2-R^2} = \left(64.218\ \text{MPa}\right)\frac{\left(\frac{85\ \text{mm}}{2}\right)^2+\left(\frac{45.000\ \text{mm}}{2}\right)^2}{\left(\frac{85\ \text{mm}}{2}\right)^2-\left(\frac{45.000\ \text{mm}}{2}\right)^2} = 114.23\ \text{MPa} \tag{35}$$

We apply Maximum-Shear-Stress Theory to find our safety factor as shown in Equation 36 and 37.

$$n \cdot \frac{2c^2}{c^2 - R^2} P = S_y \tag{36}$$

$$\implies n = \frac{S_y}{\frac{2c^2}{c^2-R^2}P} = \frac{207\ \text{MPa}}{\frac{2\times\left(\frac{85\ \text{mm}}{2}\right)^2}{\left(\frac{85\ \text{mm}}{2}\right)^2-\left(\frac{45.000\ \text{mm}}{2}\right)^2} \times (64.218\ \text{MPa})} = 1.1600 \tag{37}$$

We change the outer diameter of the hub, but this operation has little effect on our safety factor. We racked our brains and finally decided to change to a different material. Because the yield strength of aluminum is so small that it easily fails.

1.6.4 Iteration 04

Choose material To sum up the reason for the previous failure, it is because the yield strength of the material we choose is too small. Therefore, in this iteration, we will choose a material with large yield strength characteristics. Here, we decided to choose carbon steel as our material. Because our motor shaft is also carbon steel, we simply choose the same material as the motor shaft, which is AISI 4340 material, Q&T at 800°F. The physical constants and mechanical properties of the material we choose are listed in Equation 4, 5, and 6 according to Table A-5, A-21, and A-22 in the appendix.

Determine the outer diameter of the hub we design We first try $2c = 60$ mm. According to previous experience, the decisive factor for the outer diameter of the hub is torsion stress. Therefore, in this iteration, the first thing we consider is the smallest outer diameter of the hub that can withstand output of torque from motor.

We examine the torsion we need to withstand. In order to ensure our safety factor of torsion to be near 2, we should let the torsion in our hub be one fourth of the yielding strength of the material, which is near 340 MPa.

$$\tau = \frac{Tc}{J} = \frac{16T}{\pi d^3} = \frac{16 \times \left(5.934 \times 10^3\ \text{N}\cdot\text{m}\right)}{\pi \text{d}^3} = 340\ \text{MPa} \tag{38}$$

$$\implies d = 0.0446\ \text{m} = 44.6\ \text{mm} \tag{39}$$

Therefore, according to Equation 38 and 39, the diameter of hub we design based on ASME BPVC is much larger than the safety diameter, which means in this iteration, we can totally ignore the influence of torsion safety factor.

Experiment with outer diameter of hub with $2c = 60$ mm We first try $2c = 60$ mm.

The contact pressure of the interference fit is equal to

$$\begin{aligned}
P &= \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{c^2+R^2}{c^2-R^2}+v_o\right)+\frac{1}{E_i}\left(\frac{R^2+a^2}{R^2-a^2}-v_i\right)\right]} \\
&= \frac{\frac{(0.095\ \text{mm})}{2}}{\frac{(45.000\ \text{mm})}{2} \times \left[\frac{1}{207\times10^9\ \text{Pa}}\left(\frac{\left(\frac{60\ \text{mm}}{2}\right)^2+\left(\frac{45.000\ \text{mm}}{2}\right)^2}{\left(\frac{60\ \text{mm}}{2}\right)^2-\left(\frac{45.000\ \text{mm}}{2}\right)^2}+0.292\right)+\frac{1}{207\times10^9\ \text{Pa}}\left(\frac{\left(\frac{45.000\ \text{mm}}{2}\right)^2+0^2}{\left(\frac{45.000\ \text{mm}}{2}\right)^2-0^2}-0.292\right)\right]} \\
&= 95.594\ \text{MPa}
\end{aligned} \tag{40}$$

And we consider the maximum torque that the designed hub can transmit to get the minimum length that the motor shaft is forced into the hub we design, which is shown in Equation 41 and 42.

$$T = \mu\,(\pi d L)\,PR \tag{41}$$

$$\begin{aligned}\Longrightarrow L &= \frac{T}{\mu\,(\pi d)\,PR} = \frac{\left(5.934\times10^{3}\ \mathrm{N\cdot m}\right)}{\pi\times(45.000\ \mathrm{mm})\times 0.3\times(95.594\ \mathrm{MPa})\times\left(\frac{45.000\ \mathrm{mm}}{2}\right)}\\ &= 0.0651\ \mathrm{m} = 65.1\ \mathrm{mm}\approx 70\ \mathrm{mm}\end{aligned} \tag{42}$$

The length-to-diameter (L/D) ratio for interference-fit we design is equal to

$$\frac{L}{D} = \frac{70\ \mathrm{mm}}{45.000\ \mathrm{mm}} = 1.5556 \tag{43}$$

which can meet our Design Constraints 6.

And then, we check the safety factor for interference fit.

The radial stresses at the interface of shaft and hub are shown in Equation 44.

$$(\sigma_r)_{b_i} = (\sigma_r)_{b_o} = -P = -95.594\ \mathrm{MPa} \tag{44}$$

The hoop stresses at the interface of shaft and hub are shown in Equation 45 and 46.

$$(\sigma_\theta)_{b_i} = -P\frac{R^2+a^2}{R^2-a^2} = -\left(95.594\ \mathrm{MPa}\right)\frac{\left(\frac{45.000\ \mathrm{mm}}{2}\right)^2+0^2}{\left(\frac{45.000\ \mathrm{mm}}{2}\right)^2-0^2} = -95.594\ \mathrm{MPa} \tag{45}$$

$$(\sigma_\theta)_{b_0} = P\frac{c^2+R^2}{c^2-R^2} = \left(95.594\ \mathrm{MPa}\right)\frac{\left(\frac{60\ \mathrm{mm}}{2}\right)^2+\left(\frac{45.000\ \mathrm{mm}}{2}\right)^2}{\left(\frac{60\ \mathrm{mm}}{2}\right)^2-\left(\frac{45.000\ \mathrm{mm}}{2}\right)^2} = 341.41\ \mathrm{MPa} \tag{46}$$

We apply Maximum-Shear-Stress Theory to find our safety factor as shown in Equation 47 and 48.

$$n\cdot\frac{2c^2}{c^2-R^2}P = S_y \tag{47}$$

$$\Longrightarrow n = \frac{S_y}{\frac{2c^2}{c^2-R^2}P} = \frac{1360\ \mathrm{MPa}}{\frac{2\times\left(\frac{60\ \mathrm{mm}}{2}\right)^2}{\left(\frac{60\ \mathrm{mm}}{2}\right)^2-\left(\frac{45.000\ \mathrm{mm}}{2}\right)^2}\times(95.594\ \mathrm{MPa})} = 3.1121 \tag{48}$$

Also, we check the safety factor for the torsion stress as shown in Equation 49 and 50.

$$n\tau = \frac{S_y}{2} \tag{49}$$

$$\Longrightarrow n = \frac{\frac{S_y}{2}}{\tau} = \frac{\frac{1360\ \mathrm{MPa}}{2}}{\frac{16\times(5.934\times10^{3}\ \mathrm{N\cdot m})}{\pi\times(60\ \mathrm{mm})^3}} = 4.8600 \tag{50}$$

In this design, the safety factors of interference fit and torsion stress are both above 3, which means our design is very safe, reliable, and feasible.

Then, we calculate the minimum force required to press the shaft into hub as shown in Equation .

$$F = \mu\,(\pi d L)\,P = 0.3\times[\pi\times(45.000\ \mathrm{mm})\times(70\ \mathrm{mm})]\times(95.594\ \mathrm{MPa}) = 283.80\ \mathrm{kN} \tag{51}$$

In this question, we try different radial interference. It shows that when we vary 2δ from 0.050 mm to 0.125 mm, the safety factor of torsion stress will not change, the safety factor of interference fit will change from 2.3652 to 5.9130, and the approximate length at interface will vary from 49.4 mm to 123.6 mm, from which we can observe that the approximate length at interface is the critical parameter that will limit the change of radial interference. Therefore, the suggested tolerance band for the driveshaft is $[-0.050, +0.030]$ mm. If tolerance of the driveshaft diameter is ± 0.025 mm according to motor supplier, the only influence on our design is to change the length of interface to the corresponding value.

Determine the inner diameter of the wheel center This design exercise does not require us to design the key and keyway design at the hub/wheel interface. But we still need to consider the clearance fit design of the hub at the wheel end. Up to now, we have determined that the outer diameter of the hub, which is equal to 60 mm. This diameter is also the outer diameter of the shaft at the interface between hub and wheel center. Here, we decide to use close running fit, which is designed for running on accurate machines and for accurate location at moderate speeds and journal pressures (Cao et al., 2019).

According to Table 7-20, the symbol for the close running fit is H8/f6. And the basic size of the system is equal to 60 mm. According to Table A-11, the tolerance grade of H8 is $\Delta D = 0.046$ mm and the tolerance grade of f6 is $\Delta d = 0.019$ mm. Also, according to Table A-12, the fundamental deviation for f6 is $\delta_F = -0.030$ mm. Therefore, the minimum shaft outer diameter is equal to

$$d_{min} = D = 60 \text{ mm} \tag{52}$$

And the maximum shaft outer diameter is equal to

$$d_{max} = D + \Delta d = 60 \text{ mm} + 0.019 \text{ mm} = 60.019 \text{ mm} \tag{53}$$

Therefore, the range of the shaft outer diameter is $[60, 60.019]$ mm.

Then, the minimum hub inner diameter is equal to

$$D_{min} = D - \delta F = 60 \text{ mm} - (-0.030 \text{ mm}) = 60.030 \text{ mm} \tag{54}$$

And the maximum hub inner diameter is equal to

$$D_{max} = D - \delta F + \Delta D = 60 \text{ mm} - (-0.030 \text{ mm}) + 0.046 \text{ mm} = 60.076 \text{ mm} \tag{55}$$

According to Equation 54 and 55, the range of the hub inner diameter is $[60.030, 60.076]$ mm.

Hence, we can use the average interference as our dimension for the wheel center hub, which is equal to 60.053 mm. And according to Design Constraint 7, the width of the wheel center hub is equal to 10 cm. Also, we can know that the tolerance of the wheel center hub is ± 0.023 mm.

So far, we have designed all the things we need to design. We determined the inner diameter and outer diameter of the hub on the motor shaft, and the depth of the hub. Besides, we also designed the dimension of the wheel center hub. In addition, we also determined the entire length of our design parts. In general, we are quite satisfied with our design.

1.7 Final Design Summary

1.7.1 Designed System Concept

After the design of the previous four iterations, we finally determined our size and shape. We draw it as shown in Figure 3. Here, we apply a fillet radius of 2 mm to all sides to avoid the occurrence of stress concentration. Therefore, our hub design is to dig a hole one by one to satisfy our interference fit to transmit the torque. With this design, we can also avoid the stress concentration caused by shaft shoulder.

Here is our description of the designed system. The hub has a total length of 203 mm and a diameter of 60 mm. One end of the hub has a hole with a length of 70 mm and a diameter of 44.91 mm for interference fit with the shaft. And the fillet radii of all sides are 2 mm. On the other side of hub, the hub we design is acted as shaft, which is connected to the wheel center hub via clearance fit. And the dimension of the wheel center hub is 60.053 mm for diameter and 10 cm for width.

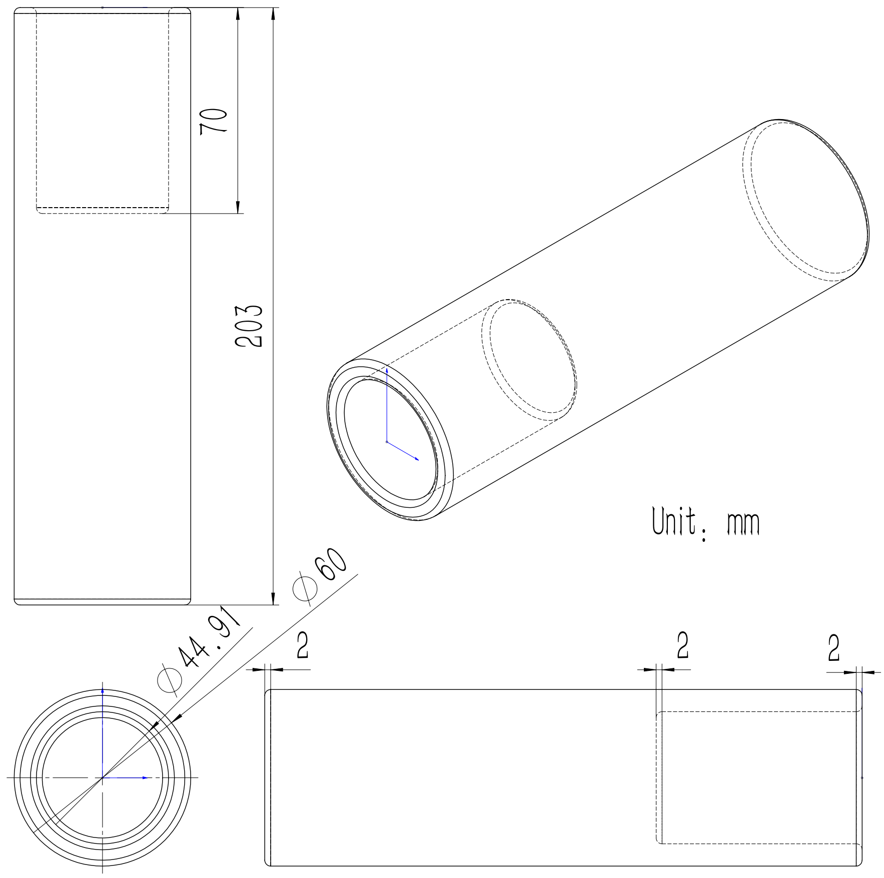

Figure 3: Draft for our design.

1.7.2 Material Selections

The material we select is the same as the material of shaft-AISI 4340 material, Q&T at 800°F, whose physical constants and mechanical properties are listed in Equation 4, 5, and 6. Because this material not only has high strength, but its cost is not very expensive. Throughout the design process, we also tried other materials, such as aluminum, but finally verified that aluminum cannot be used as the hub material we designed because its yield strength is too small.

1.7.3 Design Validation

We conduct the finite element analysis on the hub we design and also on the shaft to validate our design.

First, we analyze the equivalent stress on the shaft when there is no hub to transmit the torque, the result is shown in Figure 4.

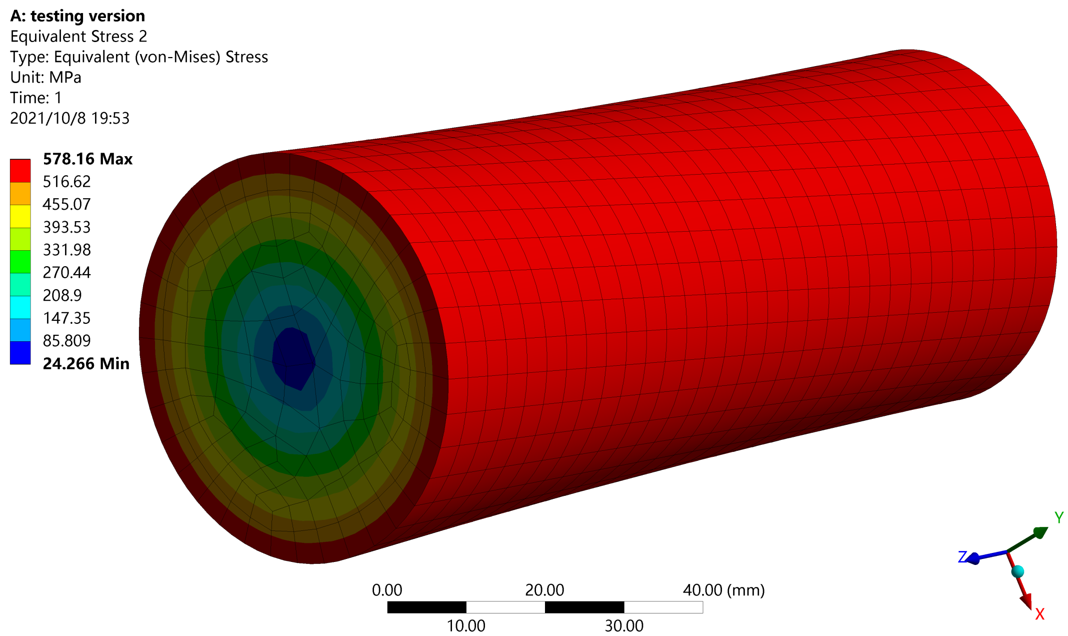

Figure 4: Equivalent stress on the shaft when there is no hub.

Next, we analyze the equivalent stresses on the hub and shaft when they are conducting the interference fit to check whether they will fail during this process, which means there is no torque transmission and the results are shown in Figure 5 and 6, respectively.

Then, we analyze the equivalent stresses on the hub and shaft when they are conducting the interference fit to check whether they will fail during this process and the results are shown in Figure 7 and 8, respectively.

It can be seen from the finite element analysis that the results of ANSYS analysis are similar to those calculated by us, except for some places where there are still some stress concentrations. But overall, when it is still running, our design can fully meet the needs of torque transmission. Our design has achieved great success.

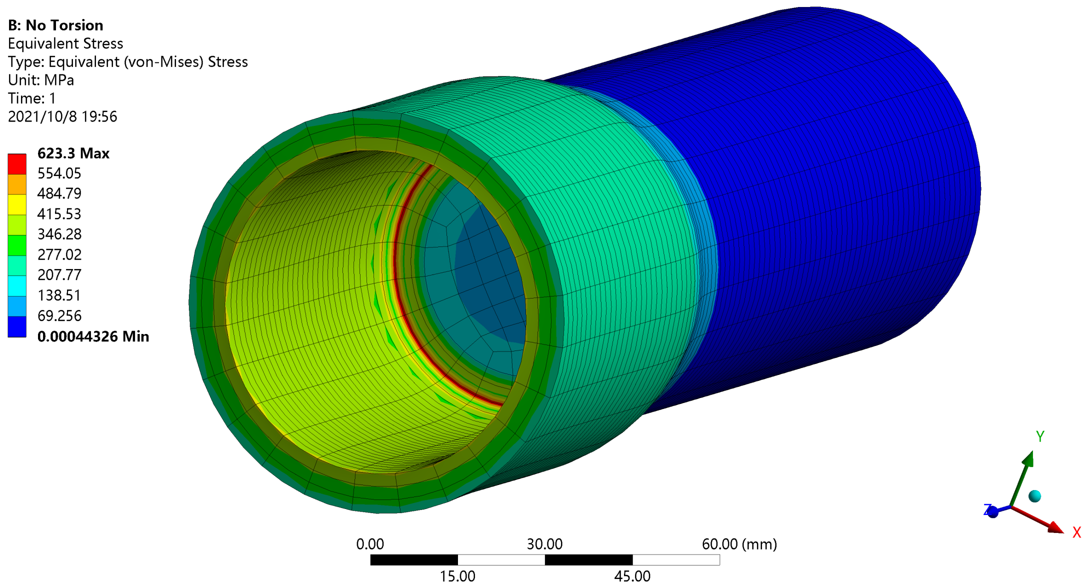

Figure 5: Equivalent stress on the hub during the interference fit.

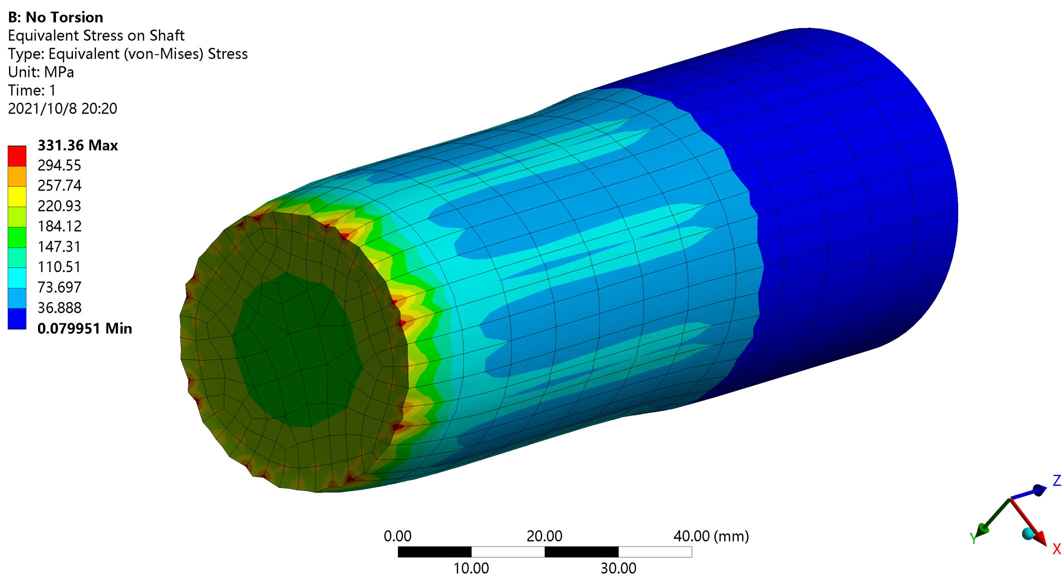

Figure 6: Equivalent stress on the shaft during the interference fit.

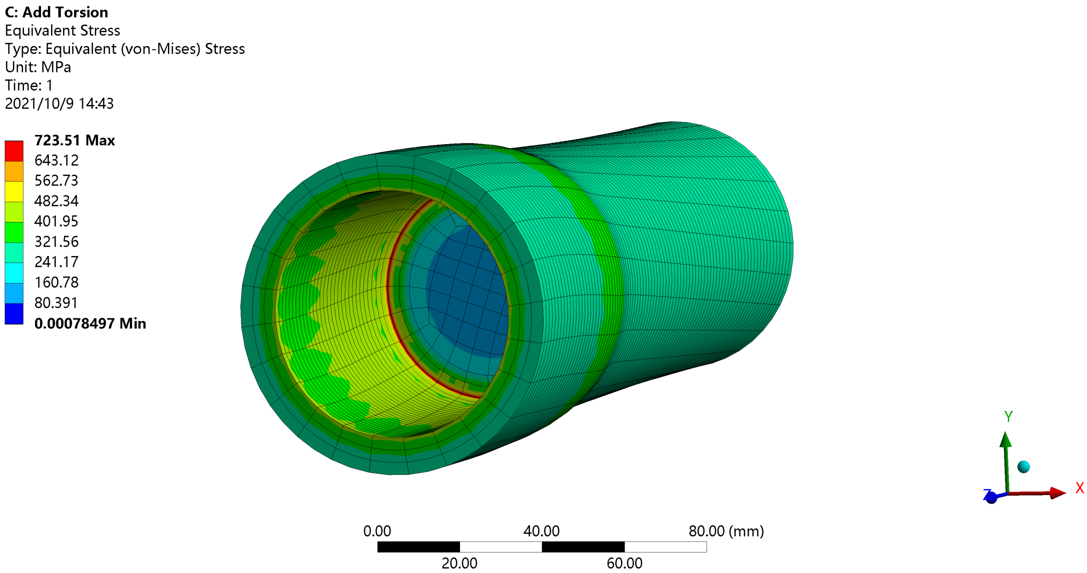

Figure 7: Equivalent stress on the hub when transmitting torque.

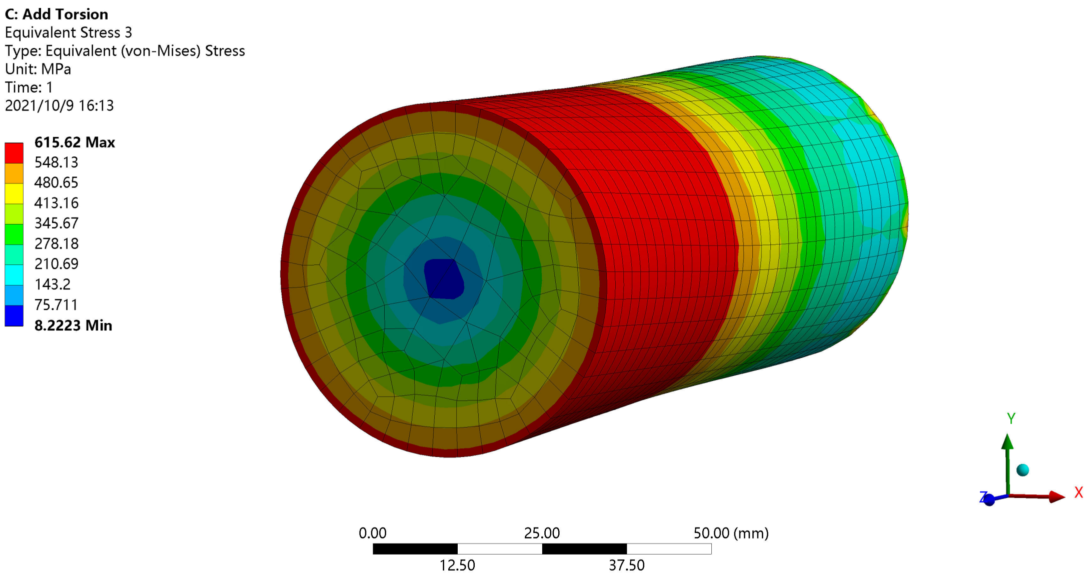

Figure 8: Equivalent stress on the shaft when transmitting torque.

1.8 Future Design Improvement

Of course, in this design exercise, we still have many parameters that have not been optimized. Our fillet radius is a random value. In future designs, we can explore the effect of different fillet radius on reducing stress concentration, especially the fillet radius of the shaft and the fillet radius at the bottom of the hub hole, because we can get through the finite element analysis, in these two places, the stress concentration generated is the largest.

In addition, we can see from our calculations that whether it is the safety factor of torsion or the safety factor of interference fit, they are all greater than 3, which is relatively safe. It seems that the material we select is relatively wasteful. Therefore, in the future design, we can choose a material so that the safety factor of torsion and the safety factor of interference fit are both around 2 to 3, which can greatly save material costs.

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