Christopher King

2018141521058

Mechanical Design 2

Class Section 01

12/03/2021

Problem 1

For a bolted assembly with six bolts, the stiffness of each bolt is kb = 3 Mlbf/in and the stiffness of the members is km = 12 Mlbf/in per bolt. An external load of 80 kips is applied to the entire joint. Assume the load is equally distributed to all the bolts. It has been determined to use 1/2 in-13 UNC SAE grade 8 bolts with rolled threads. Assume the bolts are preloaded to 75 percent of the proof load.

It is desired to find the range of torque that could apply to initially preload the bolts without expecting failure once the joint is loaded. Assume a torque coefficient of K = 0.2.

a. Determine the maximum bolt preload that can be applied without exceeding the proof strength of the bolts.

b. Determine the minimum bolt preload that can be applied while avoiding joint separation.

c. Determine the value of torque in units of ft-lbf that should be specified for preloading the bolts if it is desired to preload to the midpoint of the values found in parts (a) and (b).

Solution:

a.

1/2 in-13 UNC SAE grade 8 bolts with rolled threads. Assume a torque coefficient of K = 0.2.

Proof strength (From Table 8-9):

$$S_p = 120 \text{ kpsi}$$

Tensile stress area (From Table 8-2):

$$A_t = 0.1419 \text{ in}^2$$

The maximum bolt preload is equal to

$$F_i = S_p A_t = (120\ \text{kpsi}) \times (0.1419\ \text{in}^2) = 17.028\ \text{kips}$$

b.

The stiffness constant of the joint:

$$C = \frac{k_b}{k_b + k_m} = \frac{(3\ \text{Mlbf/in})}{(3\ \text{Mlbf/in}) + (12\ \text{Mlbf/in})} = 0.2$$

The load P is

$$P = \frac{P_{total}}{N} = \frac{80\ \text{kips}}{6} = 13.3\ \text{kips}$$

Joint separation, Eq. (8-30) with $n_0 = 1$, we can know the minimum bolt preload is equal to

$$F_i = n_0 P(1 - C) = 1 \times (13.3\ \text{kips}) \times (1 - 0.2) = 10.667\ \text{kips}$$

c.

The midpoint of the values found in parts (a) and (b) is equal to

$$\bar{F}_i = \frac{(17.028\ \text{kips}) + (10.667\ \text{kips})}{2} = 13.847\ \text{kips}$$

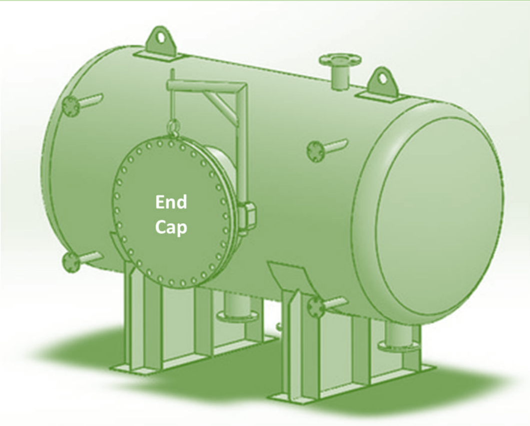
The value of torque in units of ft-lbf that should be specified for preloading the bolts is equal to

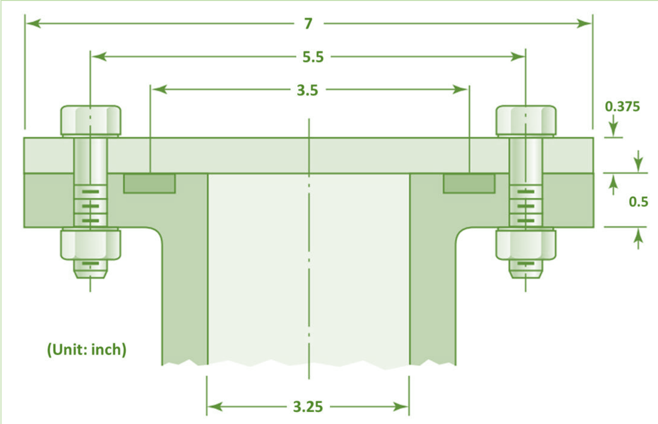
$$T = K\bar{F}_i d = 0.2 \times (13.847\ \text{kips}) \times (0.5\ \text{in}) = 115.3944\ \text{lbf} \cdot \text{ft}$$

Problem 2

A gas storage tank shown in figure below has a steel end cap bolt-tightened to a grade 30 cast iron flange. The end cap has to be removed frequently for maintenance purpose. A non-permanent connection with eight 7/16 in-14 UNC SAE grade 8 bolts was selected for the purpose. Gas pressure inside the tank results in a maximum force up to 10 kips on the end cap.

For the given bolt specifications, with two threads beyond the nut, select a suitable bolt length from the preferred sizes in Table A–17. Determine the yielding factor of safety np, the load factor nL, and the joint separation factor n0.





Solution:

Nominal major diameter of $\frac{7}{16}$ in.

Therefore, I select washer size $\frac{7}{16}$ W from Table A-33 with maximum thickness 0.083 inch.

From Table A-31, the nut height is $H = \frac{3}{8}$ in.

Because for the given bolt specifications, with two threads beyond the nut,

$$l = t_1 + t_2 + 2h_w = \left(\frac{3}{8}\text{ in}\right) + \left(\frac{1}{2}\text{ in}\right) + 2 \times (0.083\text{ inch}) = 1.0410\text{ inch}$$

$$L \geq l + H + 2p = (1.0410\text{ inch}) + \left(\frac{3}{8}\text{ in}\right) + 2 \times \left(\frac{1}{14}\text{ in}\right) = 1.5589\text{ inch}$$

The minimum length of the bolts per Table A-17 is equal to

$$L = 1.60\text{ inch}$$

The thread length of inch-series bolts is (Table 8-7)

$$L_T = 2d + \frac{1}{4} = 2 \times \left(\frac{7}{16}\text{ in}\right) + \frac{1}{4}\text{ in} = 1.1250\text{ in}$$

Length of unthreaded portion in grip:

$$l_d = L - L_T = 1.60\text{ in} - 1.1250\text{ in} = 0.4750\text{ in}$$

Length of threaded portion in grip:

$$l_t = l - l_d = 1.0410\text{ inch} - 0.4750\text{ in} = 0.5660\text{ in}$$

Area of unthreaded portion:

$$A_d = \frac{\pi d^2}{4} = \frac{\pi \times \left(\frac{7}{16}\text{ in}\right)^2}{4} = 0.1503\text{ in}^2$$

Area of threaded portion (from Table 8-1):

$$A_t = 0.1063\text{ in}^2$$

Fastener stiffness:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{(0.1503\text{ in}^2) \times (0.1063\text{ in}^2) \times (30\text{ Mpsi})}{(0.1503\text{ in}^2) \times (0.5660\text{ in}) + (0.1063\text{ in}^2) \times (0.4750\text{ in})}$$
$$= 3.5360 \times 10^6\text{ lbf/in}$$

Eq. 8-20: the spring rate or stiffness of steel frustum

$$k_1 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (30\text{ Mpsi}) \times \left(\frac{7}{16}\text{ in}\right) \times \tan 30°}{\ln \frac{\left[2 \times (0.4400\text{ in}) \times \tan 30° + (0.6563\text{ in}) - \left(\frac{7}{16}\text{ in}\right)\right]\left[(0.6563\text{ in}) + \left(\frac{7}{16}\text{ in}\right)\right]}{\left[2 \times (0.4400\text{ in}) \times \tan 30° + (0.6563\text{ in}) + \left(\frac{7}{16}\text{ in}\right)\right]\left[(0.6563\text{ in}) - \left(\frac{7}{16}\text{ in}\right)\right]}}$$
$$= 2.8527 \times 10^7\text{ lbf/in}$$

Eq. 8-20: the spring rate or stiffness of upper cast iron frustum

$$D = (0.6563\text{ in}) + 2 \times (0.4400\text{ in}) \times \tan 30° = 1.1851\text{ in}$$

$$k_2 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln\frac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)}}$$

$$= \frac{\pi \times (14.5\ \text{Mpsi}) \times \left(\frac{7}{16}\ \text{in}\right) \times \tan 30°}{\ln\frac{\left[2\times(0.0625\ \text{in})\times\tan 30° + (1.1851\ \text{in}) - \left(\frac{7}{16}\ \text{in}\right)\right]\left[(1.1851\ \text{in}) + \left(\frac{7}{16}\ \text{in}\right)\right]}{\left[2\times(0.0625\ \text{in})\times\tan 30° + (1.1851\ \text{in}) + \left(\frac{7}{16}\ \text{in}\right)\right]\left[(1.1851\ \text{in}) - \left(\frac{7}{16}\ \text{in}\right)\right]}}$$

$$= 2.3657 \times 10^8\ \text{lbf/in}$$

Eq. 8-20: the spring rate or stiffness of lower cast iron frustum

$$k_3 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln\frac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)}}$$

$$= \frac{\pi \times (14.5\ \text{Mpsi}) \times \left(\frac{7}{16}\ \text{in}\right) \times \tan 30°}{\ln\frac{\left[2\times(0.4400\ \text{in})\times\tan 30° + (0.6563\ \text{in}) - \left(\frac{7}{16}\ \text{in}\right)\right]\left[(0.6563\ \text{in}) + \left(\frac{7}{16}\ \text{in}\right)\right]}{\left[2\times(0.4400\ \text{in})\times\tan 30° + (0.6563\ \text{in}) + \left(\frac{7}{16}\ \text{in}\right)\right]\left[(0.6563\ \text{in}) - \left(\frac{7}{16}\ \text{in}\right)\right]}}$$

$$= 1.3029 \times 10^7\ \text{lbf/in}$$

Total member stiffness:

$$k_m = \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)^{-1} = 8.6180 \times 10^6\ \text{lbf/in}$$

The stiffness constant of the joint:

$$C = \frac{k_b}{k_b + k_m} = \frac{(3.5360 \times 10^6\ \text{lbf/in})}{(3.5360 \times 10^6\ \text{lbf/in}) + (8.6180 \times 10^6\ \text{lbf/in})} = 0.2909$$

Proof strength (From Table 8-9):

$$S_p = 120\ \text{kpsi}$$

Because the end cap has to be removed frequently for maintenance purpose, it is non-permanent connection.

From Eq. 8-31 and 8-32, I can know that

$$F_i = 0.75F_p = 0.75A_tS_p = 0.75 \times (0.1063\ \text{in}^2) \times (120\ \text{kpsi}) = 9.567\ \text{kips}$$

The load P is

$$P = \frac{P_{total}}{N} = \frac{10\ \text{kips}}{8} = 1.250\ \text{kips}$$

Yielding factor of safety, Eq. 8-28:

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{(120 \text{ kpsi}) \times (0.1063 \text{ in}^2)}{(0.2909) \times (1.250 \text{ kips}) + (9.567 \text{ kips})} = 1.2845$$

Overload factor of safety, Eq. 8-29:

$$n_L = \frac{S_p A_t - F_i}{CP} = \frac{(120 \text{ kpsi}) \times (0.1063 \text{ in}^2) - (9.567 \text{ kips})}{(0.2909) \times (1.250 \text{ kips})} = 8.7691$$

Separation factor of safety, Eq. 8-30:

$$n_0 = \frac{F_i}{P(1 - C)} = \frac{(9.567 \text{ kips})}{(1.250 \text{ kips}) \times (1 - 0.2909)} = 10.7939$$

Problem 3

Continued from Question 02, the gas force on end cap is cycled between 0 and 10 kips. Determine the fatigue factor of safety for the bolts using failure criteria of (a) Goodman, and (b) Gerber.

Among all considered failure scenarios, which mode is the riskiest for failure?

Solution:

$$\sigma_i = 0.75S_p = 90 \text{ kpsi}$$

$$\sigma_a = \frac{C(P_{max} - P_{min})}{2A_t} = \frac{CP}{2A_t} = \frac{(0.2909) \times (1.250 \text{ kips})}{2 \times (0.1063 \text{ in}^2)} = 1.7106 \text{ kpsi}$$

$$\sigma_m = \frac{C(P_{max} + P_{min})}{2A_t} + \sigma_i = \frac{CP}{2A_t} + \sigma_i = \frac{(0.2909) \times (1.250 \text{ kips})}{2 \times (0.1063 \text{ in}^2)} + \sigma_i = 91.711 \text{ kpsi}$$

Goodman: From Table 8-9, $S_{ut} = 150$ kpsi, and from Table 8-17, $S_e = 23.2$ kpsi

(a)

Eq. 8-45:

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} = \frac{(23.2 \text{ kpsi}) \times [(150 \text{ kpsi}) - (90 \text{ kpsi})]}{(1.7106 \text{ kpsi}) \times [(150 \text{ kpsi}) + (23.2 \text{ kpsi})]} = 4.6984$$

(b)

Gerber:

Eq. 8-46:

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right]$$

$$= \frac{1}{2 \times (1.7106 \text{ kpsi}) \times (23.2 \text{ kpsi})}$$
$$\times \Big[(150 \text{ kpsi})$$
$$\times \sqrt{(150 \text{ kpsi})^2 + 4 \times (23.2 \text{ kpsi}) \times [(23.2 \text{ kpsi}) + (90 \text{ kpsi})]}$$
$$- (150 \text{ kpsi})^2 - 2 \times (90 \text{ kpsi}) \times (23.2 \text{ kpsi})\Big] = 7.2433$$

Yielding is the riskiest for failure.