Christopher King

2018141521058

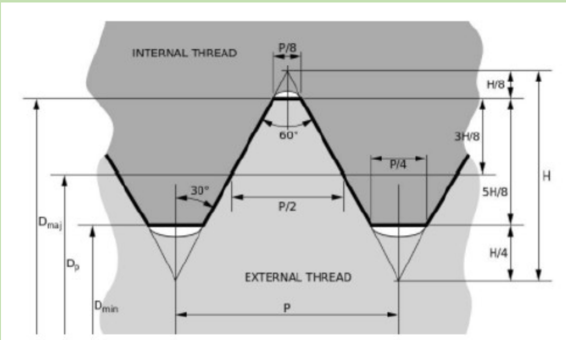
Mechanical Design 2

Class Section 01

11/28/2021

**Problem 1**

Draw the internal and external thread profiles of a ¼ -20 – UNC thread and identify the key dimensions.




**Solution:**

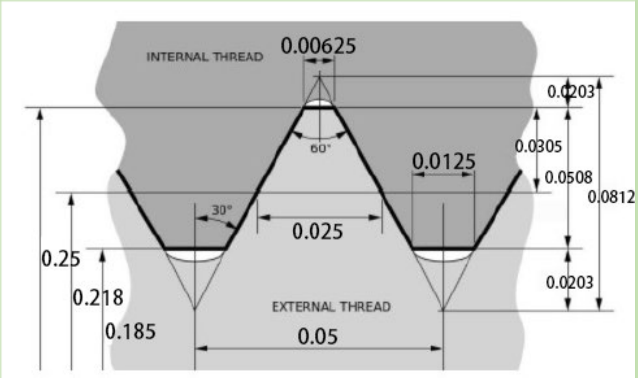
Nominal major diameter of 0.25 in and 20 threads per inch.

Tensile Stress Area: $0.0318\ \text{in}^2$.

$$p = \frac{1}{20\ /\text{inch}} = 0.05\ \text{inch}$$

$$d_r = d - 1.299038p = 0.1850481$$

$$d_p = d - 0.649519p = 0.21752405$$


INTERNAL THREAD
0.00625
0.0203
60°
0.0305
0.0125
0.0508
0.0812
30°
0.025
0.25
0.218
0.185
EXTERNAL THREAD
0.0203
0.05

**Problem 2**

An M14X2 hex-head bolt with a nut is used to clamp together two 15-mm steel plates.

a. Select a proper class R washer for the bolt and nut.

b. Determine a suitable length of the bolt, rounded up to the nearest 5 mm.

c. Determine the bolt stiffness.

d. Determine the stiffness of the member per Eq. 8-19.

e. Repeat (c) using Eq. 8-22 and Eq. 8-23.

**Solution:**

a.

Nominal major diameter of 14 mm and pitch of 2 mm.

$$d_p = 14 \text{ mm}$$

Therefore, I select washer size 14 R from Table A-33 with maximum thickness 3.50 mm.

b.

From Table A-31, the nut height is $H = 12.8$ mm.

Hence,

$$l = 2t + 2h_w = 2 \times (15 \text{ mm}) + 3.50 \text{ mm} = 33.5 \text{ mm}$$

$$L \geq l + H = 2 \times (15 \text{ mm}) + 12.8 \text{ mm} + 3.50 \text{ mm} = 46.3000 \text{ mm}$$

Rounded up to the nearest 5 mm,

$$L = 50 \text{ mm}$$

c.

The thread length of inch-series bolts is

$$L_T = 2d + 6 = 2 \times (14 \text{ mm}) + 6 \text{ mm} = 34 \text{ mm}$$

Length of unthreaded portion in grip:

$$l_d = L - L_T = 50 \text{ mm} - 34 \text{ mm} = 16 \text{ mm}$$

Length of threaded portion in grip:

$$l_t = l - l_d = 33.5 \text{ mm} - 16 \text{ mm} = 17.5 \text{ mm}$$

Area of unthreaded portion:

$$A_d = \frac{\pi d^2}{4} = \frac{\pi \times (14\ \mathrm{mm})^2}{4} = 153.9380\ \mathrm{mm}^2$$

Area of threaded portion (from Table 8-1):

$$A_t = 115\ \mathrm{mm}^2$$

Fastener stiffness:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{(153.9380\ \mathrm{mm}^2) \times (115\ \mathrm{mm}^2) \times (207\ \mathrm{GPa})}{(153.9380\ \mathrm{mm}^2) \times (14\ \mathrm{mm}) + (115\ \mathrm{mm}^2) \times (16\ \mathrm{mm})}$$
$$= 9.1723 \times 10^8\ \mathrm{N/m}$$

d.

Eq. 8-19: the spring rate or stiffness of upper frustum

$$k_1 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln\frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (207\ \mathrm{GPa}) \times (14\ \mathrm{mm}) \times \tan 30°}{\ln\frac{[2 \times (15\ \mathrm{mm}) \times \tan 30° + (21\ \mathrm{mm}) - (14\ \mathrm{mm})][(21\ \mathrm{mm}) + (14\ \mathrm{mm})]}{[2 \times (15\ \mathrm{mm}) \times \tan 30° + (21\ \mathrm{mm}) + (14\ \mathrm{mm})][(21\ \mathrm{mm}) - (14\ \mathrm{mm})]}}$$
$$= 6.2326 \times 10^9\ \mathrm{N/m}$$

Eq. 8-19: the spring rate or stiffness of middle frustum

$$D = (21\ \mathrm{mm}) + 2 \times (15\ \mathrm{mm}) \times \tan 30° = 38.32\ \mathrm{mm}$$

$$k_2 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln\frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (207\ \mathrm{GPa}) \times (14\ \mathrm{mm}) \times \tan 30°}{\ln\frac{[2 \times (1.75\ \mathrm{mm}) \times \tan 30° + (38.32\ \mathrm{mm}) - (14\ \mathrm{mm})][(38.32\ \mathrm{mm}) + (14\ \mathrm{mm})]}{[2 \times (1.75\ \mathrm{mm}) \times \tan 30° + (38.32\ \mathrm{mm}) + (14\ \mathrm{mm})][(38.32\ \mathrm{mm}) - (14\ \mathrm{mm})]}}$$
$$= 1.2539 \times 10^{11}\ \mathrm{N/m}$$

Eq. 8-19: the spring rate or stiffness of lower frustum

$$k_3 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln\frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (207\ \mathrm{GPa}) \times (14\ \mathrm{mm}) \times \tan 30°}{\ln\frac{[2 \times (16.75\ \mathrm{mm}) \times \tan 30° + (21\ \mathrm{mm}) - (14\ \mathrm{mm})][(21\ \mathrm{mm}) + (14\ \mathrm{mm})]}{[2 \times (16.75\ \mathrm{mm}) \times \tan 30° + (21\ \mathrm{mm}) + (14\ \mathrm{mm})][(21\ \mathrm{mm}) - (14\ \mathrm{mm})]}}$$
$$= 5.9375 \times 10^9\ \mathrm{N/m}$$

Total member stiffness:

$$k_m = \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)^{-1} = 2.9687 \times 10^9\ \mathrm{N/m}$$

e.

Eq. 8-22:

$$k_m = \frac{0.5774\pi E d}{2\ln\left(5\frac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} = \frac{0.5774\pi \times (207\ \mathrm{GPa}) \times (14\ \mathrm{mm})}{2\ln\left[5\frac{0.5774 \times (33.5\ \mathrm{mm}) + 0.5 \times (14\ \mathrm{mm})}{0.5774 \times (33.5\ \mathrm{mm}) + 2.5 \times (14\ \mathrm{mm})}\right]}$$
$$= 2.9689 \times 10^9\ \mathrm{N/m}$$

Eq. 8-23:

$$k_m = EdA\exp\left(\frac{Bd}{l}\right) = (207\ \mathrm{GPa}) \times (14\ \mathrm{mm}) \times 0.78715 \times \exp\left[\frac{0.62873 \times (14\ \mathrm{mm})}{(33.5\ \mathrm{mm})}\right]$$
$$= 2.9667 \times 10^9\ \mathrm{N/m}$$

(Table 8-8 (Steel): $A = 0.78715,\ B = 0.62873$)

**Problem 3**

A **30-mm** thick AISI 1020 steel plate is sandwiched between a **10-mm** thick 2024-T3 aluminum plate on the top and another **20-mm** thick 2024-T3 aluminum plate on the bottom. The plates are tightened with a bolt and nut as well as washers of class N under both. The bolt is M10 × 1.5, property class 5.8.

a. Assuming the bolt requires its length to be at least 2 threads beyond the nut after tightening. Determine the minimum length of the bolts per Table A-17.

b. Determine the bolt stiffness.

c. Determine the stiffness of the members using Eq. 8-20.

d. Determine the stiffness of the joint.

**Solution:**

a.

Nominal diameter of bolt is 10 mm.

Therefore, I select washer size 14 N from Table A-33 with maximum thickness 2.30 mm.

From Table A-31, the nut height is $H = 8.4$ mm.

Hence,

$$l = l_1 + l_2 + l_3 + h_w = (30\text{ mm}) + (10\text{ mm}) + (20\text{ mm}) + (2.30\text{ mm}) = 62.3\text{ mm}$$

$$L \geq l + H = 62.3\text{ mm} + 8.4\text{ mm} = 70.7\text{ mm}$$

The minimum length of the bolts per Table A-17 is equal to

$$L = 80\text{ mm}$$

b.

The thread length of inch-series bolts is (Table 8-7)

$$L_T = 2d + 6 = 2 \times (10\text{ mm}) + 6\text{ mm} = 26\text{ mm}$$

Length of unthreaded portion in grip:

$$l_d = L - L_T = 80\text{ mm} - 26\text{ mm} = 54\text{ mm}$$

Length of threaded portion in grip:

$$l_t = l - l_d = 62.3\text{ mm} - 54\text{ mm} = 8.3\text{ mm}$$

Area of unthreaded portion:

$$A_d = \frac{\pi d^2}{4} = \frac{\pi \times (10\ \text{mm})^2}{4} = 78.540\ \text{mm}^2$$

Area of threaded portion (from Table 8-1):

$$A_t = 58.0\ \text{mm}^2$$

Fastener stiffness:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{(78.540\ \text{mm}^2) \times (58.0\ \text{mm}^2) \times (207\ \text{GPa})}{(78.540\ \text{mm}^2) \times (8.3\ \text{mm}) + (58.0\ \text{mm}^2) \times (54\ \text{mm})}$$
$$= 2.4920 \times 10^8\ \text{N/m}$$

c.

Eq. 8-20: the spring rate or stiffness of upper aluminum frustum

$$k_1 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (71\ \text{GPa}) \times (10\ \text{mm}) \times \tan 30°}{\ln \frac{[2 \times (12.30\ \text{mm}) \times \tan 30° + (15\ \text{mm}) - (10\ \text{mm})][(15\ \text{mm}) + (10\ \text{mm})]}{[2 \times (12.30\ \text{mm}) \times \tan 30° + (15\ \text{mm}) + (10\ \text{mm})][(15\ \text{mm}) - (10\ \text{mm})]}}$$
$$= 1.4377 \times 10^9\ \text{N/m}$$

Eq. 8-20: the spring rate or stiffness of upper steel frustum

$$D = (15\ \text{mm}) + 2 \times (12.3\ \text{mm}) \times \tan 30° = 29.2\ \text{mm}$$

$$k_2 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (207\ \text{GPa}) \times (10\ \text{mm}) \times \tan 30°}{\ln \frac{[2 \times (18.8\ \text{mm}) \times \tan 30° + (29.2\ \text{mm}) - (10\ \text{mm})][(29.2\ \text{mm}) + (10\ \text{mm})]}{[2 \times (18.8\ \text{mm}) \times \tan 30° + (29.2\ \text{mm}) + (10\ \text{mm})][(29.2\ \text{mm}) - (10\ \text{mm})]}}$$
$$= 1.1876 \times 10^{10}\ \text{N/m}$$

Eq. 8-20: the spring rate or stiffness of lower steel frustum

$$D = (15\ \text{mm}) + 2 \times (20\ \text{mm}) \times \tan 30° = 29.2\ \text{mm}$$

$$k_3 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (207\ \text{GPa}) \times (10\ \text{mm}) \times \tan 30°}{\ln \frac{[2 \times (11.1\ \text{mm}) \times \tan 30° + (21\ \text{mm}) - (10\ \text{mm})][(21\ \text{mm}) + (10\ \text{mm})]}{[2 \times (11.1\ \text{mm}) \times \tan 30° + (21\ \text{mm}) + (10\ \text{mm})][(21\ \text{mm}) - (10\ \text{mm})]}}$$
$$= 2.6809 \times 10^{10}\ \text{N/m}$$

Eq. 8-20: the spring rate or stiffness of upper aluminum frustum

$$k_4 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln\frac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (71\ \text{GPa}) \times (10\ \text{mm}) \times \tan 30°}{\ln\frac{[2 \times (20\ \text{mm}) \times \tan 30° + (15\ \text{mm}) - (10\ \text{mm})][(15\ \text{mm}) + (10\ \text{mm})]}{[2 \times (20\ \text{mm}) \times \tan 30° + (15\ \text{mm}) + (10\ \text{mm})][(15\ \text{mm}) - (10\ \text{mm})]}}$$
$$= 1.2015 \times 10^9\ \text{N/m}$$

Total member stiffness:

$$k_m = \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}\right)^{-1} = 6.0629 \times 10^8\ \text{N/m}$$

d.

The joint stiffness is equal to

$$k_j = k_b + k_m = (2.4920 \times 10^8\ \text{N/m}) + (6.0629 \times 10^8\ \text{N/m}) = 8.5549 \times 10^8\ \text{N/m}$$

**Problem 4**

Assuming the bolt requires its length to be at least 2 threads beyond its grip length. Repeat Prob. #3 (a) thru (d) with the bottom aluminum plate having a threaded hole to eliminate the nut.

**Solution:**

a.

Nominal diameter of bolt is 10 mm.

Therefore, I select washer size 14 N from Table A-33 with maximum thickness 2.30 mm.

Hence,

$$l = l_1 + l_2 + \frac{d}{2} + h_w = (30\text{ mm}) + (10\text{ mm}) + \frac{(10\text{ mm})}{2} + (2.30\text{ mm}) = 47.3\text{ mm}$$

$$L \geq h + 1.5d = (30\text{ mm}) + (10\text{ mm}) + (2.30\text{ mm}) + 1.5 \times (10\text{ mm}) = 57.3\text{ mm}$$

The minimum length of the bolts per Table A-17 is equal to

$$L = 60\text{ mm}$$

b.

The thread length of inch-series bolts is (Table 8-7)

$$L_T = 2d + 6 = 2 \times (10\text{ mm}) + 6\text{ mm} = 26\text{ mm}$$

Length of unthreaded portion in grip:

$$l_d = L - L_T = 60\text{ mm} - 26\text{ mm} = 34\text{ mm}$$

Length of threaded portion in grip:

$$l_t = l - l_d = 47.3\text{ mm} - 34\text{ mm} = 13.3\text{ mm}$$

Area of unthreaded portion:

$$A_d = \frac{\pi d^2}{4} = \frac{\pi \times (10\text{ mm})^2}{4} = 78.540\text{ mm}^2$$

Area of threaded portion (from Table 8-1):

$$A_t = 58.0\text{ mm}^2$$

Fastener stiffness:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{(78.540\text{ mm}^2) \times (58.0\text{ mm}^2) \times (207\text{ GPa})}{(78.540\text{ mm}^2) \times (13.3\text{ mm}) + (58.0\text{ mm}^2) \times (34\text{ mm})}$$
$$= 3.1259 \times 10^8\text{ N/m}$$

c.

Eq. 8-20: the spring rate or stiffness of upper aluminum frustum

$$k_1 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \dfrac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (71\ \text{GPa}) \times (10\ \text{mm}) \times \tan 30°}{\ln \dfrac{[2 \times (12.30\ \text{mm}) \times \tan 30° + (15\ \text{mm}) - (10\ \text{mm})][(15\ \text{mm}) + (10\ \text{mm})]}{[2 \times (12.30\ \text{mm}) \times \tan 30° + (15\ \text{mm}) + (10\ \text{mm})][(15\ \text{mm}) - (10\ \text{mm})]}}$$
$$= 1.4377 \times 10^9\ \text{N/m}$$

Eq. 8-20: the spring rate or stiffness of upper steel frustum

$$D = (15\ \text{mm}) + 2 \times (12.3\ \text{mm}) \times \tan 30° = 29.2\ \text{mm}$$

$$k_2 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \dfrac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (207\ \text{GPa}) \times (10\ \text{mm}) \times \tan 30°}{\ln \dfrac{[2 \times (17.7\ \text{mm}) \times \tan 30° + (29.2\ \text{mm}) - (10\ \text{mm})][(29.2\ \text{mm}) + (10\ \text{mm})]}{[2 \times (17.7\ \text{mm}) \times \tan 30° + (29.2\ \text{mm}) + (10\ \text{mm})][(29.2\ \text{mm}) - (10\ \text{mm})]}}$$
$$= 1.2301 \times 10^{10}\ \text{N/m}$$

Eq. 8-20: the spring rate or stiffness of lower steel frustum

$$D = (15\ \text{mm}) + 2 \times [(60\ \text{mm}) - (30\ \text{mm}) - (10\ \text{mm}) - (2.3\ \text{mm})] \times \tan 30° = 35.4\ \text{mm}$$

$$k_3 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \dfrac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (207\ \text{GPa}) \times (14\ \text{mm}) \times \tan 30°}{\ln \dfrac{[2 \times (12.3\ \text{mm}) \times \tan 30° + (35.4\ \text{mm}) - (14\ \text{mm})][(35.4\ \text{mm}) + (14\ \text{mm})]}{[2 \times (12.3\ \text{mm}) \times \tan 30° + (35.4\ \text{mm}) + (14\ \text{mm})][(35.4\ \text{mm}) - (14\ \text{mm})]}}$$
$$= 2.1877 \times 10^{10}\ \text{N/m}$$

Eq. 8-20: the spring rate or stiffness of upper aluminum frustum

$$k_4 = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \dfrac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$
$$= \frac{\pi \times (71\ \text{GPa}) \times (10\ \text{mm}) \times \tan 30°}{\ln \dfrac{[2 \times (17.7\ \text{mm}) \times \tan 30° + (15\ \text{mm}) - (10\ \text{mm})][(15\ \text{mm}) + (10\ \text{mm})]}{[2 \times (17.7\ \text{mm}) \times \tan 30° + (15\ \text{mm}) + (10\ \text{mm})][(15\ \text{mm}) - (10\ \text{mm})]}}$$
$$= 1.2511 \times 10^9\ \text{N/m}$$

Total member stiffness:

$$k_m = \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4}\right)^{-1} = 6.1657 \times 10^8 \text{ N/m}$$

d.

The joint stiffness is equal to

$$k_j = k_b + k_m = (2.4920 \times 10^8 \text{ N/m}) + (6.0629 \times 10^8 \text{ N/m}) = 9.2916 \times 10^8 \text{ N/m}$$