Christopher King

Mechanical Design II Homework 04

Christopher King 1

2018141521058

Mechanical Design 2

Class Section 01

09/23/2021

Problem 1

A solid square rod is cantilevered at one end. The rod is 0.6 m long and supports a completely reversing transverse load at the other end of ±2 kN. The material is AISI 1080 hot-rolled steel. If the rod must support this load for 10,000 cycles with a design factor of ~1.5, what dimension should the square cross section have? Since the size is not yet known, assume a typical value of kb = 0.85 and verify its correctness later. Neglect any stress concentrations at the support end.

Solution:

For this question, we are asked to determine the dimension that the square cross section should have.

From Table A-20, I can get that the ultimate strength of AISI 1080 hot-rolled steel is equal to

$$S_{ut} = 770 \ \text{MPa} = 112 \ \text{ksi}$$

Therefore, the endurance limit is equal to

$$S_e' = 0.5S_{ut} = 0.5 \times 770 \ \text{MPa} = 385 \ \text{MPa}$$

Next, we consider to modify the endurance limit.

Surface Condition (hot-rolled):

$$k_a = aS_{ut}^b = 57.7 \times 770^{-0.718} = 0.4883$$

Size Effect:

$$k_b = 0.85$$

Loading Effect (bending):

$$k_c = 1$$

Temperature Effect (room temperature):

$$k_d = 1$$

Reliability Effect:

$$k_e = 1$$

Therefore, the modified endurance limit is equal to

$$S_e = k_a k_b k_c k_d k_e S_e' = 0.4883 \times 0.85 \times 1 \times 1 \times 1 \times 385 \ \text{MPa} = 159.79 \ \text{MPa}$$

From Figure 6-18, I can know that the fatigue strength fraction is equal to

$$f = 0.83$$

Therefore,

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.83 \times 770 \ \text{MPa})^2}{159.79 \ \text{MPa}} = 2.5561 \times 10^3 \ \text{MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3} \times \log\left(\frac{0.83 \times 770 \ \text{MPa}}{159.79 \ \text{MPa}}\right) = -0.2007$$

$$S_f = aN^b = (2.5561 \times 10^3 \ \text{MPa}) \times 10000^{-0.2007} = 402.6221 \ \text{MPa}$$

From the force diagram, I can know that

$$M_{max} = (2 \ \text{kN}) \times (0.6 \ \text{m}) = 1200 \ \text{N} \cdot \text{m}$$

$$\sigma_a = \sigma_{max} = \frac{Mc}{I} = \frac{M\left(\frac{b}{2}\right)}{\frac{bb^3}{12}} = \frac{6M}{b^3} = \frac{7200}{b^3} \ \text{Pa}$$

And

$$S_f = n\sigma_a$$

$$402.6221 \ \text{MPa} = 1.5 \times \frac{7200}{b^3} \ \text{Pa}$$

$$b = 0.0299 \ \text{m}$$

Check for the size effect:

$$d_e = 0.808\sqrt{hb} = 0.808\sqrt{b^2} = 0.808b = 0.0242 \ \text{m} = 24.2 \ \text{mm}$$

$$k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{24.2}{7.62}\right)^{-0.107} = 0.8837$$

The result doesn't match our guess.

Therefore, I conduct the iteration to determine the dimension of the beam.

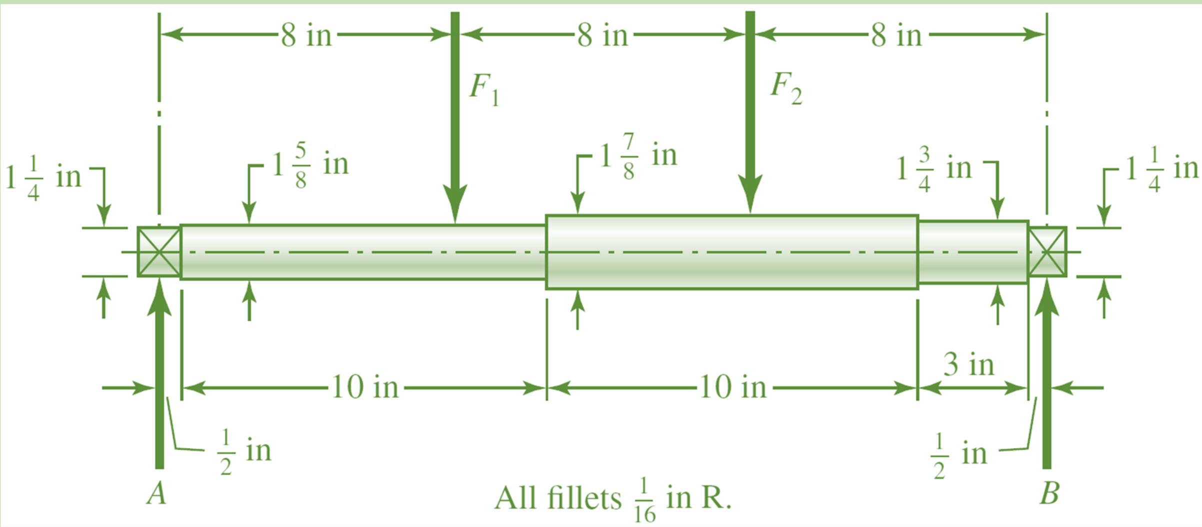
```
clc; clear all;
kb = 0.85;
for i = 1:20
    Sut = 770;
    f = 0.83;
    ka = 57.7*770^(-0.718);
    Se = ka*kb*385;
    a=(f*Sut)^2/Se;
    b=-1/3*log10((f*Sut)/Se);
    Sf = a*10000^b;
    bb = (1.5*7200/(Sf*1E6))^(1/3);
    de=0.808*bb*1000;
    kb = (de/7.62)^(-0.107);
end
```

After conducting 20 iterations, the value of k_b is converged to 0.8842 and the value of size b is converged to $0.0298\ \text{m} = 29.8\ \text{mm}$.

Problem 2

The shaft shown in the figure is machined from AISI 1040 CD steel. The shaft rotates at 1600 rpm and is supported in rolling bearings at A and B. The applied forces are F1 = 2500 lbf and F2 = 1000 lbf. Radius of all fillets is 1/16 in R.

Determine the minimum fatigue factor of safety based on achieving infinite life. If infinite life is not predicted, estimate the number of cycles to failure. Also check for yielding.



Solution:

For this question, we are asked to determine the minimum fatigue factor of safety based on achieving infinite life. If infinite life is not predicted, estimate the number of cycles to failure. Also check for yielding.

From Table A-20, I can get that the ultimate strength and yield strength of AISI 1040 CD steel is equal to

$$\begin{cases} S_{ut} = 590 \ \text{MPa} = 85 \ \text{ksi} \\ S_y = 490 \ \text{MPa} = 71 \ \text{ksi} \end{cases}$$

Therefore, the endurance limit is equal to

$$S_e' = 0.5S_{ut} = 0.5 \times 85 \ \text{ksi} = 42.5 \ \text{ksi}$$

Next, we consider to modify the endurance limit.

Surface Condition (cold drawn):

$$k_a = aS_{ut}^b = 2.70 \times 85^{-0.265} = 0.8319$$

Size Effect ($d = 1\frac{5}{8}$ in):

$$k_b = \left(\frac{d}{0.3}\right)^{-0.107} = \left(\frac{1\frac{5}{8}}{0.3}\right)^{-0.107} = 0.8346$$

Loading Effect (bending):

$$k_c = 1$$

Temperature Effect (room temperature):

$$k_d = 1$$

Reliability Effect:

$$k_e = 1$$

Therefore, the modified endurance limit is equal to

$$S_e = k_a k_b k_c k_d k_e S_e' = 0.8319 \times 0.8346 \times 1 \times 1 \times 1 \times 42.5 \ \text{ksi} = 29.5085 \ \text{ksi}$$

From the force diagram, I can know that the bending moment at the second fillet is equal to

$$M = (2000 \ \text{lbf}) \times (10.5 \ \text{in}) - (2500 \ \text{lbf}) \times (2.5 \ \text{in}) = 14750 \ \text{lbf} \cdot \text{in}$$

$$\sigma = \frac{Mc}{I} = \frac{(14750\ \text{lbf}\cdot\text{in}) \times \left(\frac{1\frac{5}{8}\ \text{in}}{2}\right)}{\frac{\pi\left(1\frac{5}{8}\ \text{in}\right)^4}{64}} = 35.0132\ \text{ksi}$$

According to Figure A-15-9, when $\frac{r}{d} = \frac{\frac{1}{16}\ \text{in}}{1\frac{5}{8}\ \text{in}} = 0.03846$ and $\frac{D}{d} = \frac{1\frac{7}{8}\ \text{in}}{1\frac{5}{8}\ \text{in}} = 1.15$, the stress concentration factor is equal to

$$K_t = 1.95$$

$$\begin{aligned}\sqrt{a} &= 0.246 - 3.08\times10^{-3}S_{ut} + 1.51\times10^{-5}S_{ut}^2 - 2.67\times10^{-8}S_{ut}^3 \\ &= 0.246 - 3.08\times10^{-3}\times85 + 1.51\times10^{-5}\times85^2 - 2.67\times10^{-8}\times85^3 \\ &= 0.0769\end{aligned}$$

$$\Rightarrow a = 0.0059$$

The fatigue stress concentration is equal to

$$K_f = 1 + \frac{(K_t - 1)}{1 + \sqrt{\frac{a}{r}}} = 1 + \frac{(1.95 - 1)}{1 + \sqrt{\frac{0.0059}{\frac{1}{16}}}} = 1.7265$$

Therefore, the fatigue factor of safety is equal to

$$n_f = \frac{S_e}{K_f\sigma} = \frac{(29.5085\ \text{ksi})}{1.7265\times(35.0132\ \text{ksi})} = 0.4881$$

Hence, I can know that infinite life is not predicted.

From Figure 6-18, I can know that the fatigue strength fraction is equal to

$$f = 0.867$$

Therefore,

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.867\times85\ \text{ksi})^2}{29.5085\ \text{ksi}} = 184.0470\ \text{ksi}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right) = -\frac{1}{3}\times\log\left(\frac{0.867\times85\ \text{ksi}}{29.5085\ \text{ksi}}\right) = -0.1325$$

$$S_f = aN^b$$

$$K_f \sigma_{rev} = aN^b$$

$$1.7265 \times (35.0132 \ \text{ksi}) = (184.0470 \ \text{ksi}) \times N^{-0.1325}$$

$$\Rightarrow N = 4459.9$$

The safety factor for yield strength is equal to

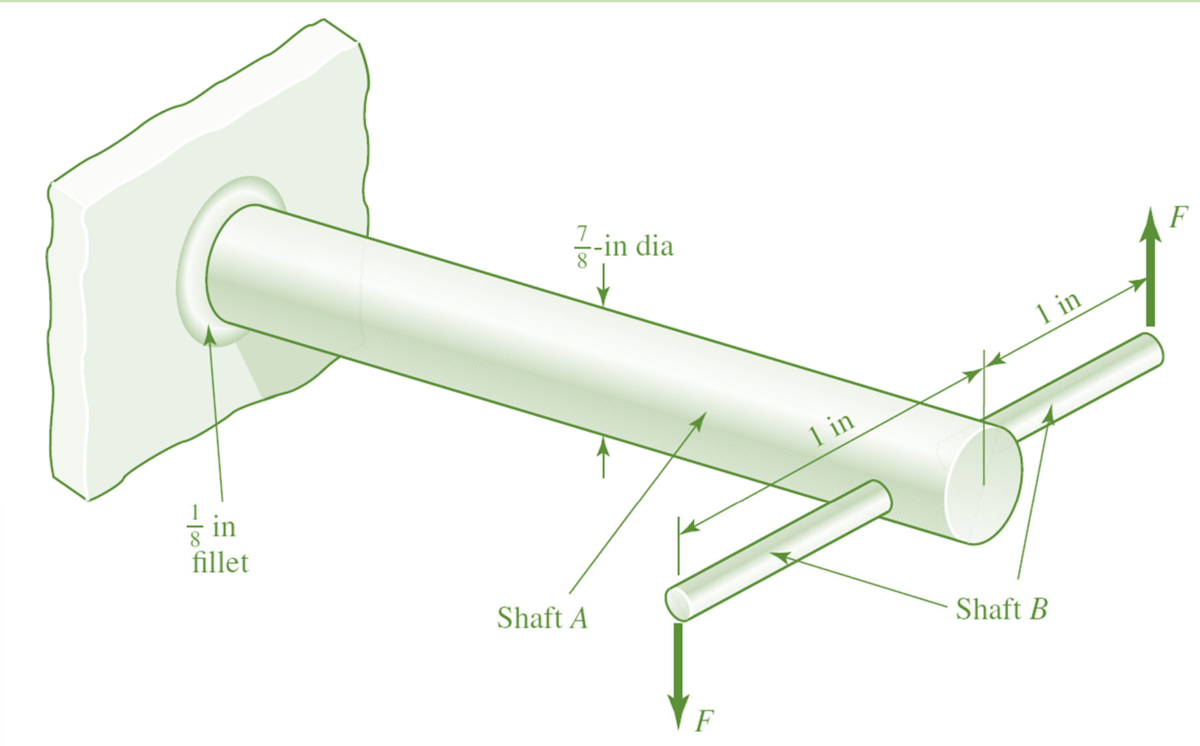
$$n_y = \frac{S_y}{K_f \sigma} = \frac{71 \ \text{ksi}}{1.7265 \times (35.0132 \ \text{ksi})} = 1.1745$$

Problem 3

Shaft A, made of AISI 1020 hot-rolled steel, is welded to a fixed support and is subjected to loading by equal and opposite forces F via shaft B. A theoretical stress concentration factor K_{ts} of 1.6 is induced in the shaft by the 1/8-in weld fillet. The length of shaft A from the fixed support to the connection at shaft B is 2 ft. The load F cycles from 150 to 500 lbf.

a. For shaft A, find the factor of safety for infinite life using the modified Goodman fatigue failure criterion.

b. Repeat part (a) using the Gerber fatigue failure criterion.



Solution:

a. For this question, we are asked to find the factor of safety for infinite life using the modified Goodman fatigue failure criterion for shaft A.

From Table A-20, I can get that the ultimate strength of AISI 1020 hot-rolled steel is equal to

$$\begin{cases} S_{ut} = 380 \text{ MPa} = 55 \text{ ksi} \\ S_y = 210 \text{ MPa} = 30 \text{ ksi} \end{cases}$$

Therefore, the endurance limit is equal to

$$S_e' = 0.5S_{ut} = 0.5 \times 55 \text{ ksi} = 27.5 \text{ ksi}$$

Next, we consider to modify the endurance limit.

Surface Condition (hot-rolled):

$$k_a = aS_{ut}^b = 14.4 \times 55^{-0.718} = 0.8106$$

Size Effect ($d = \frac{7}{8}$ in):

$$d_e = 0.370d = 0.370 \times \frac{7}{8} \text{ in} = 0.3237 \text{ in}$$

$$k_b = \left(\frac{d}{0.3}\right)^{-0.107} = \left(\frac{0.3237}{0.3}\right)^{-0.107} = 0.9919$$

Loading Effect (torsion):

$$k_c = 0.59$$

Temperature Effect (room temperature):

$$k_d = 1$$

Reliability Effect:

$$k_e = 1$$

Therefore, the modified endurance limit is equal to

$$\begin{aligned} S_{se} &= k_a k_b k_c k_d k_e S_e' = 0.8106 \times 0.9919 \times 0.59 \times 1 \times 1 \times 27.5 \text{ ksi} \\ &= 13.0445 \text{ ksi} \end{aligned}$$

And because the stress in this question is torsion,

$$\begin{aligned} \sqrt{a} &= 0.190 - 2.51 \times 10^{-3} S_{ut} + 1.35 \times 10^{-5} S_{ut}^2 - 2.67 \times 10^{-8} S_{ut}^3 \\ &= 0.246 - 3.08 \times 10^{-3} \times 55 + 1.51 \times 10^{-5} \times 55^2 - 2.67 \times 10^{-8} \\ &\times 55^3 = 0.0883 \end{aligned}$$

$$\Rightarrow a = 0.0078$$

The fatigue stress concentration is equal to

$$K_f = 1 + \frac{(K_t - 1)}{1 + \sqrt{\frac{a}{r}}} = 1 + \frac{(1.6 - 1)}{1 + \sqrt{\frac{0.0078}{\frac{1}{16}}}} = 1.4800$$

From the question, I can know that

$$\begin{cases} F_{max} = 500 \text{ lbf} \\ F_{min} = 150 \text{ lbf} \end{cases} \Rightarrow \begin{cases} T_{max} = 1000 \text{ lbf} \cdot \text{in} \\ T_{min} = 300 \text{ lbf} \cdot \text{in} \end{cases}$$

$$\Rightarrow \begin{cases} \tau_{max} = K_f \dfrac{T_{max}c}{J} = K_f \dfrac{16T_{max}}{\pi d^3} = 1.4800 \times \dfrac{16 \times (1000 \ \ \text{lbf} \cdot \text{in})}{\pi \times \left(\frac{7}{8} \ \ \text{in}\right)^3} = 11.2518 \ \ \text{ksi} \\ \tau_{min} = K_f \dfrac{T_{min}c}{J} = K_f \dfrac{16T_{min}}{\pi d^3} = 1.4800 \times \dfrac{16 \times (300 \ \ \text{lbf} \cdot \text{in})}{\pi \times \left(\frac{7}{8} \ \ \text{in}\right)^3} = 3.3755 \ \ \text{ksi} \end{cases}$$

$$\Rightarrow \begin{cases} \tau_m = \dfrac{\tau_{max} + \tau_{min}}{2} = \dfrac{(11.2518 \ \ \text{ksi}) + (3.3755 \ \ \text{ksi})}{2} = 7.3137 \ \ \text{ksi} \\ \tau_a = \dfrac{\tau_{max} - \tau_{min}}{2} = \dfrac{(11.2518 \ \ \text{ksi}) - (3.3755 \ \ \text{ksi})}{2} = 3.9381 \ \ \text{ksi} \end{cases}$$

The safety factor for yield strength is equal to

$$n_y = \frac{\frac{S_y}{2}}{\tau_{max}} = \frac{\frac{30 \ \ \text{ksi}}{2}}{11.2518 \ \ \text{ksi}} = 1.3331$$

And, in constructing the Goodman diagram, I use

$$S_{su} = 0.67 S_{ut} = 0.67 \times 55 \ \ \text{ksi} = 36.8500 \ \ \text{ksi}$$

Then implement the Modified Goodman failure criterion:

$$n_f = \frac{1}{\frac{\tau_a}{S_{se}} + \frac{\tau_m}{S_{su}}} = \frac{1}{\frac{3.9381 \ \ \text{ksi}}{13.0445 \ \ \text{ksi}} + \frac{7.3137 \ \ \text{ksi}}{36.8500 \ \ \text{ksi}}} = {\color{red}1.9985}$$

b. For this question, we are asked to find the factor of safety for infinite life using the Gerber fatigue failure criterion for shaft A.

Implement the Gerber fatigue failure criterion:

$$n_f = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}} \left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{S_{su}\tau_a}\right)^2}\right]$$

$$= \frac{1}{2} \times \left(\frac{36.8500 \ \ \text{ksi}}{7.3137 \ \ \text{ksi}}\right)^2 \times \frac{3.9381 \ \ \text{ksi}}{13.0445 \ \ \text{ksi}}$$

$$\times \left[-1 + \sqrt{1 + \left(\frac{2 \times 7.3137 \ \ \text{ksi} \times 13.0445 \ \ \text{ksi}}{36.8500 \ \ \text{ksi} \times 3.9381 \ \ \text{ksi}}\right)^2}\right] = {\color{red}2.4981}$$

Christopher King