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Mechanical Design II Homework 02

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Mechanical Design 2

Class Section 01

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Problem 1

The outer diameter of a solid aluminum shaft is in the range of 2.003-2.006 inch. Its mating hub is made of 18-8 stainless steel and has the inner diameter in the range of 2.000-2.002 inch and 3-inch outer diameter. Elastic constants of the two materials can be found in Table A–5 in the textbook.

a. Identify the range of radial interference δ, then
b. find the maximum interference pressure P, and the
c. radial and hoop stresses on both parts at the fit surface under the given P.

Solution:

a. For this question, we are asked to identify the range of radial interference δ.

$$E_i = 10.4 \ \text{Mpsi}$$
$$v_i = 0.333$$
$$E_o = 27.6 \ \text{Mpsi}$$
$$v_o = 0.305$$
$$\delta_{min} = b_{o_{min}} - b_{i_{max}} = \frac{2.003 \ \text{inch}}{2} - \frac{2.002 \ \text{inch}}{2} = 5.00 \times 10^{-4} \ \text{inch}$$
$$\delta_{max} = b_{o_{max}} - b_{i_{min}} = \frac{2.006 \ \text{inch}}{2} - \frac{2.000 \ \text{inch}}{2} = 3.00 \times 10^{-3} \ \text{inch}$$

Therefore, the range of radial interference δ is $[5.00 \times 10^{-4}, 3.00 \times 10^{-3}]$ inch.

b. For this question, we are asked to find the maximum interference pressure P.

$$c = \frac{3 \ \text{inch}}{2} = 1.5 \ \text{inch}$$
$$a = 0$$
$$R = \frac{2 \ \text{inch}}{2} = 1 \ \text{inch}$$

$$P = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{c^2+R^2}{c^2-R^2}+v_o\right)+\frac{1}{E_i}\left(\frac{R^2+a^2}{R^2-a^2}-v_i\right)\right]}$$

$$= \frac{\left(3.00\times10^{-3}\ \text{inch}\right)}{(1\ \text{inch})\times\left\{\frac{1}{(27.6\ \text{Mpsi})}\left[\frac{(1.5\ \text{inch})^2+(1\ \text{inch})^2}{(1.5\ \text{inch})^2-(1\ \text{inch})^2}+0.305\right]+\frac{1}{(10.4\ \text{Mpsi})}\left[\frac{(1\ \text{inch})^2+0^2}{(1\ \text{inch})^2-0^2}-0.333\right]\right\}}$$

$= 17.71\ \text{ksi}$

c. For this question, we are asked to find the radial and hoop stresses on both parts at the fit surface under the given P.

Radial stresses:

$$(\sigma_r)_{b_i} = (\sigma_r)_{b_o} = -P = -17.71\ \text{ksi}$$

Hoop stresses:

$$(\sigma_\theta)_{b_i} = -P\frac{R^2+a^2}{R^2-a^2} = -(17.71\ \text{ksi})\times\frac{(1\ \text{inch})^2+0^2}{(1\ \text{inch})^2-0^2} = -17.71\ \text{ksi}$$

$$(\sigma_\theta)_{b_o} = P\frac{c^2+R^2}{c^2-R^2} = -(17.71\ \text{ksi})\times\frac{(1.5\ \text{inch})^2+(1\ \text{inch})^2}{(1.5\ \text{inch})^2-(1\ \text{inch})^2} = 46.04\ \text{ksi}$$

Problem 2

Following Question 01 and calculate the followings at the interference-fit interface:

a. safety factor of the hub ID per MSS failure criteria,
b. safety factor of the hub ID per DET failure criteria, and
c. the guaranteed torque capacity limit of the fit assuming 1.25 in fit length and COF of 0.2.

Also make an assessment whether the parts will fail or not.
Yield strength of the 18-8 steel is 50 ksi.

Solution:

a. For this question, we are asked to calculate the safety factor of the hub ID per MSS failure criteria.

$$n = \frac{s_y}{(\sigma_1-\sigma_3)_{design}} = \frac{(50\ \text{ksi})}{(46.04\ \text{ksi})-(-17.71\ \text{ksi})} = 0.7842$$

b. For this question, we are asked to calculate the safety factor of the hub ID per DET failure criteria.

$$n = \frac{S_y}{\sigma_e} = \frac{S_y}{\frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}}} = \frac{S_y}{\left[\sigma_r^2-\sigma_r\sigma_\theta+\sigma_\theta^2\right]^{\frac{1}{2}}}$$

$$= \frac{(50\ \ \text{ksi})}{\left[(46.04\ \ \text{ksi})^2-(46.04\ \ \text{ksi})\times(-17.71\ \ \text{ksi})+(-17.71\ \ \text{ksi})^2\right]^{\frac{1}{2}}}$$

$$= 0.8771$$

c. For this question, we are asked to calculate the guaranteed torque capacity limit of the fit assuming 1.25 in fit length and COF of 0.2.

$$Torque = \mu(\pi dL)PR$$
$$= 0.2\times[\pi\times(2\ \ \text{inch})\times(1.25\ \ \text{inch})]\times(17.71\ \ \text{ksi})\times(1\ \ \text{inch})$$
$$= 2.782\times10^4\ \ \text{lb}\cdot\text{in}$$

d. For this question, we are asked to make an assessment whether the parts will fail or not. Because the safety factors of the hub ID by two criteria are both less than 1, the parts will fail.

Problem 3

Your mission is to make a force-fit design of a 150-mm-diameter steel shaft with a 300-mm-outside-diameter hub. The hub is 25 mm long. The designed system is intended to operate under 150degC environment.
Both hub and shaft are made of 1050 CD steel. Moduli of elasticity is 207 GPa and Poisson's ratio is 0.3.
Coefficient of friction of steel-on-steel is 0.20.
Design the fit per ANSI B4-2-1978.

a. Specify the range of the shaft outer diameter.
b. Specify the range of the hub inner diameter.
c. Calculate max and min interference.
d. How will you mark the dimensions of shaft OD and hub ID on the drawing?
e. Safety factor of the hub ID surface per DET failure criteria.
f. Guaranteed capacity for torque transmission.

Solution:

a. For this question, we are asked to specify the range of the shaft outer diameter.
According to Table 7-20, the symbol for force-fit is H7/u6.
The basic size of the system is equal to $150\ \ \text{mm}$. According to Table A-11, the tolerance grade of H7 is $\Delta D = 0.040\ \ \text{mm}$ and the tolerance grade of u6 is $\Delta d = 0.025\ \ \text{mm}$.
And according to Table A-12, the fundamental deviation for u6 is $\delta_F = +0.190\ \ \text{mm}$.
Therefore, the minimum shaft outer diameter is equal to

$$d_{min} = D + \delta_F = 150\ \ \text{mm} + 0.190\ \ \text{mm} = 150.190\ \ \text{mm}$$

And the maximum shaft outer diameter is equal to

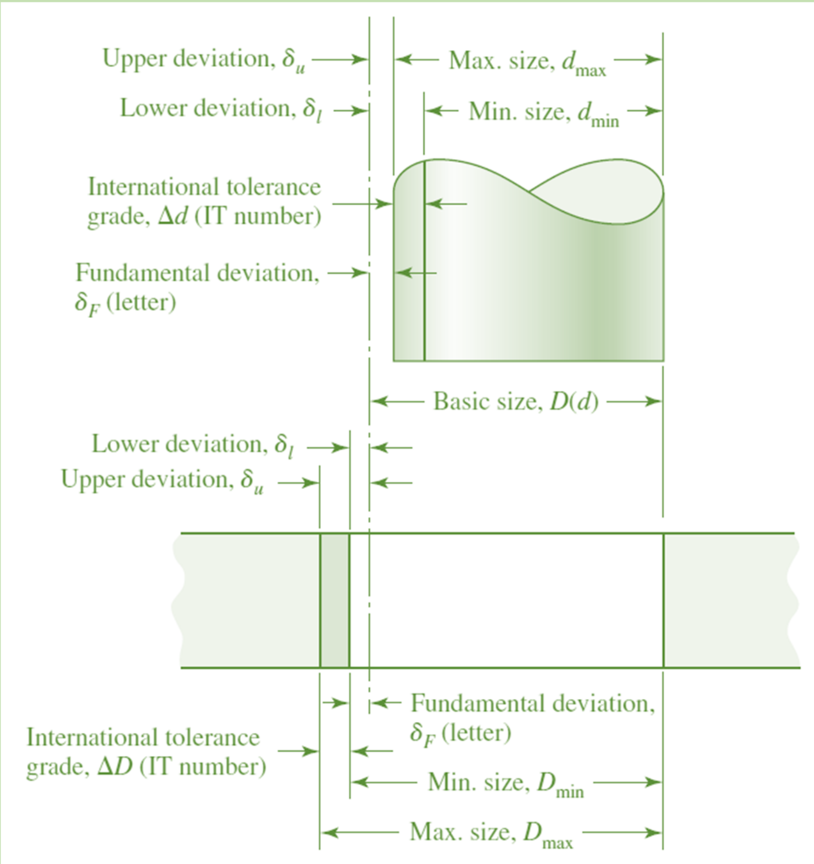
$$d_{max} = D + \delta_F + \Delta d = 150\ \ \text{mm} + 0.190\ \ \text{mm} + 0.025\ \ \text{mm} = 150.215\ \ \text{mm}$$

Therefore, the range of the shaft outer diameter is $[150.190, 150.215]\ \ \text{mm}$.

b. For this question, we are asked to specify the range of the hub inner diameter.
The minimum hub inner diameter is equal to
$$D_{min} = D = 150 \ \text{ mm}$$
And the maximum hub inner diameter is equal to
$$D_{max} = D + \Delta D = 150 \ \text{ mm} + 0.040 \ \text{ mm} = 150.040 \ \text{ mm}$$
Therefore, the range of the shaft outer diameter is $[150, 150.040] \ \text{ mm}$.

c. For this question, we are asked to calculate max and min interference.
The maximum interference is equal to
$$2\delta_{max} = d_{max} - D_{min} = 150.215 \ \text{ mm} - 150 \ \text{ mm} = 0.215 \ \text{ mm}$$
The minimum interference is equal to
$$2\delta_{min} = d_{min} - D_{max} = 150.190 \ \text{ mm} - 150.040 \ \text{ mm} = 0.150 \ \text{ mm}$$

d. For this question, we are asked to mark the dimensions of shaft OD and hub ID on the drawing.
The way to mark the dimensions of shaft OD and hub ID on the drawing is shown in figure below:



e. For this question, we are asked to determine the safety factor of the hub ID surface per DET failure criteria.
$$\delta_{max} = \frac{0.215 \ \text{ mm}}{2}$$
$$E_i = 207 \ \text{ GPa}$$

$$v_i = 0.3$$
$$E_o = 207 \ \text{GPa}$$
$$v_o = 0.3$$
$$c = \frac{300 \times 10^{-3} \ \text{m}}{2}$$
$$R = \frac{150 \times 10^{-3} \ \text{m}}{2}$$
$$a = 0$$

$$P = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{c^2+R^2}{c^2-R^2}+v_o\right)+\frac{1}{E_i}\left(\frac{R^2+a^2}{R^2-a^2}-v_i\right)\right]}$$
$$= \frac{\left(\frac{0.215 \ \text{mm}}{2}\right)}{\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)\times\left\{\frac{1}{(207\times10^9 \ \text{Pa})}\left[\frac{\left(\frac{300\times10^{-3} \ \text{m}}{2}\right)^2+\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2}{\left(\frac{300\times10^{-3} \ \text{m}}{2}\right)^2-\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2}+0.3\right]+\frac{1}{(207\times10^9 \ \text{Pa})}\left[\frac{\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2+0^2}{\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2-0^2}-0.3\right]\right\}}$$
$$= 111.26 \ \text{MPa}$$

Radial stresses:

$$(\sigma_r)_{b_i} = (\sigma_r)_{b_o} = -P = -111.26 \ \text{MPa}$$

Hoop stresses:

$$(\sigma_\theta)_{b_i} = -P\frac{R^2+a^2}{R^2-a^2} = -(111.26 \ \text{MPa})\times\frac{\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2+0^2}{\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2-0^2}$$
$$= -111.26 \ \text{MPa}$$
$$(\sigma_\theta)_{b_o} = P\frac{c^2+R^2}{c^2-R^2} = -(111.26 \ \text{MPa})\times\frac{\left(\frac{300\times10^{-3} \ \text{m}}{2}\right)^2+\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2}{\left(\frac{300\times10^{-3} \ \text{m}}{2}\right)^2-\left(\frac{150\times10^{-3} \ \text{m}}{2}\right)^2}$$
$$= 185.44 \ \text{MPa}$$

From Table A-20, I can know that the yield strength of 1050 CD steel is equal to $580 \ \text{MPa}$

$$n = \frac{S_y}{\sigma_e} = \frac{S_y}{\frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{\frac{1}{2}}} = \frac{S_y}{\left[\sigma_r^2-\sigma_r\sigma_\theta+\sigma_\theta^2\right]^{\frac{1}{2}}}$$
$$= \frac{(580 \ \text{MPa})}{\left[(185.44 \ \text{MPa})^2-(185.44 \ \text{MPa})\times(-111.26 \ \text{MPa})+(-111.26 \ \text{MPa})^2\right]^{\frac{1}{2}}}$$
$$= 2.234$$

f. For this question, we are asked to determine the guaranteed capacity for torque transmission.

$$\delta_{min} = \frac{0.150 \ \text{mm}}{2}$$

$$E_i = 207 \ \text{GPa}$$
$$v_i = 0.3$$
$$E_o = 207 \ \text{GPa}$$
$$v_o = 0.3$$
$$c = \frac{300 \times 10^{-3} \ \text{m}}{2}$$
$$R = \frac{150 \times 10^{-3} \ \text{m}}{2}$$
$$a = 0$$

$$P = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{c^2 + R^2}{c^2 - R^2} + v_o\right) + \frac{1}{E_i}\left(\frac{R^2 + a^2}{R^2 - a^2} - v_i\right)\right]}$$

$$= \frac{\left(\frac{0.150 \ \text{mm}}{2}\right)}{\left(\frac{150 \times 10^{-3} \ \text{m}}{2}\right) \times \left\{\frac{1}{(207 \times 10^9 \ \text{Pa})}\left[\frac{\left(\frac{300 \times 10^{-3} \ \text{m}}{2}\right)^2 + \left(\frac{150 \times 10^{-3} \ \text{m}}{2}\right)^2}{\left(\frac{300 \times 10^{-3} \ \text{m}}{2}\right)^2 - \left(\frac{150 \times 10^{-3} \ \text{m}}{2}\right)^2} + 0.3\right] + \frac{1}{(207 \times 10^9 \ \text{Pa})}\left[\frac{\left(\frac{150 \times 10^{-3} \ \text{m}}{2}\right)^2 + 0^2}{\left(\frac{150 \times 10^{-3} \ \text{m}}{2}\right)^2 - 0^2} - 0.3\right]\right\}}$$

$$= 77.63 \ \text{MPa}$$

$$Torque = \mu(\pi d L)PR$$
$$= 0.2 \times [\pi \times (150 \times 10^{-3} \ \text{m}) \times (25 \times 10^{-3} \ \text{m})] \times (77.63 \ \text{MPa})$$
$$\times \left(\frac{150 \times 10^{-3} \ \text{m}}{2}\right) = 1.3717 \times 10^4 \ \text{N} \cdot \text{m}$$

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