Homework 05 Solutions

Question 01 (20 points)

A simplified form of the diameter equation is shown below.

$$d = \left(\frac{16n}{\pi} \left\{ \frac{1}{S_e} \left[4(K_f \overset{0}{M_a})^2 + 3(K_{fs}T_a)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[4(K_f \overset{0}{M_m})^2 + 3(K_{fs}T_m)^2 \right]^{1/2} \right\} \right)^{1/3}$$

$M_m = M_a = 0$

$T_m = 1000\ N * m, \quad T_a = 250\ N * m$

Initial Design:

$\frac{D}{d} = 1.2 \qquad \frac{r}{d} = 0.05$

From Fig A-15-8, $K_{ts} = 1.6$, let $K_{fs} = K_{ts} = 1.6$

Shaft material: S_{ut}=1200MPa, S_y=100MPa

Assume k_b=0.9 since diameter d is unknown.

	(MPa)	Correction Factor	Parameter
S_{ut} @RT	1200		
S_e' @RT	600	0.5	Eq. 6-8
	518.9	$k_a = aS_{ut}^b = 1.58 \cdot 1200^{-0.085} = 0.865$	Ground Surface
	467.0	$k_b = 0.9$ (assumed)	Size
	275.5	$k_c = 0.59$	Loading: Torsion
	275.5	$k_d = 1$	Temperature
	239.2	$k_e = 0.868$	Reliability: 95%
S_e @RT	**239.2**		

For minimum diameter d at shoulder with infinite life, let n=1.0.

$$d = \left[\frac{16\sqrt{3}K_{fs}n}{\pi} \left(\frac{T_a}{S_e} + \frac{T_m}{S_{ut}} \right) \right]^{1/3} = \left[\frac{16 \cdot \sqrt{3} \cdot 1.6 \cdot 1.0}{\pi} \left(\frac{250}{239.2 \cdot 10^6} + \frac{1000}{1200 \cdot 10^6} \right) \right]^{1/3} = 0.0298m$$

d=0.0298m=29.8mm; use d=30 mm for below calculation

Iteration 2:

Since $\frac{D}{d} = 1.2$, $D = 1.2 \cdot 30 = 36mm$, Fillet radius $\frac{r}{d} = 0.05$, $r = 1.5mm$

Revised $K_{ts} = 1.6$, $q_s = 0.86$, $K_{fs} = 1 + 0.86(1.6 - 1) = 1.52$

$$d = \left[\frac{16\sqrt{3}K_{fs}n}{\pi} \left(\frac{T_a}{S_e} + \frac{T_m}{S_{ut}} \right) \right]^{1/3} = \left[\frac{16 \cdot \sqrt{3} \cdot 1.52 \cdot 1.0}{\pi} \left(\frac{250}{229.5 \cdot 10^6} + \frac{1000}{1200 \cdot 10^6} \right) \right]^{1/3} = 0.0295m$$

d=0.0295m≈30mm

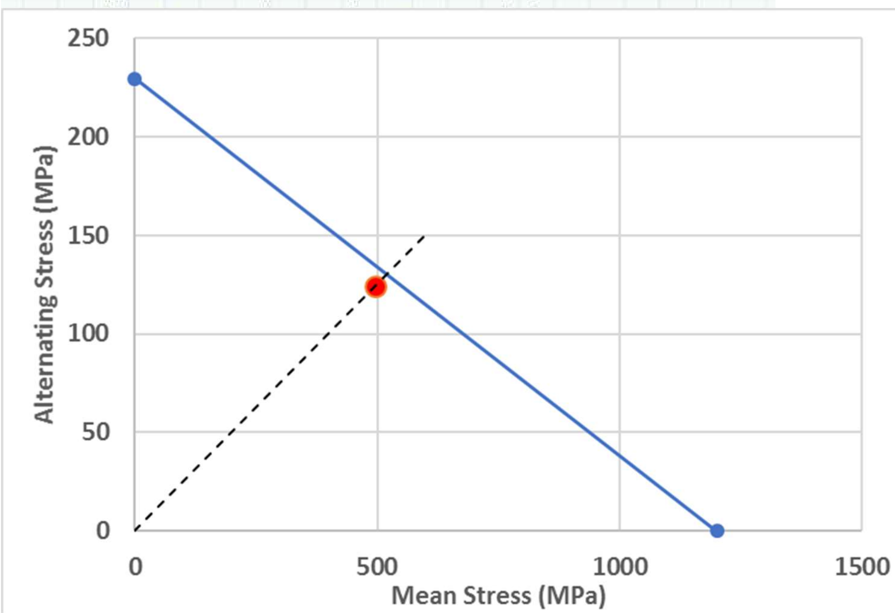
Select d at shoulder is adequate.

	(MPa)	Correction Factor	Parameter
S_{ut} @RT	1200		
S_e' @RT	600	0.5	Eq. 6-8
	518.9	$k_a = aS_{ut}^b = 1.58 \cdot 1200^{-0.085} = 0.865$	Ground Surface
	448.1	$k_b = \left(\frac{30}{7.62}\right)^{-0.107} = 0.864$	Size Eq. 6-20
	264.4	$k_c = 0.59$	Loading: Torsion
	264.4	$k_d = 1$	Temperature
	229.5	$k_e = 0.868$	Reliability: 95%
S_e @RT	**229.5**		

Calculated mean and alternating stresses shaft:

$$\sigma_a := \left(\left(\frac{32 \cdot K_f \cdot M_a}{\pi \cdot d^3}\right)^2 + 3\left(\frac{16 \cdot K_{fs} \cdot T_a}{\pi \cdot d^3}\right)^2\right)^{0.5} = 124.151\ MPa$$

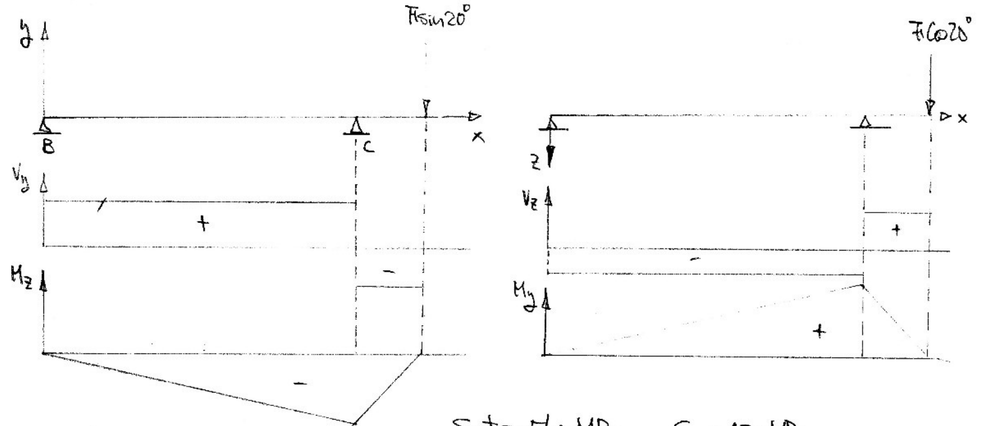
$$\sigma_m := \left(\left(\frac{32 \cdot K_f \cdot M_m}{\pi \cdot d^3}\right)^2 + 3\left(\frac{16 \cdot K_{fs} \cdot T_m}{\pi \cdot d^3}\right)^2\right)^{0.5} = 496.604\ MPa$$



The safety factor is very close to 1 as assumed.

Question 02

#2



$S_{ut} = 560\,MPa \quad S_y = 420\,MPa$

$T_A = 340\ N\cdot m = F\cos 20° \dfrac{0.15}{2} \qquad F = 4824\,N$

$M_{total} @ C = 4824 \cdot 0.1 = 482.4\ N\text{-}m$

$T_a = 0\ ; \quad T_m = 340\ N\text{-}m\ ; \quad M_a = M_{total} = 482.4\,N\text{-}m\ ; \quad M_m = 0$

Table 7-1, for sharp fillet, $k_t = 2.7 \quad k_{ts} = 2.2 \quad \left[\frac{r}{d} = 0.02,\ \frac{D}{d} = 1.5\right]$

$$d = \left[\frac{16n}{\pi}\left(\frac{2K_f M_a}{S_e} + \frac{\sqrt{3}K_{fs}T_m}{S_{ut}}\right)\right]^{1/3}$$

$k_a = 0.843, \quad k_e = 0.868$ (95% Reliability)

$k_b = 0.9$ (Assumed)

$S_e = 184\ MPa$

$$d = \left[\frac{16\cdot 2.5}{\pi}\left(\frac{2\cdot 2.7\cdot 482.4}{184\times 10^6} + \frac{\sqrt{3}\cdot 2.2\cdot 340}{560\times 10^6}\right)\right]^{1/3}$$

$= 0.0594\ m = 59.4\ mm$, Let $d = 60\ mm$

$\frac{D}{d} = 1.5 \quad D = 90\,mm \quad \frac{r}{d} = 0.02 \quad r = 1.2\,mm \quad q = 0.74 \quad q_s = 0.75$

$K_f = 1 + 0.74(2.7-1) = 2.3 \qquad K_{fs} = 1 + 0.75(2.2-1) = 1.9$

Also revise $k_b = 0.862 \qquad S_e = 164.3\ MPa$

$$d = \left[\frac{16\cdot 2.5}{\pi}\left(\frac{2\cdot 2.3\cdot 482.4}{164.3\times 10^6} + \frac{\sqrt{3}\cdot 1.9\cdot 340}{560\times 10^6}\right)\right]^{1/3} = 0.0582\ m = 58.2\ mm$$

Selected shaft $d = 60\ mm$ is adequate.

Question 03

Transmitted torque	$Trq := 2819 \cdot in \cdot lbf$
Nominal Shaft Dia	$d := 1 \cdot in$
Force on Key	$F := \dfrac{Trq}{\dfrac{d}{2}} = 5638\ lbf$
Factor of Safety	$SF := 1.1$
Key Material: 1020 CD	$S_{ut} := 68 \cdot ksi$ $\quad S_y := 57 \cdot ksi$
Per DET Criteria	$S_{sy} := 0.577 \cdot S_y = 32.889\ ksi$
Table 7-6 shows the square key dimensions for a 1" shaft dia,	$w := \dfrac{1}{4} \cdot in$ $\quad H := \dfrac{1}{4} \cdot in$
To resist failure by shear across the key, the needed key length per DET	$L := \dfrac{SF}{S_{sy}} \cdot \dfrac{F}{w} = 0.754\ in$
To resist the crushing, the needed key length	$L := \dfrac{SF}{S_y} \cdot \dfrac{F}{\dfrac{H}{2}} = 0.87\ in$

A 1/4" by 1/4" square key with minimum length of 0.87", made of 1020 CD steel should should suffice the design performance requirement.