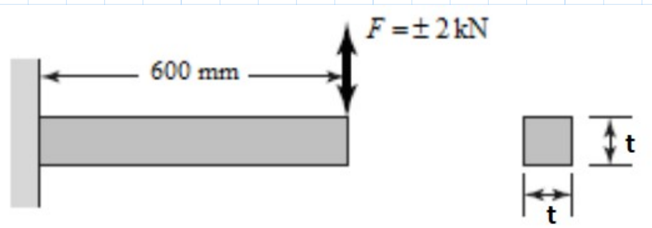
HW04 (Fall 2021)

Question 01 (30 points)

A solid square rod is cantilevered at one end. The rod is 0.6 m long and supports a completely reversing transverse load at the other end of ±2 kN. The material is AISI 1080 hot-rolled steel. If the rod must support this load for 10,000 cycles with a design factor of ~1.5, what dimension should the square cross section have? Since the size is not yet known, assume a typical value of k_b = 0.85 and verify its correctness later. Neglect any stress concentrations at the support end.

AISI 1080 HR Steel, Table A-20 $S_{ut} := 770 \cdot MPa$



$L := 0.6 \cdot m$ $F := 2000 \cdot N$

Safety Factor $n := 1.5$

Fatigue Cycles $N := 10000$

Uncorrected Endurance Strength $S_e := 0.5 \cdot S_{ut} = 385\ MPa$

Surface Condition $k_a := 57.7 \cdot \left(\frac{S_{ut}}{MPa}\right)^{-0.718} = 0.488$

Size Effect $k_b := 0.85$

Load Effect $k_c := 1$

Temperature Effect $k_d := 1$

Reliability Effect $k_e := 1$

Corrected Endurance Strength $S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_e = 159.792\ MPa$

Figure 6-18 shows Fatigue Strength Fraction $f := 0.83$

Fatigue Life Constants: $a := \frac{(f \cdot S_{ut})^2}{S_e} = 2556.13\ MPa$ $b := \frac{-1}{3} \log\left(\frac{f \cdot S_{ut}}{S_e}\right) = -0.2007$

$S_f := a \cdot N^b = 402.622\ MPa$

Max Bending Moment on rod $M_{max} := F \cdot L = 1200\ N \cdot m$

Rod X-Sec Area Moment of Inertia $I := \frac{t^4}{12}$

Fluctuating bending stress on rod $\sigma_b := \frac{M_{max} \cdot \frac{t}{2}}{I}$ $\sigma_b := \frac{M_{max} \cdot 6}{t^3}$

Limiting bending stress on rod $\sigma_b := \frac{S_f}{n} = 268.415\ MPa$

Solving rod dimension $t := \left(\frac{M_{max} \cdot 6}{\sigma_b}\right)^{\frac{1}{3}}$ $t = 29.935\ mm$

Round up the dimension $t := 30\ mm$

SInce the initial size factor was estimated, it can be updated now: Effective Diameter $d_e := 0.808 \cdot t = 24.24\ mm$

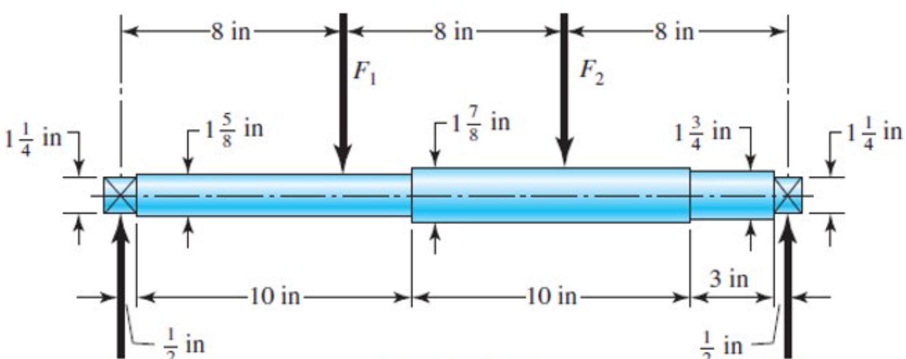
Size Effect $k_b := \left(\frac{d_e}{7.62 \cdot mm}\right)^{-0.107}$ $k_b = 0.884$

Initial guess of 0.85 was slighly conservative.

Question 02 (35 points)

The shaft shown in the figure is machined from AISI 1040 CD steel. The shaft rotates at 1600 rpm and is supported in rolling bearings at A and B. The applied forces are F1 = 2500 lbf and F2 = 1000 lbf. Radius of all fillets is 1/16 in R.

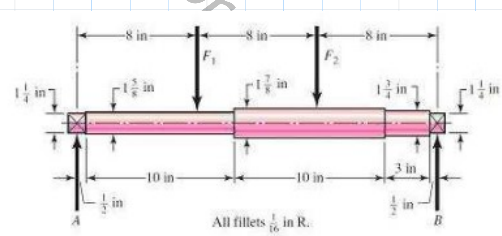
Determine the minimum fatigue factor of safety based on achieving infinite life. If infinite life is not predicted, estimate the number of cycles to failure. Also check for yielding.

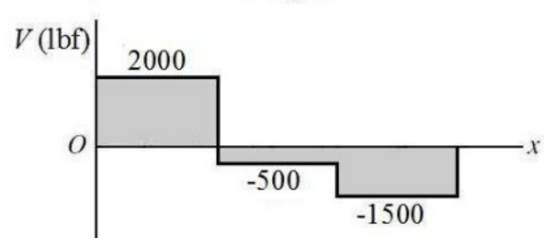


All fillets $\frac{1}{16}$ in R.

For 1040 CD Steel: (Table A-20) Yield Strenght $S_y := 71 \cdot ksi$ Ultimate Strength $S_{ut} := 85 \cdot ksi$

The shear force and bending moment diagram:





Critical point is at the shoulder fillet where BM equals to 14,750 in-lbf (highlighted with yellow marker).

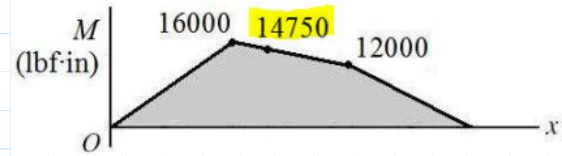
$BM := 14750 \cdot in \cdot lbf$

Shaft Diameter, Small $d := 1.625 \cdot in$

Shaft Diameter, Large $D := 1.875 \cdot in$

Section Modulus: $I := \frac{\pi \cdot d^4}{} = 0.342\ in^4$

The fully-reversed, bending stress of the rotating shaft at this location is:

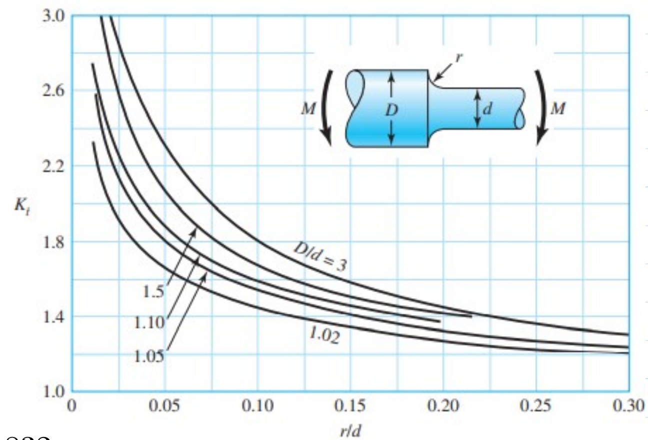


$$\sigma_b := \frac{BM \cdot d}{2 \cdot I} = 35.013\ ksi$$

Since bending stress is lower than Sy; therefore, risk of yielding should be low.

Fillet Rad r: $r := \frac{1}{16} \cdot in$ $\quad rd := \frac{r}{d} = 0.038$ $\quad Dd := \frac{D}{d} = 1.154$

From Table A-15-9 $\quad K_t := 1.95$



Notch Sensitivity Factor $\quad q := 0.76$ $\quad$ Table 6-20

Fatigue Stress Concentration:

$$K_f := 1 + q \cdot (K_t - 1) = 1.722$$

Uncorrected Endurance Strength $\quad S_e := 0.5 \cdot S_{ut} = 42.5\ ksi$

Surface Condition $\quad k_a := 2.7 \cdot \left(\frac{S_{ut}}{ksi}\right)^{-0.265} = 0.832$

Size Effect $\quad d_e := d = 1.625\ in$

$$k_b := \left(\frac{d_e}{0.3 \cdot in}\right)^{-0.107} = 0.835$$

Load Effect $\quad k_c := 1$

Temperature Effect $\quad k_d := 1$

Reliability Effect $\quad k_e := 1$

Corrected Endurance Strength $\quad S_e := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_e = 29.509\ ksi$

Safety Factor $\quad n_s := \frac{S_e}{K_f \cdot \sigma_b} = 0.489$

Safety factor of fatigue is below 1; therefore, life is not infinite.

Use S-N diagram to estimate the life:

Fatigue strength fraction f: $\quad f := 0.867$

$$a := \frac{(f \cdot S_{ut})^2}{S_e} = 184.05\ ksi \qquad b := \frac{-1}{3} \log\left(\frac{f \cdot S_{ut}}{S_e}\right) = -0.1325$$

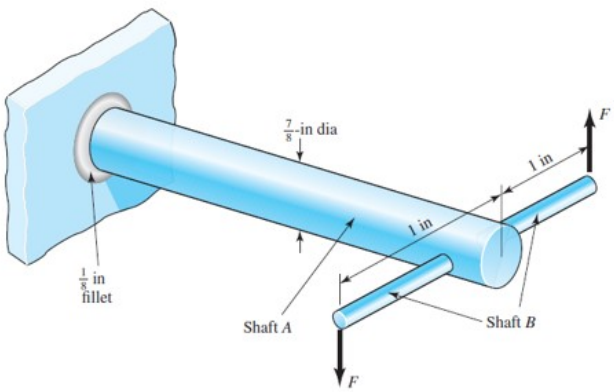
$$N := \left(\frac{K_f \cdot \sigma_b}{a}\right)^{\frac{1}{b}} = 4549$$

Estimated fatigue life is 4549 cycles.

Question 03 (35 points)

Shaft A, made of AISI 1020 hot-rolled steel, is welded to a fixed support and is subjected to loading by equal and opposite forces **F** via shaft B. A theoretical stress concentration factor K_{ts} of 1.6 is induced in the shaft by the 1/8-in weld fillet. The length of shaft A from the fixed support to the connection at shaft B is 2 ft. The load F cycles from 150 to 500 lbf.

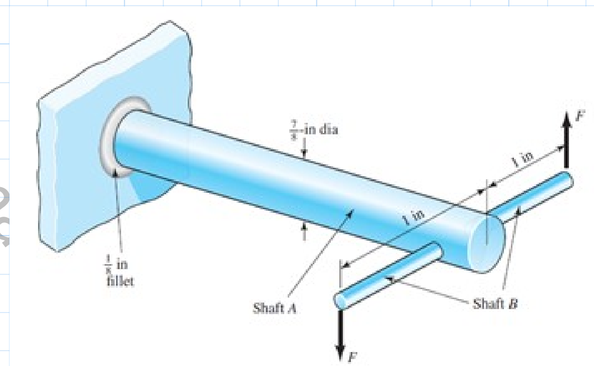
a. For shaft A, find the factor of safety for infinite life using the modified Goodman fatigue failure criterion.
b. Repeat part (a) using the Gerber fatigue failure criterion.



For 1020 HR Steel: (Table A-20)

Yield Strenght $S_y := 30 \cdot ksi$

Ultimate Strength $S_{ut} := 55 \cdot ksi$



Shaft Diameter $d := \frac{7}{8} \cdot in = 0.875\ in$

Handle Length $L := 1 \cdot in$

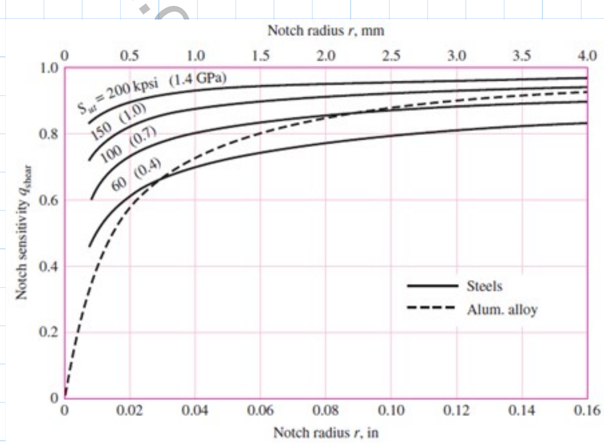
Shaft Polar Moment of Inertia $J := \frac{\pi \cdot d^4}{32} = 0.058\ in^4$

Stress Concentration @Fillet $K_{ts} := 1.6$

Notch Sensitivity Factor $q_s := 0.8$ Table 6-21

Max Torque $T_{max} := 2 \cdot 500 \cdot lbf \cdot L = 1000\ in \cdot lbf$

Min Torque $T_{min} := 2 \cdot 150 \cdot lbf \cdot L = 300\ in \cdot lbf$



$$a := \left(0.190 - 0.00251 \cdot \frac{S_{ut}}{ksi} + 1.35 \cdot 10^{-5} \cdot \left(\frac{S_{ut}}{ksi}\right)^2 - 2.67 \cdot 10^{-8} \cdot \left(\frac{S_{ut}}{ksi}\right)^3\right)^2 = 0.008$$

Fatigue Stress Concentration: $K_{fs} := 1 + \frac{K_{ts} - 1}{\ldots} = 1.443$

$$1+\sqrt{\frac{a \cdot in}{r}}$$

Shear Stress, Max, @OD $\tau_{max} := K_{fs} \cdot \frac{T_{max} \cdot d}{2 \cdot J} = 10.973\ ksi$

Shear Stress, Min, @OD $\tau_{min} := K_{fs} \cdot \frac{T_{min} \cdot d}{2 \cdot J} = 3.292\ ksi$

Mean Shear Stress $\tau_m := \frac{\tau_{max} + \tau_{min}}{2} = 7.132\ ksi$

Alternating Shear Stress $\tau_a := \frac{\tau_{max} - \tau_{min}}{2} = 3.84\ ksi$

Correction Factors

Uncorrected Endurance Strength $S_e := 0.5 \cdot S_{ut} = 27.5\ ksi$

Surface Condition $k_a := 14.4 \cdot \left(\frac{S_{ut}}{ksi}\right)^{-0.718} = 0.811$ (Hot-Rolled Surface)

Size Effect $d_e := 0.37 \cdot d = 0.324\ in$

$k_b := \left(\frac{d_e}{0.3 \cdot in}\right)^{-0.107} = 0.992$ (Non-Rotating)

Load Effect $k_c := 0.59$ (Torsional Loading)

Temperature Effect $k_d := 1$

Reliability Effect $k_e := 1$

Corrected Endurance Strength $S_{se} := k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot S_e = 13.044\ ksi$

Equation 6-54 gives $S_{su} := 0.67 \cdot S_{ut} = 36.85\ ksi$

Safety Factor per Modified Goodman: $n_s := \frac{1}{\left(\frac{\tau_a}{S_{se}}\right) + \left(\frac{\tau_m}{S_{su}}\right)} = 2.049$

Safety Factor per Gerber $n_s := \frac{1}{2} \left(\frac{S_{su}}{\tau_m}\right)^2 \cdot \frac{\tau_a}{S_{se}} \left(-1 + \sqrt{1 + \left(\frac{2 \cdot \tau_m \cdot S_{se}}{S_{su} \cdot \tau_a}\right)^2}\right) = 2.562$

Also checking against yielding per Maximum Shear Stress Theory: $n_s := \frac{S_y}{2 \cdot \tau_{max}} = 1.367$

All Safety Factors are greater than 1. Risk for either fatigue failure or static failure should be low.