HW02 (Fall 2021)

Question 01 (30 points)

The outer diameter of a solid aluminum shaft is in the range of 2.003-2.006 inch. Its mating hub is made of 18-8 stainless steel and has the inner diameter in the range of 2.000-2.002 inch and 3-inch outer diameter. Elastic constants of the two materials can be found in Table A–5 in the textbook.

a. Identify the range of radial interference δ, then
b. find the maximum interference pressure P, and the
c. radial and hoop stresses on both parts at the fit surface under the given P.

$\delta_{max} := \dfrac{2.006-2}{2} \cdot in = 0.003\ in$ $\qquad \delta_{min} := \dfrac{2.003-2.002}{2} \cdot in = 0.0005\ in$

$d_i := 0 \cdot in$ $\qquad d_o := 3 \cdot in$ $\qquad d := 2 \cdot in$

$a := \dfrac{d_i}{2} = 0\ in$ $\qquad c := \dfrac{d_o}{2} = 1.5\ in$ $\qquad R := \dfrac{d}{2} = 1\ in$

$E_o := 27600 \cdot ksi$ $\qquad \nu_o := 0.305$ $\qquad E_i := 10400 \cdot ksi$ $\qquad \nu_i := 0.333$

Max Interfacial Contact Pressure:

$\delta := \delta_{max} = 0.003\ in$

$$P := \frac{\delta}{R \cdot \left(\dfrac{1}{E_o} \left(\dfrac{c^2+R^2}{c^2-R^2} + \nu_o \right) + \dfrac{1}{E_i} \left(\dfrac{R^2+a^2}{R^2-a^2} - \nu_i \right) \right)} = 17.711\ ksi$$

$P = 17.711\ ksi$

Critical Stress @Interface (Shaft) $\qquad r := R = 1\ in$

$r_o := R = 1\ in$ $\qquad r_i := a = 0\ in$ $\qquad p_i := 0 \cdot ksi$ $\qquad p_o := P = 17.711\ ksi$

$r := 0.1 \cdot in$

$$\sigma_r := \frac{p_i \cdot r_i^{\,2} - p_o \cdot r_o^{\,2} + \left(\dfrac{r_o \cdot r_i}{r} \right)^2 (p_o - p_i)}{r_o^{\,2} - r_i^{\,2}} = -17.711\ ksi$$

$$\sigma_\theta := \frac{p_i \cdot r_i^{\,2} - p_o \cdot r_o^{\,2} - \left(\dfrac{r_o \cdot r_i}{r} \right)^2 (p_o - p_i)}{r_o^{\,2} - r_i^{\,2}} = -17.711\ ksi$$

(Hoop Stress; Shaft Side)

Critical Stress @Interface (Hub) $\qquad r := R = 1\ in$

$r_o := c = 1.5\ in$ $\qquad r_i := R = 1\ in$ $\qquad p_i := P = 17.711\ ksi$ $\qquad p_o := 0 \cdot ksi$

$$\sigma_r := \frac{p_i \cdot r_i^{\,2} - p_o \cdot r_o^{\,2} + \left(\dfrac{r_o \cdot r_i}{r} \right)^2 (p_o - p_i)}{r_o^{\,2} - r_i^{\,2}} = -17.711\ ksi$$

$$\sigma_\theta := \frac{p_i \cdot r_i^2 - p_o \cdot r_o^2 - \left(\frac{r_o \cdot r_i}{r}\right)^2 (p_o - p_i)}{r_o^2 - r_i^2} = 46.048\ ksi$$

(Hoop Stress; Hub Side)

Question 02 (20 points)

Following Question 01 and calculate the followings at the interference-fit interface:

a. safety factor of the hub ID per MSS failure criteria,
b. safety factor of the hub ID per DET failure criteria, and
c. the guaranteed torque capacity limit of the fit assuming 1.25 in fit length and COF of 0.2.

Also make an assessment whether the parts will fail or not.
Yield strength of the 18-8 steel is 50 ksi.

$\sigma_1 := \sigma_\theta = 46.048\ ksi$ $\sigma_2 := 0\ ksi$ $\sigma_3 := \sigma_r = -17.711\ ksi$

Yield Strength of 18-8 Steel $S_y := 50 \cdot ksi$

Max Shear Stress $\tau_{max} := \frac{\sigma_1 - \sigma_3}{2} = 31.879\ ksi$

Factor of Safety per MSS $n_s := \frac{S_y}{2 \cdot \tau_{max}} = 0.784$

$$\sigma_{vm} := \frac{1}{\sqrt{2}} \left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right)^{0.5} = 57.006\ ksi$$

Safety Factor Per DET $n_s := \frac{S_y}{\sigma_{vm}} = 0.877$

Safety factor less than 1. This interference fit design is unsafe.

Contact Length $L := 1.25 \cdot in$

COF @Interface $\mu := 0.2$

Total Contact Force $F := \mu \cdot (\pi \cdot 2 \cdot R \cdot L) \cdot P = (2.782 \cdot 10^4)\ lbf$

Transmitted Torque Capacity $TRQ := F \cdot R = 2318.3\ ft \cdot lbf$

Question 03 (50 points)

Your mission is to make a force-fit design of a 150-mm-diameter steel shaft with a 300-mm-outside-diameter hub. The hub is 25 mm long. The designed system is intended to operate under 150degC environment.

Both hub and shaft are made of 1050 CD steel. Moduli of elasticity is 207 GPa and Poisson's ratio is 0.3.

Coefficient of friction of steel-on-steel is 0.20.

Design the fit per ANSI B4-2-1978.

a. Specify the range of the shaft outer diameter.
b. Specify the range of the hub inner diameter.
c. Calculate max and min interference.
d. How will you mark the dimensions of shaft OD and hub ID on the drawing?
e. Safety factor of the hub ID surface per DET failure criteria.
f. Guaranteed capacity for torque transmission.

$d_i := 0 \cdot mm$ $\quad d_o := 300 \cdot mm$ $\quad d := 150 \cdot mm$ $\quad L := 25 \cdot mm$

$a := \frac{d_i}{2} = 0\ mm$ $\quad c := \frac{d_o}{2} = 150\ mm$ $\quad R := \frac{d}{2} = 75\ mm$

$E_o := 207 \cdot GPa$ $\quad \nu_o := 0.3$ $\quad E_i := 207 \cdot GPa$ $\quad \nu_i := 0.3$

Fit Type: Force-Fit Basic Diameter $D := d = 150\ mm$

Force fit: suitable for parts that can be highly stressed or for shrink fits where the heavy pressing forces required are impractical H7/u6

Table A-11

A Selection of International Tolerance Grades—Metric Series (Size Ranges Are for *Over* the Lower Limit and *Including* the Upper Limit. All Values Are in Millimeters)

Source: Preferred Metric Limits and Fits, ANSI B4.2-1978. See also BSI 4500.

Basic Sizes	Tolerance Grades IT6	IT7	IT8	IT9	IT10	IT11
0–3	0.006	0.010	0.014	0.025	0.040	0.060
3–6	0.008	0.012	0.018	0.030	0.048	0.075
6–10	0.009	0.015	0.022	0.036	0.058	0.090
10–18	0.011	0.018	0.027	0.043	0.070	0.110
18–30	0.013	0.021	0.033	0.052	0.084	0.130
30–50	0.016	0.025	0.039	0.062	0.100	0.160
50–80	0.019	0.030	0.046	0.074	0.120	0.190
80–120	0.022	0.035	0.054	0.087	0.140	0.220
120–180	0.025	0.040	0.063	0.100	0.160	0.250
180–250	0.029	0.046	0.072	0.115	0.185	0.290
250–315	0.032	0.052	0.081	0.130	0.210	0.320
315–400	0.036	0.057	0.089	0.140	0.230	0.360

Table A-11 gives the Tolerance garde of H7: $\Delta D := 0.04\ mm$

For u6: $\Delta d := 0.025 \cdot mm$

Hub ID, Max: $D_{max} := D + \Delta D = 150.04\ mm$

Hub ID, Min: $D_{min} := D = 150\ mm$

For shaft with u6 fit, its Fundamental Deviation from Table A-12: $\delta_F := 0.190 \cdot mm$

Basic Sizes	Upper-Deviation Letter c	d	f	g	h	Lower-Deviation Letter k	n	p	s	u
0–3	−0.060	−0.020	−0.006	−0.002	0	0	+0.004	+0.006	+0.014	+0.018
3–6	−0.070	−0.030	−0.010	−0.004	0	+0.001	+0.008	+0.012	+0.019	+0.023
6–10	−0.080	−0.040	−0.013	−0.005	0	+0.001	+0.010	+0.015	+0.023	+0.028
10–14	−0.095	−0.050	−0.016	−0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
14–18	−0.095	−0.050	−0.016	−0.006	0	+0.001	+0.012	+0.018	+0.028	+0.033
18–24	−0.110	−0.065	−0.020	−0.007	0	+0.002	+0.015	+0.022	+0.035	+0.041
24–30	−0.110	−0.065	−0.020	−0.007	0	+0.002	+0.015	+0.022	+0.035	+0.048
30–40	−0.120	−0.080	−0.025	−0.009	0	+0.002	+0.017	+0.026	+0.043	+0.060
40–50	−0.130	−0.080	−0.025	−0.009	0	+0.002	+0.017	+0.026	+0.043	+0.070
50–65	−0.140	−0.100	−0.030	−0.010	0	+0.002	+0.020	+0.032	+0.053	+0.087
65–80	−0.150	−0.100	−0.030	−0.010	0	+0.002	+0.020	+0.032	+0.059	+0.102
80–100	−0.170	−0.120	−0.036	−0.012	0	+0.003	+0.023	+0.037	+0.071	+0.124
100–120	−0.180	−0.120	−0.036	−0.012	0	+0.003	+0.023	+0.037	+0.079	+0.144
120–140	−0.200	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.092	+0.170
140–160	−0.210	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.100	+0.190
160–180	−0.230	−0.145	−0.043	−0.014	0	+0.003	+0.027	+0.043	+0.108	+0.210
180–200	−0.240	−0.170	−0.050	−0.015	0	+0.004	+0.031	+0.050	+0.122	+0.236

Shaft OD, Min: $d_{min} := D + \delta_F = 150.19\ mm$

Shaft OD, Max: $d_{max} := D + \delta_F + \Delta d = 150.215\ mm$

Range of Interference: $\delta_{min} := d_{min} - D_{max} = 0.15\ mm$ $\delta_{max} := d_{max} - D_{min} = 0.215\ mm$

Unilateral tolerance is commonly used for dimensional specification in interference-fit design.

So for shaft OD: 150.19 (+0.025/-0.000) mm

For hub ID: 150.040 (+0.000/-0.040) mm

Calculate Max Interfacial Contact Pressure

Worst Case of Contact Pressure $\delta := \frac{\delta_{max}}{2} = 0.1075\ mm$ $R := \frac{D}{2} = 75\ mm$

$$P := \frac{\delta}{R \cdot \left(\frac{1}{E_o}\left(\frac{c^2 + R^2}{c^2 - R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2 + a^2}{R^2 - a^2} - \nu_i\right)\right)} = 111.262\ MPa$$

Failure Assessment

Yield Strenght of 1050 CD Steel $S_y := 580 \cdot MPa$

Critical Stress @Hub ID Interface $r_o := c = 150\ mm$

$\sigma_r := -P = -111.26\ MPa$ $\sigma_\theta := P \cdot \frac{{r_o}^2 + R^2}{{r_o}^2 - R^2} = 185.44\ MPa$

$\sigma_1 := \sigma_\theta = 185.437\ MPa$ $\sigma_2 := 0\ MPa$ $\sigma_3 := \sigma_r = -111.262\ MPa$

Max Shear Stress $\tau_{max} := \frac{\sigma_1 - \sigma_3}{2} = 148.35\ MPa$

$$\tau_{max} = \frac{\ }{2}$$

Factor of Safety per MSS

$$n_s := \frac{S_y}{2 \cdot \tau_{max}} = 1.955$$

$$\sigma_{vm} := \frac{1}{\sqrt{2}} \left(\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 \right)^{0.5} = 259.612\ MPa$$

Safety Factor Per DET

$$n_s := \frac{S_y}{\sigma_{vm}} = 2.234$$

Both safety factors are ~2, the risk of design failure is low.

Calculate Min Transmitted Torque

Minimum Contact Pressure

$$\delta := \frac{\delta_{min}}{2} = 0.075\ mm$$

$$P := \frac{\delta}{R \cdot \left(\frac{1}{E_o} \left(\frac{c^2 + R^2}{c^2 - R^2} + \nu_o \right) + \frac{1}{E_i} \left(\frac{R^2 + a^2}{R^2 - a^2} - \nu_i \right) \right)} = 77.625\ MPa$$

Axial Length of Interface: $L = 25\ mm$

Friction Coefficient: $\mu := 0.2$

Total Friction Force: $F_r := \mu \cdot \left(\pi \cdot 2 \cdot R \cdot L\right) \cdot P = 182899.6\ N$

Resisting Torque: $Trq := R \cdot F_r = 13717.5\ N \cdot m$