APPENDIX A

Fluid Property Data

A.1 Specific Gravity

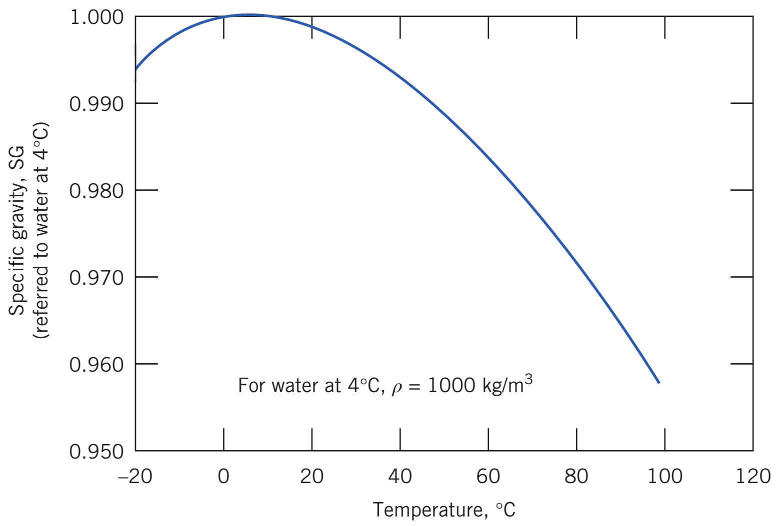
Specific gravity data for several common liquids and solids are presented in Figs. A.1*a* and A.1*b* and in Tables A.1 and A.2. For liquids specific gravity is a function of temperature. (Density data for water and air are given as functions of temperature in Tables A.7 through A.10.) For most liquids specific gravity decreases as temperature increases. Water is unique: It displays a maximum density of 1000 $\mathrm{kg/m^3}$ at 4°C. The maximum density of water is used as a reference value to calculate specific gravity. Thus

$$\mathrm{SG} \equiv \frac{\rho}{\rho_{\mathrm{H_2O}}\,(\text{at } 4^\circ\mathrm{C})}$$

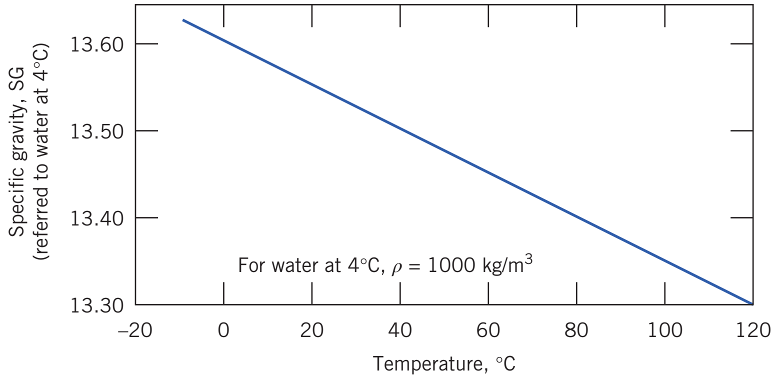
Consequently the maximum SG of water is exactly unity.

Specific gravities for solids are relatively insensitive to temperature; values given in Table A.1 were measured at 20°C.

The specific gravity of seawater depends on both its temperature and salinity. A representative value for ocean water is $\mathrm{SG} = 1.025$, as given in Table A.2.



(a) Water



(b) Mercury

Fig. A.1 Specific gravity of water and mercury as functions of temperature. (Data from Reference [1].) (The specific gravity of mercury varies linearly with temperature. The variation is given by $SG = 13.60 - 0.00240\, T$ when T is measured in °C.)

Table A.1

Specific Gravities of Selected Engineering Materials

(a) Common Manometer Liquids at 20°C	
Liquid	Specific Gravity
E.V. Hill blue oil	0.797
Meriam red oil	0.827
Benzene	0.879
Dibutyl phthalate	1.04
Monochloronaphthalene	1.20
Carbon tetrachloride	1.595
Bromoethylbenzene (Meriam blue)	1.75
Tetrabromoethane	2.95
Mercury	13.55

Source: Data from References [1–3].

Table A.1

Specific Gravities of Selected Engineering Materials (*Continued*)

(b) Common Materials

Material	Specific Gravity (—)
Aluminum	2.64
Balsa wood	0.14
Brass	8.55
Cast Iron	7.08
Concrete (cured)	2.4[a]
Concrete (liquid)	2.5[a]
Copper	8.91
Ice (0°C)	0.917
Lead	11.4
Oak	0.77
Steel	7.83
Styrofoam ($1\,\mathrm{kg/m^3}$)	0.0160
Styrofoam ($3\,\mathrm{kg/m^3}$)	0.0481
Uranium (depleted)	18.7
White pine	0.43

Source: Data from Reference [4].

[a] depending on aggregate.

Table A.2

Physical Properties of Common Liquids at 20°C

Liquid	Isentropic Bulk Modulus[a] (GN/$\mathrm{m^2}$)	Specific Gravity (—)
Benzene	1.48	0.879
Carbon tetrachloride	1.36	1.595
Castor oil	2.11	0.969
Crude oil	—	0.82–0.92
Ethanol	—	0.789
Gasoline	—	0.72
Glycerin	4.59	1.26
Heptane	0.886	0.684
Kerosene	1.43	0.82
Lubricating oil	1.44	0.88
Methanol	—	0.796
Mercury	28.5	13.55
Octane	0.963	0.702
Seawater[b]	2.42	1.025
SAE 10W oil	—	0.92
Water	2.24	0.998

Source: Data from References [1, 5, 6].

[a] Calculated from speed of sound; $1\,\mathrm{GN/m^2} = 10^9\,\mathrm{N/m^2}$.

[b] Dynamic viscosity of seawater at 20°C is $\mu = 1.08 \times 10^{-3}\,\mathrm{N \cdot s/m^2}$. (Thus, the kinematic viscosity of seawater is about 5 percent higher than that of freshwater.)

Table A.3

Properties of the U.S. Standard Atmosphere

Geometric Altitude (m)	Temperature (K)	p/p_{SL} (—)	ρ/ρ_{SL}(—)
−500	291.4	1.061	1.049
0	288.2	1.000[a]	1.000[b]
500	284.9	0.9421	0.9529
1,000	281.7	0.8870	0.9075
1,500	278.4	0.8345	0.8638
2,000	275.2	0.7846	0.8217
2,500	271.9	0.7372	0.7812
3,000	268.7	0.6920	0.7423
3,500	265.4	0.6492	0.7048
4,000	262.2	0.6085	0.6689
4,500	258.9	0.5700	0.6343
5,000	255.7	0.5334	0.6012
6,000	249.2	0.4660	0.5389
7,000	242.7	0.4057	0.4817
8,000	236.2	0.3519	0.4292
9,000	229.7	0.3040	0.3813
10,000	223.3	0.2615	0.3376
11,000	216.8	0.2240	0.2978
12,000	216.7	0.1915	0.2546
13,000	216.7	0.1636	0.2176
14,000	216.7	0.1399	0.1860
15,000	216.7	0.1195	0.1590
16,000	216.7	0.1022	0.1359
17,000	216.7	0.08734	0.1162
18,000	216.7	0.07466	0.09930
19,000	216.7	0.06383	0.08489
20,000	216.7	0.05457	0.07258
22,000	218.6	0.03995	0.05266
24,000	220.6	0.02933	0.03832
26,000	222.5	0.02160	0.02797
28,000	224.5	0.01595	0.02047
30,000	226.5	0.01181	0.01503
40,000	250.4	0.002834	0.003262
50,000	270.7	0.0007874	0.0008383
60,000	255.8	0.0002217	0.0002497
70,000	219.7	0.00005448	0.00007146
80,000	180.7	0.00001023	0.00001632
90,000	180.7	0.000001622	0.000002588

Source: Data from Reference [7].

[a] $p_{SL} = 1.01325 \times 10^5\ \mathrm{N/m^2}$ (abs).

[b] $\rho_{SL} = 1.2250\ \mathrm{kg/m^3}$.

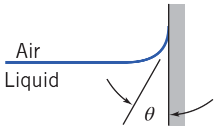
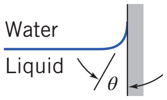
A.2 *Surface Tension*

The values of surface tension, σ, for most organic compounds are remarkably similar at room temperature; the typical range is 25 to 40 mN/m. Water is higher, at about 73 mN/m at 20°C. Liquid metals have values between 300 and 600 mN/m; mercury has a value of about 480 mN/m at 20°C. Surface tension decreases with temperature; the decrease is nearly linear with absolute temperature. Surface tension at the critical temperature is zero.

Values of σ are usually reported for surfaces in contact with the pure vapor of the liquid being studied or with air. At low pressures both values are about the same.

Table A.4

Surface Tension of Common Liquids at 20°C

Liquid	Surface Tension, σ (mN/m)[a]	Contact Angle, θ (degrees)
(a) In contact with air		
Benzene	28.9	
Carbon tetrachloride	27.0	
Ethanol	22.3	
Glycerin	63.0	
Hexane	18.4	
Kerosene	26.8	
Lube oil	25–35	
Mercury	484	140
Methanol	22.6	
Octane	21.8	
Water	72.8	~0
Source: Data from References [1, 5, 8, 9].		
(b) In contact with water		
Benzene	35.0	
Carbon tetrachloride	45.0	
Hexane	51.1	
Mercury	375	140
Methanol	22.7	
Octane	50.8	

Source: Data from References [1, 5, 8, 9].

[a] $1\ \text{mN/m} = 10^{-3}\ \text{N/m}$.

A.3 *The Physical Nature of Viscosity*

Viscosity is a measure of internal fluid friction, i.e., resistance to deformation. The mechanism of gas viscosity is reasonably well understood, but the theory is poorly developed for liquids. We can gain some insight into the physical nature of viscous flow by discussing these mechanisms briefly.

The viscosity of a Newtonian fluid is fixed by the state of the material. Thus $\mu = \mu(T, p)$. Temperature is the more important variable, so let us consider it first. Excellent empirical equations for viscosity as a function of temperature are available.

Effect of Temperature on Viscosity

a. Gases

All gas molecules are in continuous random motion. When there is bulk motion due to flow, the bulk motion is superimposed on the random motions. It is then distributed throughout the fluid by molecular collisions. Analyses based on kinetic theory predict

$$\mu \propto \sqrt{T}$$

The kinetic theory prediction is in fair agreement with experimental trends, but the constant of proportionality and one or more correction factors must be determined; this limits practical application of this simple equation.

If two or more experimental points are available, the data may be correlated using the empirical Sutherland correlation [7]

$$\mu = \frac{bT^{1/2}}{1 + S/T} \tag{A.1}$$

Constants b and S may be determined most simply by writing

$$\mu = \frac{bT^{3/2}}{S + T}$$

or

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{b}\right) T + \frac{S}{b}$$

(Compare this with $y = mx + c$.) From a plot of $T^{3/2}/\mu$ versus T, one obtains the slope, $1/b$, and the intercept, S/b. For air,

$$b = 1.458 \times 10^{-6} \frac{\text{kg}}{\text{m} \cdot \text{s} \cdot \text{K}^{1/2}}$$

$$S = 110.4\ \text{K}$$

These constants were used with Eq. A.1 to compute viscosities for the standard atmosphere in [7], the air viscosity values at various temperatures shown in Table A.10, and using appropriate conversion factors, the values shown in Table A.9.

b. Liquids

Viscosities for liquids cannot be estimated well theoretically. The phenomenon of momentum transfer by molecular collisions is overshadowed in liquids by the effects of interacting force fields among the closely packed liquid molecules.

Liquid viscosities are affected drastically by temperature. This dependence on absolute temperature may be represented by the empirical equation

$$\mu = Ae^{B/(T-C)} \tag{A.2}$$

or the equivalent form

$$\mu = A10^{B/(T-C)} \tag{A.3}$$

where T is absolute temperature.

Equation A.3 requires at least three points to fit constants A, B, and C. In theory it is possible to determine the constants from measurements of viscosity at just three temperatures. It is better practice to use more data and to obtain the constants from a statistical fit to the data.

However a curve-fit is developed, always compare the resulting line or curve with the available data. The best way is to critically inspect a plot of the curve-fit compared with the data. In general, curve-fit results will be satisfactory only when the quality of the available data and that of the empirical relation are known to be excellent.

Data for the dynamic viscosity of water are fitted well using constant values $A = 2.414 \times 10^{-5}\ \text{N} \cdot \text{s/m}^2$, $B = 247.8\ \text{K}$, and $C = 140\ \text{K}$. Reference [10] states that using these constants in Eq. A.3 predicts water viscosity within ± 2.5 percent over the temperature range from 0°C to 370°C. Equation A.3 and *Excel* were used to compute the water viscosity values at various temperatures shown in Table A.8, and using appropriate conversion factors, the values shown in Table A.7.

Note that the viscosity of a liquid decreases with temperature, while that of a gas increases with temperature.

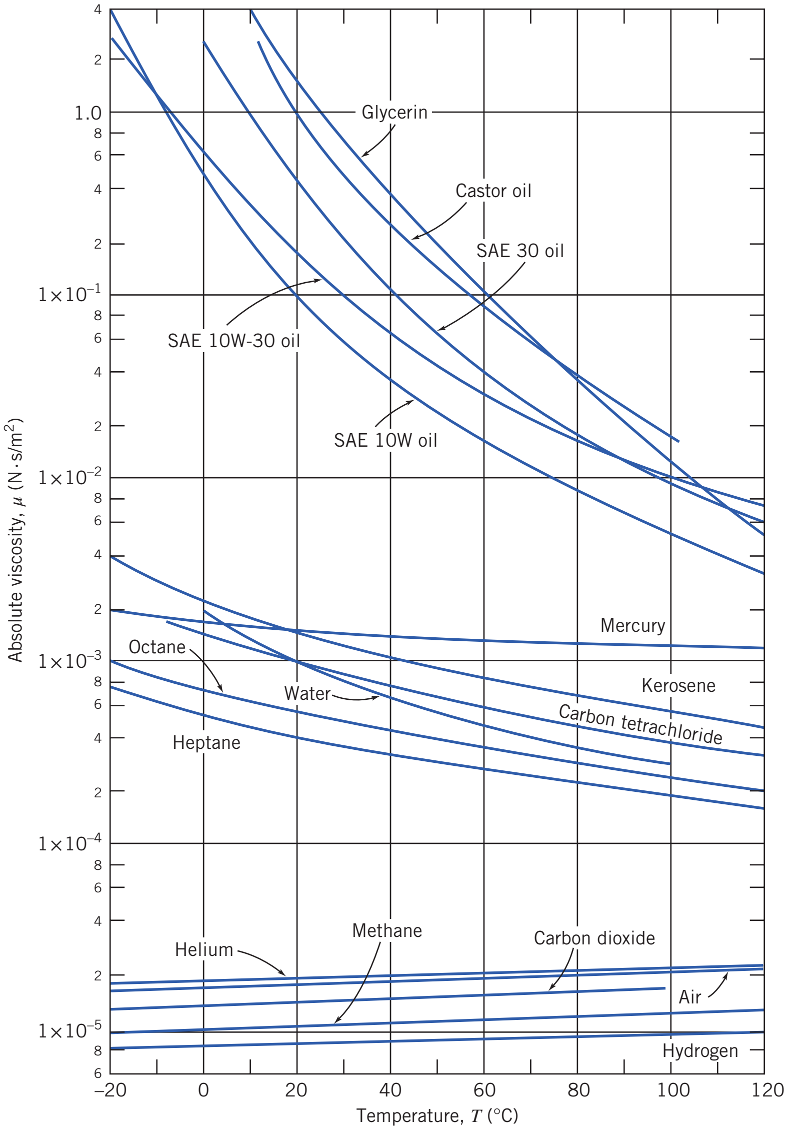


Fig. A.2 Dynamic (absolute) viscosity of common fluids as a function of temperature. (Data from References [1, 6, and 10].)

The graphs for air and water were computed from the *Excel* workbook *Absolute Viscosities*, using Eq. A.1 and Eq. A.3, respectively. The workbook can be used to compute viscosities of other fluids if constants b and S (for a gas) or A, B, and C (for a liquid) are known.

Effect of Pressure on Viscosity

a. Gases

The viscosity of gases is essentially independent of pressure between a few hundredths of an atmosphere and a few atmospheres. However, viscosity at high pressures increases with pressure (or density).

b. Liquids

The viscosities of most liquids are not affected by moderate pressures, but large increases have been found at very high pressures. For example, the viscosity of water at 10,000 atm is twice that at

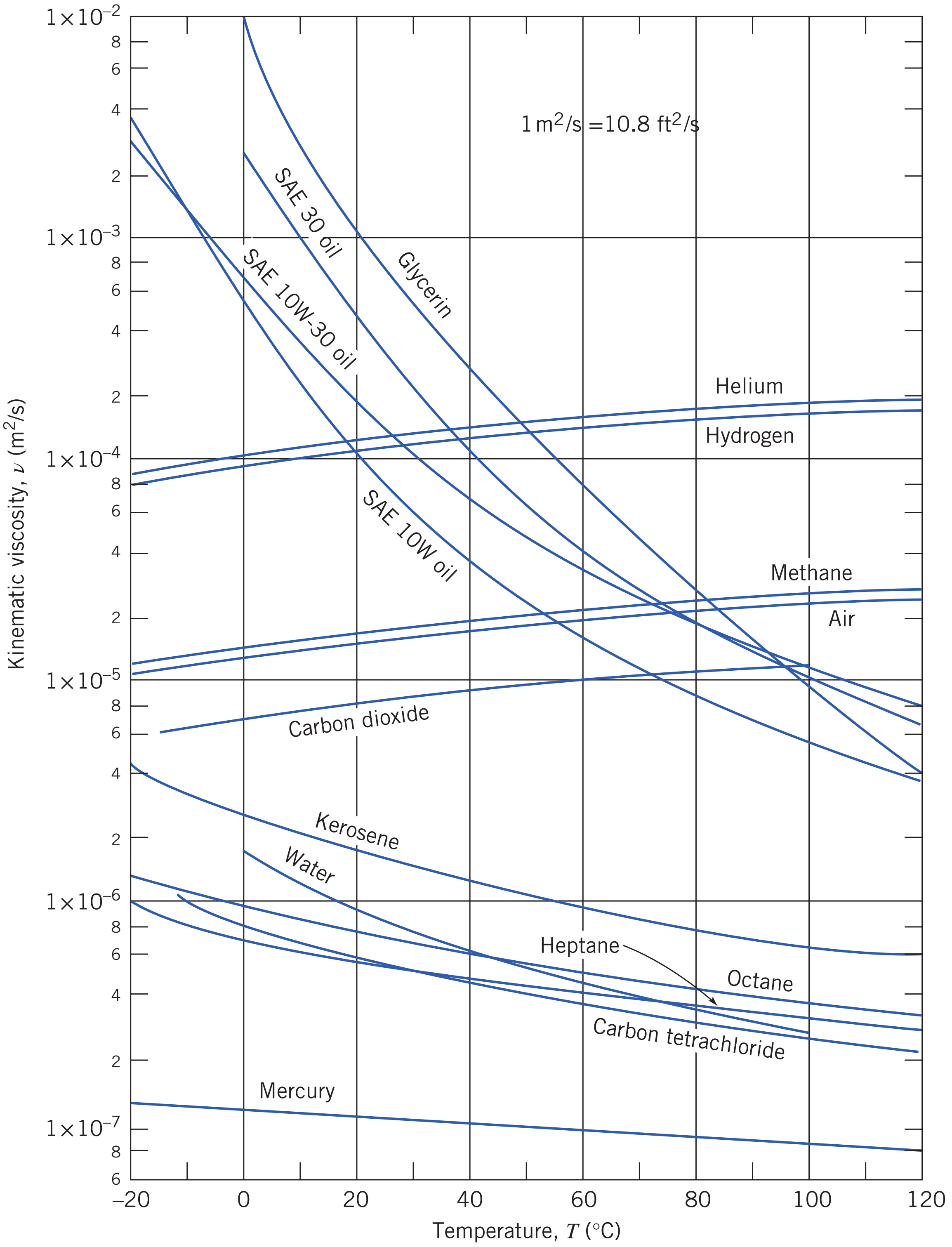


Fig. A.3 Kinematic viscosity of common fluids (at atmospheric pressure) as a function of temperature. (Data from References [1, 6, and 10].)

1 atm. More complex compounds show a viscosity increase of several orders of magnitude over the same pressure range.

More information may be found in Reid and Sherwood [11].

A.4 *Lubricating Oils*

Engine and transmission lubricating oils are classified by viscosity according to standards established by the Society of Automotive Engineers [12]. The allowable viscosity ranges for several grades are given in Table A.5.

Viscosity numbers with W (e.g., 20W) are classified by viscosity at −18°C. Those without W are classified by viscosity at 99°C.

Multigrade oils (e.g., 10W-40) are formulated to minimize viscosity variation with temperature. High polymer "viscosity index improvers" are used in blending these multigrade oils. Such additives are highly non-Newtonian; they may suffer permanent viscosity loss caused by shearing.

Special charts are available to estimate the viscosity of petroleum products as a function of temperature. The charts were used to develop the data for typical lubricating oils plotted in Figs. A.2 and A.3. For details, see [15].

Table A.5

Allowable Viscosity Ranges for Lubricants

Engine Oil	SAE Viscosity Grade	Max. Viscosity (cP)[a] at Temp. (°C)	Viscosity (cSt)[b] at 100°C	
			Min	Max
	0W	3250 at −30	3.8	—
	5W	3500 at −25	3.8	—
	10W	3500 at −20	4.1	—
	15W	3500 at −15	5.6	—
	20W	4500 at −10	5.6	—
	25W	6000 at −5	9.3	—
	20	—	5.6	<9.3
	30	—	9.3	<12.5
	40	—	12.5	<16.3
	50	—	16.3	<21.9

Axle and Manual Transmission Lubricant	SAE Viscosity Grade	Max. Temp. (°C) for Viscosity of 150,000 cP	Viscosity (cSt) at 100°C	
			Min	Max
	70W	−55	4.1	—
	75W	−40	4.1	—
	80W	−26	7.0	—
	85W	−12	11.0	—
	90	—	13.5	<24.0
	140	—	24.0	<41.0
	250	—	41.0	—

Automatic Transmission Fluid (Typical)	Maximum Viscosity (cP)	Temperature (°C)	Viscosity (cSt) at 100°C	
			Min	Max
	50000	−40	6.5	8.5
	4000	−23.3	6.5	8.5
	1700	−18	6.5	8.5

Source: Data from References [12–14].

[a] centipoise $= 1\ \mathrm{cP} = 1\ \mathrm{mPa \cdot s} = 10^{-3}\ \mathrm{Pa \cdot s}$.

[b] centistoke $= 10^{-6}\ \mathrm{m^2/s}$.

A.5 *Properties of Common Gases, Air, and Water*

Table A.6

Thermodynamic Properties of Common Gases at STP[a]

Gas	Chemical Symbol	Molecular Mass, M_m	R^b $\left(\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$	c_p $\left(\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$	c_v $\left(\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)$	$k=\frac{c_p}{c_v}$ $(-)$
Air	—	28.98	286.9	1004	717.4	1.40
Carbon dioxide	CO_2	44.01	188.9	840.4	651.4	1.29
Carbon monoxide	CO	28.01	296.8	1039	742.1	1.40
Helium	He	4.003	2077	5225	3147	1.66
Hydrogen	H_2	2.016	4124	14,180	10,060	1.41
Methane	CH_4	16.04	518.3	2190	1672	1.31
Nitrogen	N_2	28.01	296.8	1039	742.0	1.40
Oxygen	O_2	32.00	259.8	909.4	649.6	1.40
Steam[c]	H_2O	18.02	461.4	~2000	~1540	~1.30

Source: Data from References [7, 16, 17].

[a] STP = standard temperature and pressure, $T = 15^\circ\text{C}$ and $p = 101.325$ kPa (abs).

[b] $R \equiv R_u/M_m$; $R_u = 8314.3$ J/(kgmol·K); 1J = 1N · m.

[c] Water vapor behaves as an ideal gas when superheated by 55°C or more.

Table A.7

Properties of Water (SI Units)

Temperature, T (°C)	Density, ρ (kg/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)	Surface Tension, σ (N/m)	Vapor Pressure, p_v (kPa)	Bulk Modulus, E_v (GPa)
0	1000	1.76E-03	1.76E-06	0.0757	0.661	2.01
5	1000	1.51E-03	1.51E-06	0.0749	0.872	
10	1000	1.30E-03	1.30E-06	0.0742	1.23	
15	999	2.38E-05	1.23E-06	0.00504	0.247	
60	1.94	1.14E-03	1.14E-06	0.0735	1.71	
20	998	1.01E-03	1.01E-06	0.0727	2.34	2.21
25	997	8.93E-04	8.96E-07	0.0720	3.17	
30	996	8.00E-04	8.03E-07	0.0712	4.25	
35	994	7.21E-04	7.25E-07	0.0704	5.63	
40	992	6.53E-04	6.59E-07	0.0696	7.38	
45	990	5.95E-04	6.02E-07	0.0688	9.59	
50	988	5.46E-04	5.52E-07	0.0679	12.4	2.29
55	986	5.02E-04	5.09E-07	0.0671	15.8	
60	983	4.64E-04	4.72E-07	0.0662	19.9	
65	980	4.31E-04	4.40E-07	0.0654	25.0	
70	978	4.01E-04	4.10E-07	0.0645	31.2	
75	975	3.75E-04	3.85E-07	0.0636	38.6	
80	972	3.52E-04	3.62E-07	0.0627	47.4	
85	969	3.31E-04	3.41E-07	0.0618	57.8	
90	965	3.12E-04	3.23E-07	0.0608	70.1	2.12
95	962	2.95E-04	3.06E-07	0.0599	84.6	
100	958	2.79E-04	2.92E-07	0.0589	101	

Table A.8

Properties of Air at Atmospheric Pressure (SI Units)

Temperature, T (°C)	Density, ρ (kg/m^3)	Dynamic Viscosity, μ (N · s/m^2)	Kinematic Viscosity, ν (m^2/s)
0	1.29	1.72E-05	1.33E-05
5	1.27	1.74E-05	1.37E-05
10	1.25	1.76E-05	1.41E-05
15	1.23	1.79E-05	1.45E-05
20	1.21	1.81E-05	1.50E-05
25	1.19	1.84E-05	1.54E-05
30	1.17	1.86E-05	1.59E-05
35	1.15	1.88E-05	1.64E-05
40	1.13	1.91E-05	1.69E-05
45	1.11	1.93E-05	1.74E-05
50	1.09	1.95E-05	1.79E-05
55	1.08	1.98E-05	1.83E-05
60	1.06	2.00E-05	1.89E-05
65	1.04	2.02E-05	1.94E-05
70	1.03	2.04E-05	1.98E-04
75	1.01	2.06E-05	2.04E-05
80	1.00	2.09E-05	2.09E-05
85	0.987	2.11E-05	2.14E-05
90	0.973	2.13E-05	2.19E-05
95	0.960	2.15E-05	2.24E-05
100	0.947	2.17E-05	2.29E-05

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APPENDIX B

Videos for Fluid Mechanics

Referenced in the text are the following videos available at www.wiley.com/college/fox.

Chapter 2

Streamlines
Streaklines
Capillary Rise
Boundary Layer Flow
Streamlined Flow over an Airfoil
Internal Laminar Flow in a Tube
Streamlines around a Car
Laminar and Turbulent Flow

Chapter 4

Mass Conservation: Filling a Tank
Momentum Effect: A Jet Impacting a Surface

Chapter 5

An Example of Streamlines/Streaklines
Particle Motion in a Channel
Linear Deformation
Flow Past a Cylinder

Chapter 6

An Example of Irrotational Flow

Chapter 7

Geometric, Not Dynamic, Similarity: Flow Past a Block 1
Geometric, Not Dynamic, Similarity: Flow Past a Block 2

Chapter 8

The Reynolds Transition Experiment
Pipe Flow: Laminar
Pipe Flow: Transitional
The Glen Canyon Dam: A Turbulent Pipe Flow

Chapter 9

Flow around an Airfoil
Flow Separation on an Airfoil
Effect of Viscosity on Boundary Layer Growth
Examples of Boundary Layer Growth
Flow Separation: Airfoil
Flow about a Sports Car
Plate Normal to the Flow
An Object with a High Drag Coefficient
Examples of Flow around a Sphere
Vortex Trail behind a Cylinder
Flow Past an Airfoil ($\alpha = 0^\circ$)
Flow Past an Airfoil ($\alpha = 10^\circ$)
Flow Past an Airfoil ($\alpha = 20^\circ$)
Wing Tip Vortices
Leading Edge Slats

Chapter 10

Flow in an Axial Flow Compressor (Animation)

Chapter 11

A Laminar Hydraulic Jump

Chapter 12

Shock Waves due to a Projectile
Shock Waves over a Supersonic Airplane

The following videos were developed by the National Committee for Fluid Mechanics Films (NCFMF) and may be viewed at http://web.mit.edu/hml/ncfmf.html. Each of these videos goes into the subject in more depth than may be appropriate for an undergraduate class. However, selected segments of the videos are useful in bringing out important fluids phenomena.

These videos are supplied by:

Encyclopaedia Britannica Educational Corporation
331 North La Salle Street
Chicago, IL 60654

Aerodynamic Generation of Sound (44 min, principals: M. J. Lighthill, J. E. Ffowcs-Williams)
Cavitation (31 min, principal: P. Eisenberg)
Channel Flow of a Compressible Fluid (29 min, principal: D. E. Coles)
Deformation of Continuous Media (38 min, principal: J. L. Lumley)
Eulerian and Lagrangian Descriptions in Fluid Mechanics (27 min, principal: J. L. Lumley)
Flow Instabilities (27 min, principal: E. L. Mollo-Christensen)
Flow Visualization (31 min, principal: S. J. Kline)
The Fluid Dynamics of Drag (4 parts, 120 min, principal: A. H. Shapiro)
Fundamentals of Boundary Layers (24 min, principal: F. H. Abernathy)
Low-Reynolds-Number Flows (33 min, principal: Sir G. I. Taylor)
Magnetohydrodynamics (27 min, principal: J. A. Shercliff)
Pressure Fields and Fluid Acceleration (30 min, principal: A. H. Shapiro)
Rarefied Gas Dynamics (33 min, principals: F. C. Hurlbut, F. S. Sherman)
Rheological Behavior of Fluids (22 min, principal: H. Markovitz)
Rotating Flows (29 min, principal: D. Fultz)
Secondary Flow (30 min, principal: E. S. Taylor)
Stratified Flow (26 min, principal: R. R. Long)
Surface Tension in Fluid Mechanics (29 min, principal: L. M. Trefethen)
Turbulence (29 min, principal: R. W. Stewart)
Vorticity (2 parts, 44 min, principal: A. H. Shapiro)
Waves in Fluids (33 min, principal: A. E. Bryson)

Another source of fluid mechanics videos is a CD entitled “Multimedia Fluid Mechanics” by Homsy et al. It is available from Cambridge University Press, 32 Avenue of the Americas, New York, NY 10013-2473, ISBN 9780521721691. This CD contains a very large number of videos that illustrate different phenomena in fluid flow.

APPENDIX C

Selected Performance Curves for Pumps and Fans

C.1 Introduction

Many firms, worldwide, manufacture fluid machines in numerous standard types and sizes. Each manufacturer publishes complete performance data to allow application of its machines in systems. This appendix contains selected performance data for use in solving pump and fan system problems. Two pump types and one fan type are included.

Choice of a manufacturer may be based on established practice, location, or cost. Once a manufacturer is chosen, machine selection is a three-step process:

1. Select a machine type, suited to the application, from a manufacturer's full-line catalog, which gives the ranges of pressure rise (head) and flow rate for each machine type.
2. Choose an appropriate machine model and driver speed from a master selector chart, which superposes the head and flow rate ranges of a series of machines on one graph.
3. Verify that the candidate machine is satisfactory for the intended application, using a detailed performance curve for the specific machine.

It is wise to consult with experienced system engineers, either employed by the machine manufacturer or in your own organization, before making a final purchase decision.

Many manufacturers currently use computerized procedures to select a machine that is most suitable for each given application. Such procedures are simply automated versions of the traditional selection method. Use of the master selector chart and the detailed performance curves is illustrated below for pumps and fans, using data from one manufacturer of each type of machine. Literature of other manufacturers differs in detail but contains the necessary information for machine selection.

C.2 Pump Selection

Representative data are shown in Figs. C.1 through C.10 for Peerless[1] horizontal split case single-stage (series AE) pumps and in Figs. C.11 and C.12 for Peerless multistage (series TU and TUT) pumps.

Figures C.1 and C.2 are master pump selector charts for series AE pumps at 3500 and 1750 nominal rpm. On these charts, the model number (e.g., 6AE14) indicates the discharge line size (6 in. or about 150 mm nominal pipe), the pump series (AE), and the maximum impeller diameter (approximately 14 in. or 350 mm).

Figures C.3 through C.10 are detailed performance charts for individual pump models in the AE series.

Figures C.11 and C.12 are master pump selector charts for series TU and TUT pumps at 1750 nominal rpm. Data for two-stage pumps are presented in Fig. C.11, while Fig. C.12 contains data for pumps with three, four, and five stages.

Each pump performance chart contains curves of total head versus volume flow rate; curves for several impeller diameters—tested in the same casing—are presented on a single graph. Each performance chart also contains curves showing pump efficiency and driver power; the net positive suction head (*NPSH*) requirement, as it varies with flow rate, is shown by the curve at the bottom of each chart. The best efficiency point (BEP) for each impeller may be found using the efficiency curves.

Use of the master pump selector chart and detailed performance curves is illustrated in Example C.1.

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[1] Peerless Pump Company, P.O. Box 7026, Indianapolis, IN 46207-7026.

Example C.1 PUMP SELECTION PROCEDURE

Select a pump to deliver 400 m^3/h of water at 36 m total head. Choose the appropriate pump model and driver speed. Specify the pump efficiency, driver power, and *NPSH* requirement.

Given: Select a pump to deliver 400 m^3/h of water at 36 m total head.

Find:

(a) Pump model and driver speed.
(b) Pump efficiency.
(c) Driver power.
(d) *NPSH* requirement.

Solution: Use the pump selection procedure described in Section C.1. (The numbers below correspond to the numbered steps given in the procedure.)

1. Select a machine type suited to the application. (This step actually requires a manufacturer's full-line catalog, which is not reproduced here. The Peerless product line catalog specifies a maximum delivery and head of 570 m^3/h and 200 m for series AE pumps. Therefore the required performance can be obtained; assume the selection is to be made from this series.)
2. Consult the master pump selector chart. The desired operating point is not within any pump contour on the 3500 rpm selector chart (Fig. C.1). From the 1750 rpm chart (Fig. C.2), select a model 6AE14 pump. From the performance curve for the 6AE14 pump (Fig. C.6), choose a 325-mm impeller.
3. Verify the performance of the machine using the detailed performance chart. On the performance chart for the 6AE14 pump, project up from the abscissa at $Q = 400\ \mathrm{m/h}$. Project across from $H = 36$ m on the ordinate. The intersection is the pump performance at the desired operating point:

$$\eta \approx 85.8 \text{ percent} \qquad \mathcal{P} \approx 48\ \mathrm{kW}$$

From the operating point, project down to the *NPSH* requirement curve. At the intersection, read $NPSH \approx 5$ m.

This completes the selection process for this pump. One should consult with experienced system engineers to verify that the system operating condition has been predicted accurately and the pump has been selected correctly.

C.3 *Fan Selection*

Fan selection is similar to pump selection. A representative master fan selection chart is shown in Fig. C.13 for a series of Howden Buffalo[2] axial-flow fans. The chart shows the efficiency of the entire series of fans as a function of total pressure rise and flow rate. The series of numbers for each fan indicates the fan diameter in inches, the hub diameter in inches, and the fan speed in revolutions per minute. For instance, a 54-26-870 fan has a fan diameter of 54 in. or 1350 mm, a hub diameter of 26 in. or 650 mm, and should be operated at 870 rpm.

Normally, final evaluation of suitability of the fan model for the application would be done using detailed performance charts for the specific model. Instead, we use the efficiencies from Fig C.13, which are indicated by the shading of the different zones on the map. To calculate the power requirement for the fan motor, we use the following equation:

$$\mathcal{P}(\mathrm{kW}) = \frac{Q(\mathrm{m}^3/\mathrm{s}) \times \Delta p(\mathrm{mm}\cdot\mathrm{H_2O})}{\eta}$$

A sample fan selection is presented in Example C.2.

---

[2] Howden Buffalo Inc., 2029 W. DeKalb St., Camden, SC 29020.

Example C.2 FAN SELECTION PROCEDURE

Select an axial flow fan to deliver 850 m^3/min of standard air at 32 mm. H_2O total pressure. Choose the appropriate fan model and driver speed. Specify the fan efficiency and driver power.

Given: Select an axial flow fan to deliver 850 m^3/min of standard air at 32 mm H_2O total head.

Find:

(a) Fan size and driver speed.
(b) Fan efficiency.
(c) Driver power.

Solution: Use the fan selection procedure described in Section C.1. (The numbers below correspond to the numbered steps given in the procedure.)

1. Select a machine type suited to the application. (This step actually requires a manufacturer's full-line catalog, which is not reproduced here. Assume the fan selection is to be made from the axial fan data presented in Fig. C.13.)
2. Consult the master fan selector chart. The desired operating point is within the contour for the 48-21-860 fan on the selector chart (Fig. C.13). To achieve the desired performance requires driving the fan at 860 rpm.
3. Verify the performance of the machine using a detailed performance chart. To determine the efficiency, we consult Fig C.13 again. We estimate an efficiency of 85 percent. To determine the motor power requirement, we use the equation given above:

$$\mathscr{P} = \frac{Q\Delta p}{\eta} = 8.50\frac{\text{m}^3}{\text{min}} \times \frac{\text{min}}{60\ \text{s}} \times 32\ \text{mm H}_2\text{O} \times 999\frac{\text{kg}}{\text{m}^3} \times 9.81\frac{\text{m}}{\text{s}^2} \times \frac{1}{0.85} \times \frac{\text{m}}{1000\ \text{mm}} = 5.23\ \text{kW}$$

This completes the fan selection process. Again, one should consult with experienced system engineers to verify that the system operating condition has been predicted accurately and the fan has been selected correctly.

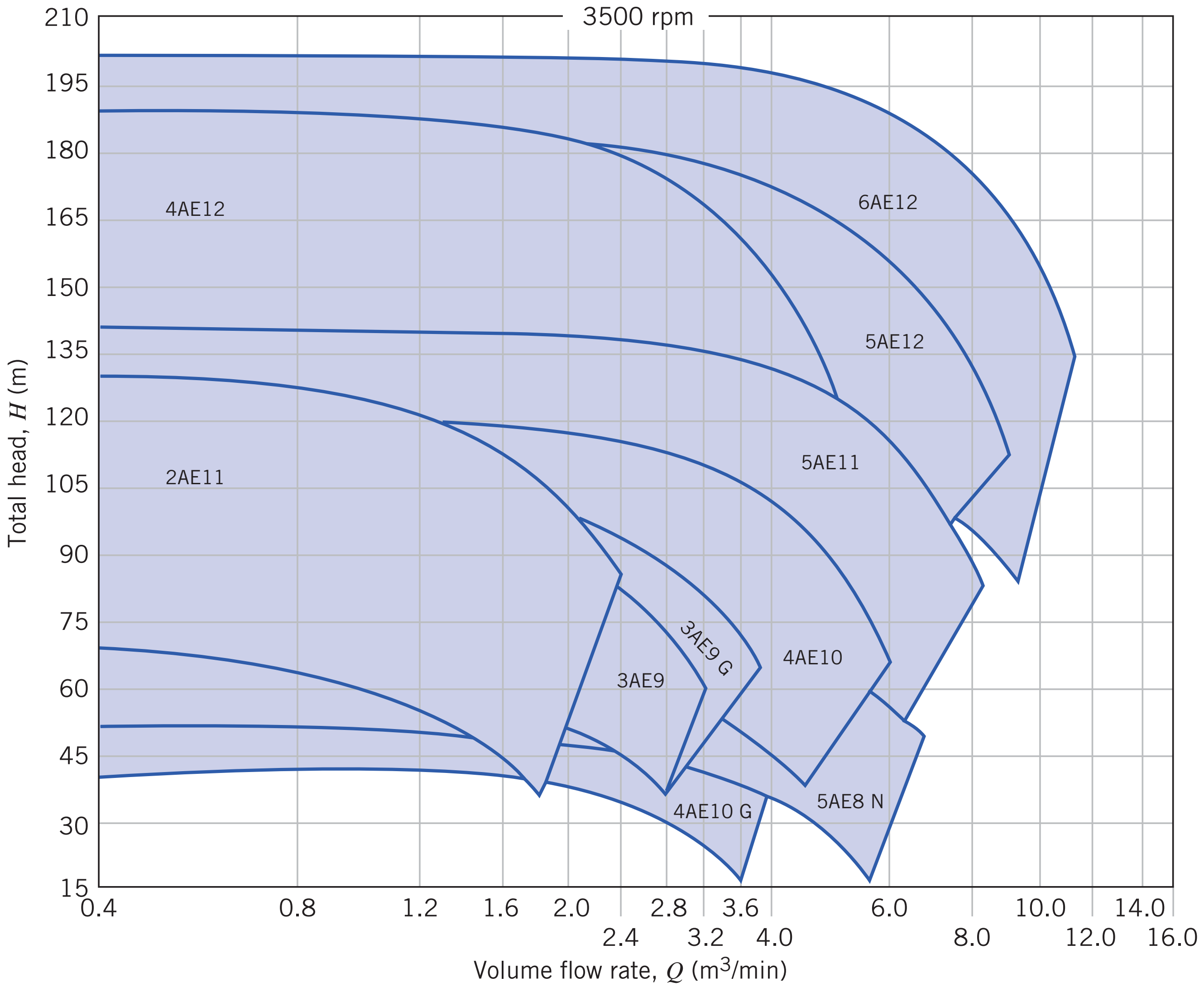

Fig. C.1 Selector chart for Peerless horizontal split case (series AE) pumps at 3500 nominal rpm.

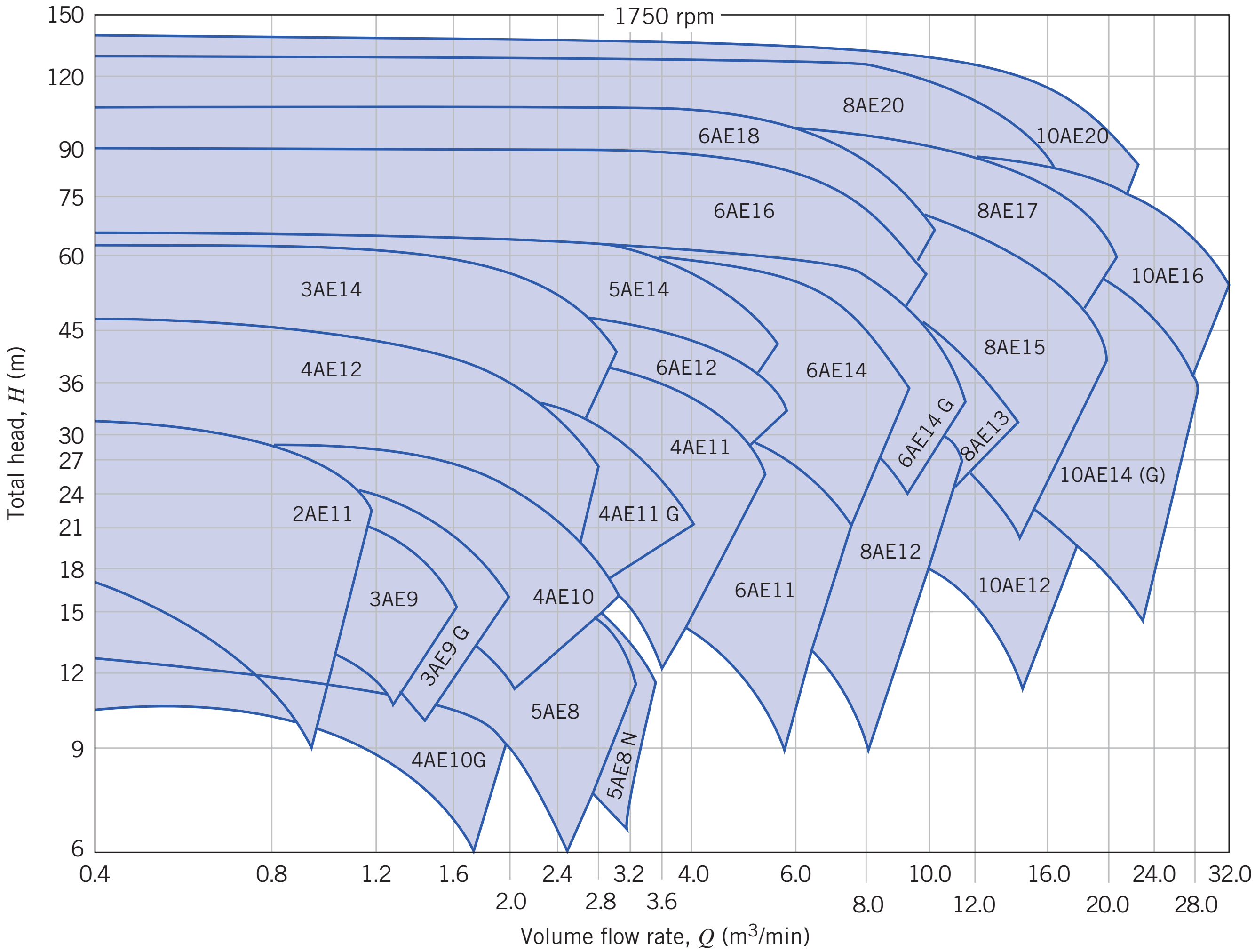

Fig. C.2 Selector chart for Peerless horizontal split case (series AE) pumps at 1750 nominal rpm.

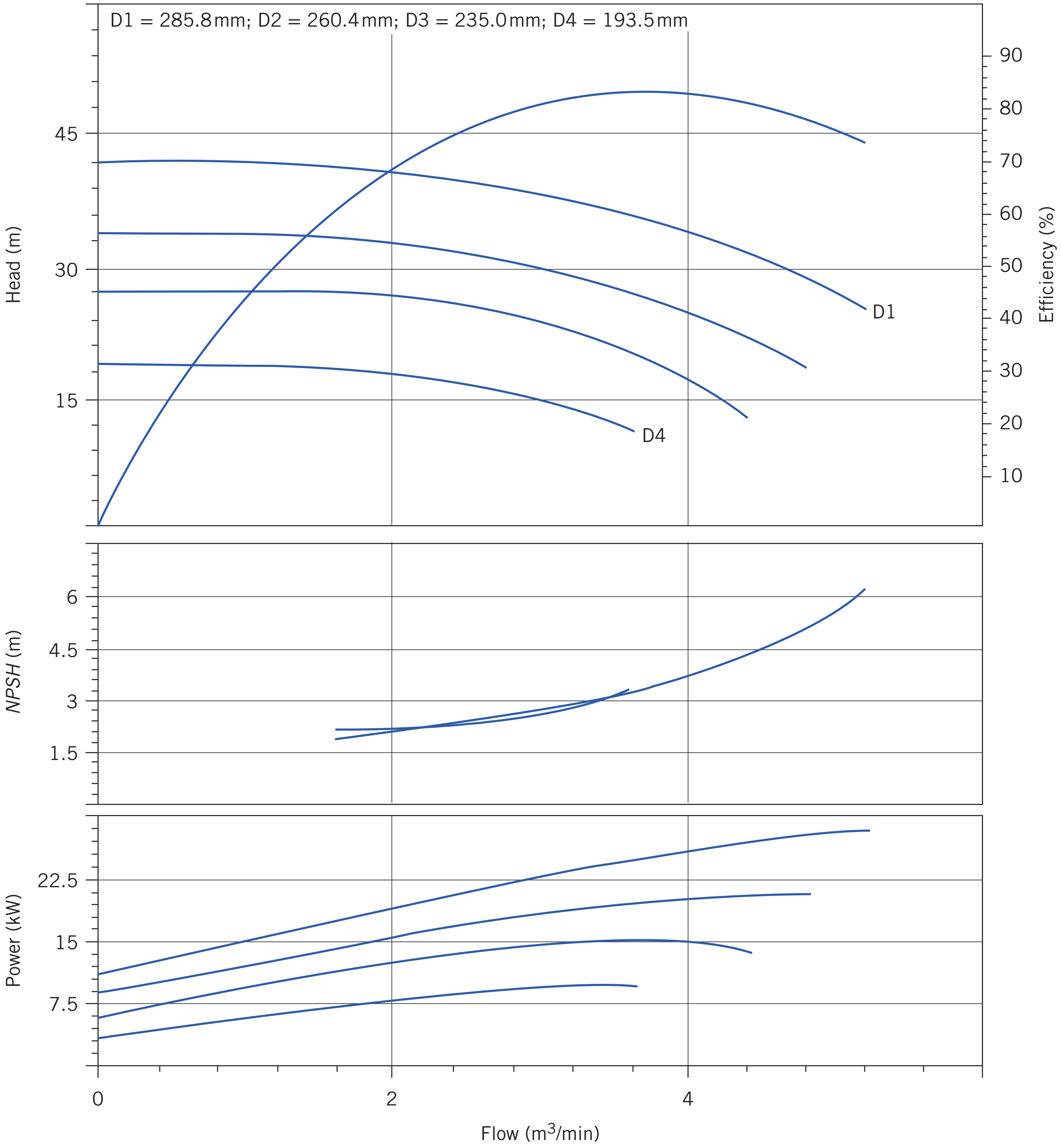

Fig. C.3 Performance curve for Peerless 4AE11 pump at 1750 rpm.

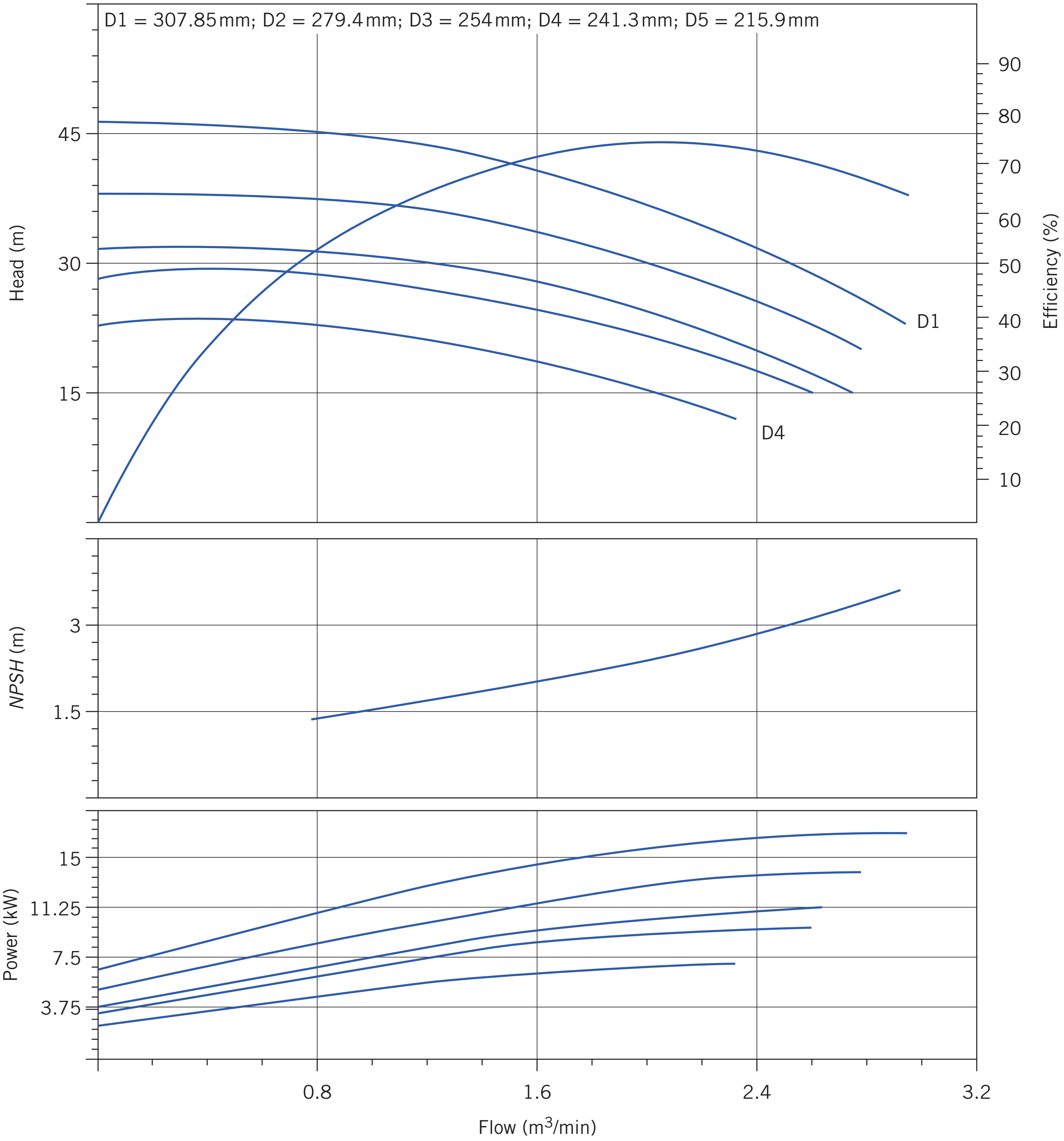

Fig. C.4 Performance curve for Peerless 4AE12 pump at 1750 rpm.

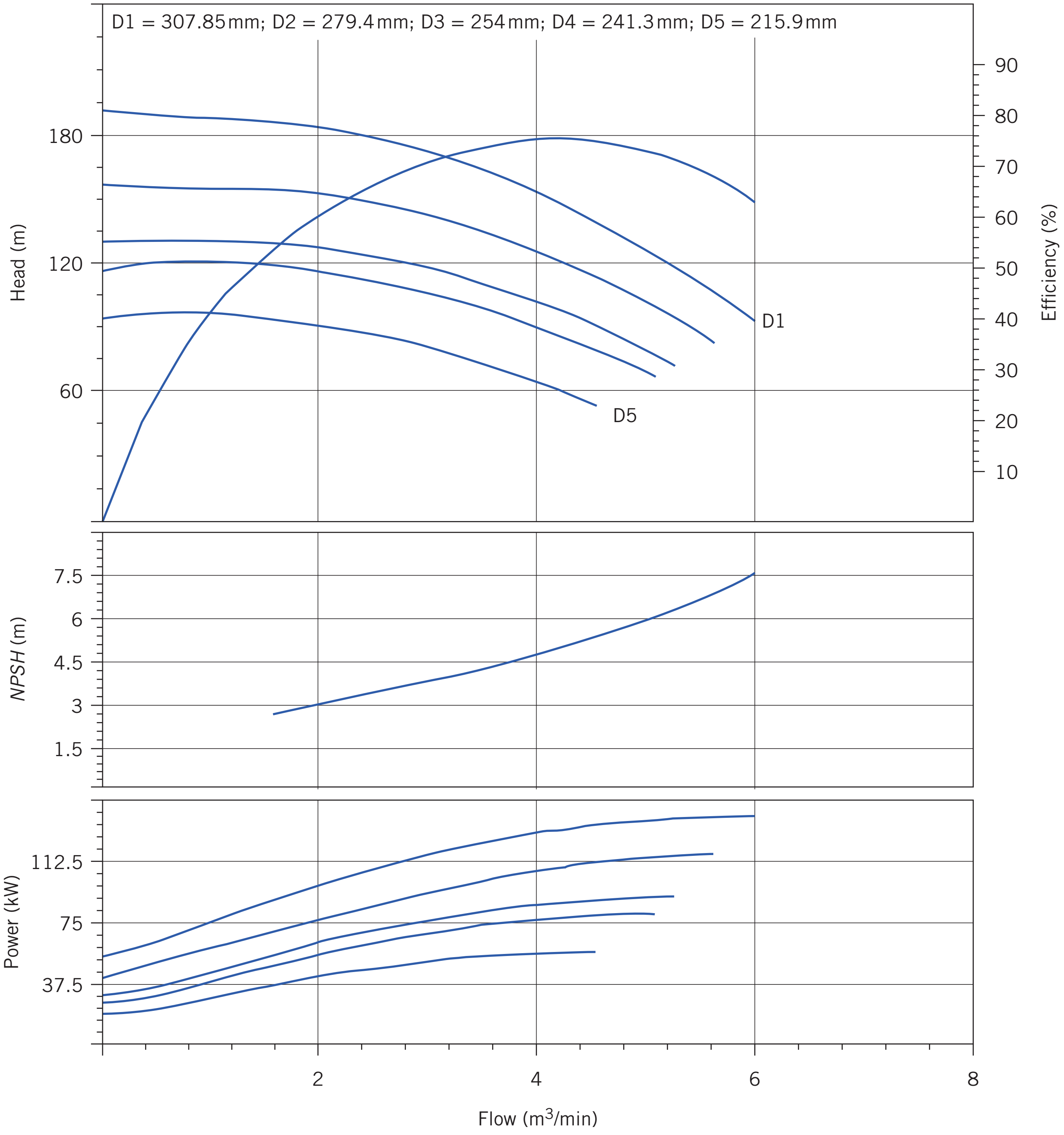

Fig. C.5 Performance curve for Peerless 4AE12 pump at 3550 rpm.

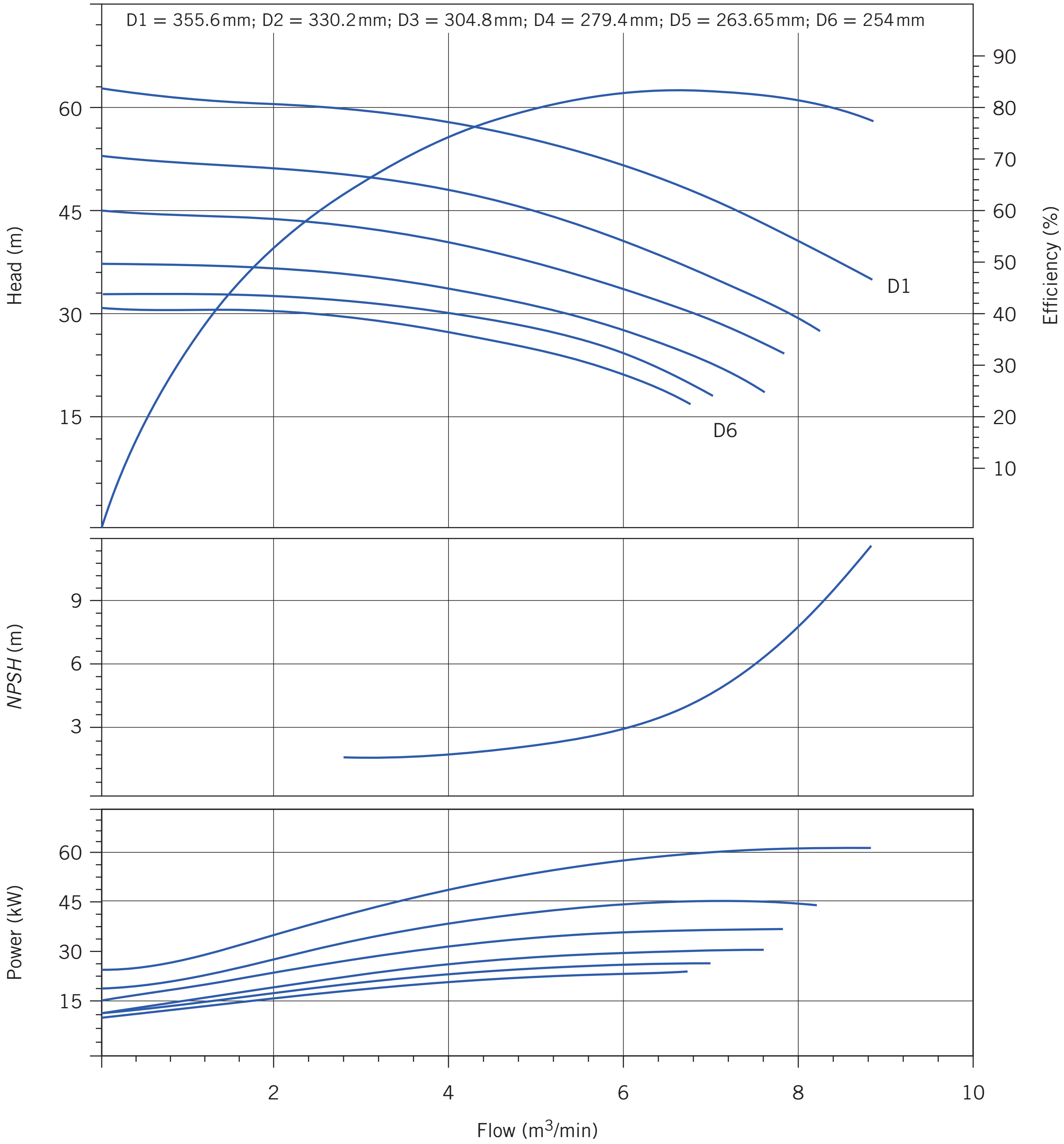

Fig. C.6 Performance curve for Peerless 6AE14 pump at 1750 rpm.

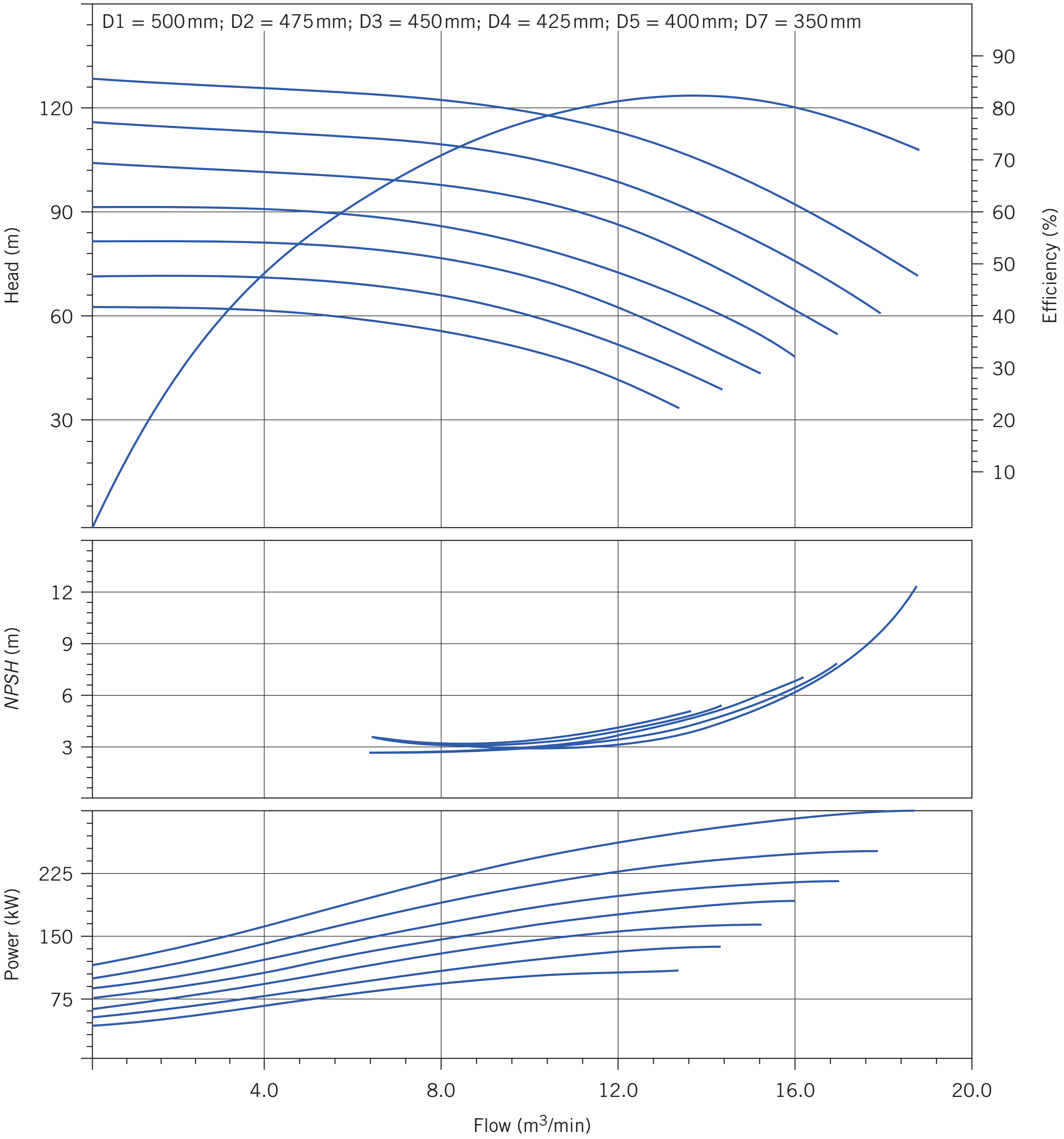

Fig. C.7 Performance curve for Peerless 8AE20G pump at 1770 rpm.

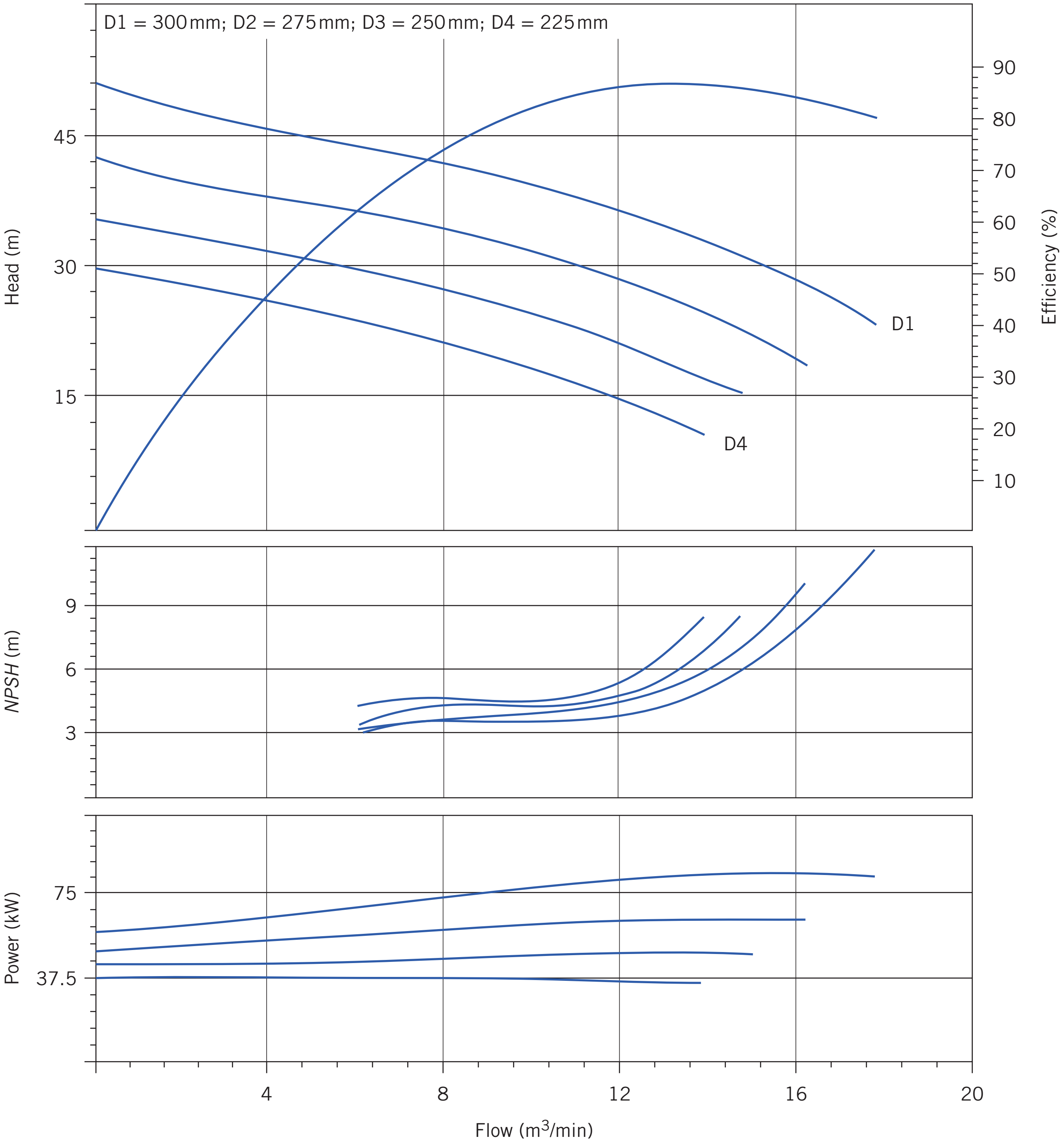

Fig. C.8 Performance curve for Peerless 10AE12 pump at 1760 rpm.

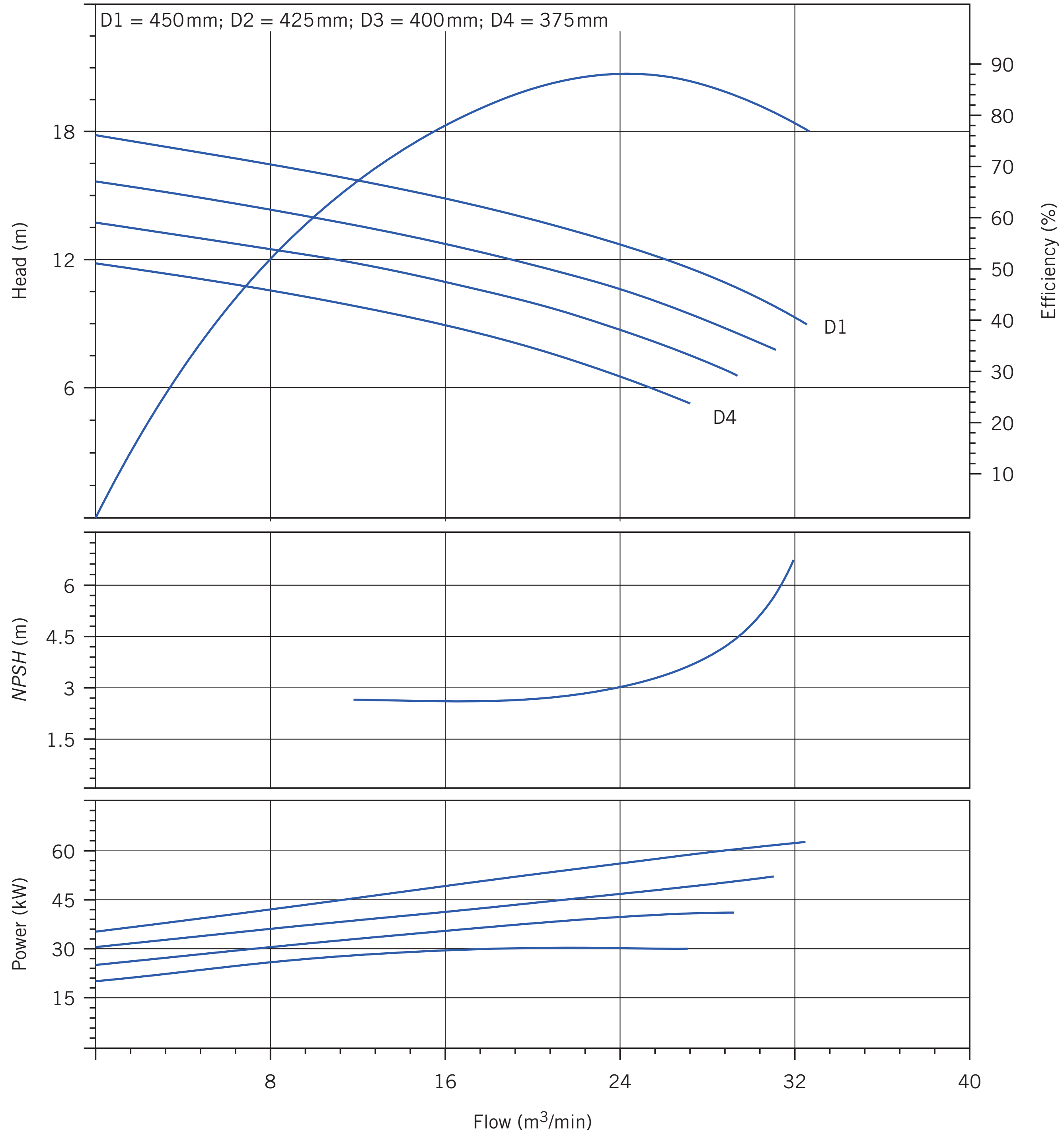

Fig. C.9 Performance curve for Peerless 16A18B pump at 705 rpm.

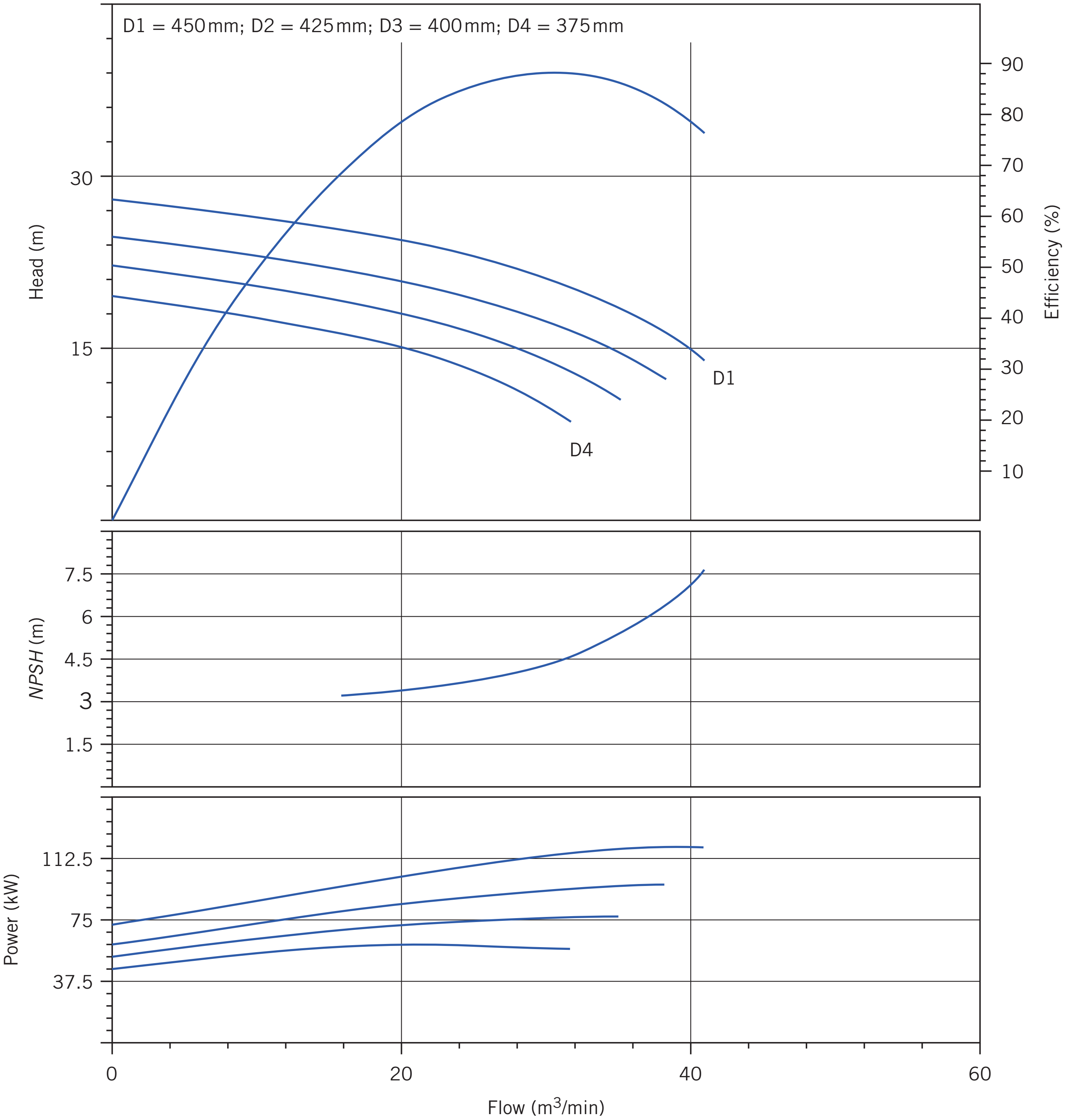

Fig. C.10 Performance curve for Peerless 16A18B pump at 880 rpm.

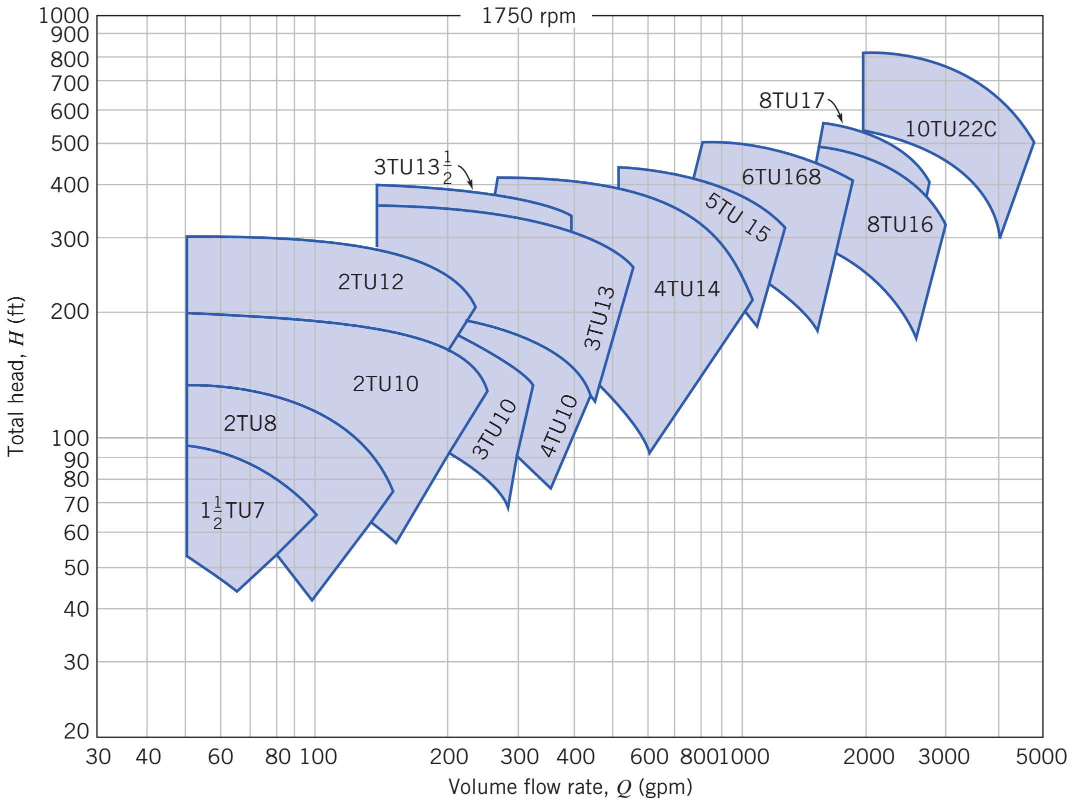

Fig. C.11 Selector chart for Peerless two-stage (series TU and TUT) pumps at 1750 nominal rpm.

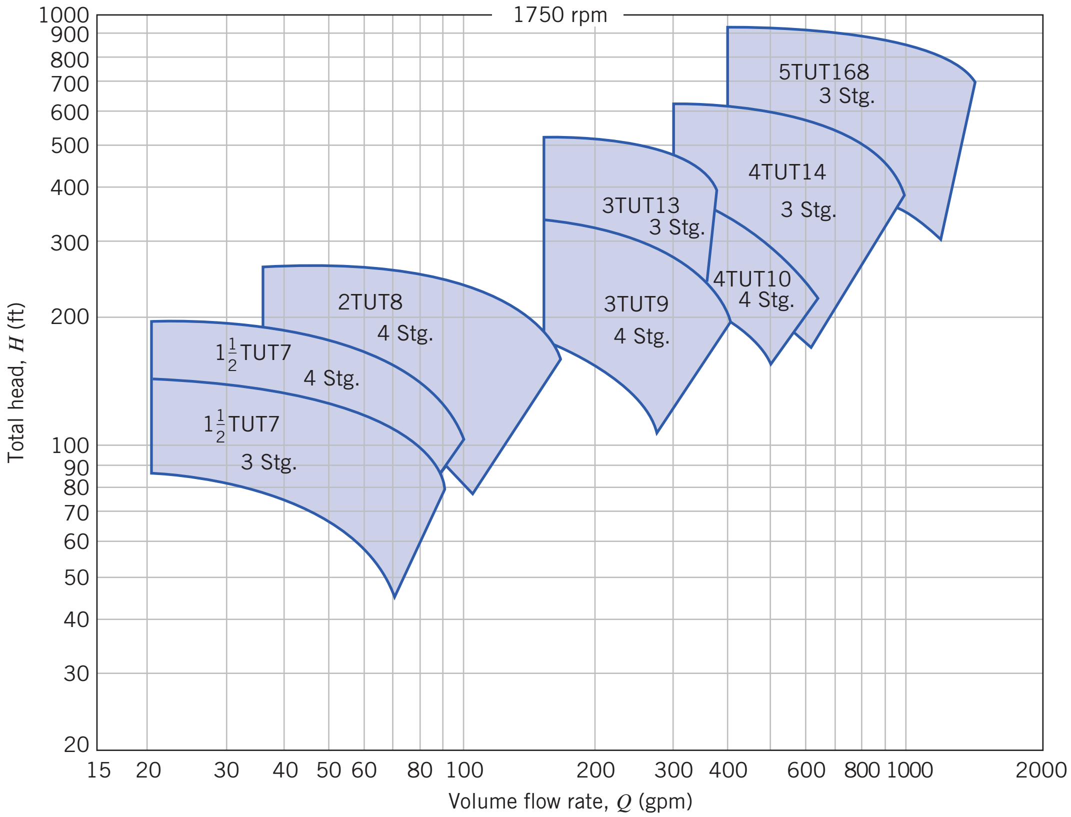

Fig. C.12 Selector chart for Peerless multistage (series TU and TUT) pumps at 1750 nominal rpm.

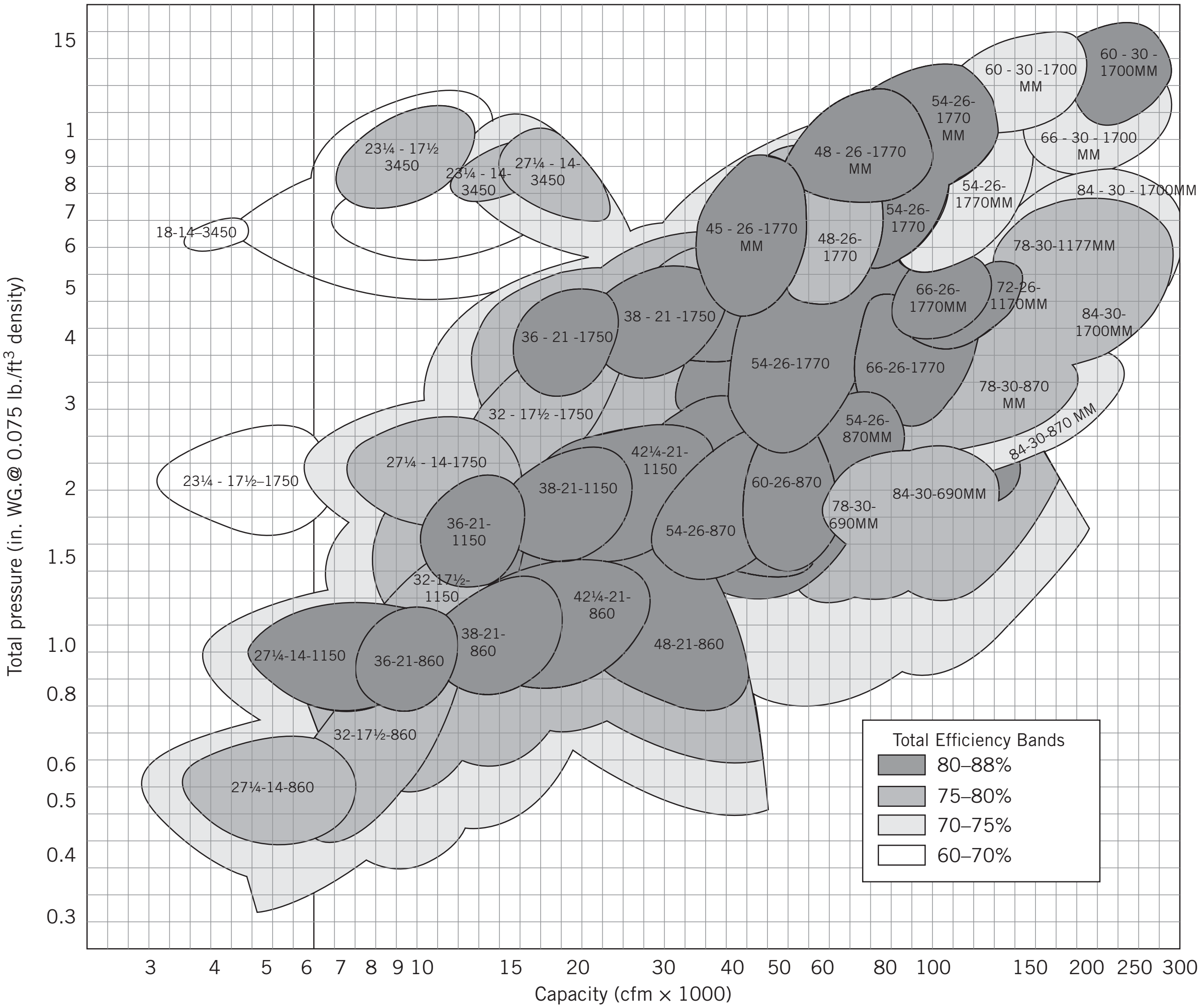

Fig. C.13 Master fan selection chart for Howden Buffalo axial fans.

REFERENCES

1. Peerless Pump literature:
   - Horizontal Split Case Single Stage Double Suction Pumps, Series AE, Brochure B-1200, 2003.
   - Horizontal Split Case, Multistage Single Suction Pumps, Types TU, TUT, 60 Hertz, Performance Curves, Brochure B-1440, 2003.
   - RAPID v8.25.6, March 2007.
2. Buffalo Forge literature:
   - Axivane Axial Fan Optimum Efficiency Selection Chart, n.d.

APPENDIX D

Flow Functions for Computation of Compressible Flow

D.1 *Isentropic Flow*

Isentropic flow functions are computed using the following equations:

$$\frac{p_0}{p} = \left[1 + \frac{k-1}{2}M^2\right]^{k/(k-1)} \qquad (12.21\text{a}/12.30\text{a})$$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad (12.21\text{b}/12.30\text{b})$$

$$\frac{\rho_0}{\rho} = \left[1 + \frac{k-1}{2}M^2\right]^{1/(k-1)} \qquad (12.21\text{c}/12.30\text{c})$$

$$\frac{A}{A*} = \frac{1}{M}\left[\frac{1 + \frac{k-1}{2}M^2}{\frac{k+1}{2}}\right]^{(k+1)/2(k-1)} \qquad (12.30\text{d})$$

Representative values of the isentropic flow functions for $k = 1.4$ are presented in Table D.1 and plotted in Fig. D.1.

Table D.1

Isentropic Flow Functions (one-dimensional flow, ideal gas, $k = 1.4$)

M	T/T_0	p/p_0	ρ/ρ_0	A/A^*
0.00	1.0000	1.0000	1.0000	∞
0.50	0.9524	0.8430	0.8852	1.340
1.00	0.8333	0.5283	0.6339	1.000
1.50	0.6897	0.2724	0.3950	1.176
2.00	0.5556	0.1278	0.2301	1.688
2.50	0.4444	0.05853	0.1317	2.637
3.00	0.3571	0.02722	0.07623	4.235
3.50	0.2899	0.01311	0.04523	6.790
4.00	0.2381	0.006586	0.02766	10.72
4.50	0.1980	0.003455	0.01745	16.56
5.00	0.1667	0.001890	0.01134	25.00

This table was computed from the *Excel* workbook *Isentropic Relations*. The workbook contains a more detailed, printable version of the table and can be easily modified to generate data for a different Mach number range, or for a different gas.

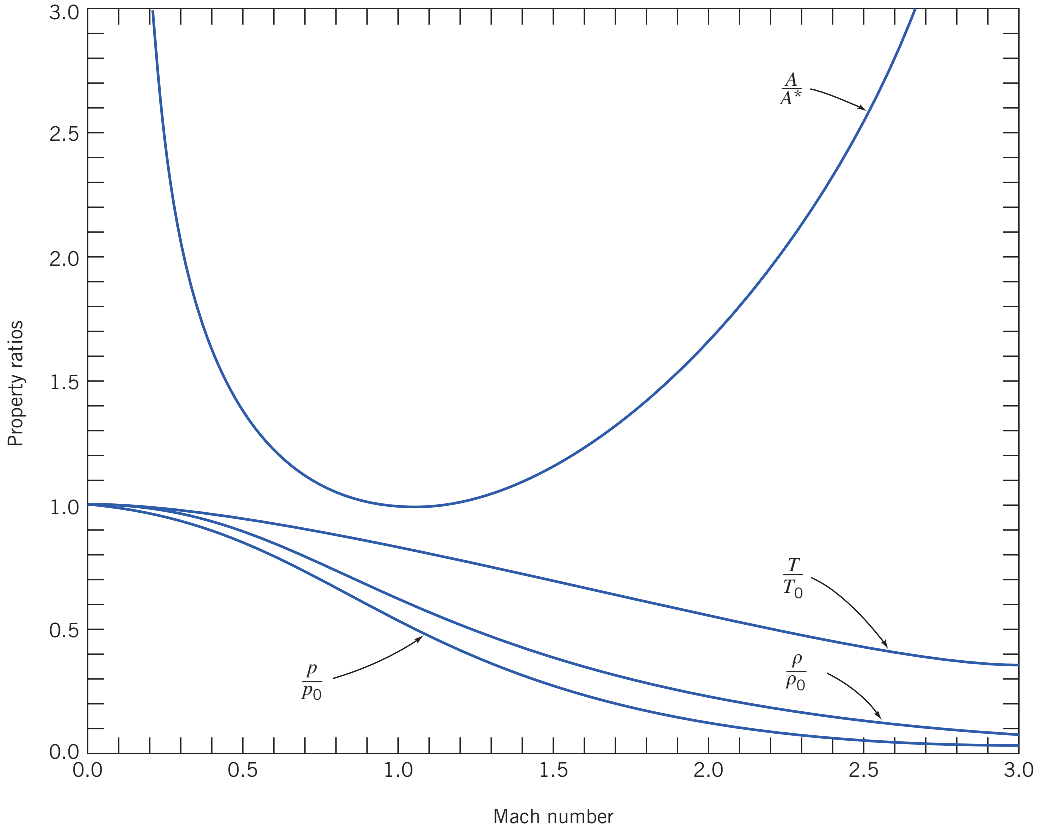

Fig. D.1 Isentropic flow functions.

This graph was generated from the *Excel* workbook. The workbook can be modified easily to generate curves for a different gas.

D.2 *Normal Shock*

Normal-shock flow functions are computed using the following equations:

$$M_2^2 = \frac{M_1^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_1^2 - 1} \tag{12.43a}$$

$$\frac{p_{0_2}}{p_{0_1}} = \frac{\left[\dfrac{\dfrac{k+1}{2}M_1^2}{1+\dfrac{k-1}{2}M_1^2}\right]^{k/(k-1)}}{\left[\dfrac{2k}{k+1}M_1^2 - \dfrac{k-1}{k+1}\right]^{1/(k-1)}} \tag{12.43b}$$

$$\frac{T_2}{T_1} = \frac{\left(1+\dfrac{k-1}{2}M_1^2\right)\left(kM_1^2 - \dfrac{k-1}{2}\right)}{\left(\dfrac{k+1}{2}\right)^2 M_1^2} \tag{12.43c}$$

$$\frac{p_2}{p_1} = \frac{2k}{k+1}M_1^2 - \frac{k-1}{k+1} \tag{12.43d}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{\dfrac{k+1}{2}M_1^2}{1+\dfrac{k-1}{2}M_1^2} \tag{12.43e}$$

Representative values of the normal-shock flow functions for $k = 1.4$ are presented in Table D.2 and plotted in Fig. D.2.

Table D.2

Normal-Shock Flow Functions (one-dimensional flow, ideal gas, $k = 1.4$)

M_1	M_2	$p0_2/p0_1$	T_2/T_1	p_2/p_1	ρ_2/ρ_1
1.00	1.000	1.000	1.000	1.000	1.000
1.50	0.7011	0.9298	1.320	2.458	1.862
2.00	0.5774	0.7209	1.687	4.500	2.667
2.50	0.5130	0.4990	2.137	7.125	3.333
3.00	0.4752	0.3283	2.679	10.33	3.857
3.50	0.4512	0.2130	3.315	14.13	4.261
4.00	0.4350	0.1388	4.047	18.50	4.571
4.50	0.4236	0.09170	4.875	23.46	4.812
5.00	0.4152	0.06172	5.800	29.00	5.000

This table was computed from the *Excel* workbook *Normal-Shock Relations*. The workbook contains a more detailed, printable version of the table and can be modified easily to generate data for a different Mach number range, or for a different gas.

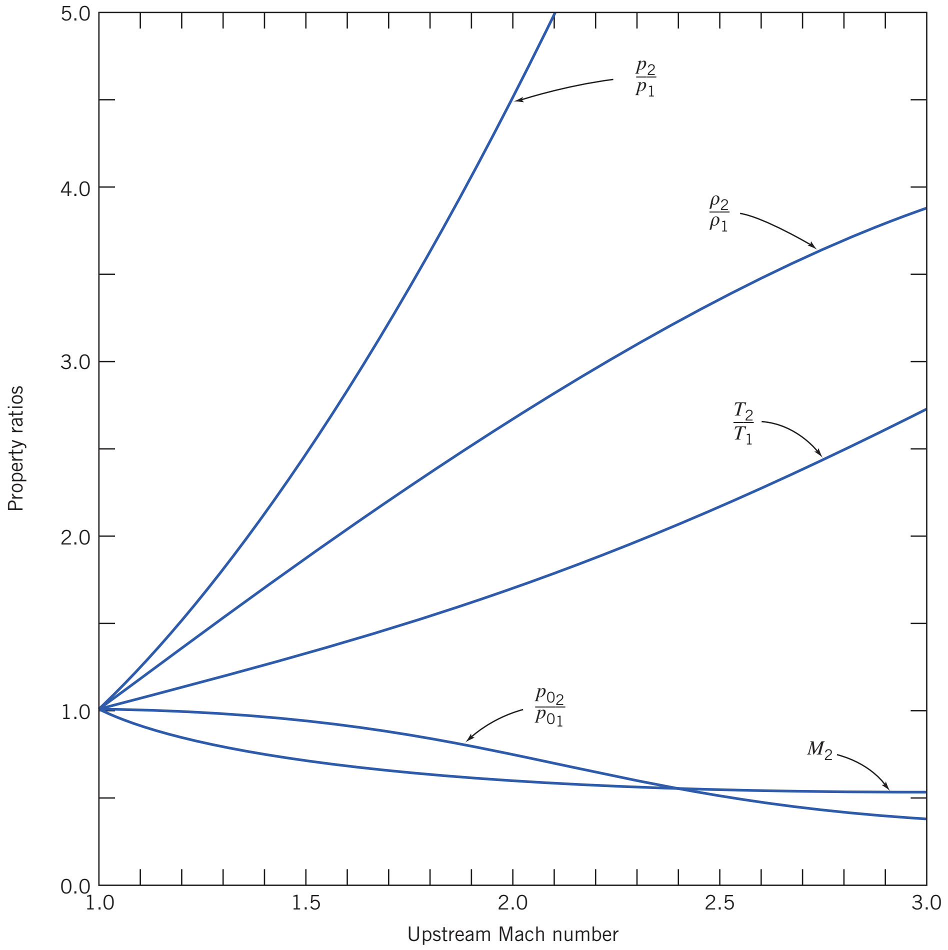

Fig. D.2 Normal-shock flow functions.

This graph was generated from the *Excel* workbook. The workbook can be modified easily to generate curves for a different gas.

APPENDIX E

Analysis of Experimental Uncertainty

E.1 Introduction

Experimental data often are used to supplement engineering analysis as a basis for design. Not all data are equally good; the validity of data should be documented before test results are used for design. Uncertainty analysis is the procedure used to quantify data validity and accuracy.

Analysis of uncertainty also is useful during experiment design. Careful study may indicate potential sources of unacceptable error and suggest improved measurement methods.

E.2 Types of Error

Errors always are present when experimental measurements are made. Aside from gross blunders by the experimenter, experimental error may be of two types. Fixed (or systematic) error causes repeated measurements to be in error by the same amount for each trial. Fixed error is the same for each reading and can be removed by proper calibration or correction. Random error (nonrepeatability) is different for every reading and hence cannot be removed. The factors that introduce random error are uncertain by their nature. The objective of uncertainty analysis is to estimate the probable random error in experimental results.

We assume that equipment has been constructed correctly and calibrated properly to eliminate fixed errors. We assume that instrumentation has adequate resolution and that fluctuations in readings are not excessive. We assume also that care is used in making and recording observations so that only random errors remain.

E.3 Estimation of Uncertainty

Our goal is to estimate the uncertainty of experimental measurements and calculated results due to random errors. The procedure has three steps:

1. Estimate the uncertainty interval for each measured quantity.
2. State the confidence limit on each measurement.
3. Analyze the propagation of uncertainty into results calculated from experimental data.

Below we outline the procedure for each step and illustrate applications with examples.

Step 1 *Estimate the measurement uncertainty interval.* Designate the measured variables in an experiment as $x_1, x_2, \ldots, x_n$. One possible way to find the uncertainty interval for each variable would be to repeat each measurement many times. The result would be a distribution of data for each variable. Random errors in measurement usually produce a *normal* (*Gaussian*) frequency distribution of measured values. The data scatter for a normal distribution is characterized by the standard deviation, σ. The uncertainty interval for each measured variable, x_i, may be stated as $\pm n\sigma_i$, where $n = 1$, 2, or 3.

However, the most typical situation in engineering work is a "single-sample" experiment, where only one measurement is made for each point [1]. A reasonable estimate of the measurement uncertainty due to random error in a single-sample experiment usually is plus or minus half the smallest scale division (the *least count*) of the instrument. However, this approach also must be used with caution, as illustrated in the following example.

Example E.1 UNCERTAINTY IN BAROMETER READING

The observed height of the mercury barometer column is $h = 752.6$ mm. The least count on the vernier scale is 0.1 mm, so one might estimate the probable measurement error as ±0.05 mm.

A measurement probably could not be made this precisely. The barometer sliders and meniscus must be aligned by eye. The slider has a least count of 1 mm. As a conservative estimate, a measurement could be made to the nearest millimeter. The probable value of a single measurement then would be expressed as 752.6 ± 0.5 mm. The relative uncertainty in barometric height would be stated as

$$u_h = \pm \frac{0.5\ \text{mm}}{752.6\ \text{mm}} = \pm 0.000664 \quad \text{or} \quad \pm 0.0664\ \text{percent}$$

Comments:

1. An uncertainty interval of ±0.1 percent corresponds to a result specified to three significant figures; this precision is sufficient for most engineering work.
2. The measurement of barometer height was precise, as shown by the uncertainty estimate. But was it accurate? At typical room temperatures, the observed barometer reading must be reduced by a temperature correction of nearly 3 mm! This is an example of a fixed error that requires a correction factor.

When repeated measurements of a variable are available, they are usually normally distributed data, for which over 99 percent of measured values of x_i lie within $\pm 3\sigma_i$ of the mean value, 95 percent lie within $\pm 2\sigma_i$, and 68 percent lie within $\pm \sigma_i$ of the mean value of the data set [2]. Thus it would be possible to quantify expected errors within any desired *confidence limit* if a statistically significant set of data were available.

The method of repeated measurements usually is impractical. In most applications it is impossible to obtain enough data for a statistically significant sample owing to the excessive time and cost involved. However, the normal distribution suggests several important concepts:

1. Small errors are more likely than large ones.
2. Plus and minus errors are about equally likely.
3. No finite maximum error can be specified.

Step 2 *State the confidence limit on each measurement.* The uncertainty interval of a measurement should be stated at specified odds. For example, one may write $h = 752.6 \pm 0.5$ mm (20 to 1). This means that one is willing to bet 20 to 1 that the height of the mercury column actually is within ±0.5 mm of the stated value. It should be obvious [3] that "... the specification of such odds can only be made by the experimenter based on ... total laboratory experience. There is no substitute for sound engineering judgment in estimating the uncertainty of a measured variable."

The confidence interval statement is based on the concept of standard deviation for a normal distribution. Odds of about 370 to 1 correspond to $\pm 3\sigma$; 99.7 percent of all future readings are expected to fall within the interval. Odds of about 20 to 1 correspond to $\pm 2\sigma$ and odds of 3 to 1 correspond to $\pm \sigma$ confidence limits. Odds of 20 to 1 typically are used for engineering work.

Step 3 *Analyze the propagation of uncertainty in calculations.* Suppose that measurements of independent variables, $x_1, x_2, \ldots, x_n$, are made in the laboratory. The relative uncertainty of each independently measured quantity is estimated as u_i. The measurements are used to calculate some result, R, for the experiment. We wish to analyze how errors in the x_is *propagate* into the calculation of R from measured values.

In general, R may be expressed mathematically as $R = R(x_1, x_2, \ldots, x_n)$. The effect on R of an error in measuring an individual x_i may be estimated by analogy to the derivative of a function [4]. A variation, δx_i, in x_i would cause variation δR_i in R,

$$\delta R_i = \frac{\partial R}{\partial x_i} \delta x_i$$

The relative variation in R is

$$\frac{\delta R_i}{R} = \frac{1}{R}\frac{\partial R}{\partial x_i}\delta x_i = \frac{x_i}{R}\frac{\partial R}{\partial x_i}\frac{\delta x_i}{x_i} \tag{E.1}$$

Equation E.1 may be used to estimate the relative uncertainty in the result due to uncertainty in x_i. Introducing the notation for relative uncertainty, we obtain

$$u_{R_i} = \frac{x_i}{R}\frac{\partial R}{\partial x_i} u_{x_i} \tag{E.2}$$

How do we estimate the relative uncertainty in R caused by the combined effects of the relative uncertainties in all the x_is? The random error in each variable has a range of values within the uncertainty interval. It is unlikely that all errors will have adverse values at the same time. It can be shown [1] that the best representation for the relative uncertainty of the result is

$$u_R = \pm\left[\left(\frac{x_1}{R}\frac{\partial R}{\partial x_1}u_1\right)^2 + \left(\frac{x_2}{R}\frac{\partial R}{\partial x_2}u_2\right)^2 + \cdots + \left(\frac{x_n}{R}\frac{\partial R}{\partial x_n}u_n\right)^2\right]^{1/2} \tag{E.3}$$

Example E.2 UNCERTAINTY IN VOLUME OF CYLINDER

Obtain an expression for the uncertainty in determining the volume of a cylinder from measurements of its radius and height. The volume of a cylinder in terms of radius and height is

$$\rlap{-}V = \rlap{-}V(r,h) = \pi r^2 h$$

Differentiating, we obtain

$$d\rlap{-}V = \frac{\partial \rlap{-}V}{\partial r}dr + \frac{\partial \rlap{-}V}{\partial h}dh = 2\pi rh\,dr + \pi r^2\,dh$$

since

$$\frac{\partial \rlap{-}V}{\partial r} = 2\pi rh \quad \text{and} \quad \frac{\partial \rlap{-}V}{\partial h} = \pi r^2$$

From Eq. E.2, the relative uncertainty due to radius is

$$u_{\rlap{-}V,r} = \frac{\delta \rlap{-}V_r}{\rlap{-}V} = \frac{r}{\rlap{-}V}\frac{\partial \rlap{-}V}{\partial r}u_r = \frac{r}{\pi r^2 h}(2\pi rh)u_r = 2u_r$$

and the relative uncertainty due to height is

$$u_{\rlap{-}V,h} = \frac{\delta \rlap{-}V_h}{\rlap{-}V} = \frac{h}{\rlap{-}V}\frac{\partial \rlap{-}V}{\partial h}u_h = \frac{h}{\pi r^2 h}(\pi r^2)u_h = u_h$$

The relative uncertainty in volume is then

$$u_{\rlap{-}V} = \pm\left[(2u_r)^2 + (u_h)^2\right]^{1/2} \tag{E.4}$$

Comment:
The coefficient 2, in Eq. E.4, shows that the uncertainty in measuring cylinder radius has a larger effect than the uncertainty in measuring height. This is true because the radius is squared in the equation for volume.

E.4 *Applications to Data*

Applications to data obtained from laboratory measurements are illustrated in the following examples.

Example E.3 UNCERTAINTY IN LIQUID MASS FLOW RATE

The mass flow rate of water through a tube is to be determined by collecting water in a beaker. The mass flow rate is calculated from the net mass of water collected divided by the time interval,

$$\dot{m} = \frac{\Delta m}{\Delta t} \tag{E.5}$$

where $\Delta m = m_f - m_e$. Error estimates for the measured quantities are

$$\begin{aligned} \text{Mass of full beaker, } m_f &= 400 \pm 2\ \text{g}(20 \text{ to } 1) \\ \text{Mass of empty beaker, } m_e &= 200 \pm 2\ \text{g}(20 \text{ to } 1) \\ \text{Collection time interval, } \Delta t &= 10 \pm 0.2\ \text{s}(20 \text{ to } 1) \end{aligned}$$

The relative uncertainties in measured quantities are

$$u_{m_f} = \pm \frac{2\ \text{g}}{400\ \text{g}} = \pm 0.005$$

$$u_{m_e} = \pm \frac{2\ \text{g}}{200\ \text{g}} = \pm 0.01$$

$$u_{\Delta t} = \pm \frac{0.2\ \text{s}}{10\ \text{s}} = \pm 0.02$$

The relative uncertainty in the measured value of net mass is calculated from Eq. E.3 as

$$\begin{aligned} u_{\Delta m} &= \pm \left[\left(\frac{m_f}{\Delta m} \frac{\partial \Delta m}{\partial m_f} u_{m_f} \right)^2 + \left(\frac{m_e}{\Delta m} \frac{\partial \Delta m}{\partial m_e} u_{m_e} \right)^2 \right]^{1/2} \\ &= \pm \{[(2)(1)(\pm 0.005)]^2 + [(1)(-1)(\pm 0.01)]^2\}^{1/2} \\ u_{\Delta m} &= \pm 0.0141 \end{aligned}$$

Because $\dot{m} = \dot{m}(\Delta m, \Delta t)$, we may write Eq. E.3 as

$$u_{\dot{m}} = \pm \left[\left(\frac{\Delta m}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta m} u_{\Delta m} \right)^2 + \left(\frac{\Delta t}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta t} u_{\Delta t} \right)^2 \right]^{1/2} \tag{E.6}$$

The required partial derivative terms are

$$\frac{\Delta m}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta m} = 1 \quad \text{and} \quad \frac{\Delta t}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta t} = -1$$

Substituting into Eq. E.6 gives

$$u_{\dot{m}} = \pm \{[(1)(\pm 0.0141)]^2 + [(-1)(\pm 0.02)]^2\}^{1/2}$$

$$u_{\dot{m}} = \pm 0.0245 \quad \text{or} \quad \pm 2.45 \text{ percent } (20 \text{ to } 1)$$

Comment:
The 2 percent uncertainty interval in time measurement makes the most important contribution to the uncertainty interval in the result.

Example E.4 UNCERTAINTY IN THE REYNOLDS NUMBER FOR WATER FLOW

The Reynolds number is to be calculated for flow of water in a tube. The computing equation for the Reynolds number is

$$Re = \frac{4\dot{m}}{\pi \mu D} = Re(\dot{m}, D, \mu) \tag{E.7}$$

We have considered the uncertainty interval in calculating the mass flow rate. What about uncertainties in μ and D? The tube diameter is given as $D = 6.35$ mm. Do we assume that it is exact? The diameter might be measured to the nearest 0.1 mm. If so, the relative uncertainty in diameter would be estimated as

$$u_D = \pm \frac{0.05\ \text{mm}}{6.35\ \text{mm}} = \pm 0.00787 \quad \text{or} \quad \pm 0.787\ \text{percent}$$

The viscosity of water depends on temperature. The temperature is estimated as $T = 24 \pm 0.5°\text{C}$. How will the uncertainty in temperature affect the uncertainty in μ? One way to estimate this is to write

$$u_{\mu(T)} = \pm \frac{\delta \mu}{\mu} = \frac{1}{\mu} \frac{d\mu}{dT} (\pm \delta T) \tag{E.8}$$

The derivative can be estimated from tabulated viscosity data near the nominal temperature of 24°C. Thus

$$\frac{d\mu}{dT} \approx \frac{\Delta \mu}{\Delta T} = \frac{\mu(25°\text{C}) - \mu(23°\text{C})}{(25-23)°\text{C}} = (0.000890 - 0.000933) \frac{\text{N}\cdot\text{s}}{\text{m}^2} \times \frac{1}{2°\text{C}}$$

$$\frac{d\mu}{dT} = -2.15 \times 10^{-5}\ \text{N}\cdot\text{s}/(\text{m}^2 \cdot °\text{C})$$

It follows from Eq. E.8 that the relative uncertainty in viscosity due to temperature is

$$u_{\mu(T)} = \frac{1}{0.000911} \frac{\text{m}^2}{\text{N}\cdot\text{s}} \times -2.15 \times 10^{-5} \frac{\text{N}\cdot\text{s}}{\text{m}^2 \cdot °\text{C}} \times (\pm 0.5°\text{C})$$

$$u_{\mu(T)} = \pm 0.0118 \quad \text{or} \quad \pm 1.18\ \text{percent}$$

Tabulated viscosity data themselves also have some uncertainty. If this is ±1.0 percent, an estimate for the resulting relative uncertainty in viscosity is

$$u_\mu = \pm [(\pm 0.01)^2 + (\pm 0.0118)^2]^{1/2} = \pm 0.0155 \quad \text{or} \quad \pm 1.55\ \text{percent}$$

The uncertainties in mass flow rate, tube diameter, and viscosity needed to compute the uncertainty interval for the calculated Reynolds number now are known. The required partial derivatives, determined from Eq. E.7, are

$$\frac{\dot{m}}{Re} \frac{\partial Re}{\partial \dot{m}} = \frac{\dot{m}}{Re} \frac{4}{\pi \mu D} = \frac{Re}{Re} = 1$$

$$\frac{\mu}{Re} \frac{\partial Re}{\partial \mu} = \frac{\mu}{Re} (-1) \frac{4\dot{m}}{\pi \mu^2 D} = -\frac{Re}{Re} = -1$$

$$\frac{D}{Re} \frac{\partial Re}{\partial D} = \frac{D}{Re} (-1) \frac{4\dot{m}}{\pi \mu D^2} = -\frac{Re}{Re} = -1$$

Substituting into Eq. E.3 gives

$$u_{Re} = \pm \left\{ \left[\frac{\dot{m}}{Re} \frac{\partial Re}{\partial \dot{m}} u_{\dot{m}} \right]^2 + \left[\frac{\mu}{Re} \frac{\partial Re}{\partial \mu} u_\mu \right]^2 + \left[\frac{D}{Re} \frac{\partial Re}{\partial D} u_D \right]^2 \right\}^{1/2}$$

$$u_{Re} = \pm \left\{ [(1)(\pm 0.0245)]^2 + [(-1)(\pm 0.0155)]^2 + [(-1)(\pm 0.00787)]^2 \right\}^{1/2}$$

$$u_{Re} = \pm 0.0300 \quad \text{or} \quad \pm 3.00\ \text{percent}$$

Comment:
Examples E.3 and E.4 illustrate two points important for experiment design. First, the mass of water collected, Δm, is calculated from two measured quantities, m_f and m_e. For any stated uncertainty interval in the measurements of m_f and m_e, the *relative* uncertainty in Δm can be decreased by making Δm larger. This might be accomplished by using larger containers or a longer measuring interval, Δt, which also would reduce the relative uncertainty in the measured Δt. Second, the uncertainty in tabulated property data may be significant. The data uncertainty also is increased by the uncertainty in measurement of fluid temperature.

Example E.5 UNCERTAINTY IN AIR SPEED

Air speed is calculated from pitot tube measurements in a wind tunnel. From the Bernoulli equation,

$$V = \left(\frac{2gh\rho_{\text{water}}}{\rho_{\text{air}}} \right)^{1/2} \tag{E.9}$$

where h is the observed height of the manometer column.

The only new element in this example is the square root. The variation in V due to the uncertainty interval in h is

$$\frac{h}{V}\frac{\partial V}{\partial h} = \frac{h}{V}\frac{1}{2}\left(\frac{2gh\rho_{\text{water}}}{\rho_{\text{air}}} \right)^{-1/2} \frac{2g\rho_{\text{water}}}{\rho_{\text{air}}}$$

$$\frac{h}{V}\frac{\partial V}{\partial h} = \frac{h}{V}\frac{1}{2}\frac{1}{V}\frac{2g\rho_{\text{water}}}{\rho_{\text{air}}} = \frac{1}{2}\frac{V^2}{V^2} = \frac{1}{2}$$

Using Eq. E.3, we calculate the relative uncertainty in V as

$$u_V = \pm \left[\left(\frac{1}{2} u_h \right)^2 + \left(\frac{1}{2} u_{\rho_{\text{water}}} \right)^2 + \left(-\frac{1}{2} u_{\rho_{\text{air}}} \right)^2 \right]^{1/2}$$

If $u_h = \pm 0.01$ and the other uncertainties are negligible,

$$u_V = \pm \left\{ \left[\frac{1}{2} (\pm 0.01) \right]^2 \right\}^{1/2}$$

$$u_V = \pm 0.00500 \quad \text{or} \quad \pm 0.500 \text{ percent}$$

Comment:
The square root reduces the relative uncertainty in the calculated velocity to half that of u_h.

E.5 *Summary*

A statement of the probable uncertainty of data is an important part of reporting experimental results completely and clearly. The American Society of Mechanical Engineers requires that all manuscripts submitted for journal publication include an adequate statement of uncertainty of experimental data [5]. Estimating uncertainty in experimental results requires care, experience, and judgment, in common with many endeavors in engineering. We have emphasized the need to quantify the uncertainty of measurements, but space allows including only a few examples. Much more information is available in the references that follow (e.g., [4, 6, 7]). We urge you to consult them when designing experiments or analyzing data.

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