Sichuan University - Pittsburgh Institute

ME 1071: Applied Fluids

Lecture 9 Introduction to Compressible Flow

Spring 2021

Weekly Study Plan

Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	**Apr. 6**	**Exam I**
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 25	Chapter 11: Flow in Open Channels
9	Apr. 27	Chapter 11: Flow in Open Channels
10	May. 11	Exam II Review
11	**May. 18**	**Exam II**
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review **(Final Exam at Jun. 26)**

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Outlines

- **Introduction**
- **A Brief Review of Thermodynamics**
  - Perfect gas, internal energy and enthalpy
  - First law of thermodynamics
  - Entropy and the second law of thermodynamics
  - Isentropic relations
- **Governing Equations of Inviscid, Compressible Flow**
- **Speed of Sound**
- **Definition of Total (Stagnation) Conditions**

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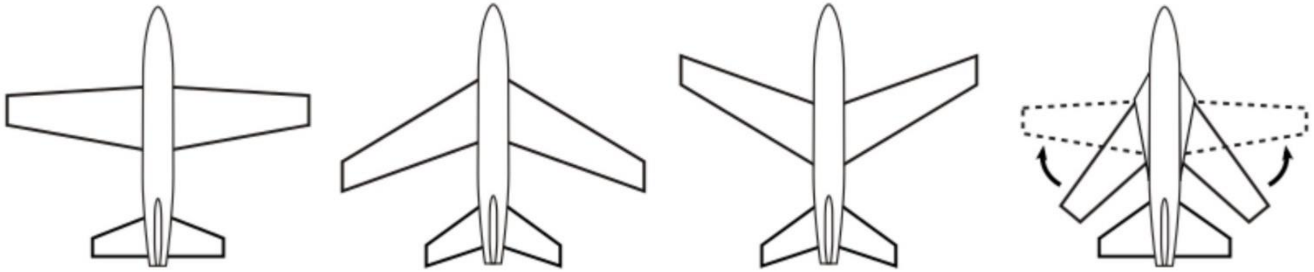
Introduction

Tupolev Tu-160

- **Some history of high-speed flight**

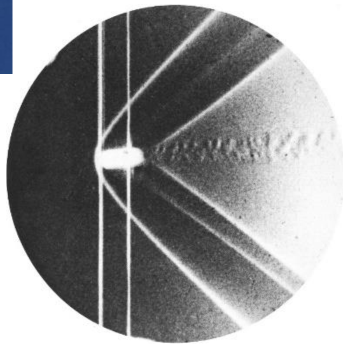


Straight — **Before 1935**

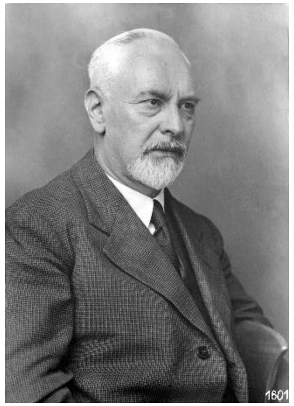
Swept — **After 1935** →

Forward swept

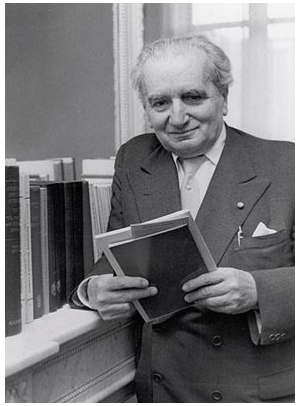
Variable sweep (swing-wing)



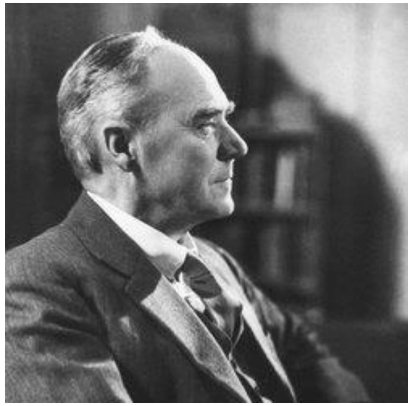
Ernst Mach's 1887 shadowgraph of a bow shockwave around a supersonic bullet



Ludwig Prandtl



Theodore Von Karman



G. I. Taylor

Bell_X-1 (in flight operated by Yeager, 1947)

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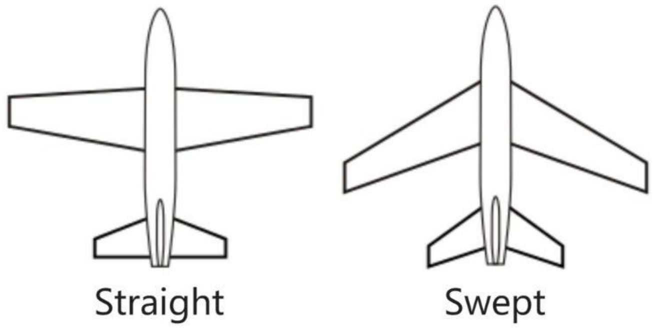
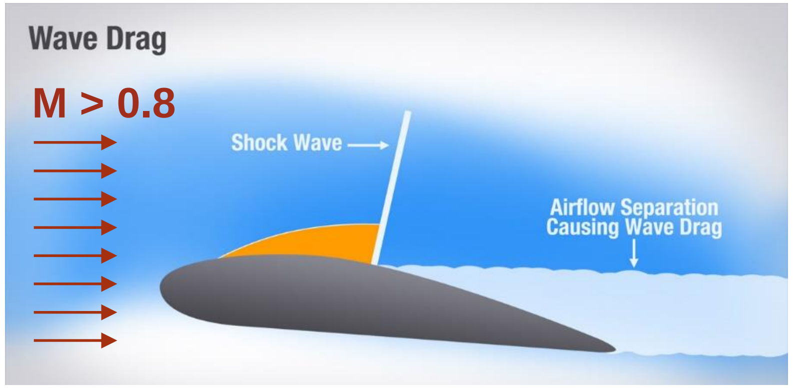
Introduction

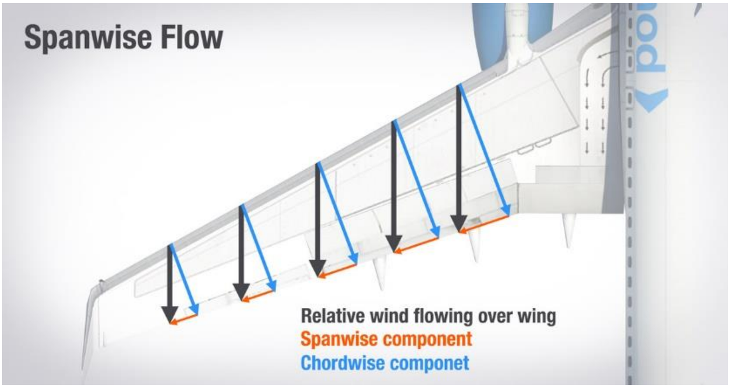
- **Some history of high-speed flight**



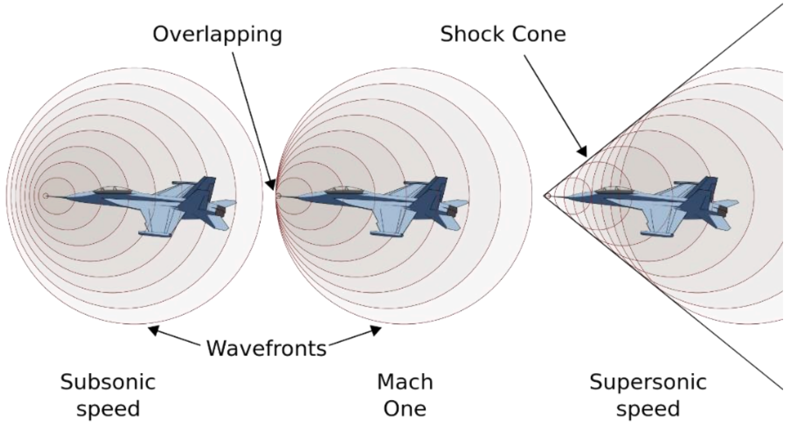
- (平直翼 vs 后掠翼)



Wave drag induced by local supersonic flow



Spanwise flow of swept wing



- **Sound barrier**
  - As aircraft approach the speed of sound, shock waves build up on the wings, interfering with the airflow that produces lift and keeps the plan in the air. The shock waves might just rip the aircraft apart.
  - The sound barrier is just an engineering limit at that time.
  - Special aerodynamic design (thin airfoil, swept and short wings etc.), strong materials and powerful propulsion.

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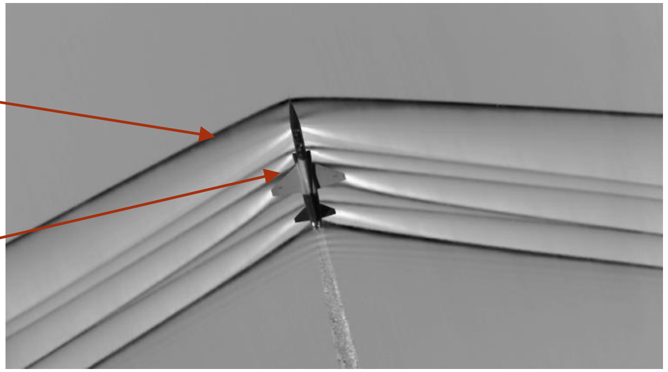
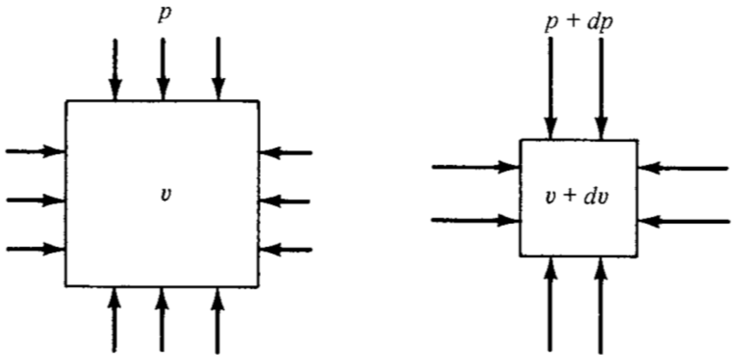
Introduction

- **Compressible flow**
  - Fluid density changes significantly
  - In high speed flow that Mach number > 0.3, the density change due to velocity is > 5%



- **Compressibility**
  - The relative volume change of a fluid or solid as a response to a pressure change
  - Thermodynamic fluid property that can be found in handbooks



$$\tau = -\frac{1}{v}\frac{dv}{dp},\ [\mathrm{Pa}^{-1}] \qquad \tau = \frac{1}{\rho}\frac{d\rho}{dp} \Rightarrow d\rho = \rho\tau dp$$

Isothermal: $\tau_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T$ Isentropic: $\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s$

- **Mach number:** the ratio of local velocity *V* to the local speed of sound *c* $\boxed{M \equiv \frac{V}{c}}$

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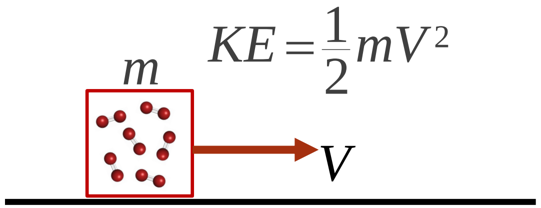
A Brief Review of Thermodynamics

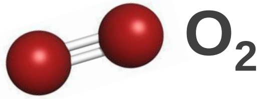
- Perfect gas
  - A gas which the intermolecular forces are neglected
  - Equation of state

$$\boxed{p=\rho RT,\ R=\frac{R_u}{M_m},\ for\ air\ R=287\ \mathrm{J/(kg\cdot K)}}$$

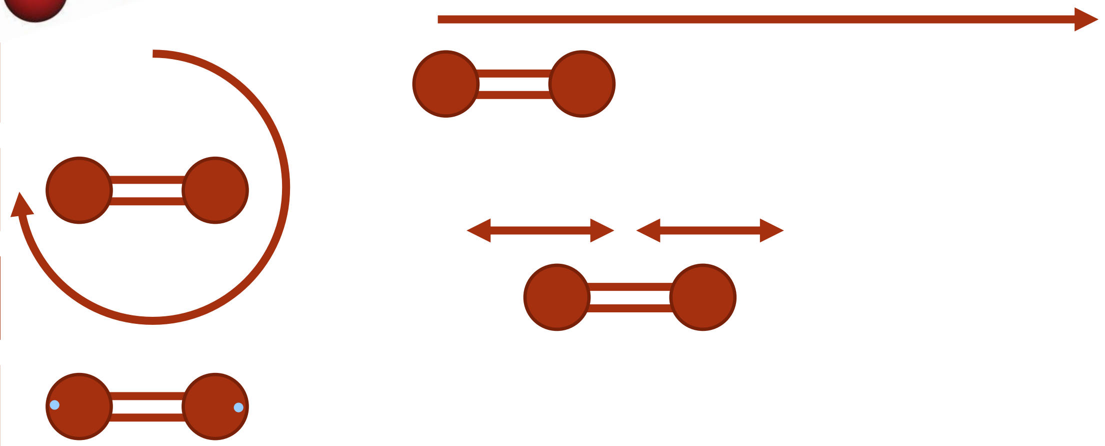


- Kinetic energy 动能
  - Macro system motions

- Internal energy (e)
  - Summation of the energies of all molecules (in random motions) in a system.



Internal energy
- Translational 平移
- Rotational 旋转
- Vibrational 振动
- Electronical 电子运动



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A Brief Review of Thermodynamics

- Enthalpy
  - The sum of the system's internal energy and the product of its pressure and volume.

$$\boxed{h = e + pv}$$

  - For a perfect gas, both e and h are functions of temperature only

$$e = e(T) = c_v T \qquad h = h(T) = c_p T$$

  - Calorically perfect gas: a perfect gas where c_v and c_p are constant.
  - Mayer's relation for ideal gas: $\boxed{c_p - c_v = R}$
  - Heat capacity ratio: $k \equiv c_p / c_v$

$$\boxed{c_p = \frac{kR}{k-1}} \qquad \boxed{c_v = \frac{R}{k-1}}$$

  - For air at standard conditions (1 atm, 25 ℃), k = 1.4.

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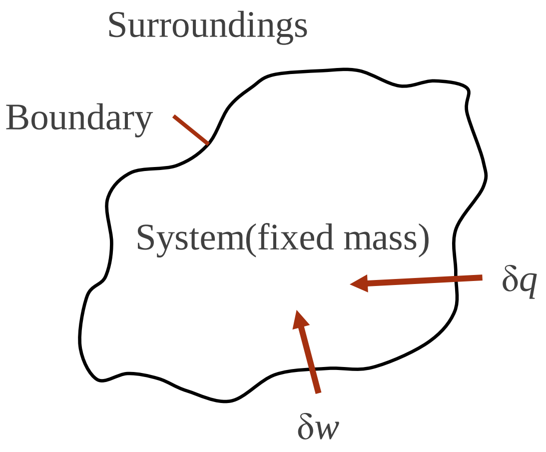
A Brief Review of Thermodynamics

- **First law of thermodynamics**



δq : An incremental amount of heat added to the system across the boundary

δw : The work done on the system by the surroundings

de : The change in energy of the system

$$\boxed{de = \delta q + \delta w = \delta q - p dv}$$

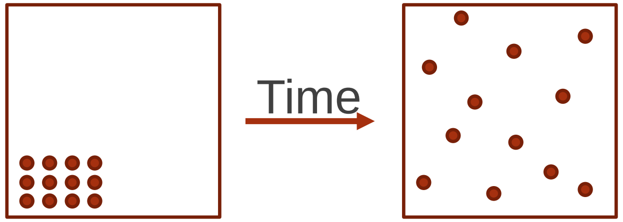
- Adiabatic process
  - No heat is added to or taken away from the system.
- Reversible process
  - No dissipative phenomena (effects of viscosity, thermal conductivity and mass diffusion) occur.
- Isentropic process
  - Both adiabatic and reversible.

A Brief Review of Thermodynamics

- **Entropy**
  - The degree of randomness
  - The change in entropy $ds = \frac{\delta q_{rev}}{T}$
  - δq_{rev} requires the difference in temperature between the heat source and the system has to approach zero, i.e., the transfer would have to occur infinitely slowly (quasi-statically).

$$\boxed{ds = \frac{\delta q}{T} + ds_{irrev}, \quad ds_{irrev} \geq 0}$$

- **Second law of thermodynamics**
  - The total entropy of an isolated system can never decrease over time, and is constant if and only if all processes are reversible.



$$\boxed{ds \geq 0}$$

Disorder is more probable than order.

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A Brief Review of Thermodynamics

- **Entropy change of a calorically perfect gas in a reversible process**
  - Reversible work $\delta w_{rev} = -pdv$
  - Entropy change $ds = \delta q_{rev} / T$
  - From the first law of thermodynamics and internal energy $de = \delta q_{rev} + \delta w_{rev} = Tds - pdv = c_v dT$
  - Enthalpy change $dh = de + pdv + vdp = c_p dT$

$$\boxed{p = \rho RT = \frac{RT}{v}}$$

$$ds = c_v \frac{dT}{T} + p\frac{dv}{T} \quad\Rightarrow\quad ds = c_v \frac{dT}{T} + R\frac{dv}{v} \quad\Rightarrow\quad s_2 - s_1 = c_v \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$ds = c_p \frac{dT}{T} - v\frac{dp}{T} \quad\Rightarrow\quad ds = c_p \frac{dT}{T} - R\frac{dp}{p} \quad\Rightarrow\quad s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

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A Brief Review of Thermodynamics

- **Isentropic relations**
  - Isentropic process is a process that is both adiabatic and reversible.

$$\underbrace{s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} = 0 \qquad s_2 - s_1 = c_v \ln\frac{T_2}{T_1} - R\ln\frac{v_2}{v_1} = 0}$$

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^k = \left(\frac{T_2}{T_1}\right)^{k/(k-1)}}$$

  - A large amount of practical compressible flow problems can be considered as isentropic.
  - The flow outside the boundary layer can be assumed to be isentropic.
  - Within the boundary layer, entropy increases due to strong dissipative mechanisms of viscosity, thermal conduction and diffusion.

- **Third law of thermodynamics**
  - The entropy of a system approaches a constant value as its temperature approaches absolute zero.

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Governing Equations of Inviscid, Compressible Flow

- **Continuity**

$$\boxed{\frac{\partial}{\partial t}\oiiint_V \rho dV + \oiint_S \rho \mathbf{V}\cdot\mathbf{dS} = 0}$$

- **Momentum**

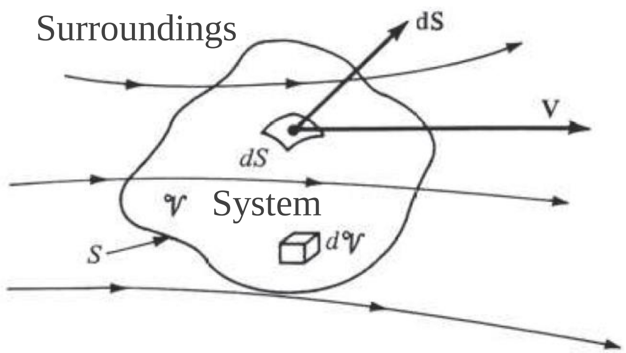
$$\boxed{\frac{\partial}{\partial t}\oiiint_V \rho \mathbf{V} dV + \oiint_S (\rho \mathbf{V}\cdot\mathbf{dS})\mathbf{V} = -\oiint_S p\mathbf{dS} + \oiiint_V \rho \mathbf{f} dV}$$

- **Energy (inviscid)**

$$\boxed{\frac{\partial}{\partial t}\oiiint_V \rho\left(e+\frac{V^2}{2}\right)dV + \oiint_S (\rho \mathbf{V}\cdot\mathbf{dS})\left(e+\frac{V^2}{2}\right) = \oiiint_V \dot{q}\rho dV - \oiint_S (p\mathbf{dS})\cdot\mathbf{V} + \oiiint_V \rho(\mathbf{f}\cdot\mathbf{V})dV}$$

- **Equation of state** $\boxed{p=\rho RT}$

- **Internal energy** $\boxed{e=c_v T}$



Bernoulli's equation does NOT hold for compressible flow!

Five unknowns

p, $\mathbf{V}$, ρ, e, and T

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The Basic Normal Shock Equations

- **Continuity**

$$\boxed{\rho_1 u_1 = \rho_2 u_2}$$

- **Momentum**

$$\boxed{p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2}$$

- **Energy**

$$\boxed{h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}}$$

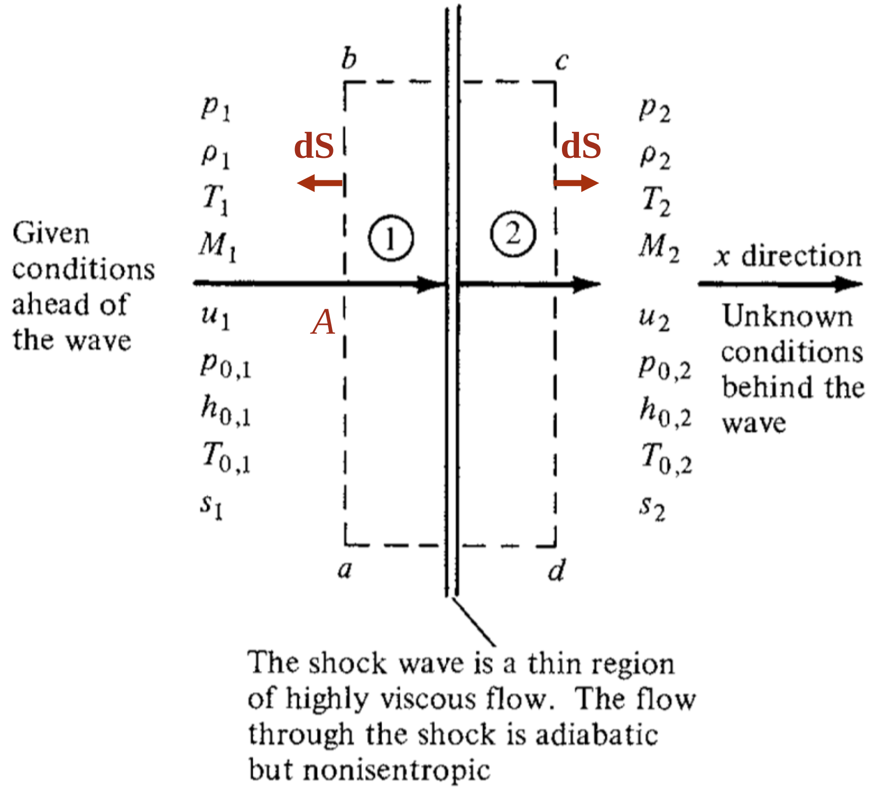
- **Enthalpy**

$$\boxed{h_2 = c_p T_2}$$

- **Equation of state**

$$\boxed{p_2 = \rho_2 R T_2}$$

- **Assumptions**
  - Steady flow, $\partial / \partial t = 0$.
  - Adiabatic, $\dot{q} = 0$.
  - No viscous effects.
  - No body forces, $\mathbf{f} = 0$.
  - **+ One-dimensional flow**



Five unknowns

p_2, u_2, ρ_2, h_2, and T_2

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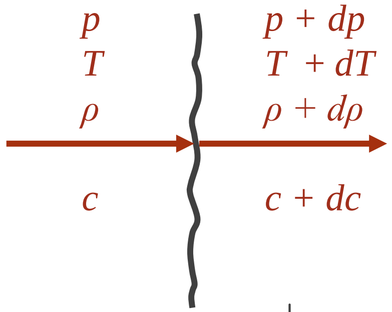
Speed of Sound

- **What is sound?**
  - In physics, sound is mechanical waves propagating in gases, liquids, and solids.
  - For gaseous media, sound is the propagating energy wave induced by variations in the local temperature, pressure and density.
  - Propagated by molecular collisions.
  - Depend on temperature only.

- dp, dT, $d\rho$, and dc are infinitesimal. The flow through the sound wave is adiabatic and reversible —— isentropic.
- From the continuity equation of one-dimensional flow



$$\boxed{c=\sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}}$$

$$\rho_1 u_1 = \rho_2 u_2 \longrightarrow \rho c = (\rho + d\rho)(c + dc) \xrightarrow{\text{Product of two differentials is neglected.}} c = -\rho\frac{dc}{d\rho}$$

- From the momentum equation of one-dimensional flow

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \longrightarrow p + \rho c = (p + dp) + (\rho + d\rho)(c + dc)^2 \longrightarrow dc = \frac{dp + c^2 d\rho}{-2c\rho}$$

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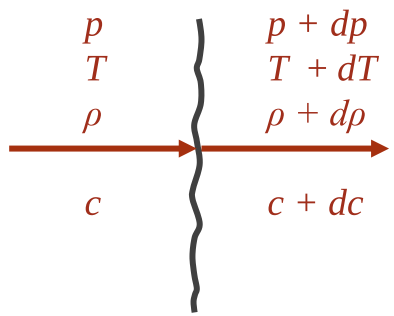
Speed of Sound

- Additional assumption: **calorically perfect gas**



$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^k \rightarrow \frac{p}{\rho^k} = const = a \rightarrow \left(\frac{\partial p}{\partial \rho}\right)_s = ak\rho^{k-1} = \frac{kp}{\rho} \rightarrow \boxed{c = \sqrt{kp/\rho} = \sqrt{\gamma RT}}$$

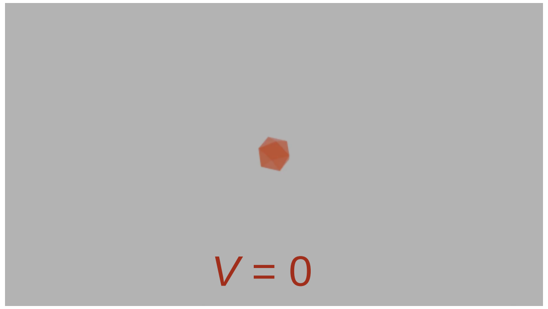
- The speed of sound in a calorically perfect gas is a function of T only.
- At sea level c = 340.9 m/s.
- **Speed of sound and compressibility**

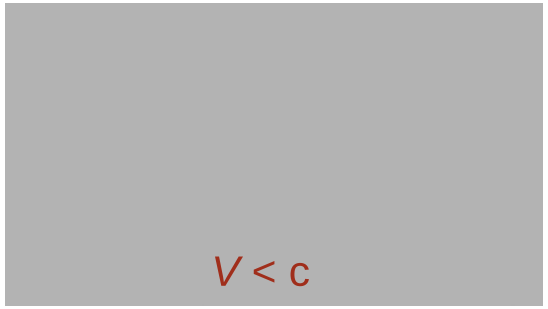
$$\tau_s = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_s = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial p}\right)_s = \frac{1}{\rho c^2} \longrightarrow c = \sqrt{\frac{1}{\tau_s \rho}}$$

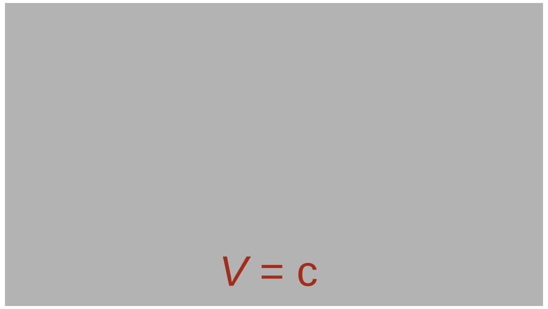
- The speed of sound in a theoretically incompressible fluid is infinite so that $M = V/\infty = 0$.
- Speed of sound in different medium: $c_{\text{solid}} > c_{\text{fluid}} > c_{\text{gas}}$.

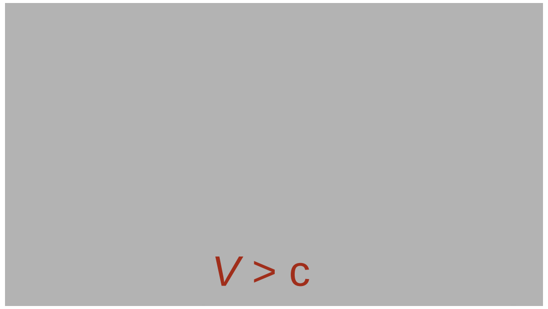
Speed of Sound

- **The propagation pattern of the disturbances**
  - The propagation of disturbance to the upstream is related to the moving speed of the object.

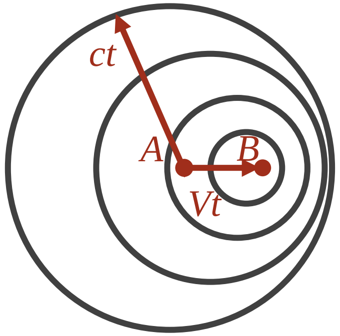




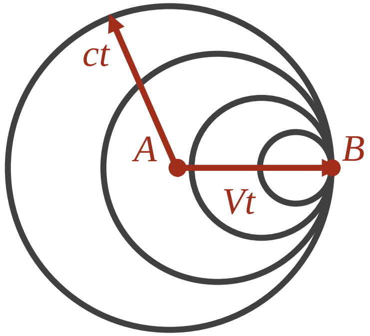




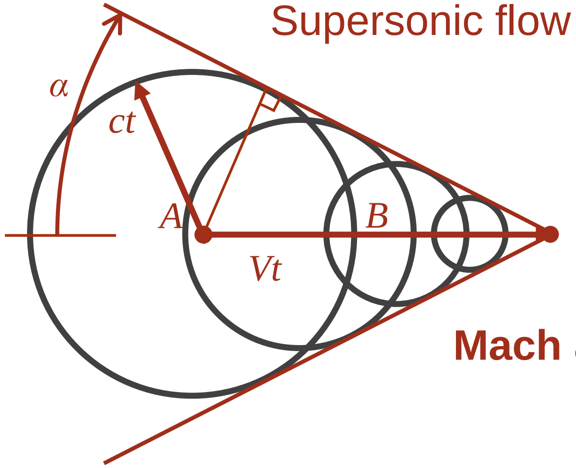
Subsonic flow $V < c$



Sonic flow $V = c$



Supersonic flow $V > c$



$$\sin\alpha = \frac{ct}{Vt} = \frac{c}{V} = \frac{1}{M}$$

Mach angle $\boxed{\alpha = \sin^{-1}\frac{1}{M}}$

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Speed of Sound

- **Additional physical meaning of the Mach number**
  - The ratio of kinetic energy to internal energy of a moving fluid element is

$$\frac{V^2/2}{e}=\frac{V^2/2}{c_v T}=\frac{V^2/2}{RT/(k-1)}=\frac{kV^2/2}{c^2/(k-1)}=\frac{k(k-1)}{2}M^2$$

  - The Mach number is a measure of the directed motion of the gas compared with the random thermal motion of the molecules.

- **Example 1**

  At a point in an airflow the pressure, temperature, and velocity are 1 atm, 320 K, and 1000 m/s. Calculate the Mach number at this point.

- **Example 2**

  Calculate the ratio of kinetic energy to internal energy at a point in an airflow where the Mach number is: (a) M = 2, and (b) M = 20.

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Definition of Total (Stagnation) Conditions

- Stagnation: local velocity is zero
- The condition that the velocity of fluid element adiabatically or isentropically slows down to zero.

$$\boxed{p,\ \rho,\ h,\ T}\ (\boldsymbol{V}) \xrightarrow[\text{or isentropically}]{\text{Slow down adiabatically}} \boxed{p_0,\ \rho_0,\ h_0,\ T_0}\ (\boldsymbol{V}=0)$$

- **Total enthalpy of a steady, adiabatic, inviscid flow**
  - **Assumption:** body forces are negligible

$$\boxed{\rho\frac{D(e+V^2/2)}{Dt} = \cancel{\dot{q}\rho} - \nabla\cdot(p\mathbf{V}) + \cancel{\rho(\mathbf{f}\cdot\mathbf{V})} + \cancel{\dot{Q}'_{\text{viscous}}} + \cancel{\dot{W}'_{\text{viscous}}}}$$

$+$

$$\rho\frac{D(p/\rho)}{Dt} = \frac{Dp}{Dt} - \frac{p}{\rho}\underbrace{\frac{D\rho}{Dt}}_{\textcircled{1}} = \underbrace{\frac{Dp}{Dt}}_{\textcircled{2}} + p\nabla\cdot\mathbf{V} = \frac{\partial p}{\partial t} + \underbrace{\mathbf{V}\cdot\nabla p + p\nabla\cdot\mathbf{V}}_{\textcircled{3}} = \frac{\partial p}{\partial t} + \nabla\cdot(p\mathbf{V})$$

$=$

$$\rho\frac{D(e+p/\rho+V^2/2)}{Dt} = \frac{\partial p}{\partial t} \quad\Rightarrow\quad \rho\frac{D(h+V^2/2)}{Dt} = 0 \quad\Rightarrow\quad \boxed{h+\frac{V^2}{2} = h_0 = \text{const}}$$

Continuity

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{V} = 0$$

Vector identity (向量恒等式)

$$\nabla\cdot(p\mathbf{V}) = p\nabla\cdot\mathbf{V} + \mathbf{V}\cdot\nabla p$$

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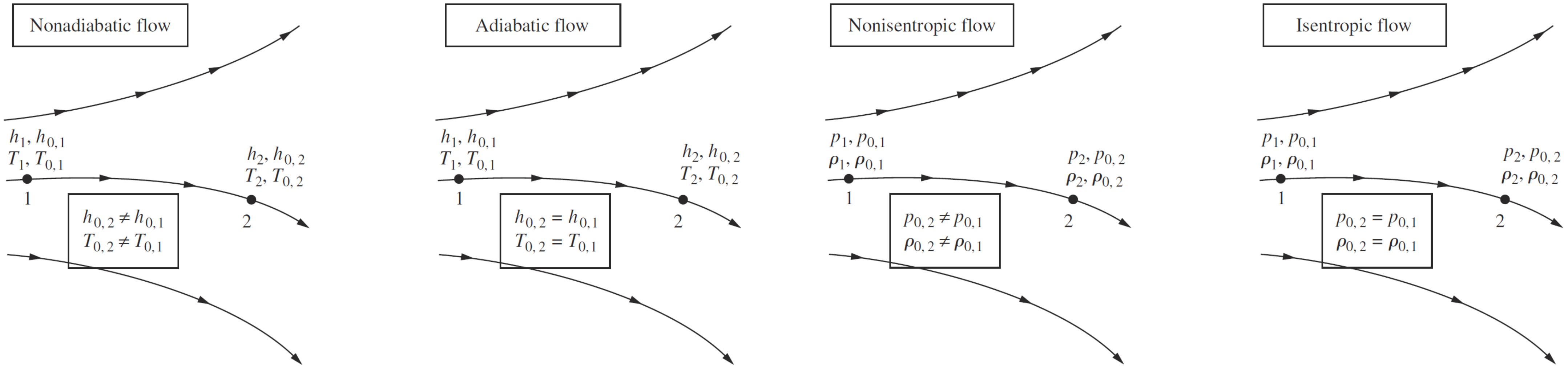
Definition of Total (Stagnation) Conditions

- **Total enthalpy of a steady, adiabatic, inviscid flow**
  - **Assumption:** body forces are negligible
  - h_0 is equal to its freestream value throughout the entire flow

$$\boxed{h+\frac{V^2}{2}=h_0=\text{const}}$$

  - Calorically perfect gas: $T_0 = \text{const}$



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Definition of Total (Stagnation) Conditions

- **Total enthalpy of a steady, adiabatic, inviscid flow**
  - **Assumption:** body forces are negligible
  - h_0 is equal to its freestream value throughout the entire flow

$$\boxed{h + \frac{V^2}{2} = h_0 = \text{const}}$$

- **Example**

At a point in an airflow the pressure, temperature, and velocity are 1 atm, 320 K, and 1000 m/s. Calculate the total temperature and total pressure at this point.

Assumptions: calorically perfect gas.

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Special Forms of the Energy Equation

- **Total condition (总状态、滞止状态)**
  - The condition that the velocity of fluid element adiabatically or isentropically slows down to zero.

V: $\boxed{h,\ T}$ $\xrightarrow{\text{Slow down adiabatically}}$ $V=0$: $\boxed{h_0,\ T_0}$

V: $\boxed{p,\ \rho}$ $\xrightarrow{\text{Slow down isentropically}}$ $V=0$: $\boxed{p_0,\ \rho_0}$

Starts from $\boxed{h+\dfrac{V^2}{2}=h_0=\text{const}} \longrightarrow \boxed{T_0=\text{const}}$ $\dfrac{T_0}{T}=1+\dfrac{k-1}{2}M^2 \longrightarrow$

$$\left\{\begin{aligned} \frac{p_0}{p}&=\left(1+\frac{k-1}{2}M^2\right)^{k/(k-1)} \\ \frac{\rho_0}{\rho}&=\left(1+\frac{k-1}{2}M^2\right)^{1/(k-1)} \end{aligned}\right.$$

$c=\sqrt{kRT}$

- **Sonic condition (音速状态) / Critical Condition**
  - The condition that the velocity of fluid element adiabatically or isentropically approaches to sonic velocity.

V: $\boxed{h,\ T}$ $\xrightarrow{\text{Approaches adiabatically}}$ $V^*=c^*$: $\boxed{h^*,\ T^*}$

V: $\boxed{p,\ \rho}$ $\xrightarrow{\text{Approaches isentropically}}$ $V^*=c^*$: $\boxed{p^*,\ \rho^*}$

$$\frac{T^*}{T_0}=\frac{2}{k+1}$$

$$\frac{p^*}{p_0}=\left(\frac{2}{k+1}\right)^{k/(k-1)} \qquad \frac{\rho^*}{\rho_0}=\left(\frac{2}{k+1}\right)^{1/(k-1)}$$

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Special Forms of the Energy Equation

- **Energy equation for steady, adiabatic, inviscid, one-dimensional flow**

$$\boxed{h_1+\frac{V_1^2}{2}=h_2+\frac{V_2^2}{2}}$$

- Calorically perfect gas, $h = C_pT$

$$c_pT_1+\frac{V_1^2}{2}=c_pT_2+\frac{V_2^2}{2} \longrightarrow \frac{kRT_1}{k-1}+\frac{V_1^2}{2}=\frac{kRT_2}{k-1}+\frac{V_2^2}{2} \longrightarrow \boxed{\frac{c_1^2}{k-1}+\frac{V_1^2}{2}=\frac{c_2^2}{k-1}+\frac{V_2^2}{2}}$$

- Total condition $V_2=0 \longrightarrow \boxed{\frac{c^2}{k-1}+\frac{V^2}{2}=\frac{c_0^2}{k-1}}$

- Sonic condition $M=V/c^*=1 \longrightarrow \boxed{\frac{c^2}{k-1}+\frac{V^2}{2}=\frac{k+1}{2(k-1)}c^{*2}}$

- We can further derive the relations for isentropic stagnation conditions

$$\frac{T_0}{T}=1+\frac{k-1}{2}M^2 \qquad \frac{p_0}{p}=\left(1+\frac{k-1}{2}M^2\right)^{k/(k-1)} \qquad \frac{\rho_0}{\rho}=\left(1+\frac{k-1}{2}M^2\right)^{1/(k-1)}$$

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Special Forms of the Energy Equation

- **Energy equation for steady, adiabatic, inviscid, one-dimensional flow**

- We have derived the relations for stagnation conditions

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)} \qquad \frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)}$$

- At sonic conditions

$$\frac{T^*}{T_0} = \frac{2}{k+1} = 0.833 \qquad \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = 0.528 \qquad \frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)} = 0.634$$

- Characteristic Mach number $M^* \equiv V/c^*$ $\quad c^* = \sqrt{kRT^*}$

$$\boxed{\frac{c^2}{k-1} + \frac{V^2}{2} = \frac{k+1}{2(k-1)} c^{*2}} \longrightarrow \boxed{M^{*2} = \frac{(k+1)M^2}{2+(k-1)M^2}} \begin{cases} M^* < 1 & \text{if } M < 1 \\ M^* = 1 & \text{if } M = 1 \\ M^* > 1 & \text{if } M > 1 \\ M^* \to \sqrt{\dfrac{k+1}{k-1}} = 2.645 & \text{if } M \to \infty \end{cases}$$

Bounded values for CFD simulations!

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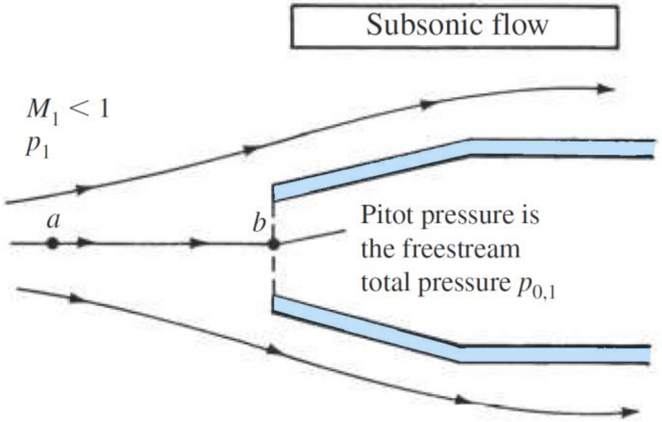
Special Forms of the Energy Equation

Why are we focusing on total conditions?



$$\frac{p_{0,1}}{p_1} = \left(1 + \frac{k-1}{2} M_1^{\,2}\right)^{k/(k-1)} \qquad \frac{T_{0,1}}{T_1} = 1 + \frac{k-1}{2} M_1^{\,2}$$

$$c = \sqrt{kRT_1}$$

- Using the above equations ,the measured total pressure $p_{0,1}$ and the corrected outside temperature, the flight velocity can be estimated as $V = Mc$.

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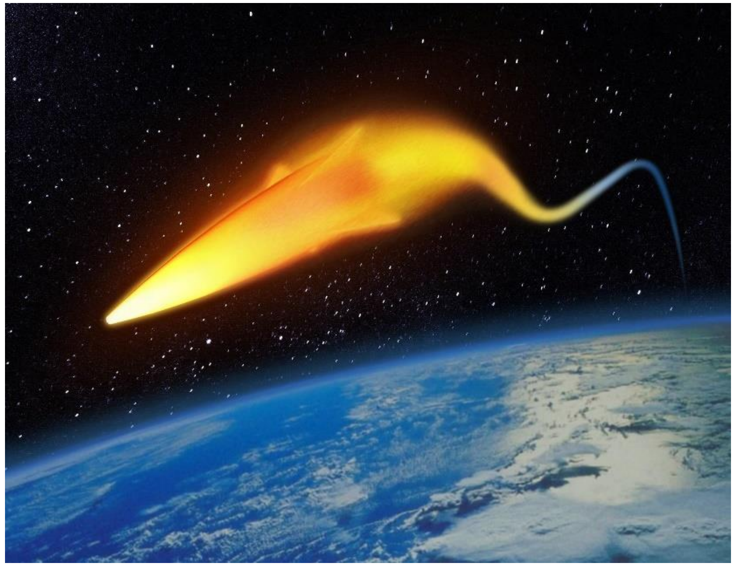
Special Forms of the Energy Equation

Why are we focusing on total conditions?



$$\frac{T_0}{T_\infty} = 1 + \frac{k-1}{2} M_\infty^{\ 2}$$

- Heat transfer must be considered in high speed flights.
- Total temperature is a simple and effective approximation when designing the insulation or cooling system at the primary stage.

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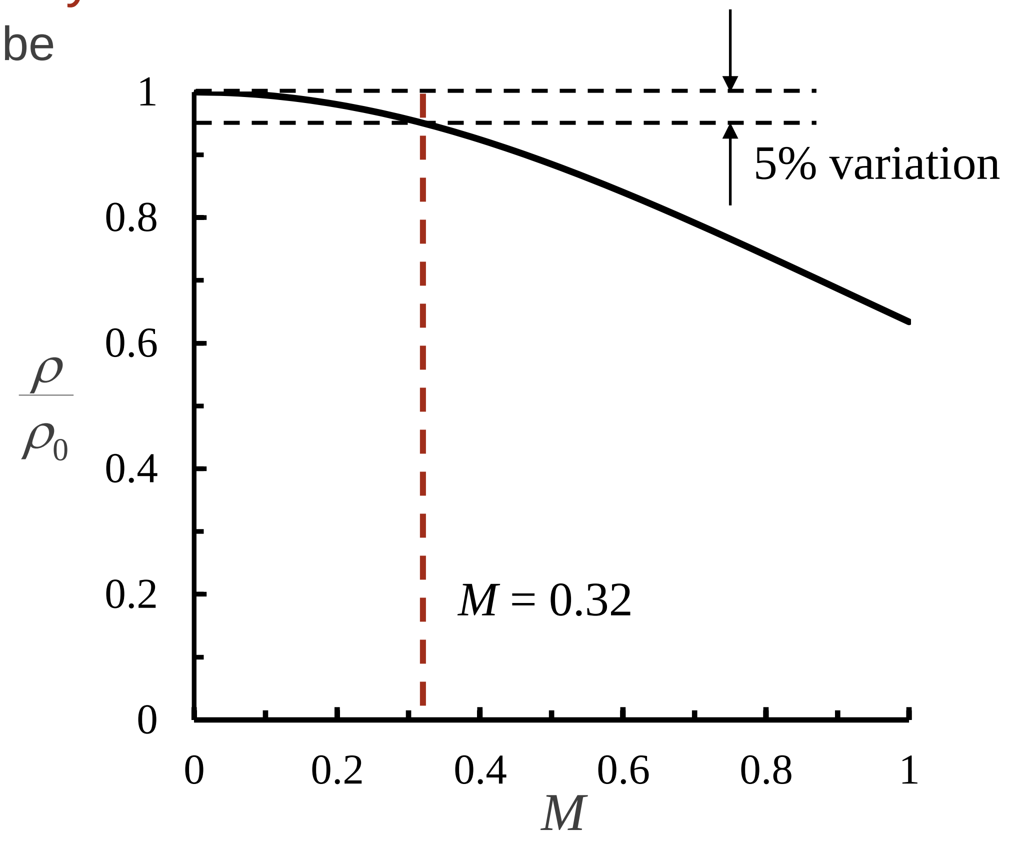
When Is a Flow Compressible?

- Compressible flow: fluid density changes significantly.
  - The relation between density and Mach number can be described by:

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)}$$

- To ensure density change < 5%, M must be less than 0.3.

- When M < 0.3, the flow can be treated as incompressible; otherwise, the compressibility must be considered.



Isentropic variation of density with Mach number

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When Is a Flow Compressible?

- **Example**
  - Consider the flow of air through a nozzle starting in the reservoir at nearly zero velocity and standard sea level values of p_0 = 1 atm and T_0 = 288 K, and expanding to a velocity of 107 m/s at the nozzle exit. Calculate the pressure at the nozzle exit assuming first incompressible flow and then compressible flow.

How about expanding to a velocity of 275 m/s?

incompressible

From Bernoulli's equation

$$p = p_0 - \frac{1}{2}\rho_0 V^2 = p_0 - \frac{1}{2}\frac{p_0}{RT_0}V^2 = 101325 - 0.5\frac{101325}{287\times 288}107^2 = 94307 \text{ Pa}$$

54972 Pa

compressible

$$T = T_0 - \frac{V^2}{2c_p} = 288 - \frac{107^2}{2(1.4\times 287/(1.4-1))} = 282.3 \text{ K}$$

250.4 K

$$p = p_0\left(\frac{T}{T_0}\right)^{k/(k-1)} = 101325\left(\frac{282.3}{288}\right)^{3.5} = 94478 \text{ Pa}$$

88096 Pa

$$M = V/c = V/\sqrt{kRT} = 107/\sqrt{1.4\times 287\times 282.3} = 0.317$$

0.866

The flow can be treated as incompressible.

The flow is compressible.

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Homework

Problem 12.9

Carbon dioxide flows at a speed of 10 m/s in a pipe and then through a nozzle where the velocity is 50 m/s . What is the change in gas temperature between pipe and nozzle? Assume this is an adiabatic flow of a perfect gas.

$$0 = (h_1 + \frac{V_1^2}{2}) - (h_2 + \frac{V_2^2}{2})$$

$$h_2 - h_1 = \frac{1}{2}\left(V_1^2 - V_2^2\right) = \frac{1}{2}(10^2 - 50^2) = -1.2\ kJ/kg$$

Assume a constant specific heat

$$h_2 - h_1 = c_p(T_2 - T_1)$$

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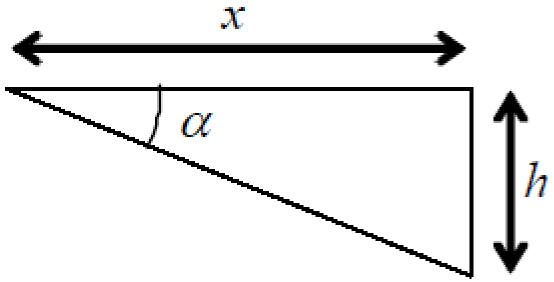
Homework

Problem 12.9

A supersonic aircraft flies at 3 km altitude at a speed of 1000 m/s on a standard day. How long after passing directly above a ground observer is the sound of the aircraft heard by the ground observer?



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Homework

Problem 12.9

Compute the air density in the undisturbed air, and at the stagnation point of an aircraft flying at 250 m/s in air at 28 kPa and 250℃. What is the percentage increase in density? Can we approximate this as an incompressible flow?

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)}$$

$$\frac{\rho_0 - \rho}{\rho} \times 100\% = \left(\frac{\rho_0}{\rho} - 1\right) \times 100\%$$

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Homework

This assignment is due by **6pm on May 27th**.

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.

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