ME 1071: Applied Fluids

Lecture 7 Flow in Open Channels

Spring 2021

Weekly Study Plan

Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	**Apr. 6**	**Exam I**
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 25	Chapter 11: Flow in Open Channels
9	Apr. 27	Chapter 11: Flow in Open Channels
10	May. 11	Exam II Review
11	**May. 18**	**Exam II**
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review

海纳百川 有容乃大

Outlines

- **Flow in Open Channel**
- **The Hydraulic Jump**

海纳百川 有容乃大

Outlines

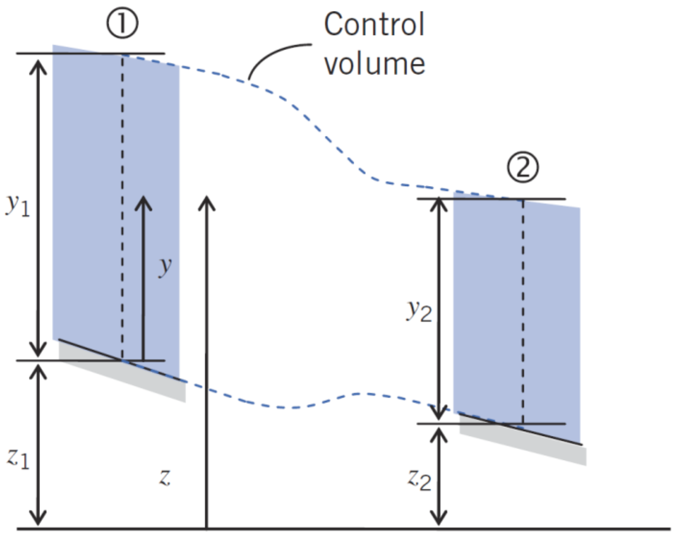
- **Flow in Open Channel**
- **The Hydraulic Jump**

海纳百川 有容乃大

Energy Equation for Open-Channel Flows

Assumptions

1. Steady flow.
2. Incompressible flow.
3. Uniform velocity at a section.
4. Gradually varying depth so that pressure distribution is hydrostatic.
5. Small bed slope.
6. $W_s = W_{shear} = W_{other} = 0$.



Energy Equation for Open-Channel Flow

$$\boxed{\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_l}$$

Total Head or Energy Head

$$H = \frac{V^2}{2g} + y + z$$

$$H_1 - H_2 = H_l$$

Specific Energy

$$E = \frac{V^2}{2g} + y$$

$$E_1 - E_2 + z_1 - z_2 = H_l$$

海纳百川 有容乃大

Energy Equation for Open-Channel Flows

The Specific Energy

- E indicates actual energy (kinetic plus potential/pressure per unit mass flow rate) being carried by the flow

$$E = \frac{V^2}{2g} + y$$

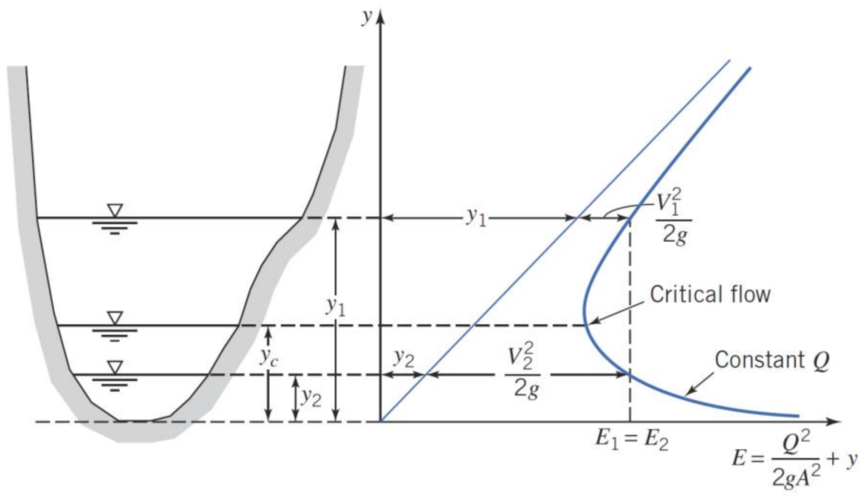


Fig. 11.7 Specific energy curve for a given flow rate.

Critical Depth (Fr = 1)

$$\boxed{Q^2 = \frac{gA_c^3}{b_{sc}}} \qquad \boxed{V_c = \sqrt{gy_{hc}}}$$

Minimum Specific Energy

- the specific energy is at its minimum at critical conditions, i.e., **Fr = 1**.

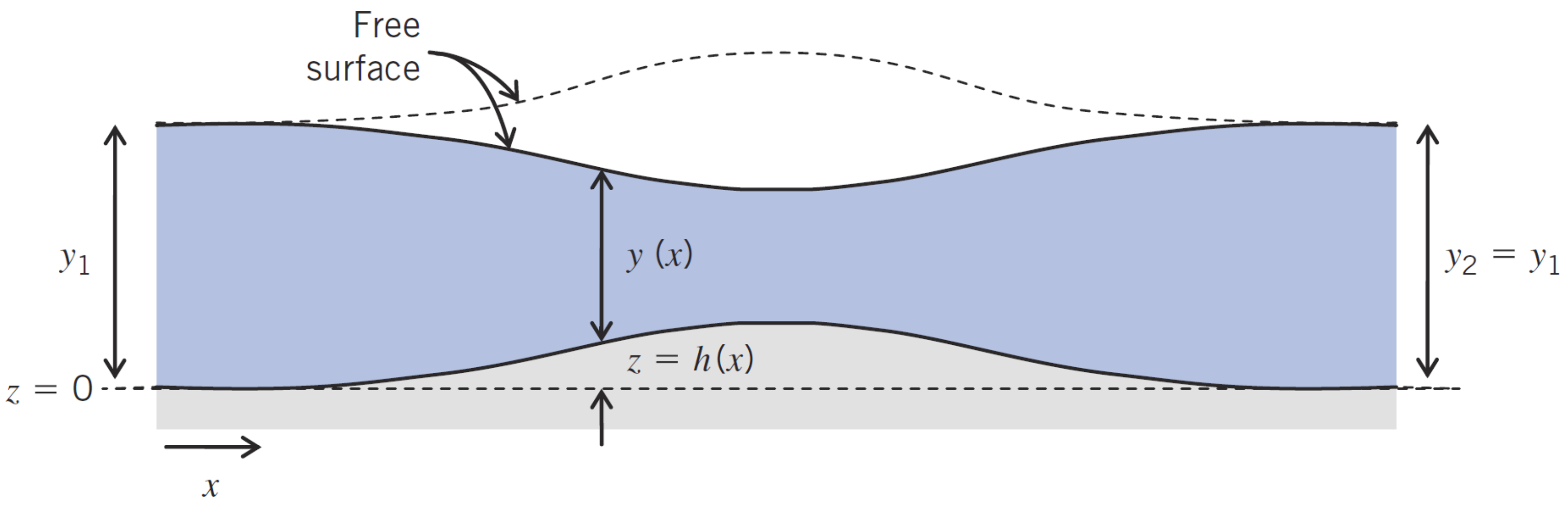
$$\boxed{y_c = \left[\frac{Q^2}{gb^2}\right]^{1/3} \qquad E_{\min} = \frac{3}{2}y_c \qquad (Rectangular\ channel)}$$

海纳百川 有容乃大

Localized Effect of Area Change

Flow over a Bump

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 = \frac{V^2}{2g} + y + z = \text{const}$$



海纳百川 有容乃大

Localized Effect of Area Change

Flow over a Bump

海纳百川 有容乃大

Localized Effect of Area Change

Example

A rectangular channel 2 m wide has a flow of 2.4 m^3/s at a depth of 1.0 m. Determine whether critical depth occurs at:

a) A section where a bump of height h = 0.20 m is in the channel bed.

b) A side wall constriction that reduces the channel width to 1.7 m.

c) A combined bump and side wall constrictions.

$$y_c = \left[\frac{Q^2}{gb^2}\right]^{1/3}$$

$$E_1 = y_1 + \frac{Q^2}{2gA^2} = y_1 + \frac{Q^2}{2gb^2y_1^2}$$

$$E_{min} = \frac{3}{2}y_c$$

海纳百川 有容乃大

The Hydraulic Jump

Phenomenon

海纳百川 有容乃大

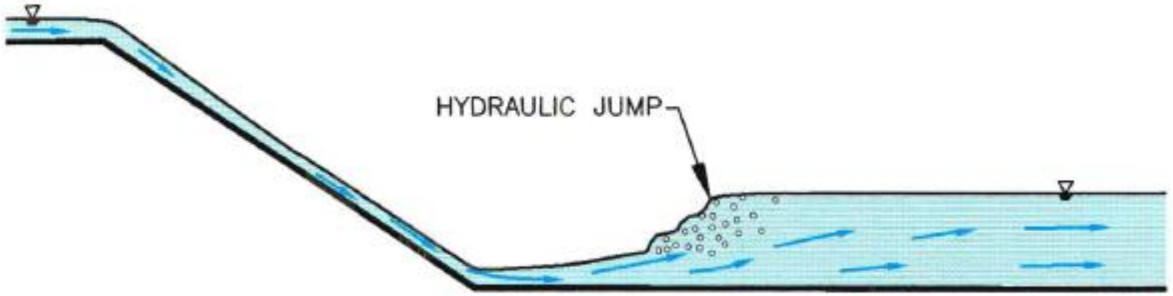
The Hydraulic Jump

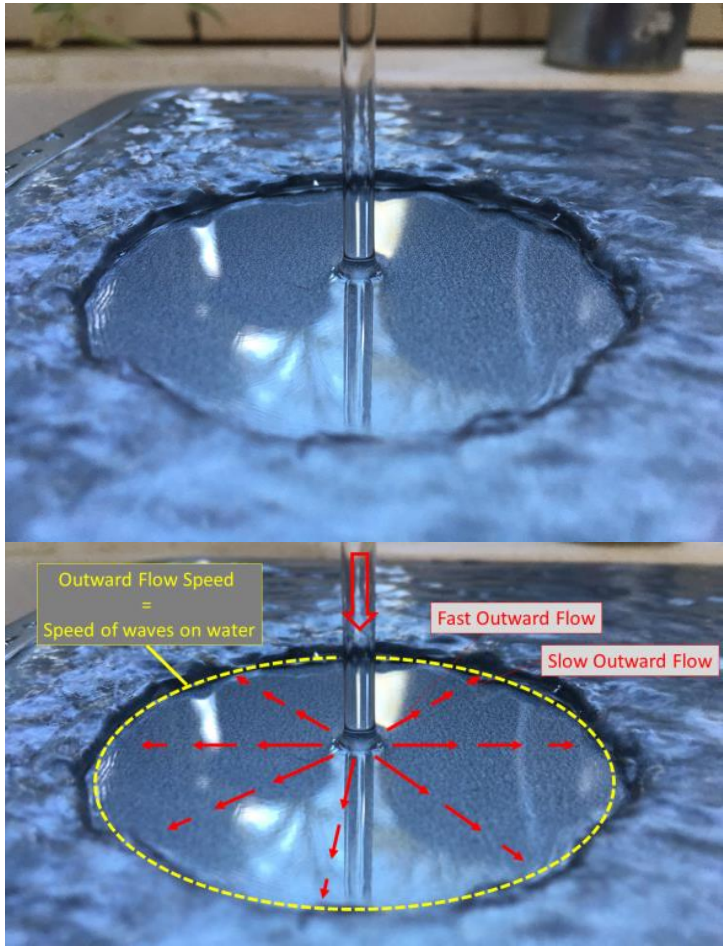
Subcritical Flow Fr <1

- Disturbances move upstream, smooth adjustment

Supercritical Flow Fr >1

- Disturbances cannot move upstream
- The transition from supercritical to subcritical flow occurs abruptly through a hydraulic jump.

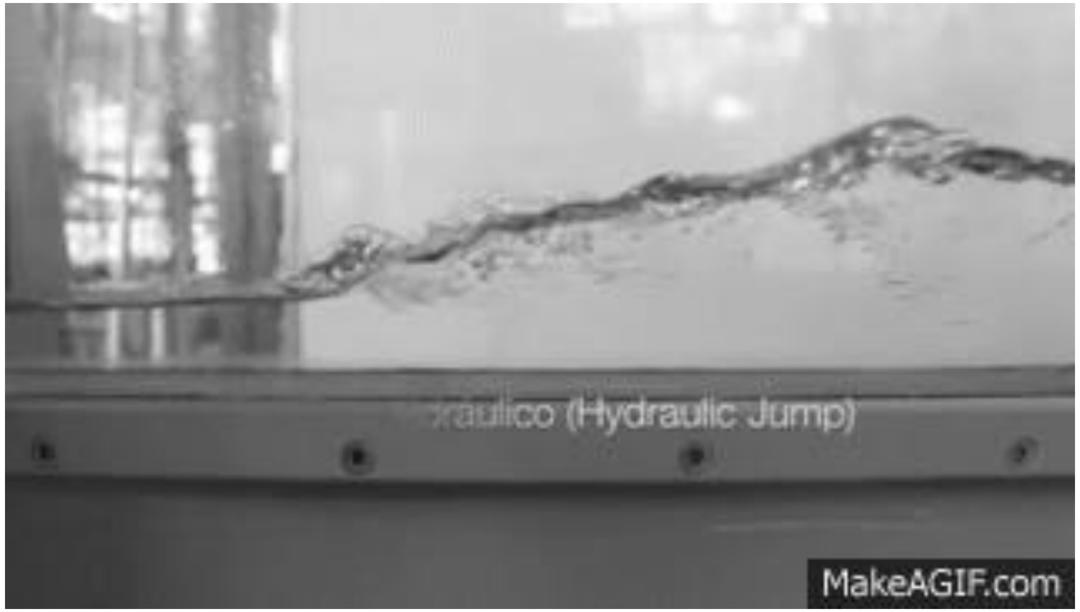


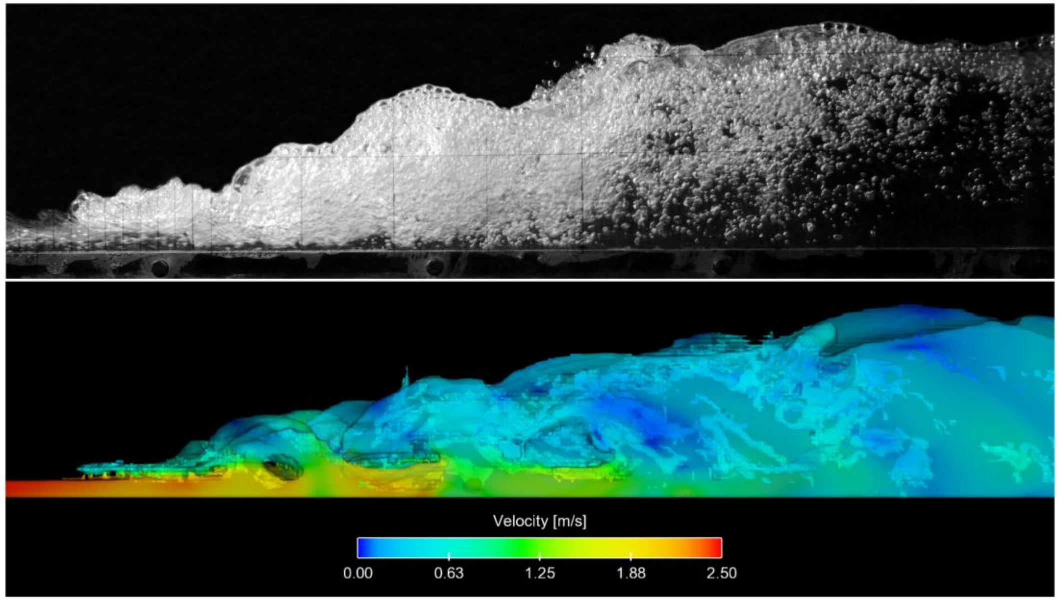


海纳百川 有容乃大

The Hydraulic Jump

More Examples of Hydraulic Jump





海纳百川 有容乃大

The Hydraulic Jump

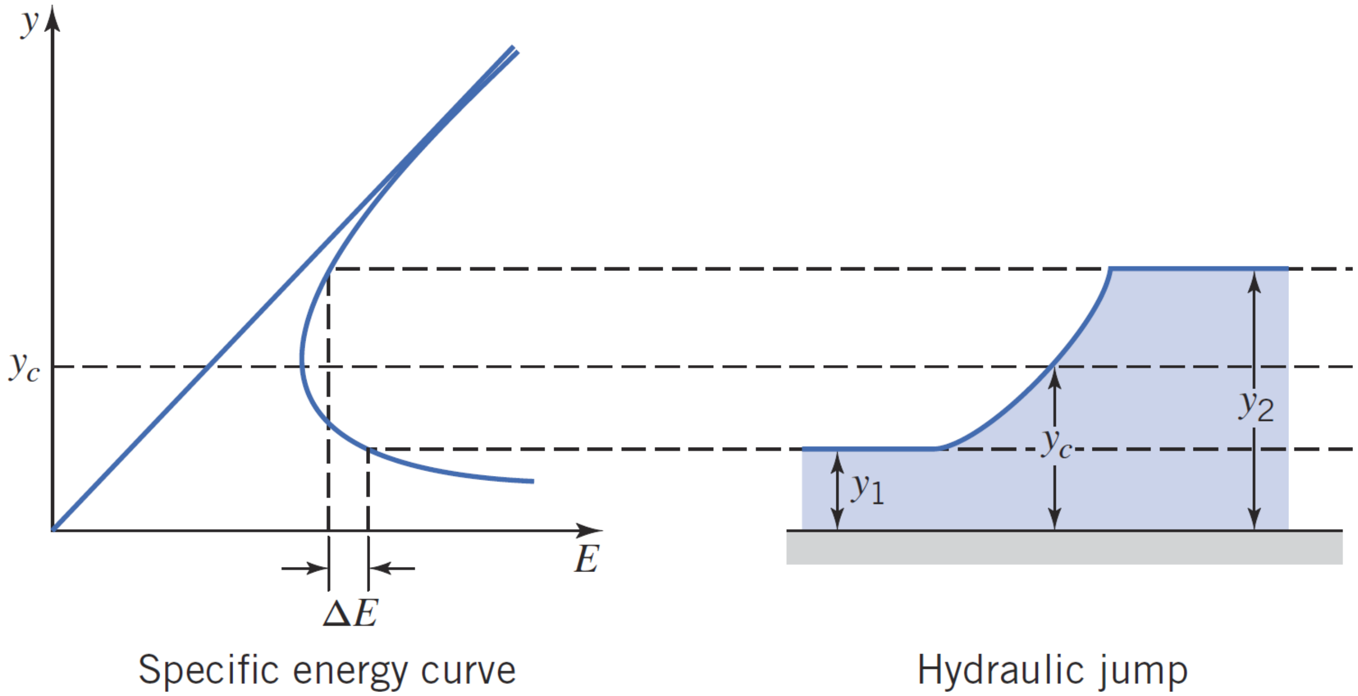
More Examples of Hydraulic Jump

海纳百川 有容乃大

The Hydraulic Jump

Specific Energy Curve for Hydraulic Jump

- Unlike the changes due to phenomena such as a bump, the abrupt change in depth involves a significant loss of mechanical energy through turbulent mixing.



海纳百川 有容乃大

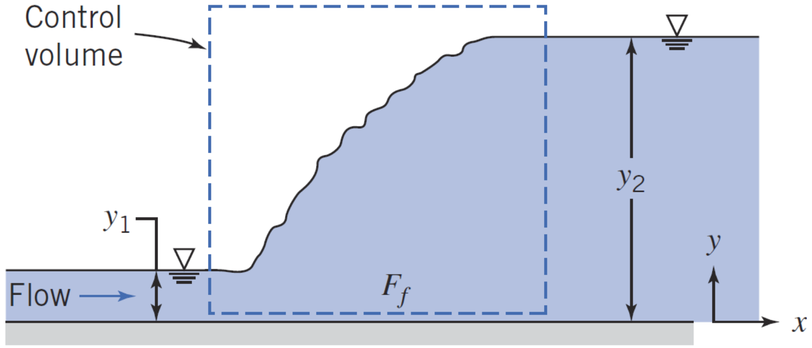
The Hydraulic Jump

Governing Equations for Hydraulic Jump

$$\boxed{Continuity \quad V_1 y_1 = V_2 y_2}$$

$$\boxed{Momentum \quad \frac{V_1^2 y_1}{g} + \frac{y_1^2}{2} = \frac{V_2^2 y_2}{g} + \frac{y_2^2}{2}}$$

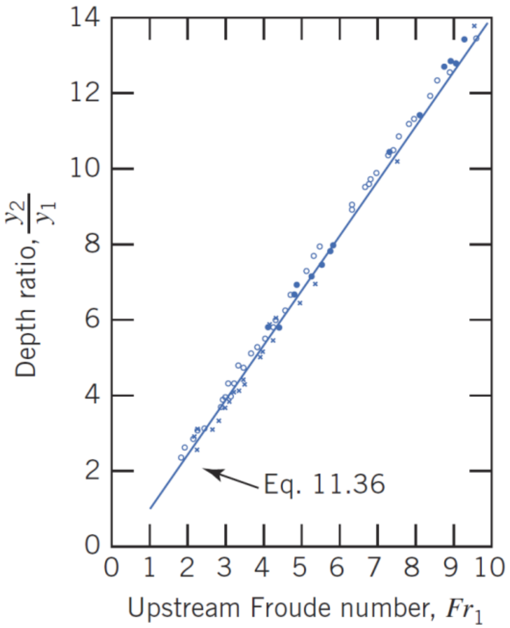
$$\boxed{Energy \quad E_1 = \frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + H_l = E_2 + H_l}$$



Depth Increase Across a Hydraulic Jump

- The ratio of downstream to upstream depths across a hydraulic jump is only a function of the upstream Froude number.

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1 + 8Fr_1^2} - 1\right], \; Fr_1 > 1}$$

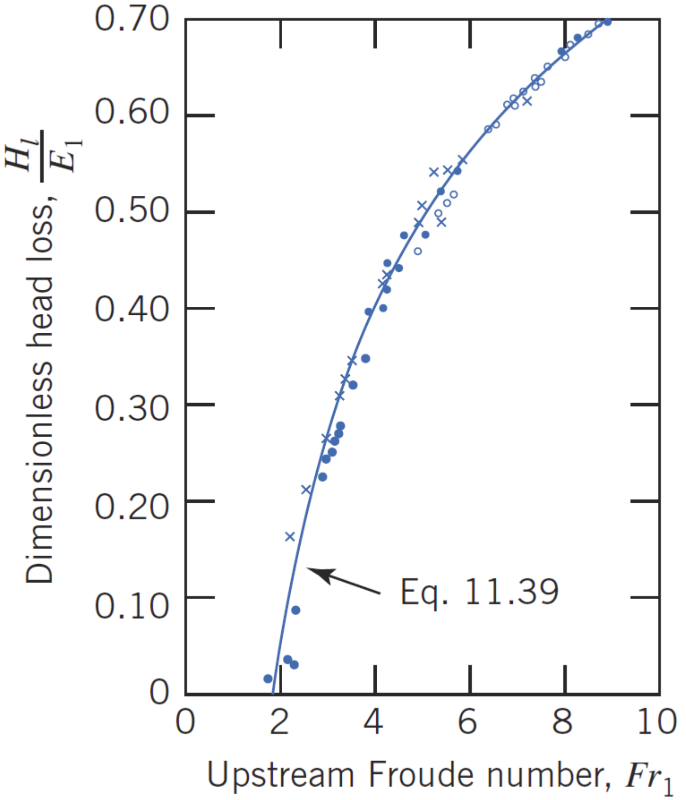


海纳百川 有容乃大

The Hydraulic Jump

Head Loss Across a Hydraulic Jump

- The head loss is only a function of the upstream Froude number.
- Hydraulic jump can occur only in supercritical flow.
- Flow downstream from a jump always is subcritical.



$$\boxed{H_l = \frac{[y_2 - y_1]^3}{4y_1y_2}, \quad y_2 > y_1}$$

$$\boxed{\frac{H_l}{E_1} = \frac{\left[\sqrt{1+8Fr_1^2} - 3\right]^3}{8\left[\sqrt{1+8Fr_1^2} - 1\right]\left[Fr_1^2 + 2\right]}, \quad Fr_1 > 1}$$

Energy Dissipation Ratio

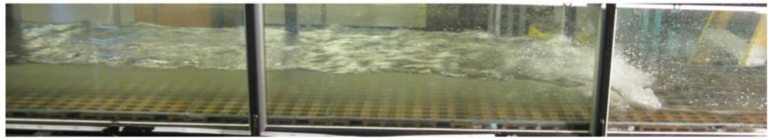
海纳百川 有容乃大

The Hydraulic Jump

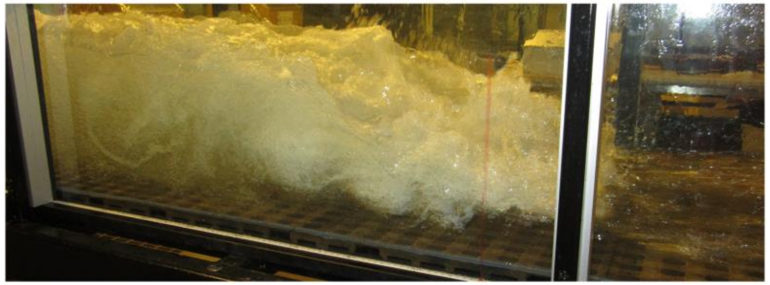
Hydraulic Jump Classification

(a)

(b)



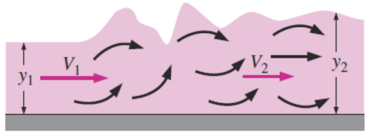
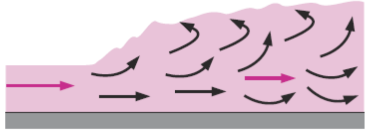
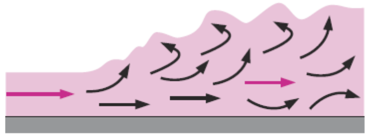
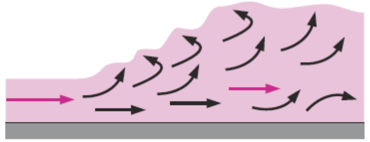
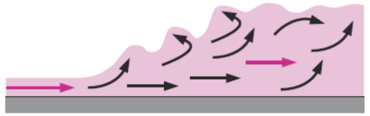
(c)



(d)

Classification of hydraulic jumps

Source: U.S. Bureau of Reclamation (1955).

Upstream Fr_1	Depth Ratio y_2/y_1	Fraction of Energy Dissipation	Description	Surface Profile
<1	1	0	*Impossible jump.* Would violate the second law of thermodynamics.	
1–1.7	1–2	<5%	*Undular jump* (or *standing wave*). Small rise in surface level. Low energy dissipation. Surface rollers develop near Fr = 1.7.	
1.7–2.5	2–3.1	5–15%	*Weak jump.* Surface rising smoothly, with small rollers. Low energy dissipation.	
2.5–4.5	3.1–5.9	15–45%	*Oscillating jump.* Pulsations caused by entering jets at the bottom generate large waves that can travel for miles and damage earth banks. Should be avoided in the design of stilling basins.	
4.5–9	5.9–12	45–70%	*Steady jump.* Stable, well-balanced, and insensitive to downstream conditions. Intense eddy motion and high level of energy dissipation within the jump. Recommended range for design.	
>9	>12	70–85%	*Strong jump.* Rough and intermittent. Very effective energy dissipation, but may be uneconomical compared to other designs.	

有容乃大

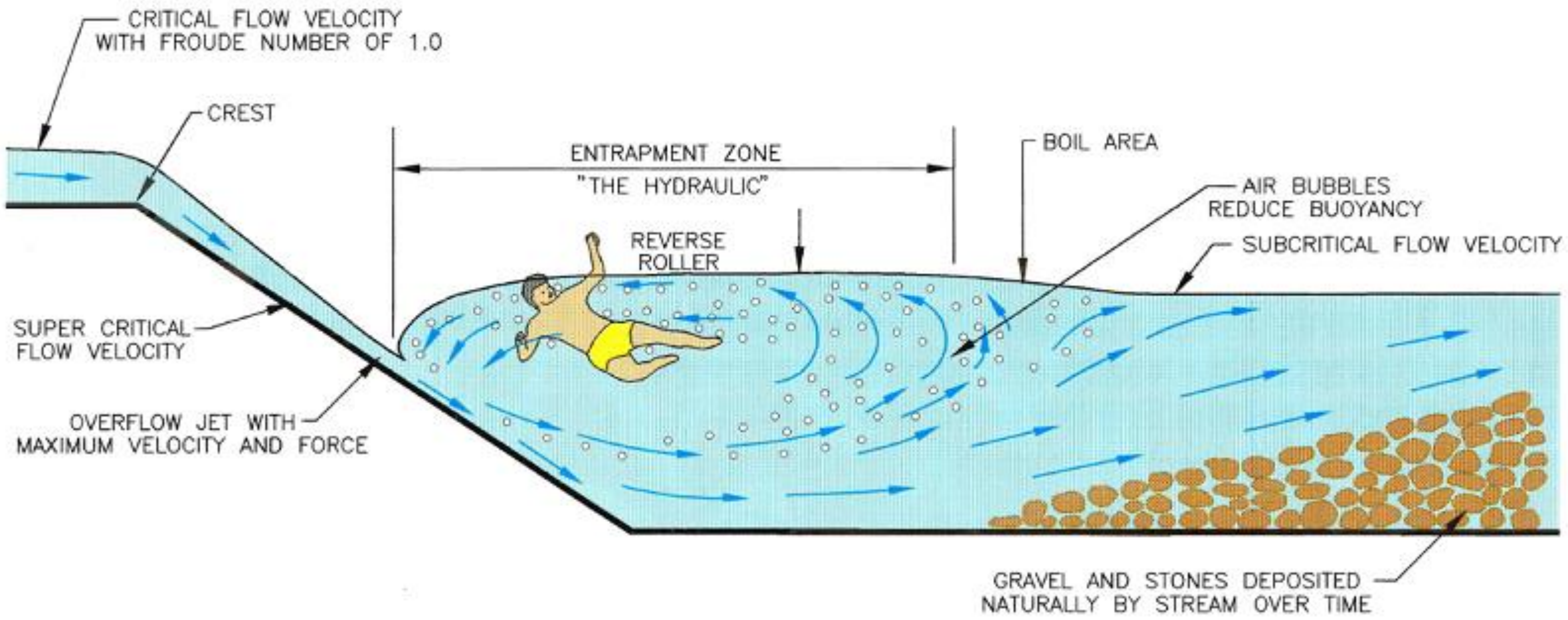
The Hydraulic Jump

Hydraulic Jump Variation

The Hydraulic Jump

The Dangerous Hydraulic Jump

- Strong Jumps may pose a significant public safety hazard.

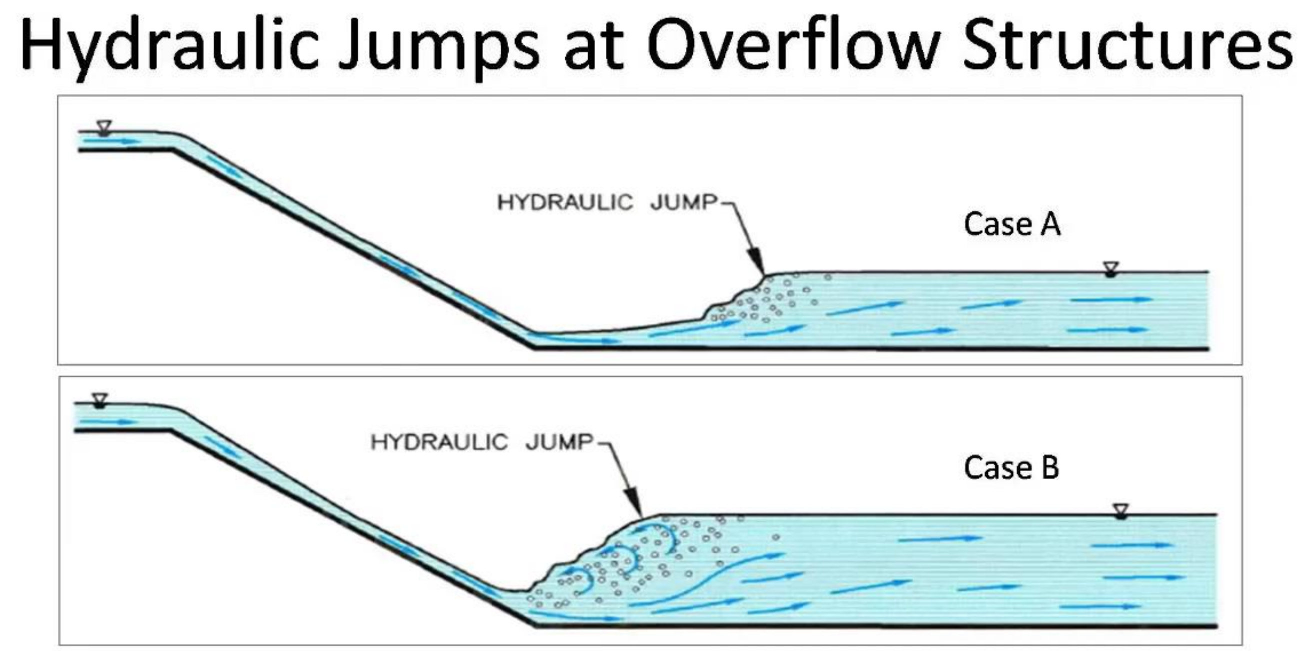


海纳百川 有容乃大

The Hydraulic Jump

The Dangerous Hydraulic Jump

- Strong Jumps may pose a significant public safety hazard.



Figures Courtesy of Wright Water Engineers Inc.

海纳百川 有容乃大

The Hydraulic Jump

An Application of Hydraulic Jump

- Strong Jumps can dissipate the energy of the flow.

海纳百川 有容乃大

The Hydraulic Jump

Example

A hydraulic jump occurs in a rectangular channel 3 m wide. The water depth before the jump is 0.6 m, and after the jump is 1.6 m. Compute (a) the flow rate in the channel (b) the critical depth (c) the head loss in the jump?

$$\frac{y_2}{y_1} = \frac{1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

$$Fr = \frac{V}{\sqrt{gy}}$$

$$y_c = \left[\frac{Q^2}{gb^2}\right]^{1/3}$$

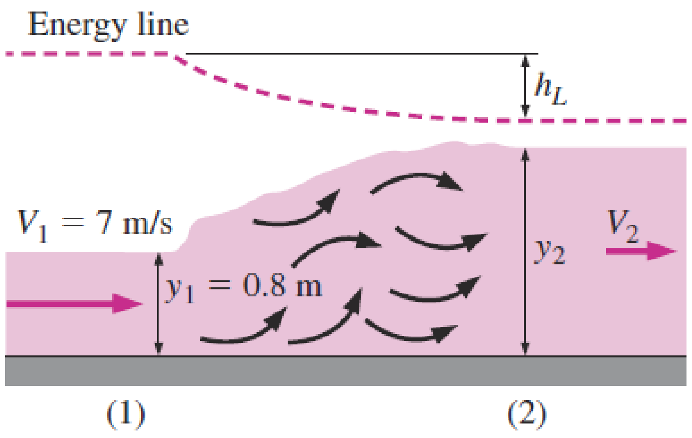
$$H_l = \frac{[y_2 - y_1]^3}{4y_1y_2}$$

海纳百川 有容乃大

The Hydraulic Jump

Example

Water discharging into a 10-m-wide rectangular horizontal channel from a sluice gate is observed to have undergone a hydraulic jump. The flow depth and velocity before the jump are 0.8 m and 7 m/s, respectively. Determine (a) the flow depth and the Froude number after the jump, (b) the head loss and the dissipation ratio, and (c) the wasted power production potential due to the hydraulic jump.

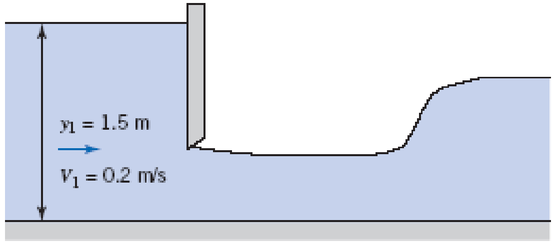


海纳百川 有容乃大

Homework

Problem 11.24

Flow through a sluice gate is shown. Estimate the water depth and velocity after the gate (well before the hydraulic jump)



$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + y_2$$

$$\frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2$$

$$V = q/y_2$$

$$\frac{V_1^2}{2g} + y_1 = \frac{q^2}{2gy_2^2} + y_2$$

$$Fr_1 = V_1/\sqrt{gy_1} \qquad Fr_2 = V_2/\sqrt{gy_2}$$

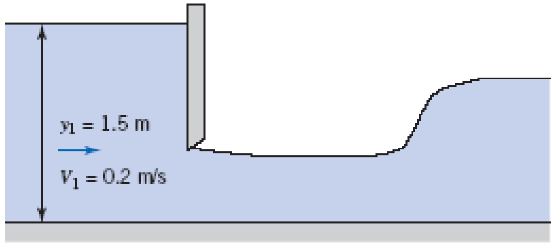
Homework

Problem 11.30

Estimate the depth of water before and after the jump for the hydraulic jump downstream of the sluice gate.

$$Fr > 1 \ supercritical$$

$$\frac{y_3}{y_2} = \frac{1}{2}\left(-1 + \sqrt{1 + 8Fr_2^2}\right)$$



海纳百川 有容乃大

Homework

This assignment is due by **6pm on April 29th**.

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.