Sichuan University - Pittsburgh Institute

ME 1071: Applied Fluids

Lecture 6 Flow in Open Channels

Spring 2021

Weekly Study Plan

Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	**Apr. 6**	**Exam I**
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 25	Chapter 11: Flow in Open Channels
9	Apr. 27	Chapter 11: Flow in Open Channels
10	May. 11	Exam II Review
11	**May. 18**	**Exam II**
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review

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Outlines

- **Basic Concepts and Definitions**
- **Energy Equation for Open-Channel Flows**
- **Localized Effect of Area Change (Frictionless Flow)**

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Basic Concepts and Definitions

Open-Channel Flow

- A type of liquid flow within a conduit or in channel with a free surface
- Natural or human-made channel flows
- Flood, agricultural irrigation, hydroelectric power, etc.

Amazon River

Dujiangyan Irrigation System

The Three Gorges Dam

Hoover Dam

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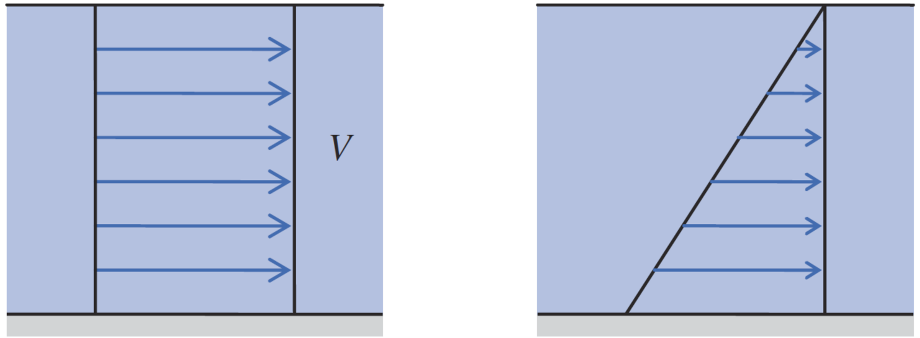
Basic Concepts and Definitions

Open-Channel Flow

- Flows for which the local effects of area change predominate and frictional forces may be neglected.
- Flow with an abrupt change in depth.
- Flow at what is called normal depth.
- Gradually varied flow.

Analysis Assumption

- Gravity is driving force
- One dimensional and steady
- Uniform velocity
- Pressure distributions approximated as hydrostatics



(*a*) Approximate velocity profile

(*b*) Approximate pressure distribution (gage)

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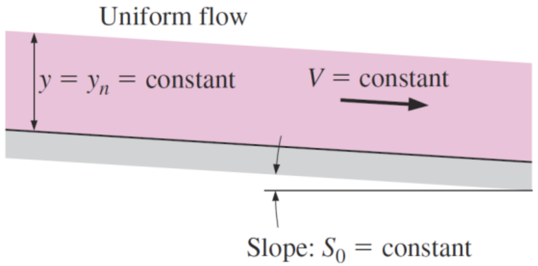
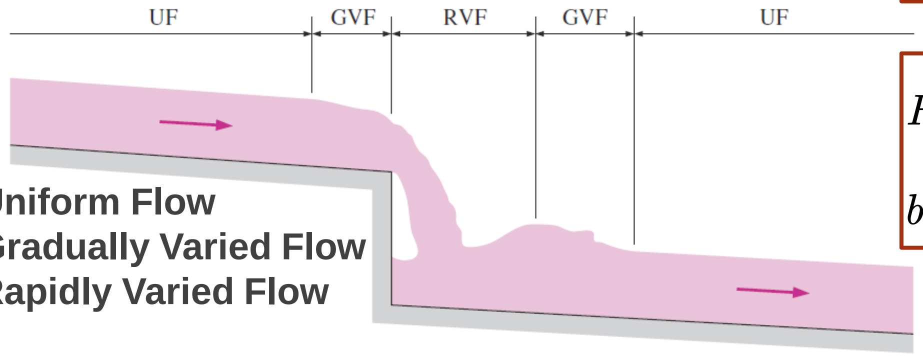
Basic Concepts and Definitions

Channel Geometry





Uniform Flow
Gradually Varied Flow
Rapidly Varied Flow

Hydraulic radius: $R_h = \dfrac{A}{P}$

A: cross section area

P: wetted perimeter

Hydraulic diameter: $D_h = \dfrac{4A}{P}$

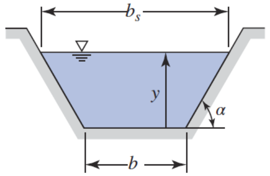
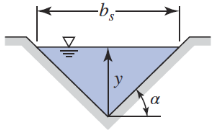
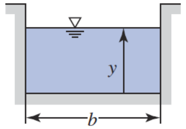
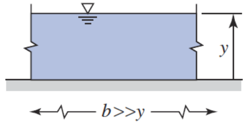
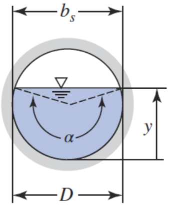
Hydraulic Depth: $y_h = \dfrac{A}{b_s}$

b_s*: the width at the surface*

For Open Channel Flow, if a Reynolds is based on the hydraulic radius

Laminar $\mathrm{Re} \lesssim 500$, *Turbulent* $\mathrm{Re} \gtrsim 2500$

Table 11.1
Geometric Properties of Common Open-Channel Shapes

Shape	Section	Flow Area, A	Wetted Perimeter, P	Hydraulic Radius, R_h
Trapezoidal		$y(b + y \cot \alpha)$	$b + \dfrac{2y}{\sin \alpha}$	$\dfrac{y(b + y\cot\alpha)}{b + \dfrac{2y}{\sin\alpha}}$
Triangular		$y^2 \cot \alpha$	$\dfrac{2y}{\sin \alpha}$	$\dfrac{y \cos \alpha}{2}$
Rectangular		by	$b + 2y$	$\dfrac{by}{b + 2y}$
Wide Flat		by	b	y
Circular		$(\alpha - \sin\alpha)\dfrac{D^2}{8}$	$\dfrac{\alpha D}{2}$	$\dfrac{D}{4}\left(1 - \dfrac{\sin\alpha}{\alpha}\right)$

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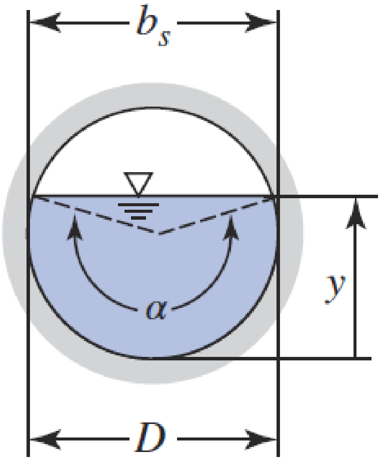
Basic Concepts and Definitions

Example

Find out the flow area A, wetted perimeter P and hydraulic radius R_h for the open-channel flow in a circular conduit.

Hint: consider (1) $y < D/2$; (2) $y > D/2$



Homework 11.1

$$(\alpha - \sin\alpha)\frac{D^2}{8} \qquad \frac{\alpha D}{2} \qquad \frac{D}{4}\left(1 - \frac{\sin\alpha}{\alpha}\right)$$

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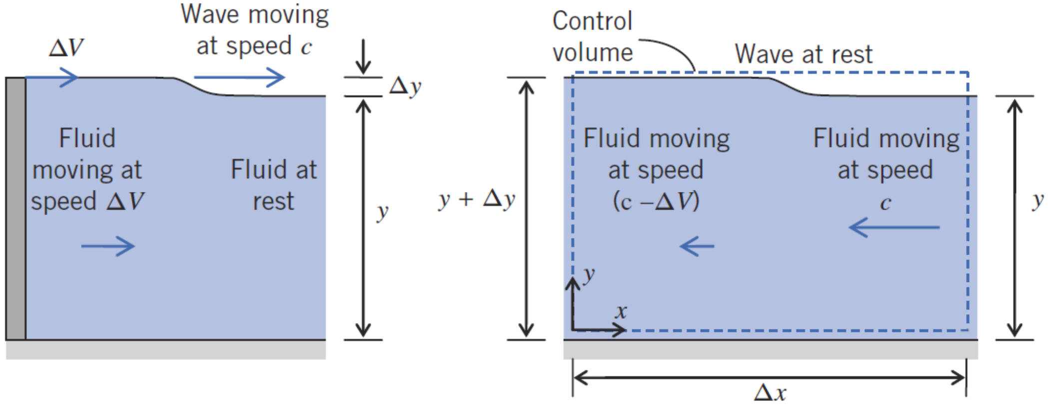
Basic Concepts and Definitions

Speed of Surface Waves and the Froude Number

Assumptions

1. Steady flow.
2. Incompressible flow.
3. Uniform velocity at each section.
4. Hydrostatic pressure distribution at each section.
5. Frictionless flow.



(a) Absolute coordinates　　(b) Coordinates at rest relative to wave

Continuity

$$(c - \Delta V)\{(y + \Delta y)b\} - cyb = 0 \qquad \Delta V = c\frac{\Delta y}{y + \Delta y}$$

Momentum

$$F_{S_x} = \sum_{CS} u\rho \vec{V} \cdot \vec{A}$$
$$= -(c - \Delta V)\rho\{(c - \Delta V)(y + \Delta y)b\} - c\rho\{-cyb\}$$

$$F_{S_x} = F_{R_{\text{left}}} - F_{R_{\text{right}}} = (p_c A)_{\text{left}} - (p_c A)_{\text{right}}$$
$$= \frac{\rho g b}{2}(y + \Delta y)^2 - \frac{\rho g b}{2} y^2$$

$$\rightarrow c^2 = gy\left(1 + \frac{\Delta y}{2y}\right)\left(1 + \frac{\Delta y}{y}\right)$$

$$\boxed{c = \sqrt{gy}}$$

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Basic Concepts and Definitions

Example

You are enjoying a summer's afternoon relaxing in a rowboat on a pond. You decide to find out how deep the water is by splashing your oar and timing how long it takes the wave you produce to reach the edge of the pond. (The pond is artificial; so it has approximately the same depth even to the shore.) From floats installed in the pond, you know you're 6.0 m from shore, and you measure the time for the wave to reach the edge to be 1.5 s. Estimate the pond depth. Does it matter if it's a freshwater pond or if it's filled with seawater?

$$y = \frac{L^2}{g\Delta t^2}$$

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Basic Concepts and Definitions

Speed of Surface Waves and the Froude Number

The Froude number

$$\boxed{Retangular\ channels:\ Fr = \frac{V}{\sqrt{gy}}}$$

$$\boxed{Nonretangular\ channels:\ Fr = \frac{V}{\sqrt{gy_h}}}$$

- ***Fr*<1** Flow is *subcritical*, *tranquil*, or *streaming*.

Disturbances can travel upstream; downstream conditions can affect the flow upstream. The flow can gradually adjust to the disturbance.

- ***Fr*=1** Flow is *critical*.
- ***Fr*>1** Flow is *supercritical*, *rapid*, or *shooting*.

No disturbance can travel upstream; downstream conditions cannot be felt upstream. The flow may “violently” respond to the disturbance because the flow has no chance to adjust to the disturbance before encountering it.

Outlines

- Basic Concepts and Definitions
- **Energy Equation for Open-Channel Flows**
- Localized Effect of Area Change (Frictionless Flow)

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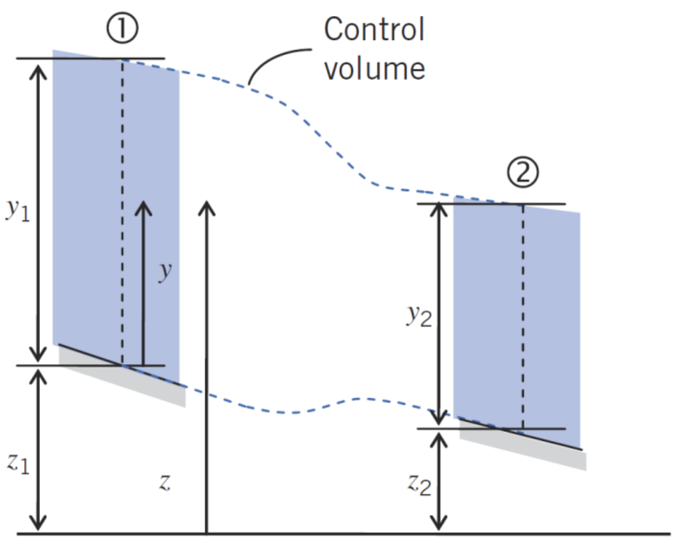
Energy Equation for Open-Channel Flows

Assumptions

1. Steady flow.
2. Incompressible flow.
3. Uniform velocity at a section.
4. Gradually varying depth so that pressure distribution is hydrostatic.
5. Small bed slope.
6. $W_s = W_{shear} = W_{other} = 0$.



Energy Equation for Open-Channel Flow

$$\boxed{\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_l}$$

Total Head or Energy Head

$$H = \frac{V^2}{2g} + y + z$$

$$H_1 - H_2 = H_l$$

Specific Energy

$$E = \frac{V^2}{2g} + y$$

$$E_1 - E_2 + z_1 - z_2 = H_l$$

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Energy Equation for Open-Channel Flows

The Specific Energy

- E indicates actual energy (kinetic plus potential/pressure per unit mass flow rate) being carried by the flow

$$E = \frac{V^2}{2g} + y$$

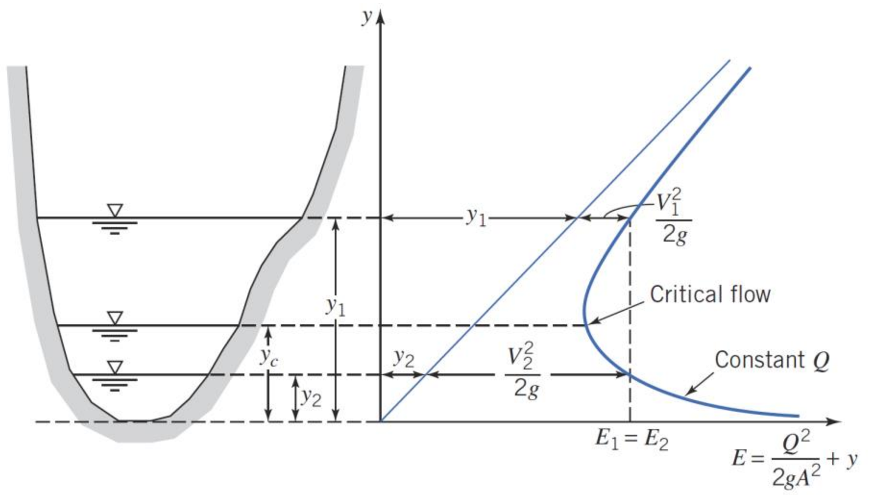

Fig. 11.7 Specific energy curve for a given flow rate.

Critical Depth (Fr = 1)

$$\boxed{Q^2 = \frac{gA_c^3}{b_{sc}}} \qquad \boxed{V_c = \sqrt{g y_{hc}}}$$

Minimum Specific Energy

- the specific energy is at its minimum at critical conditions, i.e., **Fr = 1**.

$$\boxed{y_c = \left[\frac{Q^2}{gb^2}\right]^{1/3} \qquad E_{\min} = \frac{3}{2} y_c \qquad (Rectangular\ channel)}$$

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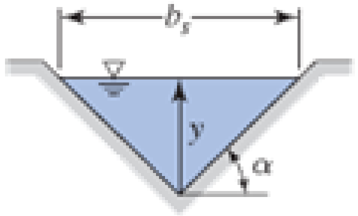
Energy Equation for Open-Channel Flows

Example

A triangular section channel of $\alpha = 60°$ has a flow rate of 300 m³/s. Find the critical depth for this flow rate. Verify the Fr is 1.



$$Q^2 = \frac{gA_c^3}{b_{sc}} = \frac{g[y_c^2 \cot\alpha]^3}{2y_c \cot\alpha} = \frac{1}{2} g y_c^5 \cot^2\alpha$$

$$y_c = \left[\frac{2Q^2 \tan^2\alpha}{g}\right]^{1/5}$$

$$Fr = \frac{V_c}{\sqrt{g y_h}} = \frac{Q}{y_c^2 \cot\alpha}\sqrt{b_{sc}/gA_c}$$

$$Fr = \frac{Q}{y_c^2 \cot\alpha}\sqrt{2y_c \cot\alpha / g y_c^2 \cot\alpha} = \frac{Q}{y_c^2 \cot\alpha}\sqrt{2/g y_c}$$

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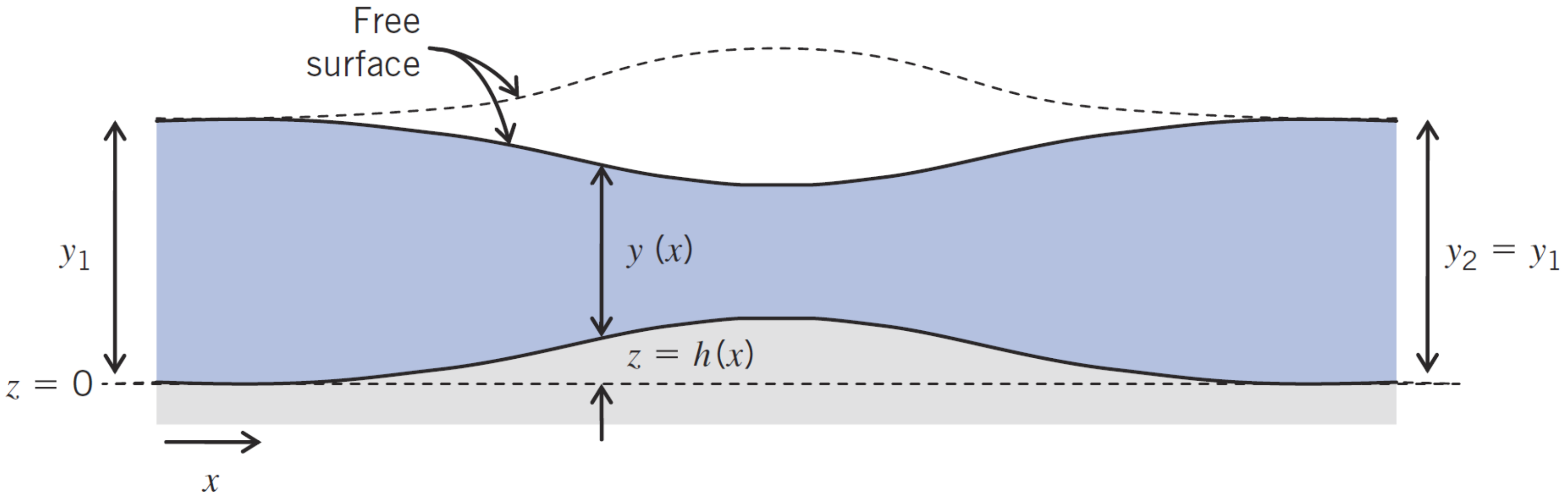
Localized Effect of Area Change

Flow over a Bump

$$\frac{V_1^2}{2g}+y_1+z_1=\frac{V_2^2}{2g}+y_2+z_2=\frac{V^2}{2g}+y+z=\text{const}$$



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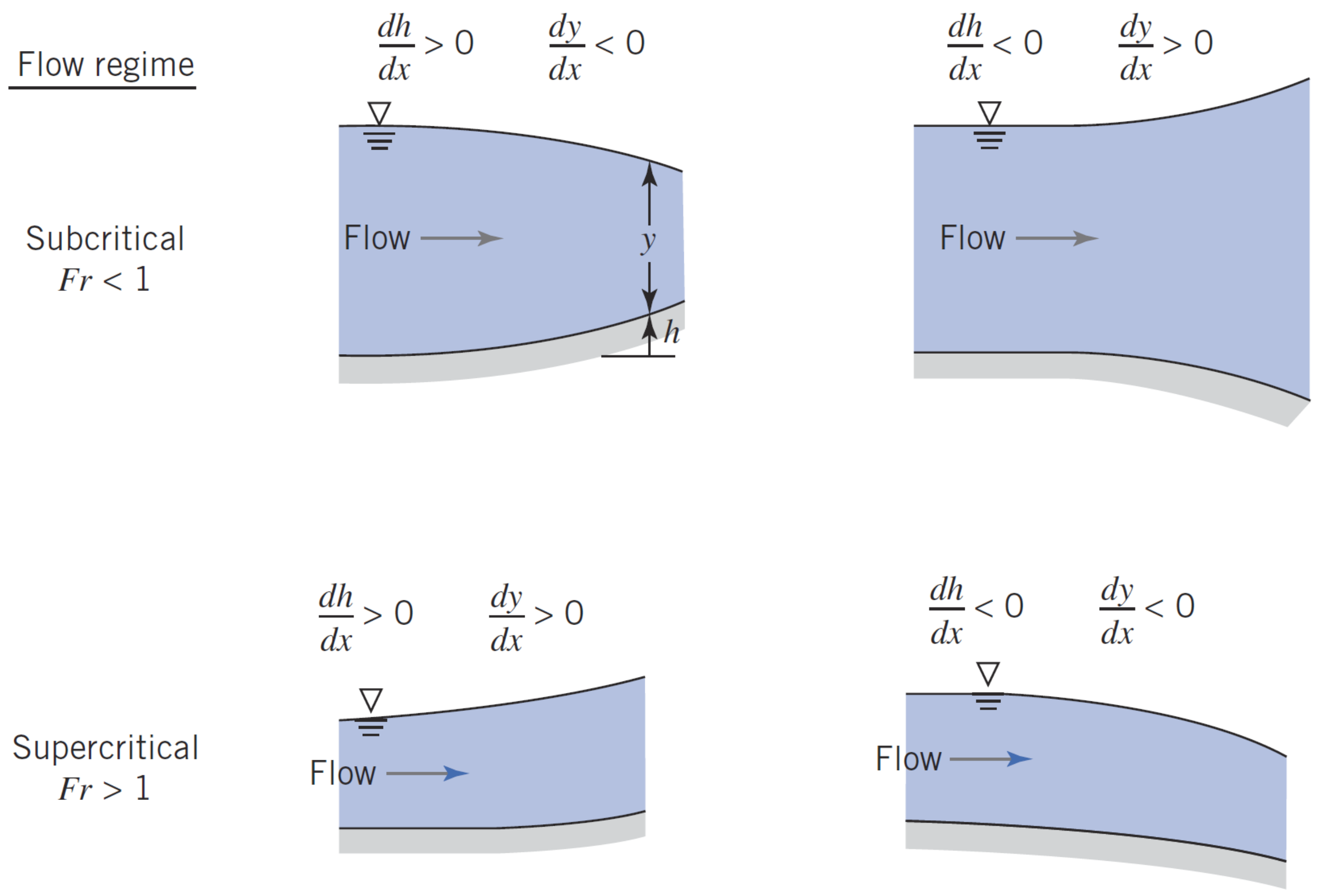
Localized Effect of Area Change

Flow over a Bump

$$\frac{dy}{dx} = \frac{1}{Fr^2 - 1}\frac{dh}{dx}$$



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Localized Effect of Area Change

Example

A rectangular channel 2 m wide has a flow of 2.4 m^3/s at a depth of 1.0 m. Determine whether critical depth occurs at:

a) A section where a bump of height h = 0.20 m is in the channel bed.

b) A side wall constriction that reduces the channel width to 1.7 m.

c) A combined bump and side wall constrictions.

$$y_c = \left[\frac{Q^2}{gb^2}\right]^{1/3}$$

$$E_1 = y_1 + \frac{Q^2}{2gA^2} = y_1 + \frac{Q^2}{2gb^2y_1^2}$$

$$E_{min} = \frac{3}{2}y_c$$

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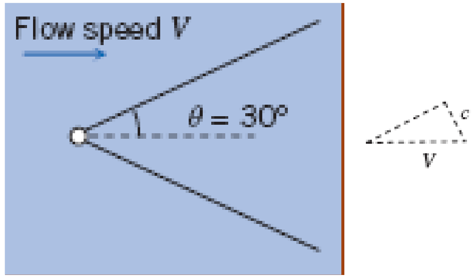
Homework

Problem 11.7

Surface waves are caused by a sharp object that just touches the free surface of a stream of flowing water, forming the wave pattern shown. The stream depth is 150 mm. Determine the flow speed and Froude Number. Note that the wave travels at speed c normal to the wave front, as shown in the velocity diagram.

$$c = \sqrt{gy}$$

$$Fr = \frac{V}{c}$$



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Homework

Problem 11.19

A rectangular channel 3 m wide carries a discharge of 0.57m^3/s at 0.27 m depth. A smooth bump 0.06 m high is placed on the floor of the channel. Estimate the local change in flow depth caused by the bump.

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+y_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+y_2+h$$

$$\frac{V_1^2}{2g}+y_1-h=\frac{V_2^2}{2g}+y_2$$

$$V=\frac{Q}{b\,y}$$

$$\frac{Q^2}{2gb^2y_1^2}+y_1-h=\frac{Q^2}{2gb^2y_2^2}+y_2$$

$$Fr_1=V_1/\sqrt{gy_1} \qquad Fr_2=V_2/\sqrt{gy_2}$$

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Homework

This assignment is due by **6pm on April 27th**.

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.

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