Sichuan University - Pittsburgh Institute

ME 1071: Applied Fluids

Lecture 4 External Incompressible Viscous Flow

Spring 2021

Weekly Study Plan

Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	**Apr. 6**	**Exam I**
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 27	Chapter 9: External Incompressible Viscous Flow
9	May. 4	Chapter 11: Flow in Open Channels
10	May. 11	Chapter 11/Exam II Review
11	**May. 18**	**Exam II**
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review

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Outlines

- Introduction
- Boundary Layers
- External Flow Solution
  - Laminar Flow
  - Momentum Integral Equation
- Pressure Gradient Flow

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- **Introduction**
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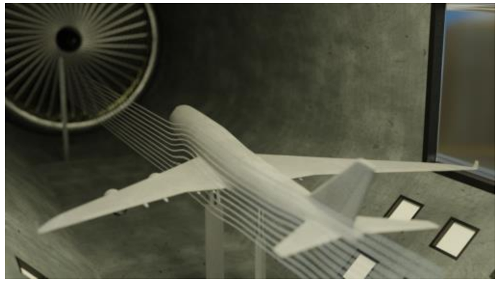
Introduction

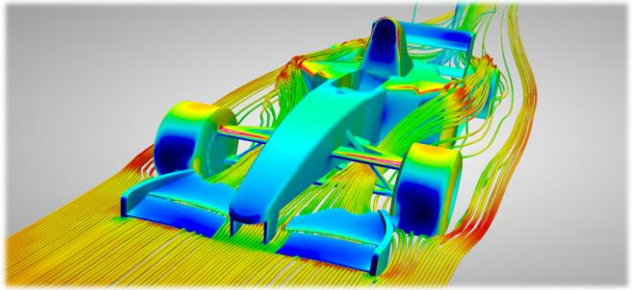
External Flow

- External flows are flows over bodies immersed in an unbounded fluid
- Boundary layers develop freely, without constraints imposed by adjacent surfaces



Lift and Drag

- Birds
- Flight and ground vehicles

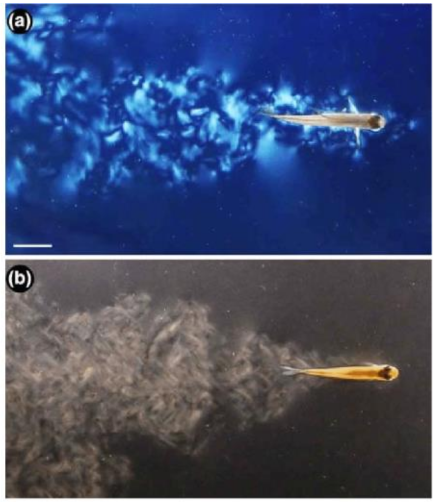


Energy Harvesting

- Wind turbine

Submerged Bodies

- Fish
- Boats
- Submarine



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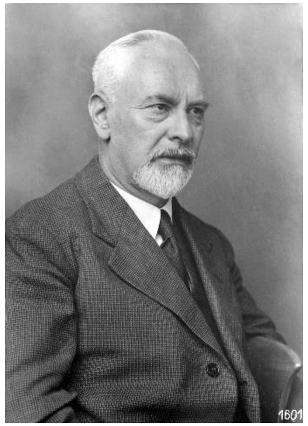
Boundary Layer

Concept

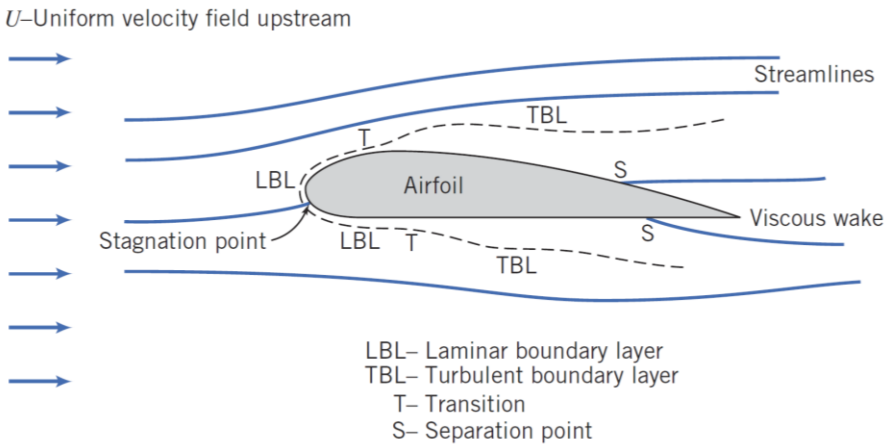
- The thin region of flow adjacent to a surface, where the flow is retarded by the influence of friction between a solid surface and the fluid.
- Using Prandtl's concept of a boundary layer adjacent to an aerodynamic surface, the Navier-Stokes equations can be reduced to a more tractable (容易处理的) form called the boundary-layer equations.

Two Important Regions

- Only in the thin region adjacent to a solid boundary (the boundary layer) is the effect of viscosity important.
- In the region outside of the boundary layer, the effect of viscosity is negligible and the fluid may be treated as inviscid.



Ludwig Prandtl



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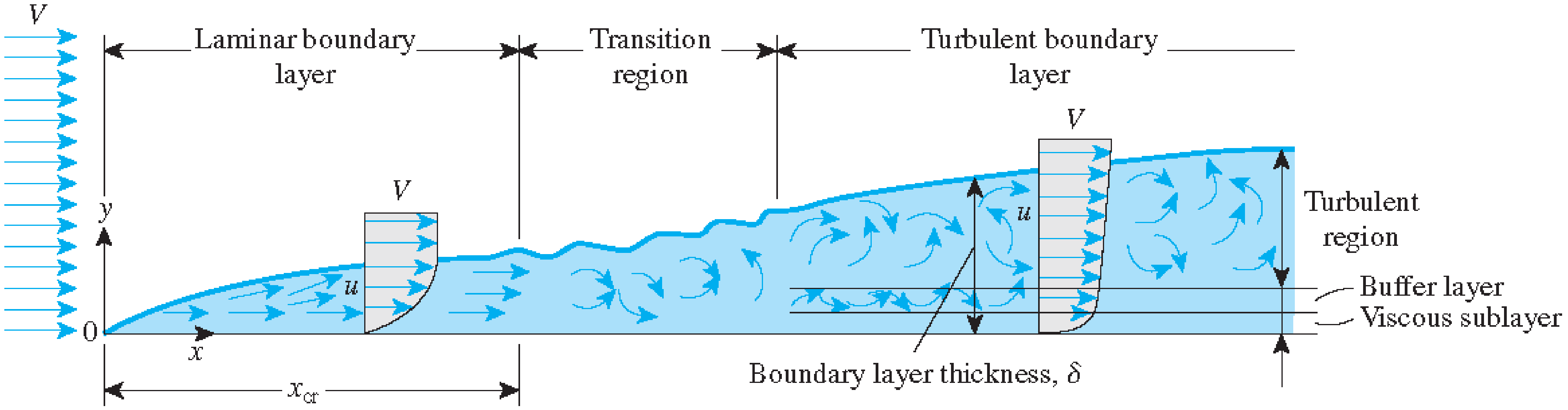
Boundary Layer

Regions for Boundary Layer Development over a Flat Plate

- The simplest possible boundary layer
- constant pressure field and zero pressure gradient
- Laminar from the leading edge and transits to turbulent downstream

$$\boxed{\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} \approx 500,000}$$



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Boundary Layer

Disturbance Thickness δ

- δ is defined as the distance above the wall where $u = 0.99U$

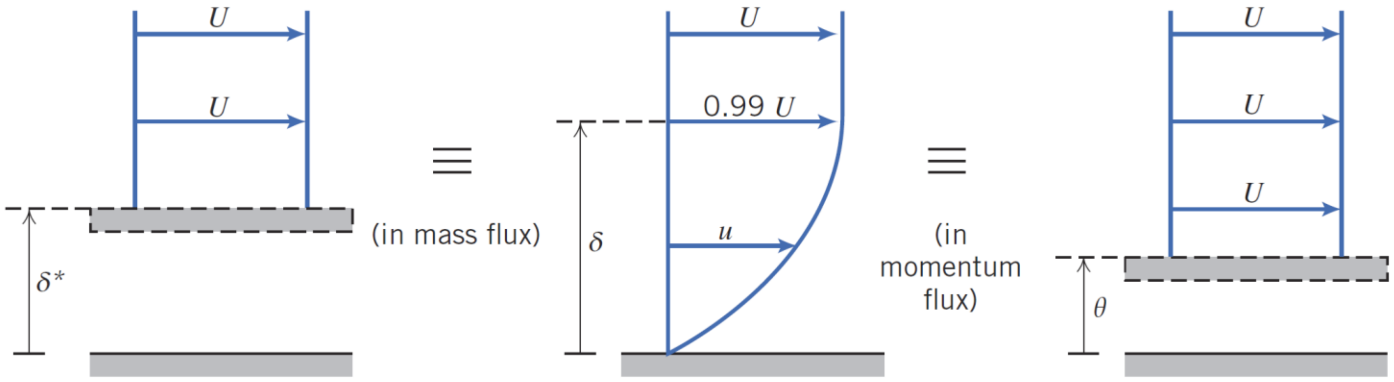
$$\boxed{u = 0.99U}$$

Displacement Thickness δ*

$$\boxed{\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \left(1 - \frac{u}{U}\right) dy}$$

Momentum Thickness θ

$$\boxed{\theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy}$$



(a) Displacement thickness, δ*   (b) Disturbance thickness, δ   (c) Momentum thickness, θ

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Boundary Layer

Engineering Analysis Assumptions

1. $u \rightarrow U$ at $y = \delta$
2. $du/dy \rightarrow 0$ at $y = \delta$
3. $v \ll U$ within boundary layer
4. Pressure variation across the thin boundary layer is negligible.

All these thicknesses (disturbance, displacement, momentum) are larger in turbulent than in laminar boundary layer.

Example

A laboratory wind tunnel has a test section that is 305 mm square. Boundary-layer velocity profiles are measured at two cross-sections and displacement thicknesses are evaluated from the measured profiles. At section 1, where the freestream speed is U_1=26m/s, the displacement thickness is δ^*_1=1.5 mm. At section 2, located downstream from section , δ^*_2=2.1mm. Calculate the change in static pressure between sections 1 and 2. Express the result as a fraction of the freestream dynamic pressure at section 1. Assume standard atmosphere conditions.

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Laminar Boundary Layer Equations

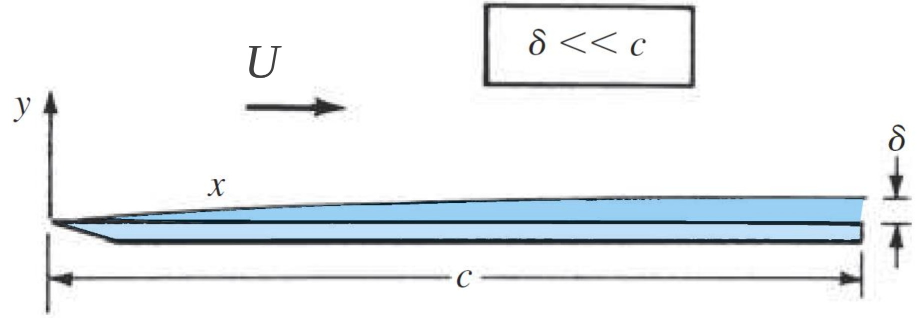
The two-dimensional, steady flow, boundary-layer equations

$$\text{Continuity:} \qquad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$x\ \text{momentum:} \qquad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

$$y\ \text{momentum:} \qquad \frac{\partial p}{\partial y} = 0$$

$$\text{Energy:} \qquad \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{dp_e}{dx} + \mu \left(\frac{\partial u}{\partial y}\right)^2$$



Assumption 1: δ << c
Assumption 2: $1/Re = O(\delta^2)$
Assumption 3: Mach number is not too large!

To complete the system

$$p = \rho R T$$

$$h = c_p T$$

The boundary conditions

$$\text{At the wall:} \qquad y = 0 \quad u = 0 \quad v = 0 \quad T = T_w$$

$$\text{At the boundary-layer edge:} \qquad y \to \infty \quad u \to U \quad T \to T_e$$

Solutions:
(1) Classical solutions;
(2) Numerical solutions.

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Laminar Flow over a Flat Plate

Incompressible boundary-layer equations

General 2-D steady equations

$$\text{Continuity:} \quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$x \text{ momentum:} \quad \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp_e}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

$$y \text{ momentum:} \quad \frac{\partial p}{\partial y} = 0$$

$$\text{Energy:} \quad \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{dp_e}{dx} + \mu \left(\frac{\partial u}{\partial y}\right)^2$$

Incompressible flow over a flat plate

Constant ρ $\longrightarrow$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Constant ρ, μ
dp / *dx* = 0 $\longrightarrow$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

Kinematic viscosity
(运动黏度)
$\nu \equiv \mu / \rho$

$\longrightarrow$

$$\frac{\partial p}{\partial y} = 0$$

How to solve these coupled equations?

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Laminar Flow over a Flat Plate

Transformation of the incompressible boundary-layer equations

- Transform the independent variables (x, y) to (ξ, λ)

$$\xi = x \quad and \quad \lambda = y\sqrt{\frac{U}{\nu x}}$$

$$\frac{\partial \xi}{\partial x} = 1 \qquad \frac{\partial \xi}{\partial y} = 0 \qquad \frac{\partial \lambda}{\partial y} = \sqrt{\frac{U}{\nu x}}$$

Chain rule

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} + \frac{\partial \lambda}{\partial x}\frac{\partial}{\partial \lambda}$$

$$\frac{\partial}{\partial y} = \sqrt{\frac{U}{\nu x}}\frac{\partial}{\partial \lambda}$$

$$\frac{\partial^2}{\partial y^2} = \frac{U}{\nu x}\frac{\partial^2}{\partial \lambda^2}$$

Define stream function (satisfies the continuity equation)

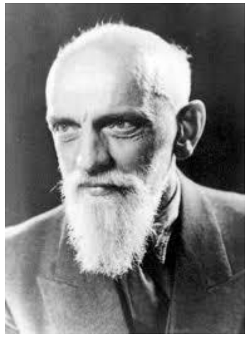
$$\psi = \sqrt{\nu x U}\, f(\lambda)$$

$$u = \frac{\partial \psi}{\partial y} = U f'(\lambda) \qquad v = -\frac{\partial \psi}{\partial x} = -\frac{1}{2}\sqrt{\frac{\nu U}{x}} f - \sqrt{\nu x U}\frac{\partial \lambda}{\partial x} f'$$

$$U f'\left(U\frac{\partial \lambda}{\partial x} f''\right) - \left(\frac{1}{2}\sqrt{\frac{\nu U}{x}} f + \sqrt{\nu x U}\frac{\partial \eta}{\partial x} f'\right) U\sqrt{\frac{U}{\nu x}} f'' = \nu U \frac{U}{\nu x} f'''$$

$$\boxed{2f''' + f f'' = 0}$$

Blasius' equation
(布拉休斯方程)



H. Blasius

Partial differential equations become an ordinary differential equation (ODE)!
(偏微分方程组变为单个常微分方程）

Laminar Flow over a Flat Plate

Solving Blasius's equation

- Third-order, nonlinear, ordinary differential equation (can be solved using Runge-Kutta method)

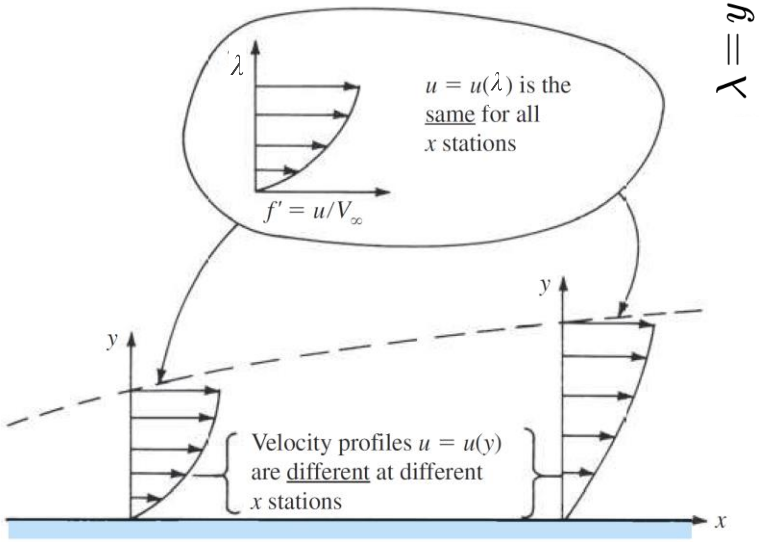
$$\boxed{2f''' + ff'' = 0}$$

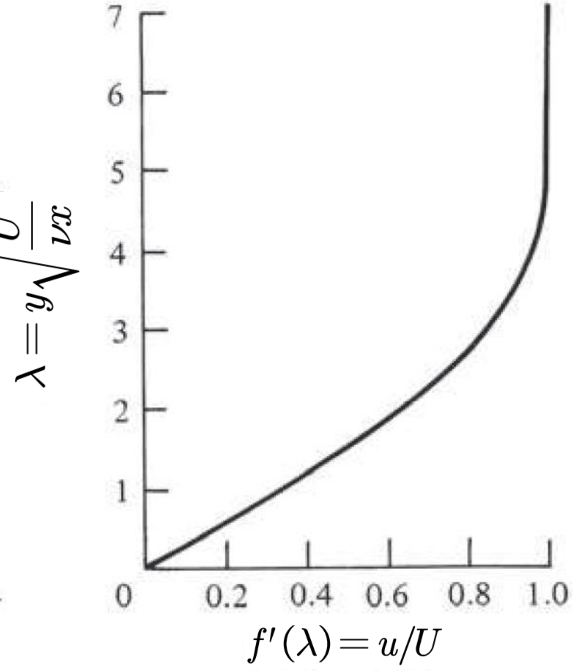
Boundary conditions:

At $\lambda = 0$: $f = 0$, $f' = 0$; *At $\lambda \to \infty$: $f' = 1$;*

At $\lambda = 0$: $f''(0)$ needs to be assumed for shooting method.

- **Self-similar solution**
  - Solutions are the same when against a similarity variable λ.
  - The governing equations can be re
    to one or more ODEs.
  - Occurs only for certain types of flo





Incompressible velocity profile for a flat plate; solution of the Blasius equation.

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Laminar Flow over a Flat Plate

Self-similar solution

- Local skin-friction coefficient

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2\mu}{\rho U^2}\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{2\mu}{\rho U^2}U\sqrt{\frac{U}{\nu x}}f''(0)$$

$$= 2\sqrt{\frac{\mu}{\rho U x}}f''(0) = \frac{2f''(0)}{\sqrt{\mathrm{Re}_x}}$$

$$\boxed{c_f = \frac{0.664}{\sqrt{\mathrm{Re}_x}}}$$

$$\boxed{\tau_w = \frac{0.332\rho U^2}{\sqrt{\mathrm{Re}_x}}}$$

- Skin friction drag coefficient (c ≡ chord length)

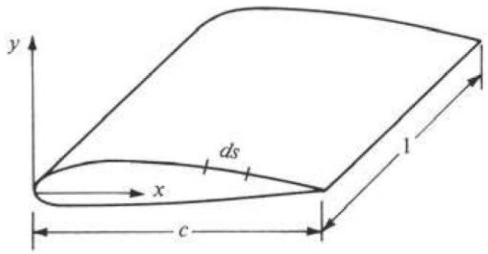
$$C_f = \frac{1}{c}\int_0^c c_f dx = \frac{0.664}{c}\int_0^c x^{-1/2}dx = \frac{1.328}{c}\sqrt{\frac{\mu c}{\rho U}}$$

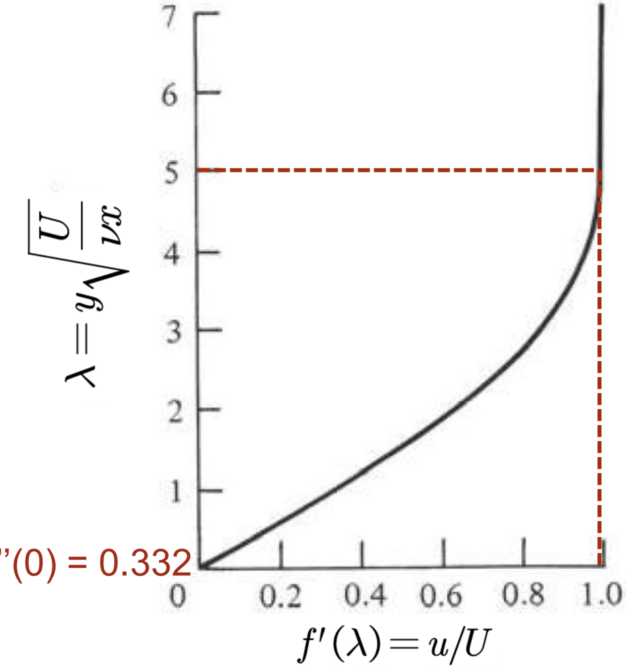
$$\boxed{c_f = \frac{1.328}{\sqrt{\mathrm{Re}_c}}}$$

- Boundary-layer thickness

$$\lambda = y\sqrt{\frac{U}{\nu x}} = \delta\sqrt{\frac{U}{\nu x}} \approx 5.0$$

$$\boxed{\delta = \frac{5.0x}{\sqrt{\mathrm{Re}_x}}}$$





Incompressible velocity profile for a flat plate; solution of the Blasius equation.

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Laminar Flow over a Flat Plate

Self-similar solution

- Displacement thickness

$$\delta^* = \int_0^{y_1}\left(1-\frac{u}{U}\right)dy = \sqrt{\frac{\nu x}{U}}\int_0^{\lambda_1}[1-f'(\lambda)]d\lambda = \sqrt{\frac{\nu x}{U}}[\lambda_1 - f(\lambda_1)] \xrightarrow[\text{when } \lambda > 5]{\lambda - f(\lambda) = 1.72} \boxed{\delta^* = \frac{1.72x}{\sqrt{\mathrm{Re}_x}}}$$

- Momentum thickness

$$\theta = \int_0^{y_1}\frac{u}{U}\left(1-\frac{u}{U}\right)dy \xrightarrow{\text{Variable transformation}} \theta = \sqrt{\frac{\nu x}{U}}\int_0^{\lambda_1} f'[1-f']d\lambda \xrightarrow{= 0.664} \boxed{\theta = \frac{0.664x}{\sqrt{\mathrm{Re}_x}}}$$

- Relative relations

$$\left.\begin{aligned}\delta &= \frac{5.0x}{\sqrt{\mathrm{Re}_x}}\\ \delta^* &= \frac{1.72x}{\sqrt{\mathrm{Re}_x}}\\ \theta &= \frac{0.664x}{\sqrt{\mathrm{Re}_x}}\end{aligned}\right\} \quad \delta^* = 0.34\delta \qquad \theta = 0.13\delta$$

$$\left.\begin{aligned}C_f &= \frac{1.328}{\sqrt{\mathrm{Re}_c}}\\ \theta_{x=c} &= \frac{0.664c}{\sqrt{\mathrm{Re}_c}}\end{aligned}\right\} \quad C_f = \frac{2\theta_{x=c}}{c}$$

The integrated skin-friction coefficient for the flat plate is proportional to θ at the trailing edge.

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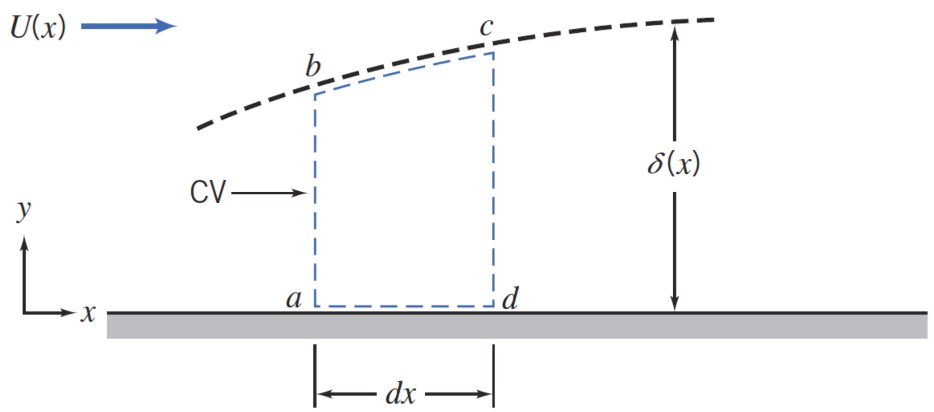
Momentum Integral Equation

Provides Alternative Approximation to Blasius Solution

$$\tau_w = \frac{\partial}{\partial x} U^2 \int_0^\delta \rho \frac{u}{U}\left(1 - \frac{u}{U}\right) dy + U\frac{dU}{dx}\int_0^\delta \rho\left(1 - \frac{u}{U}\right) dy$$

$$\boxed{\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}}$$

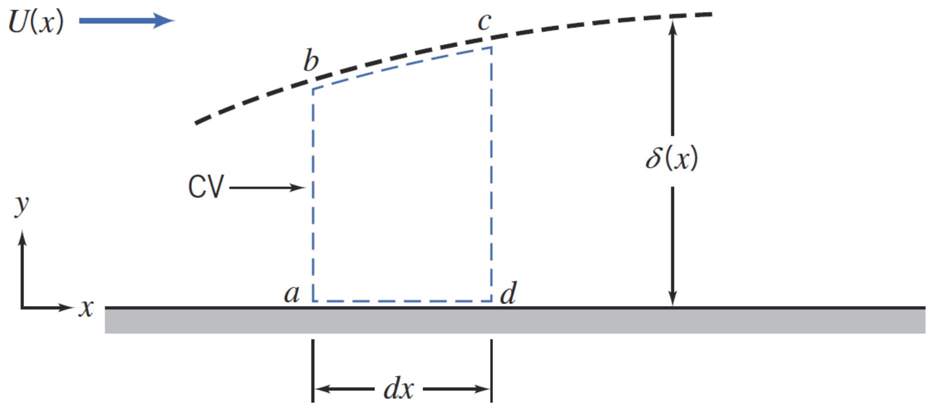


- Obtained by applying continuity and *x* momentum to a differential control volume. The flow is steady, incompressible, two-dimensional with no body forces parallel to the surface. Equation holds for both laminar and turbulent flows.

Momentum Integral Equation

Equation is used to estimate the boundary-layer thickness as a function of *x*:

$$\boxed{\frac{\tau_w}{\rho} = \frac{d}{dx}(U^2\theta) + \delta^* U \frac{dU}{dx}}$$



1. Obtain a first approximation to the freestream velocity distribution, *U*(*x*). The pressure in the boundary layer is related to the freestream velocity, *U*(*x*), using the Bernoulli equation
2. Assume a reasonable velocity-profile shape inside the boundary layer
3. Derive an expression for τ_w using the results obtained from item 2

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Momentum Integral Equation

Flow with Zero Gradient

$$\tau_w = \frac{\partial}{\partial x} U^2 \int_0^\delta \rho \frac{u}{U}\left(1 - \frac{u}{U}\right) dy + U\frac{dU}{dx}\int_0^\delta \rho\left(1 - \frac{u}{U}\right) dy \xrightarrow{U(x) = U = \text{constant}} \tau_w = \rho U^2 \frac{d}{dx}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

Laminar Flow

$$\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2 \qquad \tau_w = \frac{2\mu U}{\delta} \qquad \frac{\delta}{x} = \sqrt{\frac{30\mu}{\rho U x}} = \frac{5.48}{\sqrt{\mathrm{Re}_x}} \qquad C_f = \frac{0.730}{\sqrt{\mathrm{Re}_x}}$$

Turbulent Flow

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} \qquad \frac{\delta}{x} = 0.382\left(\frac{\nu}{Ux}\right)^{1/5} = \frac{0.382}{\mathrm{Re}_x{}^{1/5}} \qquad C_f = \frac{0.0594}{\mathrm{Re}_x{}^{1/5}}$$

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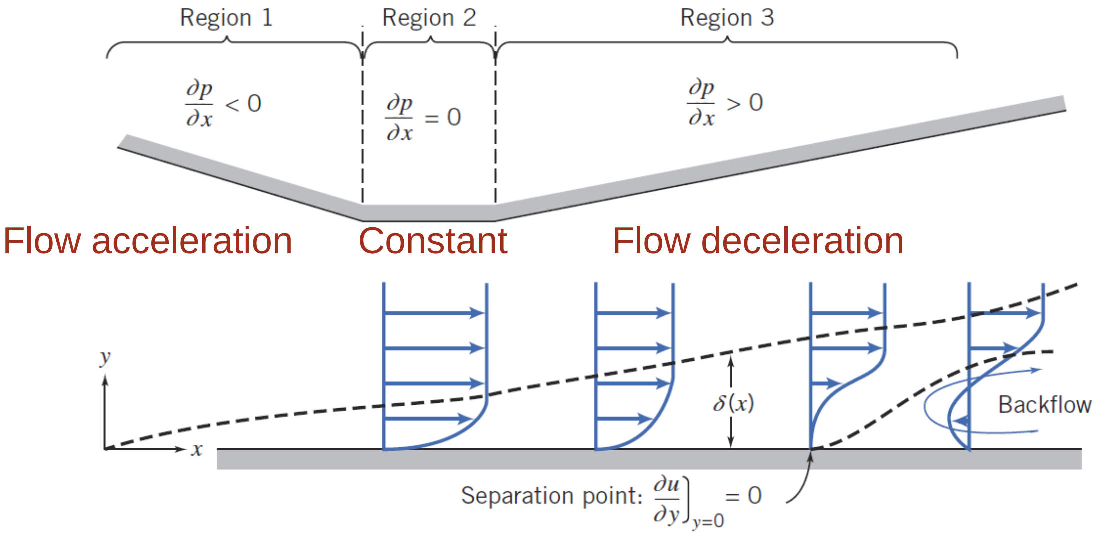
Pressure Gradient Flow

Pressure Gradient ∂p/∂x

- Favorable pressure gradient: the pressure decreases in the flow direction it tends to overcome the slowing of fluid particles caused by friction in the boundary layer.
- Adverse pressure gradient is one in which pressure increases in the flow direction, it will cause fluid particles in the boundary-layer to slow down at a greater rate than that due to boundary-layer friction alone.



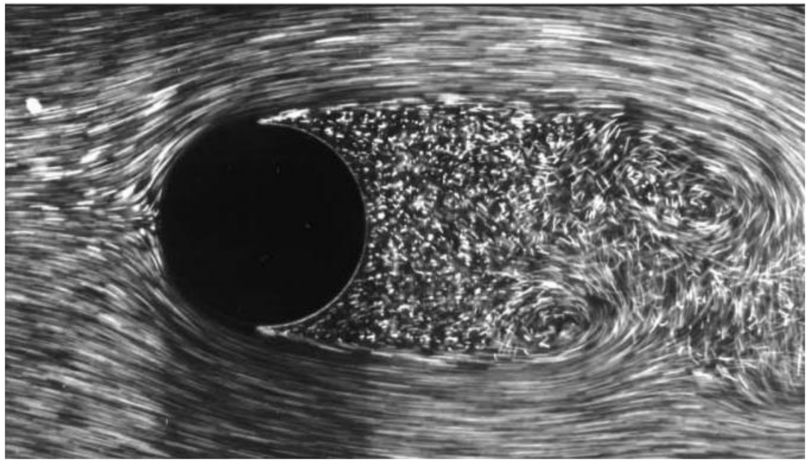


Fig. 9.6 Boundary-layer flow with pressure gradient (boundary-layer thickness exaggerated for clarity).

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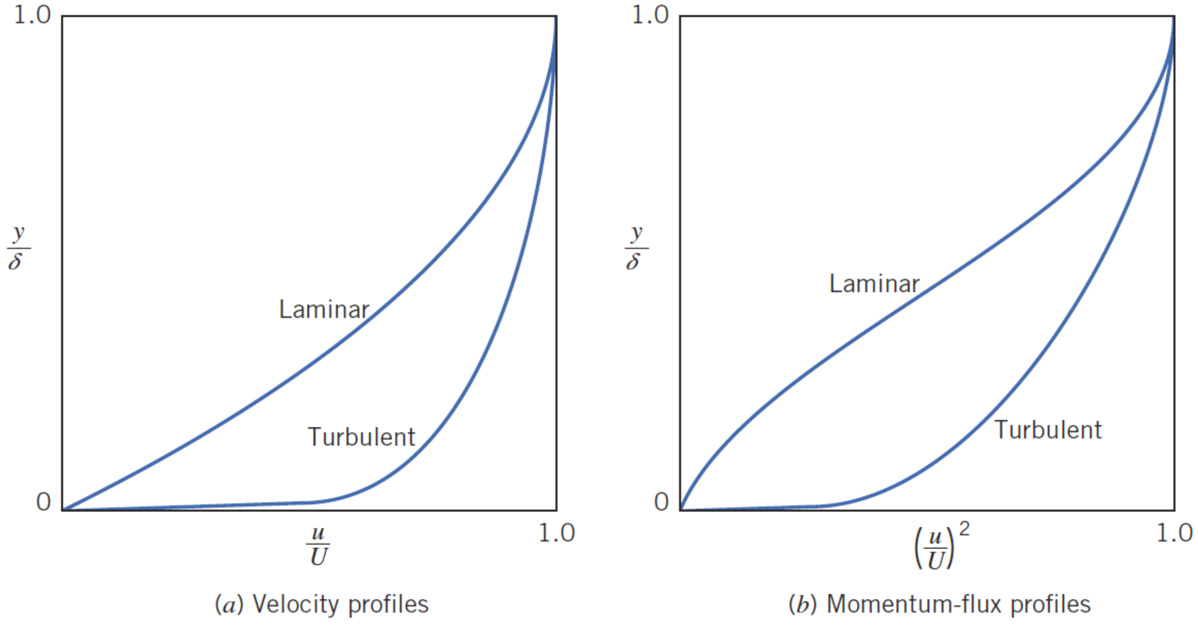
Pressure Gradient Flow

Velocity Profiles

- The momentum flux within the turbulent boundary layer is greater than within the laminar.
- Separation occurs when the momentum of fluid layers near the surface is reduced to zero by the combined action of pressure and viscous forces.
- The turbulent layer is better able to resist separation in an adverse pressure gradient



Nondimensional profiles for flat plate boundary-layer flow.

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Homework

Problem 9.4

For flow around a sphere the boundary layer becomes turbulent around $Re_D \approx 2.5 \times 10^5$. Find the speeds at which (a) an American golf ball (D = 43 mm), (b) a British golf ball (D = 41.1 mm), and (c) a soccer ball (D = 222 mm) develop turbulent boundary layers. Assume standard atmospheric conditions.

$$Re_D = \rho \frac{VD}{\mu} = \frac{VD}{\nu}$$

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Homework

Problem 9.10

A simplistic boundary-layer model is

$$\frac{u}{U} = \sqrt{2}\frac{y}{\delta} \qquad 0 < y < \delta/2$$

$$\frac{u}{U} = \left(2 - \sqrt{2}\right)\frac{y}{\delta} + \left(\sqrt{2} - 1\right) \qquad \delta/2 < y < \delta$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

$$u(y = 0) = 0 \qquad u(y = \delta) = U \qquad \frac{du}{dy}(y = \delta) = 0$$

$$\frac{\delta^*}{\delta} = \frac{1}{\delta}\int_0^\delta \left(1 - \frac{u}{U}\right) dy \qquad \frac{y}{\delta} = \eta$$

$$\frac{\theta}{\delta} = \frac{1}{\delta}\int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

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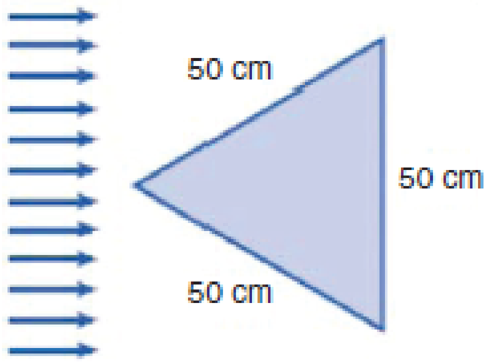
Homework

Problem 9.32

Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5 m/s air flow. The air is 20° C and 1 atm.



$$Re_L = \frac{UL}{v}$$

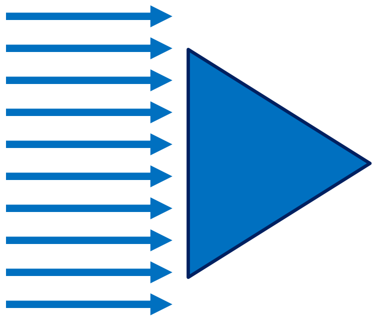
$$F_D = \int_0^L \tau_w \, dA = \int_0^L \tau_w \, W\left(\frac{x}{L}\right) dx$$

$$C_f = \frac{\tau_w}{1/2\rho U^2} = \frac{0.730}{\sqrt{Re_x}}$$

Homework

Problem 9.33

Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5 m/s air flow, except that the base rather than the tip faces the flow. Would you expect this to be larger than, the same as, or lower than the drag for Problem 9.32?



$$Re_L = \frac{UL}{v}$$

$$F_D = \int_0^L \tau_w \, dA = \int_0^L \tau_w \, W \left(1 - \frac{x}{L}\right) dx$$

$$C_f = \frac{\tau_w}{1/2\rho U^2} = \frac{0.730}{\sqrt{Re_x}}$$

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Homework

This assignment is due by **6pm on April 15th**.

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.

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