Sichuan University - Pittsburgh Institute

ME 1071: Applied Fluids

Lecture 2 Internal Incompressible Viscous Flow

Spring 2021

Outlines

- **Shear Stress Distribution in Fully Developed Pipe Flow**
- **Turbulent Velocity Profiles in Fully Developed Pipe Flow**
- **Energy Considerations in Pipe Flow**
- **Calculation of Head Loss**
  - **Major Losses**
  - **Minor Losses**
- **Fluid Systems**

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Shear Stress Distribution

Laminar flow in a pipe

- Zero shear stress at centerline
- Largest shear stress at the wall surface

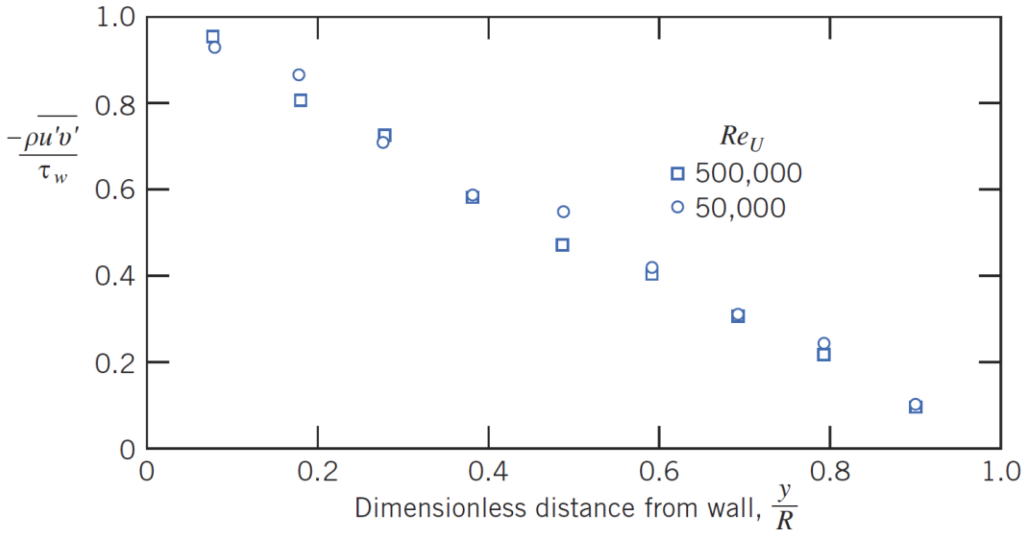
$$\tau_{rx} = \mu \frac{du}{dr} = \frac{r}{2}\left(\frac{\partial p}{\partial x}\right)$$

Turbulent flow in a pipe

- No analytical solution
- u' and v' are negatively correlated

$$\tau = \tau_{\text{lam}} + \tau_{\text{turb}} = \mu \frac{d\bar{u}}{dy} \underline{- \rho \overline{u'v'}}$$

Reynolds Stress/ Turbulent shear



- The Reynolds stress drops to zero close to the wall
- Wall layer: the region very close to the wall, viscous stress is dominant
- Both viscous and Reynolds stress are important in the region between the wall layer and the central portion of the pipe

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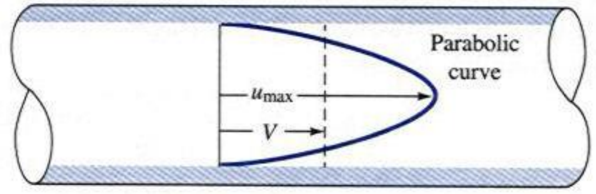
Turbulent Velocity Profiles

Turbulent vs Laminar

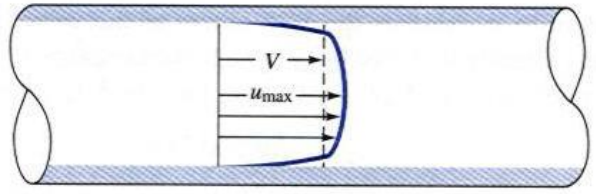
- Except for flows of very viscous fluids in small diameter ducts, internal flows generally are turbulent.
- For turbulent flow, no universal relationship between the stress field and the mean velocity field.
- turbulent flow profiles relies on experimental data

Re < 2300
Analytical solution



(a)

Re > 10^4
Relies on experimental data



(b)

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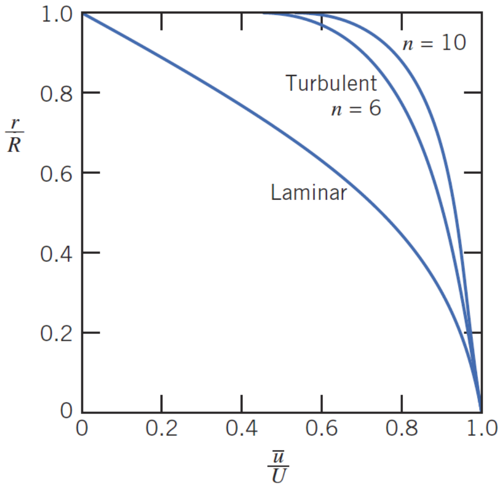
Turbulent Velocity Profiles

Fully Developed Pipe Flow

$$\tau = \tau_{\text{lam}} + \tau_{\text{turb}} = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'}$$

- Mean turbulent velocity profiles are similar by dimensional analysis
- Denote $u_* \equiv (\tau_w/\rho)^{1/2}$ and $\bar{u}$ as mean velocity, $y = R - r$
- Denote $u^+ = \dfrac{\bar{u}}{u_*}$, $y^+ = \dfrac{y u_*}{\nu}$
- The empirical (One-seventh) power-law

$$\frac{\bar{u}}{U} = \left(\frac{y}{R}\right)^{1/7} = \left(1 - \frac{r}{R}\right)^{1/7}$$



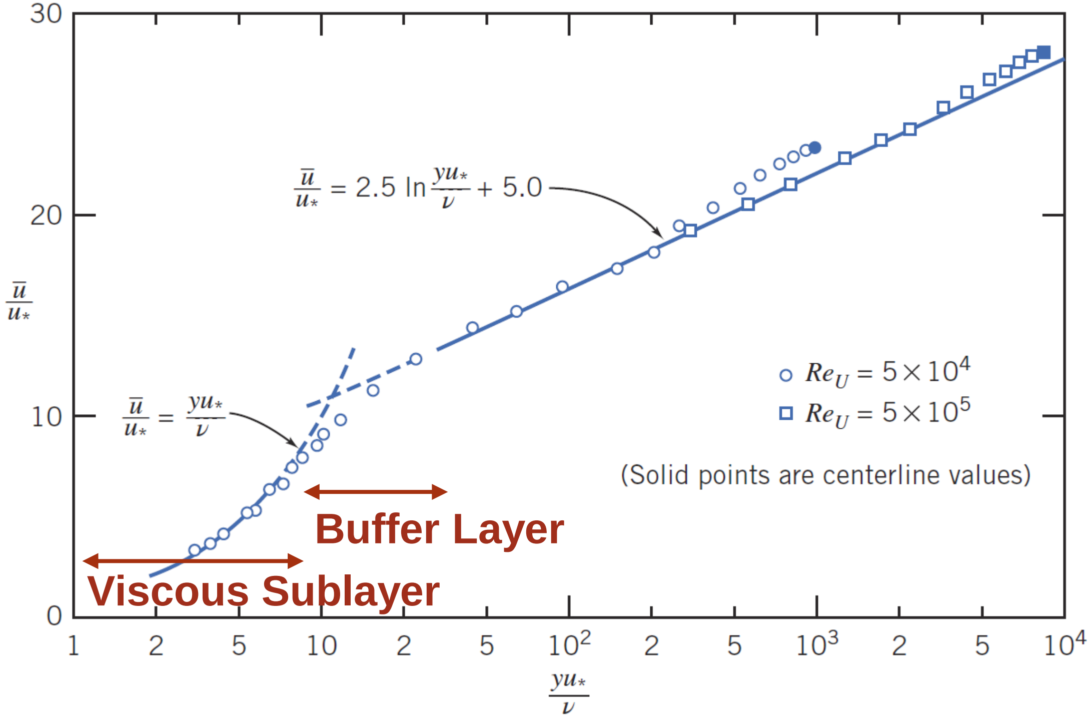


Fig. 8.9 Turbulent velocity profile for fully developed flow in a smooth pipe. (Data from Laufer [5].)

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Energy Considerations in Pipe Flow

The Energy Grade Line (EGL)

- A measure of the total mechanical energy: pressure, kinetic and potential

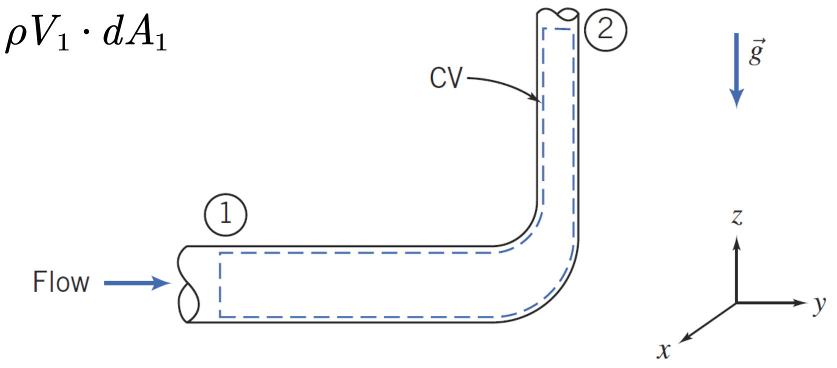
$$EGL = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

$$\dot{Q} - \cancel{\dot{W}_S} - \cancel{\dot{W}_{shear}} - \cancel{\dot{W}_{other}} = \cancel{\frac{\partial}{\partial t}} \int_{CV} e\rho dV + \int_{CS} (e + pv)\rho \vec{V} \cdot d\vec{A} \longleftarrow e = u + \frac{V^2}{2} + gz$$

$$\dot{Q} = \dot{m}(u_2 - u_1) + \dot{m}\left(\frac{p_2}{\rho} - \frac{p_1}{\rho}\right) + \dot{m}g(z_2 - z_1) + \int_{A_2} \frac{V_2^2}{2} \rho V_2 \cdot dA_2 - \int_{A_1} \frac{V_1^2}{2} \rho V_1 \cdot dA_1$$

$$\alpha = \frac{\int_A \frac{V^2}{2} \rho V \cdot dA}{\dot{m}\bar{V}^2}, \text{ kinetic energy coefficient}$$

$$\boxed{\left(\frac{p_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2\right) = u_2 - u_1 - \frac{\delta Q}{dm} = h_{l_T}}$$



Total head loss was caused by loss of mechanical energy and heat to thermal energy

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Calculation of Head Loss

Major Losses: Friction Factor

Total head loss h_{l_T}
- h_l Major losses, due to frictional effect in fully developed flow in constant-area tubes
- h_{l_m} resulting from entrances, fittings, area changes, and so on

- For fully developed flow through a constant-area pipe

$$\left(\frac{p_1}{\rho}+\alpha_1\frac{V_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\alpha_2\frac{V_2^2}{2}+gz_2\right)=u_2-u_1-\frac{\delta Q}{dm}=h_{l_T}\xrightarrow{\alpha_1\frac{V_1^2}{2}=\alpha_2\frac{V_2^2}{2},\ h_{l_m}=0}\boxed{\frac{p_1-p_2}{\rho}=g(z_2-z_1)+h_l}$$

- If the pipe is horizontal

$$\boxed{\frac{p_1-p_2}{\rho}=\frac{\Delta p}{\rho}=h_l}$$

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Calculation of Head Loss

Major Losses: Friction Factor

- Laminar flow (analytical)

$$\frac{p_1 - p_2}{\rho} = \frac{\Delta p}{\rho} = h_l \xrightarrow{Q = \frac{\pi \Delta p D^4}{128 \mu L}} \boxed{h_l = \left(\frac{64}{\mathrm{Re}}\right) \frac{L}{D} \frac{\bar{V}^2}{2}}$$

- Turbulent flow (experimental)
  - By dimensional analysis, the pressure drop is known to depend on pipe diameter, *D*, pipe length, *L*, pipe roughness, *e*, average flow velocity, *V*, fluid density, *ρ*, and fluid viscosity, *μ*.

$$\Delta p = \Delta p\left(D, L, e, \bar{V}, \rho, \mu\right)$$

$$\downarrow$$

$$\frac{\Delta p}{\rho \bar{V}^2} = \phi_1\left(\mathrm{Re}, \frac{L}{D}, \frac{e}{D}\right) \longrightarrow \frac{h_l}{\bar{V}^2/2} = \frac{L}{D}\phi_2\left(\mathrm{Re}, \frac{e}{D}\right) \xrightarrow{\text{friction factor } f = \phi_2\left(\mathrm{Re}, \frac{e}{D}\right)} \boxed{h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}}$$

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Calculation of Head Loss

Moody Chart

friction factor

$$\text{Laminar flow}\ \ f = 64/\mathrm{Re}$$

$$\text{Turbulent flow}\ \ f = \phi_2\left(\mathrm{Re},\ \frac{e}{D}\right)$$

$$\frac{1}{\sqrt{f}} = -2.0\,log\left(\frac{e/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} = -1.8\log\left(\left(\frac{e/D}{3.7}\right)^{1.11} + \frac{2.51}{Re}\right)\quad (Re > 3000)$$

$$f = \frac{0.316}{Re^{0.25}}\quad \left(Re \le 10^5\right)\ \text{smooth pipe}$$

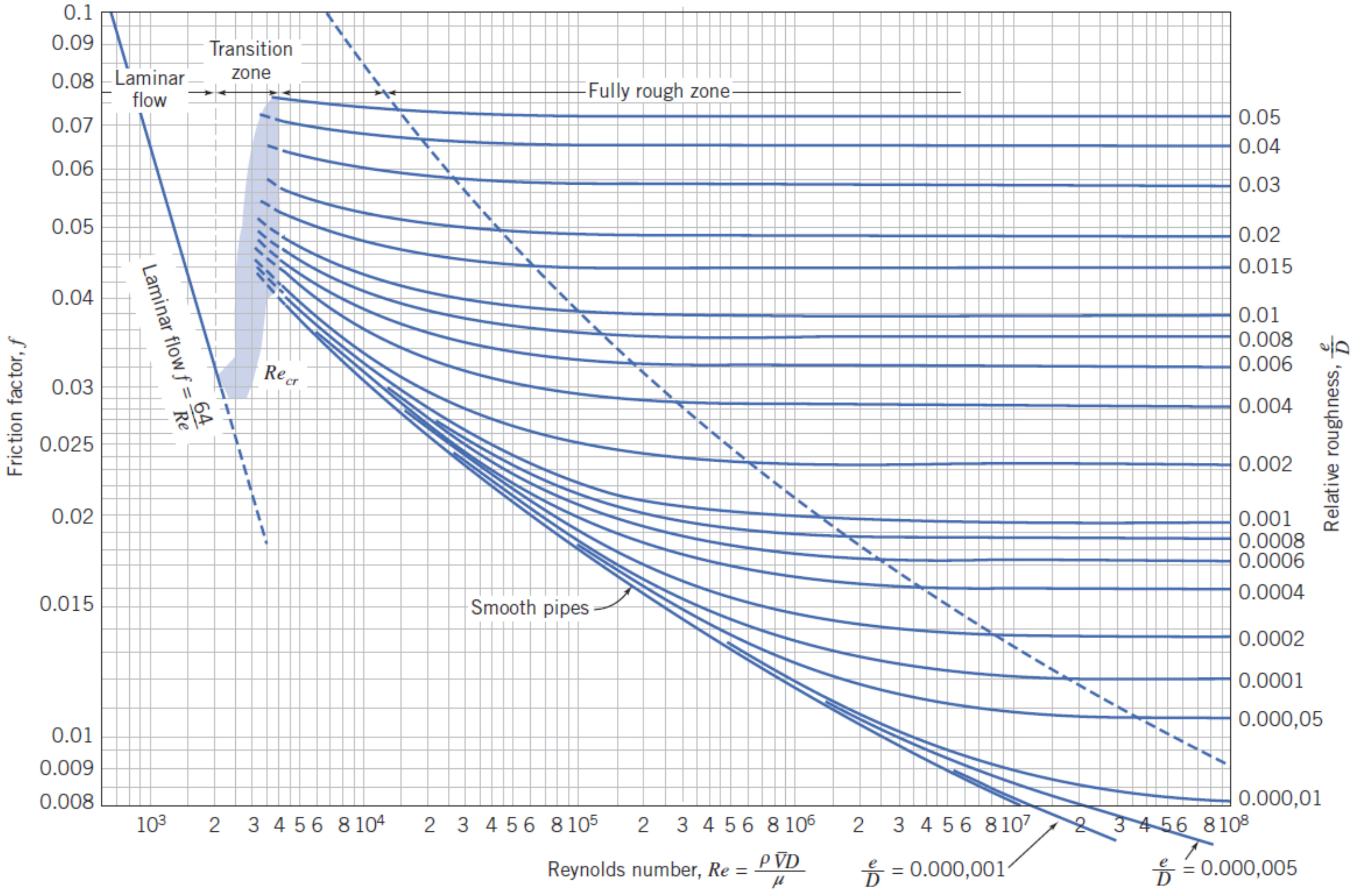


Fig. 8.13 Friction factor for fully developed flow in circular pipes. (Data from Moody [8].)

Calculation of Head Loss

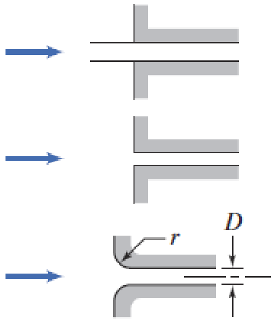
Minor Losses

1. Inlets and Exits

- Poor inlet and outlet designs lead to considerable head loss.
- Sharp corners $\rightarrow$ *vena contracta*
- Values for K can be found in Table 8.2 for a few common inlet geometries.

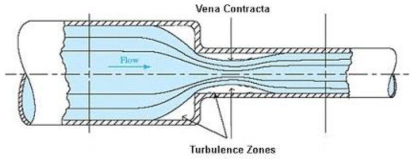
$$h_{l_m} = \frac{K\bar{V}^2}{2}$$

Table 8.2
Minor Loss Coefficients for Pipe Entrances



Entrance Type	Minor Loss Coefficient, K^a
Reentrant	0.5 – 1.0 (depending on length of pipe entrance)
Square-edged	0.5
Rounded	r/D: 0.02, 0.06, ≥ 0.15; K: 0.3, 0.2, 0.04

[a] Based on $h_{l_m} = K(\bar{V}^2/2)$, where $\bar{V}$ is the mean velocity in the pipe.
Source: Data from Reference [12].



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Calculation of Head Loss

Minor Losses

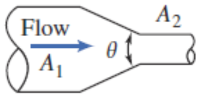
2. Enlargements and Contractions

- Sudden changes in area lead to head loss based on velocity changes.
- Installing a nozzle or diffuser helps mitigate head loss and values of K for common nozzle area ratios can be seen in Table 8.3
- For diffusers, we will use the pressure recovery coefficient and relate it to the ideal pressure recovery coefficient

$$\boxed{h_{l_m} = (C_{pi} - C_p)\frac{\bar{V}^2}{2},\ C_p = \frac{p_2 - p_1}{\bar{V}_1^2/2},\ C_{pi} = 1 - \frac{1}{AR^2}}$$

Table 8.3

Loss Coefficients (K) for Gradual Contractions: Round and Rectangular Ducts

		Included Angle, θ, Degrees						
	A_2/A_1	10	15–40	50–60	90	120	150	180
	0.50	0.05	0.05	0.06	0.12	0.18	0.24	0.26
	0.25	0.05	0.04	0.07	0.17	0.27	0.35	0.41
	0.10	0.05	0.05	0.08	0.19	0.29	0.37	0.43

Note: Coefficients are based on $h_{l_m} = K(\bar{V}_2^2/2)$.
Source: Data from ASHRAE [12].

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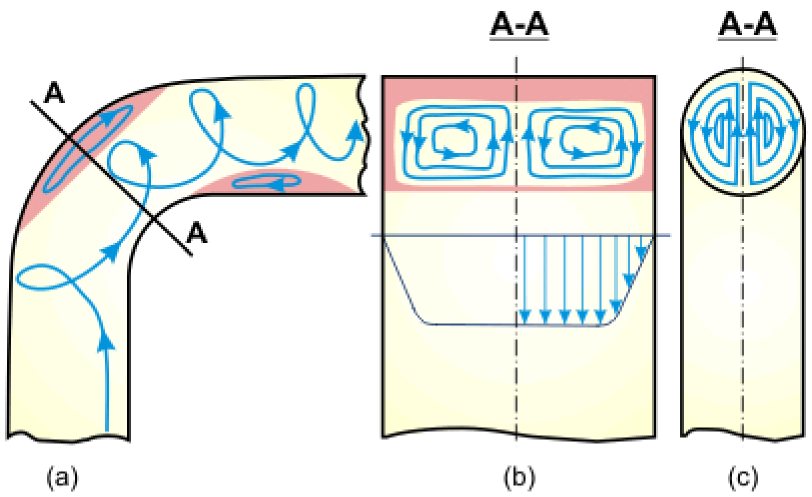
Calculation of Head Loss

Minor Losses

3. Pipe Bends

- The head loss in a bend is larger than friction effects in a straight section of equal length.
- The additional loss is primarily the result of secondary flow.
- Values of K for different pipe bend geometries can be seen in Table 8.4

$$\boxed{h_{l_m} = \frac{K\bar{V}^2}{2}}$$



Secondary flow in a pipe bend

Table 8.4

Representative Loss Coefficients for Fittings and Valves

Fitting	Geometry	*K*	Fitting	Geometry	*K*
90° elbow	Flanged regular	0.3	Globe valve	Open	10
	Flanged long radius	0.2	Angle valve	Open	5
	Threaded regular	1.5	Gate valve	Open	0.20
	Threaded long radius	0.7		75% open	1.10
	Miter	1.30		50% open	3.6
	Miter with vanes	0.20		25% open	28.8
45° Elbow	Threaded regular	0.4	Ball valve	Open	0.5
	Flanged long radius	0.2		1/3 closed	5.5
Tee, dividing line flow	Threaded	0.9		2/3 closed	200
	Flanged	0.2	Water meter		7
Tee, branching flow	Threaded	2.0	Coupling		0.08
	Flanged	1.0			

Source: Data from References [12] and [34].

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Calculation of Head Loss

Minor Losses

4. Valves and Fittings

- Values of K for different types of valves and fittings can be seen in Table 8.4

$$h_{l_m} = \frac{K\bar{V}^2}{2}$$

Table 8.4

Representative Loss Coefficients for Fittings and Valves

Fitting	Geometry	K	Fitting	Geometry	K
90° elbow	Flanged regular	0.3	Globe valve	Open	10
	Flanged long radius	0.2	Angle valve	Open	5
	Threaded regular	1.5	Gate valve	Open	0.20
	Threaded long radius	0.7		75% open	1.10
	Miter	1.30		50% open	3.6
	Miter with vanes	0.20		25% open	28.8
45° Elbow	Threaded regular	0.4	Ball valve	Open	0.5
	Flanged long radius	0.2		1/3 closed	5.5
Tee, dividing	Threaded	0.9		2/3 closed	200
line flow	Flanged	0.2	Water meter		7
Tee, branching	Threaded	2.0	Coupling		0.08
flow	Flanged	1.0			

Source: Data from References [12] and [34].

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Fluid Systems

Pumping, Fans and Blowers

- The driving force for maintaining the flow against friction is a pump for liquids or a fan or blower for gases

$$\dot{W}_{pump} = \dot{m}\left[\left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{discharge} - \left(\frac{p}{\rho} + \frac{\bar{V}^2}{2} + gz\right)_{suction}\right] = \dot{m}\Delta h_{pump}$$

- Pump efficiency = work done / power input $\eta = \dot{W}_{pump} / \dot{W}_{in}$
- Pumps, Fans and Blowers can be accounted for as a negative loss

$$\left(\frac{p_1}{\rho} + \alpha_1\frac{V_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2\frac{V_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump}$$

It is interesting to note that a pump adds energy to the fluid in the form of a gain in pressure—the everyday, invalid perception is that pumps add kinetic energy to the fluid.

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Fluid Systems

Noncircular Ducts

Hydraulic Diameter

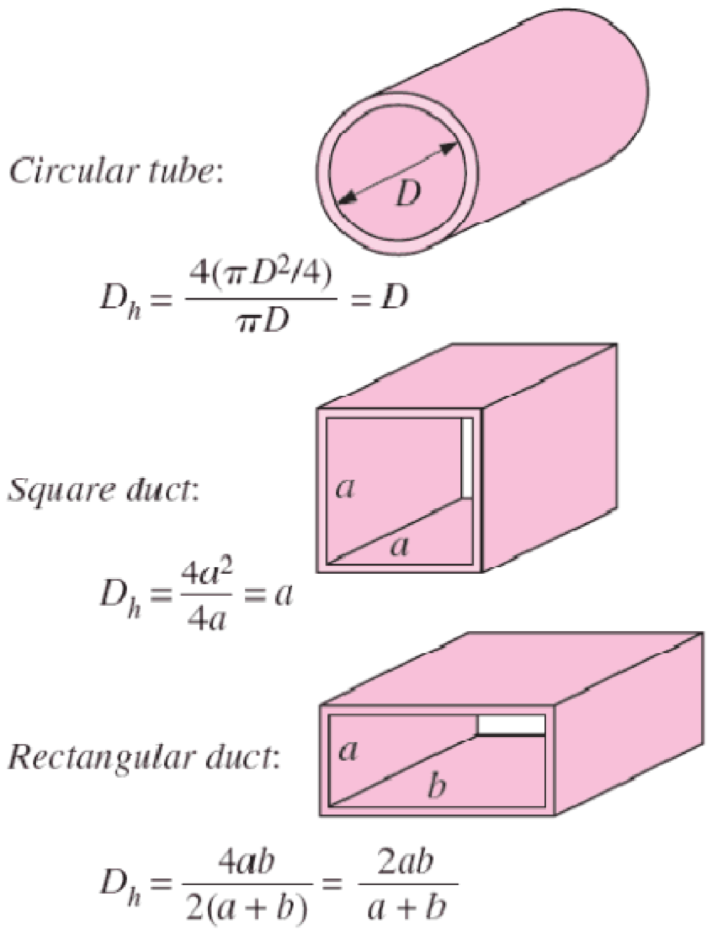
$$D_h = \frac{4A}{P}$$

- A is the cross-sectional area
- P is the wetted perimeter
- For a Rectangular Duct of width b and height h

$$D_h = \frac{4A}{P} = \frac{4bh}{2(b+h)} = \frac{2bh}{b+h}$$

- For a Circular Duct of width b and height h

$$D_h = \frac{4A}{P} = \frac{4\pi D^2}{4\pi D} = D$$



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Fluid Systems

Pipe Flow Solutions (Single-Path Systems)

1. Find Δp for a given L, D and Q (Example 8.5)

   $Q \rightarrow Re \rightarrow f \rightarrow h_{l_T} \rightarrow \Delta p$

2. Find L for a given Δp, D and Q (Example 8.6)

   $Q \rightarrow Re \rightarrow f, \Delta p \rightarrow h_{l_T} \rightarrow L$

3. Find Q for a given Δp, L and D (Example 8.7)

   $\Delta p \rightarrow h_{l_T} \rightarrow Guess\ f \rightarrow \bar{V} \rightarrow Re \rightarrow f \rightarrow if\ equal \rightarrow \bar{V} \rightarrow Q$

4. Find D for a given Δp, L and Q (Example 8.8)

   $\Delta p \rightarrow h_{l_T} \rightarrow Guess\ D, Q \rightarrow f \rightarrow h_{l_T} \rightarrow if\ equal \rightarrow D$

$$\boxed{\left(\frac{p_1}{\rho} + \alpha_1 \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\bar{V}_2^2}{2} + gz_2\right) = h_{l_T} - \Delta h_{pump}}$$

$$h_{l_T} = \sum h_l + \sum h_{l_m}$$

$$\boxed{h_l = f \frac{L}{D} \frac{\bar{V}^2}{2}}$$

$$\boxed{h_{l_m} = \frac{K\bar{V}^2}{2}}$$

friction factor

Laminar flow $f = 64/\mathrm{Re}$

Turbulent flow $f = \phi_2\left(\mathrm{Re}, \frac{e}{D}\right)$

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Example 8.5 pipe flow into a reservoir: pressure drop unknown

A 100-m length of smooth horizontal pipe is attached to a large reservoir. A pump is attached to the end of the pipe to pump water into the reservoir at a volume flow rate of 0.01 m^3/s. What pressure must the pump produce at the pipe to generate this flow rate? The inside diameter of the smooth pipe is 75 mm.

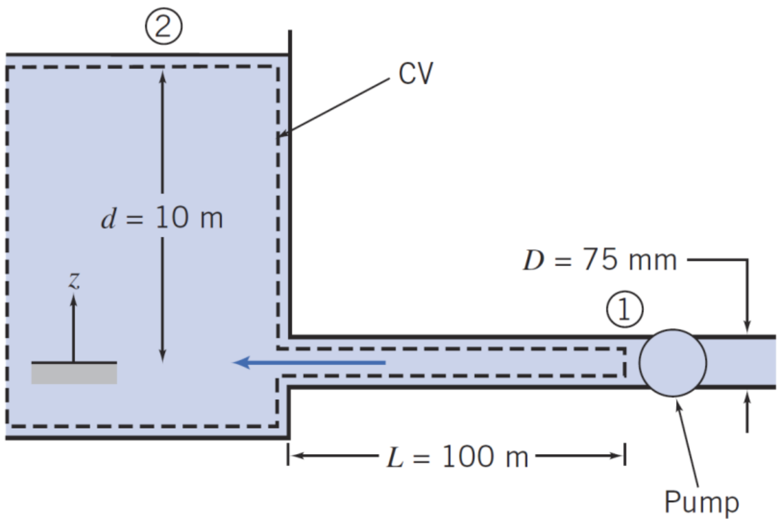
Find Δp for a given L, D and Q $\quad Q \rightarrow Re \rightarrow f \rightarrow h_{l_T} \rightarrow \Delta p$

$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + g z_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + g z_2\right) = h_{l_T}$$

$$h_{l_T} = \sum h_l + \sum h_{l_m} \qquad h_l = f \frac{L}{D} \frac{\bar{V}^2}{2} \qquad h_{l_m} = \frac{K \bar{V}^2}{2}$$

$$\alpha_1 \approx 1.0, \; K = 1.0, \; f = 0.0162$$

$$p_{pump} = \Delta p = \rho \left(g d + f \frac{L}{D} \frac{\bar{V}^2}{2}\right) = 153 \text{ kPa gage}$$



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Homework

Problem 8.104

Water flows from a tank with a very short outlet pipe. Estimate the exit flow rate. How could the flow rate be increased?

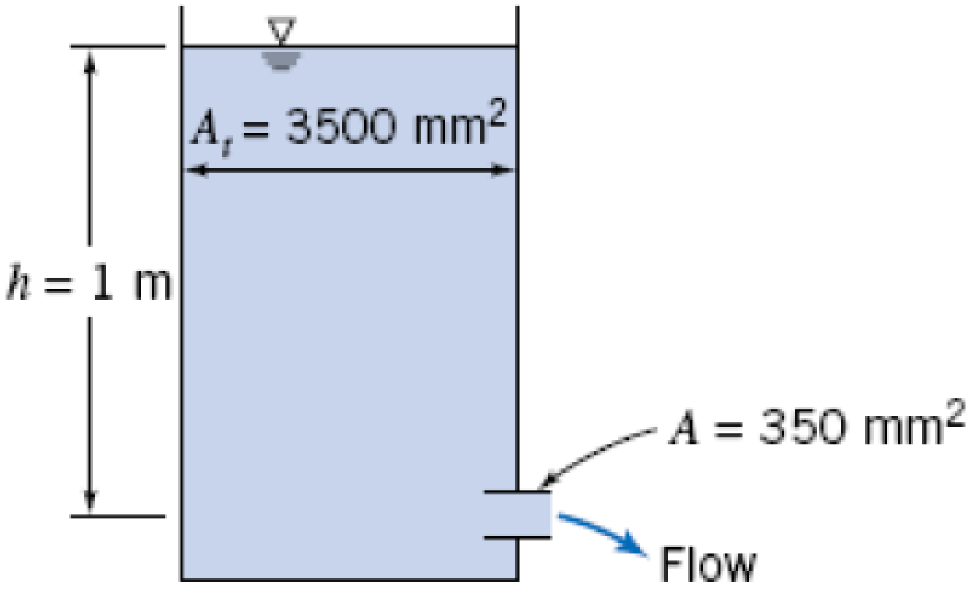
$$h_{l_T} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$h_{l_T} = \left(\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{\bar{V}_2^2}{2}\right)$$

$$h_{l_T} = h_l + h_{lm} = h_{lm} = \frac{K\bar{V}_2^2}{2}$$

$$\left(\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{\bar{V}_2^2}{2}\right) = \frac{K\bar{V}_2^2}{2}$$

$$V_1 = V_2\frac{A_1}{A_2}$$



Homework

Problem 8.105

A pool is to be filled that has a 1.5 m diameter and is 0.76 m deep. The pool is located 5.5 m above the water source which travels through a 15 m long, 1.6 cm diameter hose that is very smooth. Neglecting minor losses, how long will it take to fill if the water pressure at the source is 414 kPa?

$$h_{l_T} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$h_{l_T} = \left(\frac{p_1}{\rho}\right) - (gz_2)$$

$$h_{l_T} = h_l + h_{lm} = h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$\left(\frac{p_1}{\rho}\right) - (gz_2) = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$\bar{V} = \sqrt{\frac{2D\left(\frac{p_1}{\rho} - gz_2\right)}{f\,\mathrm{L}}} = \sqrt{\frac{2(0.016\,m)\left(\frac{414000}{1000} - (9.81)(5.5 + .76)\right)}{f(15\text{ m})}} = \frac{0.87}{\sqrt{f}}$$

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{1000\bar{V}(0.016\,m)}{0.00101} = 15841.6\bar{V}$$

$$\frac{1}{\sqrt{f}} = -2.0\,log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) = -2.0\,log\left(\frac{2.51}{Re\sqrt{f}}\right)$$

Guess f=0.015:

$$\bar{V} = \frac{0.87}{\sqrt{0.015}} = 7.10\frac{m}{s} \quad \rightarrow \quad Re = 15841.6(7.10) = 1.1 \times 10^5 \quad \rightarrow \quad f = 0.0177$$

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Homework

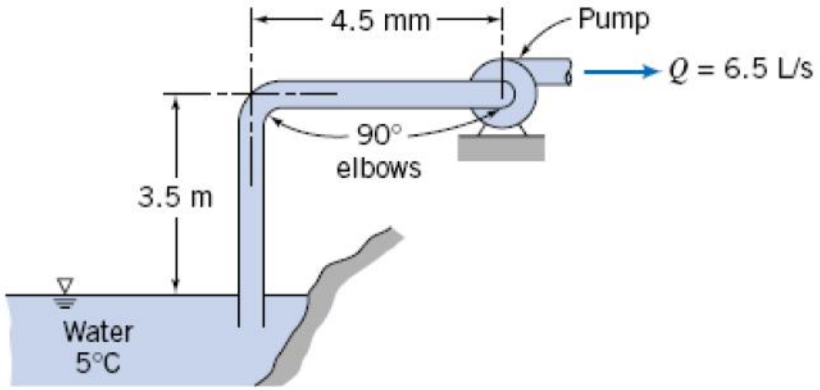
Problem 8.163

Determine the smallest standard commercial steel pipe that will allow for a static pressure to be greater than -6m H_2O gage.

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$\sum h_l + \sum h_{l_m} = -\left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right) = \frac{V_2^2}{2}\left[f\frac{L}{D} + K_{ent} + 2K_{elb}\right]$$

$$\frac{p_2}{\rho g} = -z_2 - \frac{V_2^2}{2g}\left[1 + f\frac{L}{D} + K_{ent} + 2K_{elb}\right] = -3.5 - \frac{V_2^2}{19.62}\left[3.15 + 8\frac{f}{D}\right]$$



$$D = 0.0254\, m \rightarrow V = 12.83\frac{m}{s} \rightarrow Re = 2.96 \times 10^5, \frac{e}{D} = .00181 \rightarrow f = .024$$

$$\frac{p_2}{\rho g} = -93.35 \neq -6$$

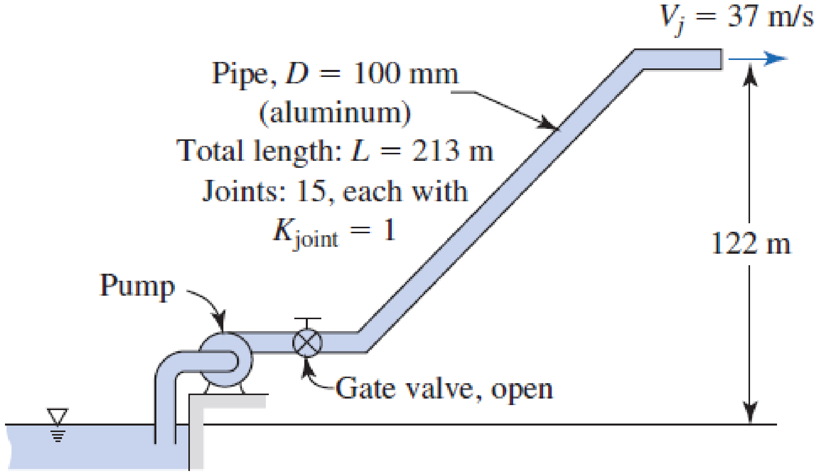
Homework

Problem 8.176

Calculate the minimum pressure needed at the pump outlet for a 38 L/s flow rate and the input power required if the pumping efficiency is 70%.

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_3}{\rho} + \frac{\bar{V}_3^2}{2} + gz_3\right) + \Delta h_{pump}$$

$$\Delta h_{pump} = gz_3 + \frac{\bar{V}_3^2}{2} + \frac{V^2}{2}\left[f\frac{L}{D} + K_{ent} + K_{90^\circ} + 2K_{45^\circ} + 15 + K_v\right]$$



$$Q \rightarrow V_3 = 4.84\frac{m}{s} \rightarrow Re = 4.25\times 10^5, \frac{e}{D} = .000015 \rightarrow f$$

$$\Delta h_{pump} = (9.81)(122) + \frac{37^2}{2} + \frac{4.84^2}{2}\left[f\frac{213}{.1} + .75 + .7 + 2(.2) + 15 + .2\right]$$

$$\dot{W}_{p,th} = \dot{m}\Delta h_{pump} = \rho Q \Delta h_{pump}$$

$$\dot{W}_p = \frac{\dot{W}_{p,th}}{\eta}$$

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right) + \Delta h_{pump}$$

$$p_2 = \rho\Delta h_{pump}$$

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This assignment is due by **6pm on March 25th**.

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.

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