Sichuan University - Pittsburgh Institute

ME 1071: Applied Fluids

Lecture 13 Final Exam Review

Spring 2021

Weekly Study Plan

Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	**Apr. 6**	**Exam I**
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 25	Chapter 11: Flow in Open Channels
9	Apr. 27	Chapter 11: Flow in Open Channels
10	May. 11	Exam II Review
11	**May. 18**	**Exam II**
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review **(Final Exam at Jun. 26)**

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Final Exam

Saturday June 26th from 10:00 am to 12:00 am

4 long answer questions and 1 opening are included! No hints but all the important equations and tables will be provided as supplemental materials.

Closed-book, calculators are allowed but no smart phones or tablets.

Work must be done by hand on the solution form provided, no upload to BB is needed for this time.

Each page of the solution form must be signed with name and student ID.

Will cover all the contents of Lectures 6, 7, 9, 10 and 11.

Outlines

- **Open Channel Flow**
  - Froude Number
  - Critical Flow
  - Area Change
  - Hydraulic Jump
- **Introduction to Compressible Flow**
  - Speed of Sound
  - Total (Stagnation) Conditions and Critical Conditions
  - Aera Variation and Choked Flow
  - Normal Shock

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Basic Concepts and Definitions

Speed of Surface Waves and the Froude Number

The Froude number

$$\boxed{Retangular\ channels:\ Fr = \frac{V}{\sqrt{gy}}}$$

$$\boxed{Nonretangular\ channels:\ Fr = \frac{V}{\sqrt{gy_h}}}$$

- ***Fr*<1** Flow is *subcritical*, *tranquil*, or *streaming*.

Disturbances can travel upstream; downstream conditions can affect the flow upstream. The flow can gradually adjust to the disturbance.

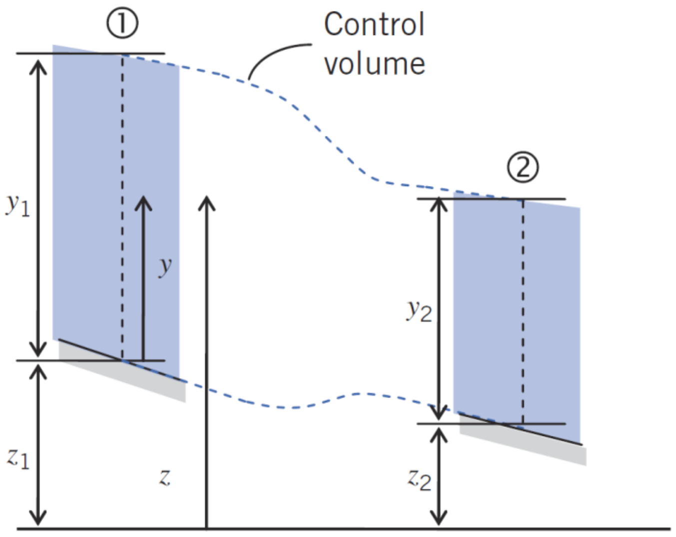
- ***Fr*=1** Flow is *critical*.
- ***Fr*>1** Flow is *supercritical*, *rapid*, or *shooting*.

No disturbance can travel upstream; downstream conditions cannot be felt upstream. The flow may “violently” respond to the disturbance because the flow has no chance to adjust to the disturbance before encountering it.

Energy Equation for Open-Channel Flows

Assumptions

1. Steady flow.
2. Incompressible flow.
3. Uniform velocity at a section.
4. Gradually varying depth so that pressure distribution is hydrostatic.
5. Small bed slope.
6. $W_s = W_{shear} = W_{other} = 0$.



Energy Equation for Open-Channel Flow

$$\boxed{\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_l}$$

Total Head or Energy Head

$$H = \frac{V^2}{2g} + y + z$$

$$H_1 - H_2 = H_l$$

Specific Energy

$$E = \frac{V^2}{2g} + y$$

$$E_1 - E_2 + z_1 - z_2 = H_l$$

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Energy Equation for Open-Channel Flows

The Specific Energy

- E indicates actual energy (kinetic plus potential/pressure per unit mass flow rate) being carried by the flow

$$E = \frac{V^2}{2g} + y$$

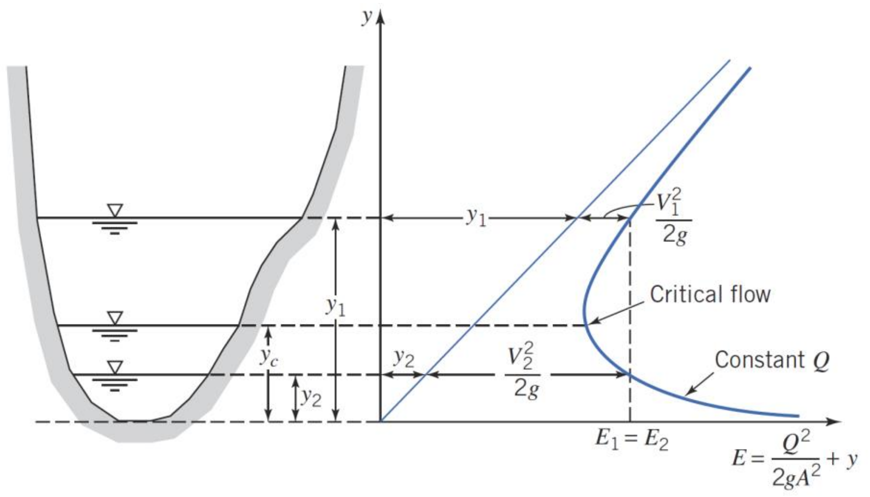

Fig. 11.7 Specific energy curve for a given flow rate.

Critical Depth (Fr = 1)

$$\boxed{Q^2 = \frac{gA_c^3}{b_{sc}}} \qquad \boxed{V_c = \sqrt{gy_{hc}}}$$

Minimum Specific Energy

- the specific energy is at its minimum at critical conditions, i.e., **Fr = 1**.

$$\boxed{y_c = \left[\frac{Q^2}{gb^2}\right]^{1/3} \qquad E_{\min} = \frac{3}{2}y_c \qquad (Rectangular\ channel)}$$

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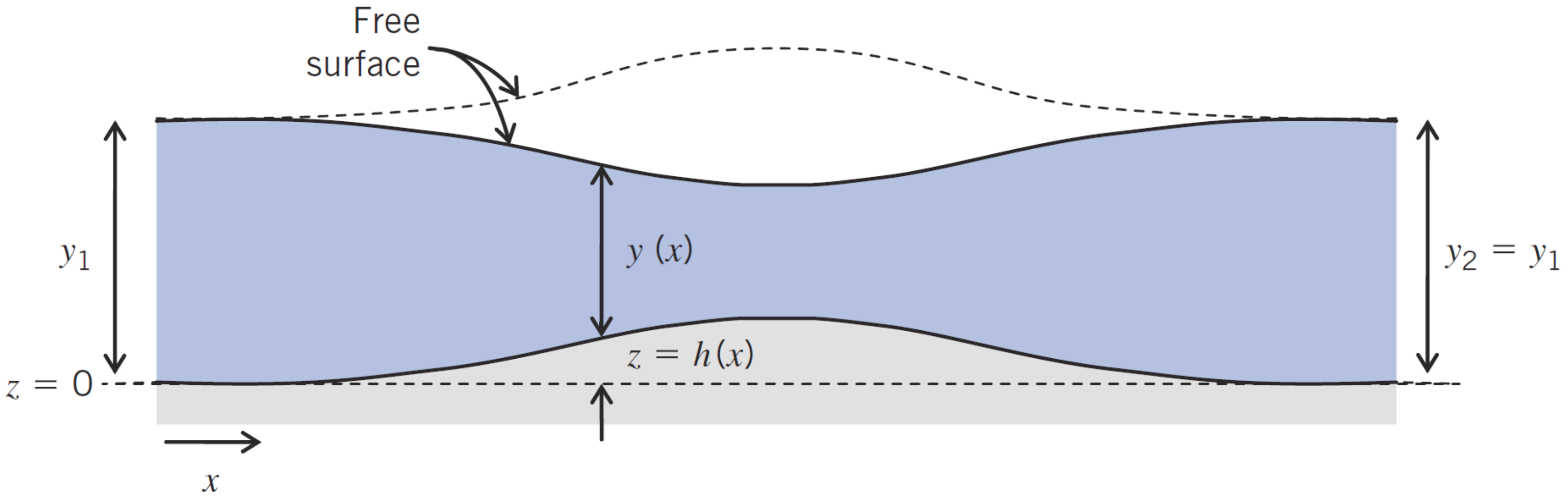
Localized Effect of Area Change

Flow over a Bump

$$\frac{V_1^2}{2g}+y_1+z_1=\frac{V_2^2}{2g}+y_2+z_2=\frac{V^2}{2g}+y+z=\text{const}$$



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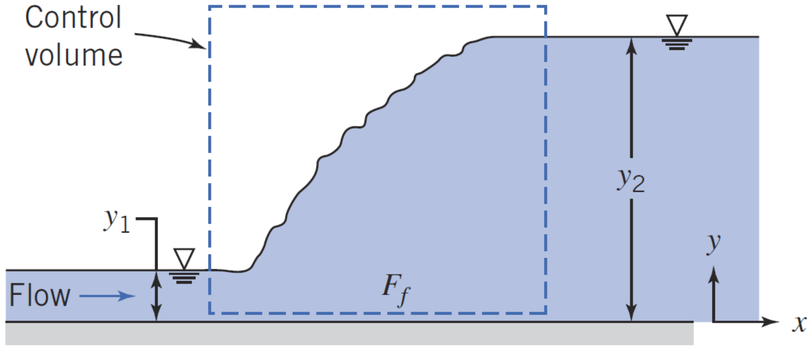
The Hydraulic Jump

Governing Equations for Hydraulic Jump

$$\boxed{Continuity \quad V_1 y_1 = V_2 y_2}$$

$$\boxed{Momentum \quad \frac{V_1^2 y_1}{g} + \frac{y_1^2}{2} = \frac{V_2^2 y_2}{g} + \frac{y_2^2}{2}}$$

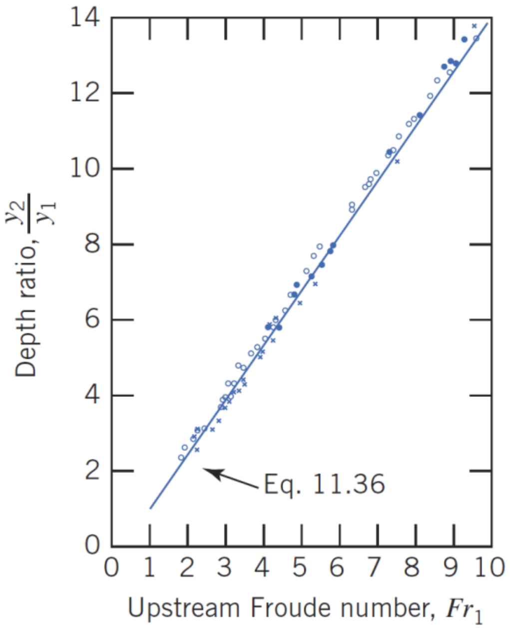
$$\boxed{Energy \quad E_1 = \frac{V_1^2}{2g} + y_1 = \frac{V_2^2}{2g} + y_2 + H_l = E_2 + H_l}$$



Depth Increase Across a Hydraulic Jump

- The ratio of downstream to upstream depths across a hydraulic jump is only a function of the upstream Froude number.

$$\boxed{\frac{y_2}{y_1} = \frac{1}{2}\left[\sqrt{1 + 8Fr_1^2} - 1\right], \; Fr_1 > 1}$$



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The Hydraulic Jump

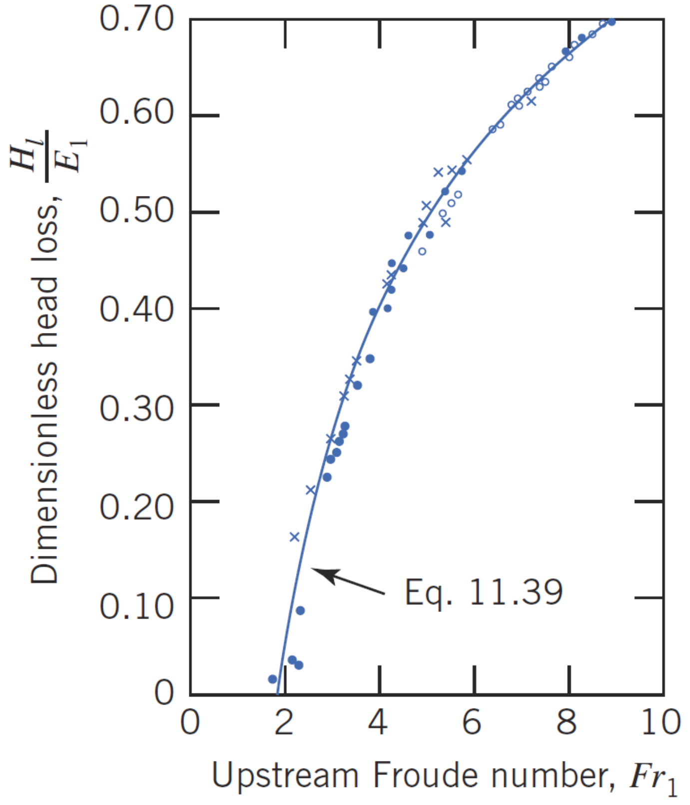
Head Loss Across a Hydraulic Jump

- The head loss is only a function of the upstream Froude number.
- Hydraulic jump can occur only in supercritical flow.
- Flow downstream from a jump always is subcritical.

$$\boxed{H_l = \frac{[y_2 - y_1]^3}{4y_1 y_2}, \; y_2 > y_1}$$

$$\boxed{\frac{H_l}{E_1} = \frac{\left[\sqrt{1+8Fr_1^2} - 3\right]^3}{8\left[\sqrt{1+8Fr_1^2} - 1\right]\left[Fr_1^2 + 2\right]}, \; Fr_1 > 1}$$

Energy Dissipation Ratio



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Outlines

- **Open Channel Flow**
  - Froude Number
  - Critical Flow
  - Area Change
  - Hydraulic Jump
- **Introduction to Compressible Flow**
  - Speed of Sound
  - Total (Stagnation) Conditions and Critical Conditions
  - Aera Variation and Choked Flow
  - Normal Shock

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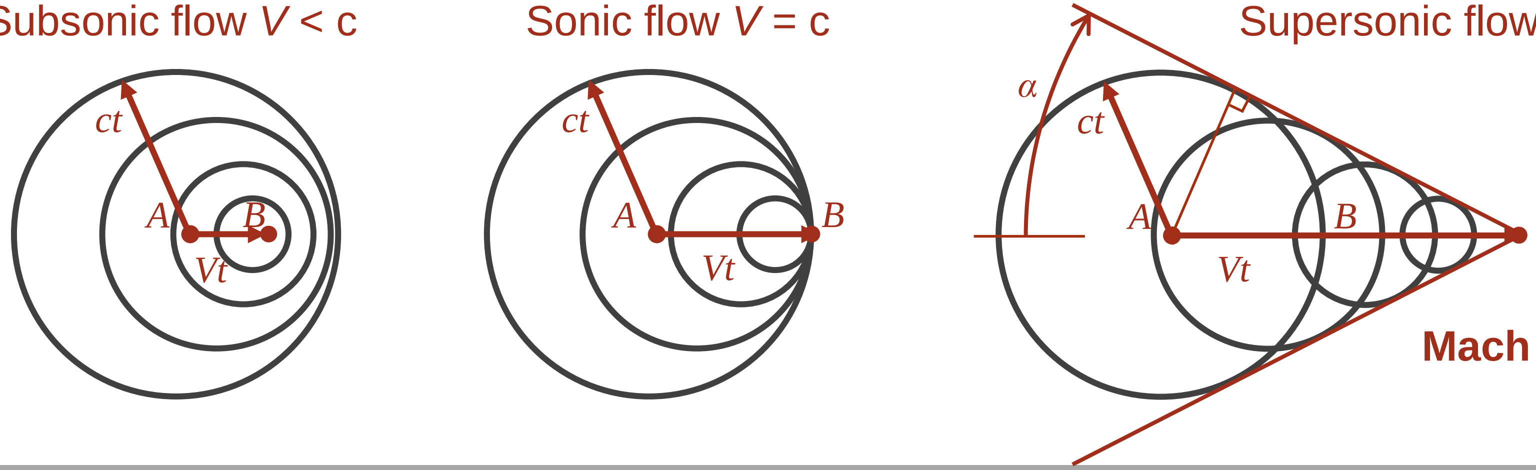
Speed of Sound

- **For calorically perfect gas**

$$\boxed{c = \sqrt{kp/\rho} = \sqrt{kRT}}$$

- The speed of sound in a calorically perfect gas is a function of T only.
- At sea level c = 340.9 m/s.
- **The propagation pattern of the disturbances**
  - The propagation of disturbance to the upstream is related to the moving speed of the object.

Subsonic flow $V < c$ Sonic flow $V = c$ Supersonic flow $V > c$



$$\sin\alpha = \frac{ct}{Vt} = \frac{c}{V} = \frac{1}{M}$$

Mach angle $\boxed{\alpha = \sin^{-1}\frac{1}{M}}$

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Special Forms of the Energy Equation

- **Total condition (总状态、滞止状态)**
  - The condition that the velocity of fluid element adiabatically or isentropically slows down to zero.

$$\boxed{h,\ T} \xrightarrow{\text{Slow down adiabatically}} \boxed{h_0,\ T_0}$$

$V \qquad\qquad V = 0$

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\boxed{p,\ \rho} \xrightarrow{\text{Slow down isentropically}} \boxed{p_0,\ \rho_0}$$

$V \qquad\qquad V = 0$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)}$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)}$$

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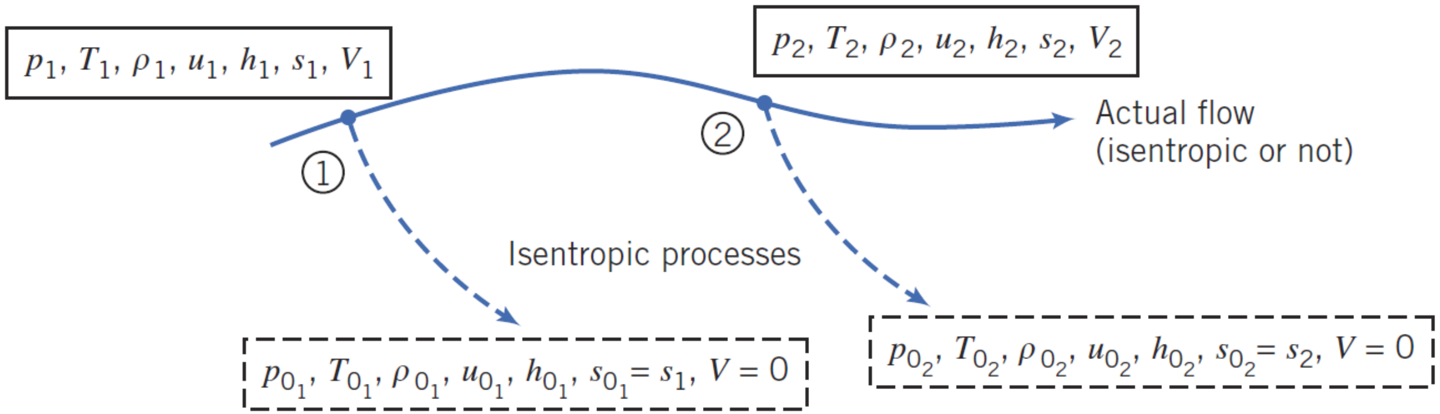
Critical Conditions

- **Sonic condition (音速状态) / Critical Condition**
  - The condition that the velocity of fluid element adiabatically or isentropically approaches to sonic velocity (M = 1).

$$\underset{h,\,T}{V} \xrightarrow{\text{Approaches adiabatically}} \underset{h^*,\,T^*}{V^* = c^*} \qquad \underset{p,\,\rho}{V} \xrightarrow{\text{Approaches isentropically}} \underset{p^*,\,\rho^*}{V^* = c^*}$$

$$\frac{T^*}{T_0} = \frac{2}{k+1} \qquad c^* = \sqrt{kRT^*} = \sqrt{\frac{2k}{k+1}RT_0}$$

$$\frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \qquad \frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)}$$



Local isentropic stagnation properties.

The critical conditions are similar to the stagnation conditions, except that the final velocity is brought to sonic velocity (M = 1) instead of zero velocity.

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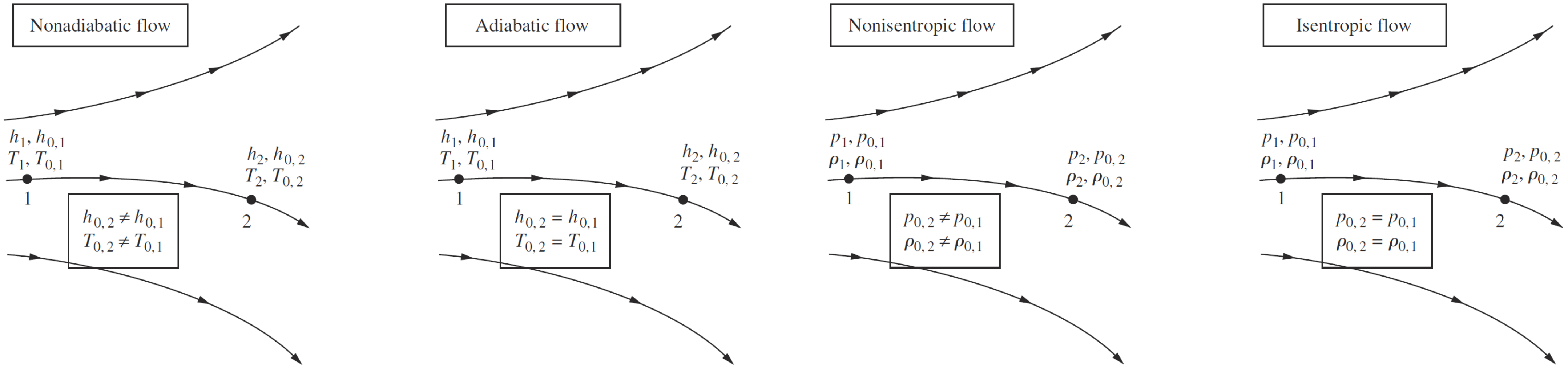
Definition of Total (Stagnation) Conditions

- **Total enthalpy of a steady, adiabatic, inviscid flow**
  - **Assumption:** body forces are negligible
  - h_0 is equal to its freestream value throughout the entire flow

$$\boxed{h+\frac{V^2}{2}=h_0=\text{const}}$$

  - Calorically perfect gas: $T_0 = \text{const}$



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Special Forms of the Energy Equation

- **Energy equation for steady, adiabatic, inviscid, one-dimensional flow**
- We have derived the relations for stagnation conditions

$$\frac{T_0}{T} = 1 + \frac{k-1}{2}M^2 \qquad \frac{p_0}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)} \qquad \frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2}M^2\right)^{1/(k-1)}$$

- At sonic conditions

$$\frac{T^*}{T_0} = \frac{2}{k+1} = 0.833 \qquad \frac{p^*}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} = 0.528 \qquad \frac{\rho^*}{\rho_0} = \left(\frac{2}{k+1}\right)^{1/(k-1)} = 0.634$$

- Characteristic Mach number $M^* \equiv V/c^*$ $c^* = \sqrt{kRT^*}$

$$\boxed{\frac{c^2}{k-1} + \frac{V^2}{2} = \frac{k+1}{2(k-1)}c^{*2}} \longrightarrow \boxed{M^{*2} = \frac{(k+1)M^2}{2+(k-1)M^2}}$$

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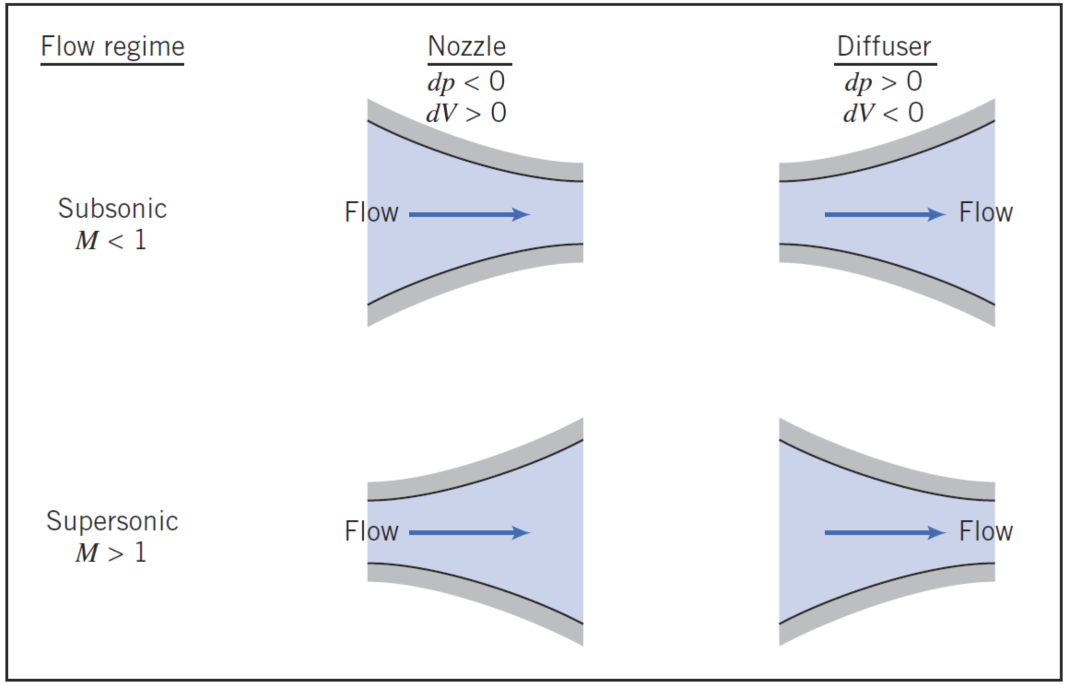
Isentropic Flow with Area Variation

- **Flow variation induced by area change**

Area-velocity relation

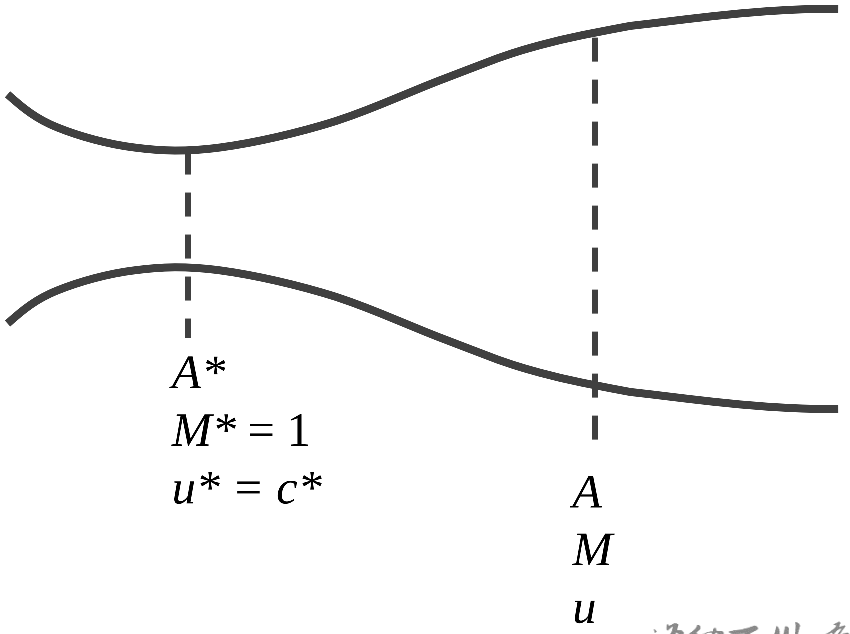
$$\boxed{\frac{dA}{A}=(M^2-1)\frac{du}{u}}$$



Nozzle and diffuser shapes as a function of initial Mach number.

Area-Mach number relation

$$\boxed{\left(\frac{A}{A^*}\right)^2=\frac{1}{M^2}\left[\frac{2}{k+1}\left(1+\frac{k-1}{2}M^2\right)\right]^{(k+1)/(k-1)}}$$



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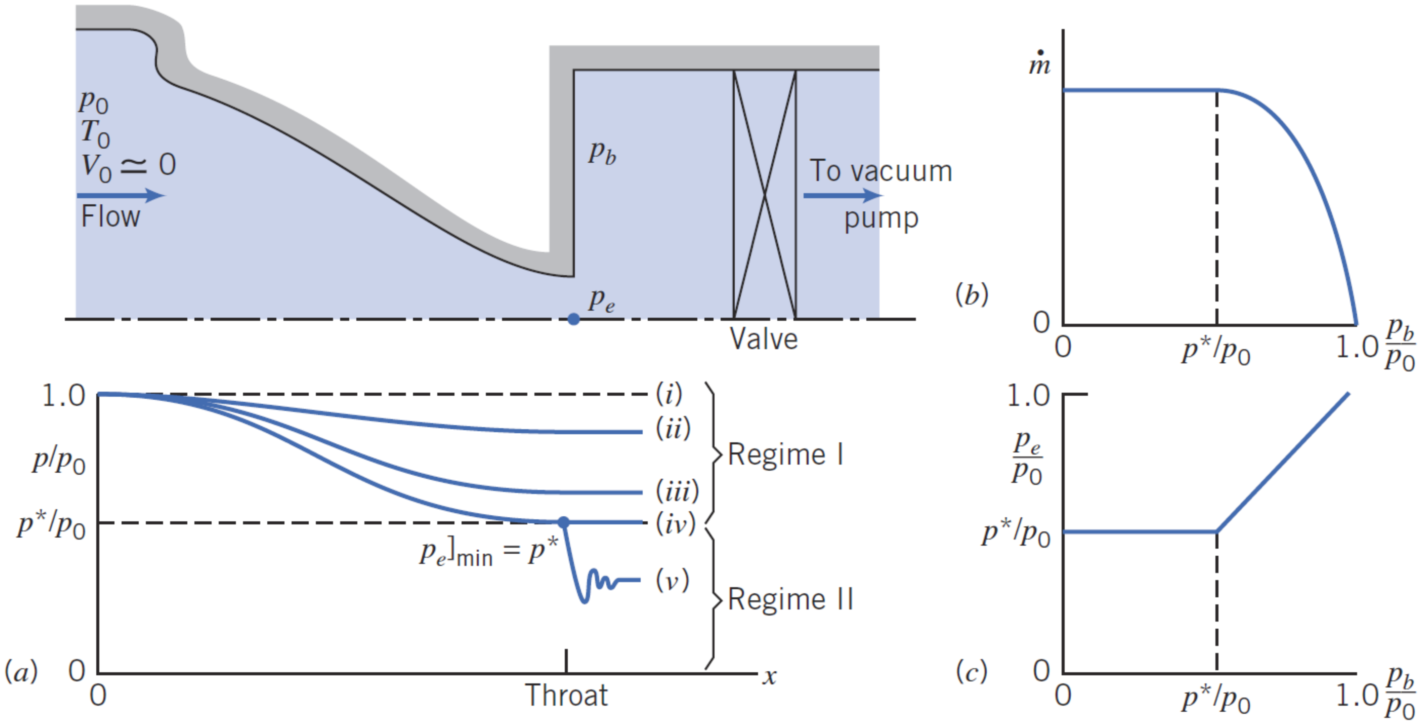
Choked Flow

- **The Flow Rate of an Isentropic Flow with Area Variation**
  - The limiting of the mass flow rate is called choking of the flow, this happens when the Mach number at the throat equals to 1.



$$\boxed{\dot{m}_{\text{choked}} = A_e\, p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}}$$

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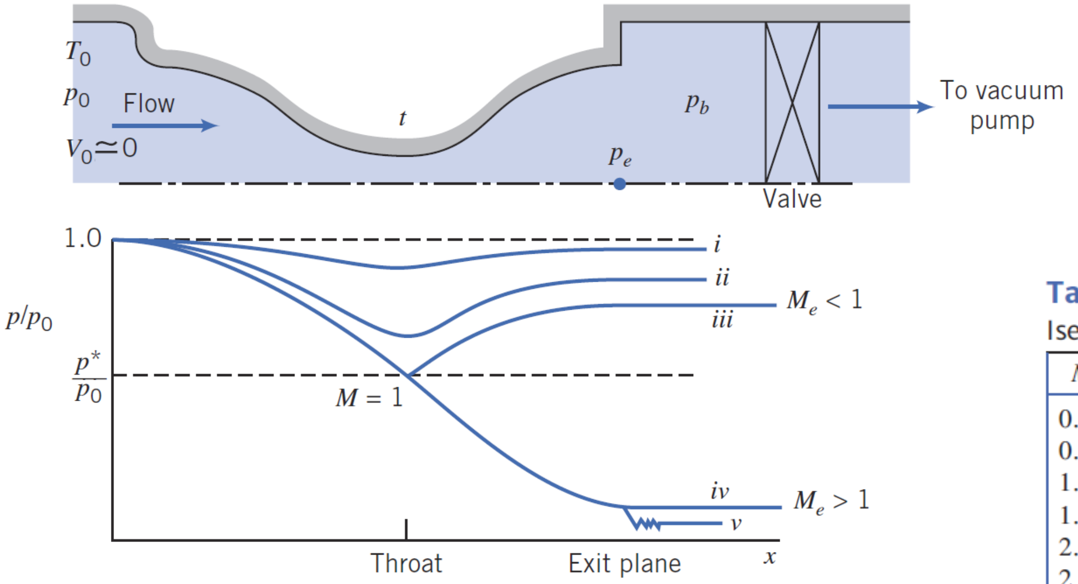
Choked Flow

- **Throat**: the minimum area of the convergent-divergent duct.



Pressure distributions for isentropic flow in a converging-diverging nozzle.

$$\dot{m}_{\text{choked}} = A_t\, p_0 \sqrt{\frac{k}{RT_0}} \left(\frac{2}{k+1}\right)^{(k+1)/2(k-1)}$$

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{k+1}\left(1+\frac{k-1}{2}M^2\right)\right]^{(k+1)/(k-1)}$$

Table D.1

Isentropic Flow Functions (one-dimensional flow, ideal gas, $k = 1.4$)

M	T/T_0	p/p_0	ρ/ρ_0	A/A^*
0.00	1.0000	1.0000	1.0000	∞
0.50	0.9524	0.8430	0.8852	1.340
1.00	0.8333	0.5283	0.6339	1.000
1.50	0.6897	0.2724	0.3950	1.176
2.00	0.5556	0.1278	0.2301	1.688
2.50	0.4444	0.05853	0.1317	2.637
3.00	0.3571	0.02722	0.07623	4.235
3.50	0.2899	0.01311	0.04523	6.790
4.00	0.2381	0.006586	0.02766	10.72
4.50	0.1980	0.003455	0.01745	16.56
5.00	0.1667	0.001890	0.01134	25.00

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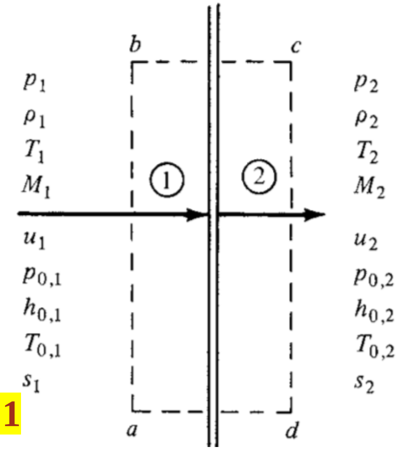
Calculation of Normal Shock-Wave Properties

- **Assumptions**
  - Steady flow, $\partial/\partial t = 0$.
  - Adiabatic, $\dot{q}=0$.
  - No viscous effects.
  - No body forces, $\mathbf{f} = 0$.

+ One-dimensional flow, $M_1 >= 1$



Continuity	$\rho_1 u_1 = \rho_2 u_2$
Momentum	$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$
Energy	$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$
Enthalpy	$h_2 = c_p T_2$
Equation of state	$p_2 = \rho_2 R T_2$

$$M_2^{\,2} = \frac{1+[(k-1)/2]M_1^{\,2}}{kM_1^{\,2}-(k-1)/2}$$

$$T_{0,1} = T_{0,2}$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(k+1)M_1^2}{2+(k-1)M_1^2}$$

$$\frac{p_{0,2}}{p_{0,1}} = e^{-(s_2-s_1)/R}$$

$$\frac{p_2}{p_1} = 1+\frac{2k}{k+1}(M_1^2-1)$$

$$\frac{p_{0,2}}{p_{0,1}} = \left[1+\frac{2k}{k+1}(M_1^2-1)\right]^{\frac{-1}{k-1}}\left[\frac{(k+1)M_1^2}{2+(k-1)M_1^2}\right]^{\frac{k}{k-1}}$$

$$\frac{h_2}{h_1} = \frac{T_2}{T_1} = \left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}$$

$$s_2 - s_1 = c_p \ln\left\{\left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}\right\} - R\ln\left[1+\frac{2k}{k+1}(M_1^2-1)\right] \geq 0$$

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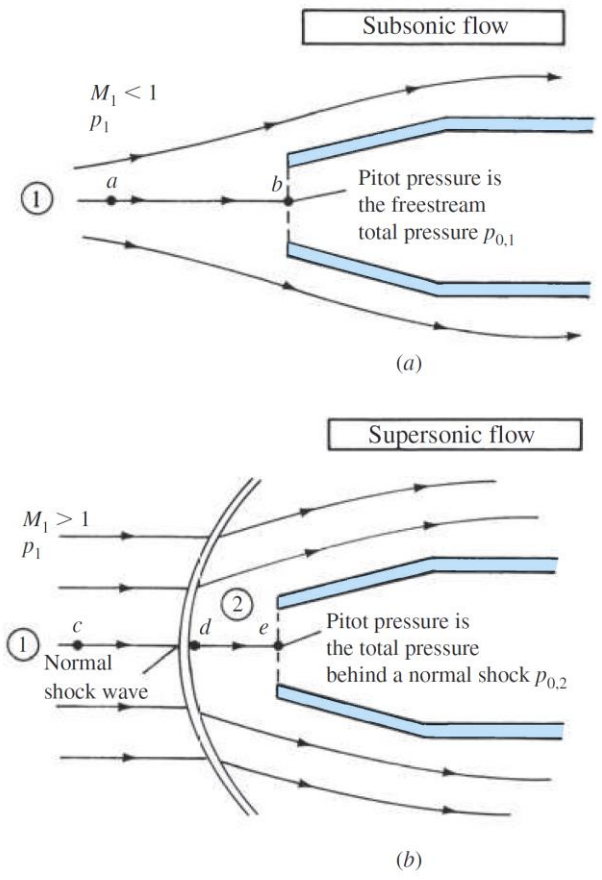
Measurement of Velocity in Compressible Flows

- Velocity measurement using a pitot tube



- For **subsonic compressible flow**

a → b is isentropic process.

$$\frac{p_{0,1}}{p_1} = \left(1 + \frac{k-1}{2} M_1^2\right)^{k/(k-1)}$$

$$\downarrow$$

$$M_1^2 = \frac{2}{k-1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(k-1)/k} - 1\right]$$

$$\downarrow$$

$$u_1^2 = \frac{2c_1^2}{k-1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(k-1)/k} - 1\right]$$

Comparing to incompressible flow, additional knowledge of c_1 is needed for subsonic compressible flow.

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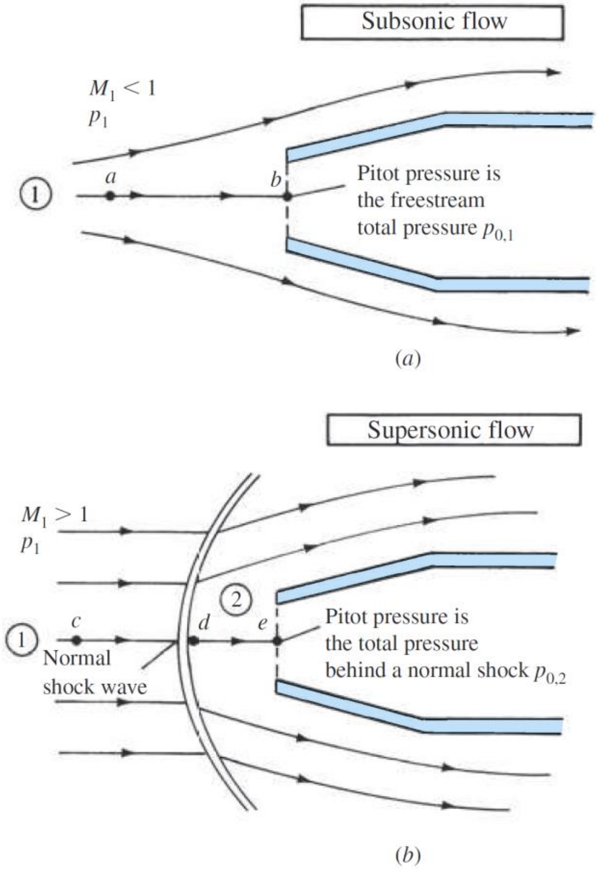
Measurement of Velocity in Compressible Flows

- Velocity measurement using a pitot tube



- For **supersonic compressible flow**

c → the point before the shock wave is isentropic process.
d → e is also isentropic process.
However, the point before the shock wave → d is nonisentropic.

$$\frac{p_{0,2}}{p_1} = \frac{p_{0,2}}{p_2}\frac{p_2}{p_1}$$

$$\frac{p_{0,2}}{p_2} = \left(1+\frac{k-1}{2}M_2^2\right)^{k/(k-1)} \qquad M_2^{\,2} = \frac{1+[(k-1)/2]M_1^{\,2}}{kM_1^{\,2}-(k-1)/2} \qquad \frac{p_2}{p_1} = 1+\frac{2k}{k+1}(M_1^2-1)$$

$$\frac{p_{0,2}}{p_1} = \left(\frac{(k+1)^2 M_1^{\,2}}{4kM_1^{\,2}-2(k-1)}\right)^{k/(k-1)}\left(1+\frac{2k}{k+1}(M_1^2-1)\right)$$

The Rayleigh Pitot tube fomula

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Homework Review

Problem 12.63

Testing of a demolition explosion is to be evaluated. Sensors indicate that the shock wave generated at the instant of explosion is 30 MPa (abs). If the explosion occurs in air at 20℃ and 1010 kPa, find the speed of the shock wave, and the temperature and speed of the air just after the shock passes. As an approximation assume k=1.4. (Why is this an approximation?)

$$\frac{p_2}{p_1}=1+\frac{2k}{k+1}(M_1^2-1)$$

$$\frac{T_2}{T_1}=\left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}$$

$$M_2^{\ 2}=\frac{1+[(k-1)/2]M_1^{\ 2}}{kM_1^{\ 2}-(k-1)/2}$$

$$V_S=V_1=M_1\sqrt{kRT_1}$$

$$V_S-V=V_2=M_2\sqrt{kRT_2}$$

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Homework Review

Problem 12.67

Air undergoes a normal shock. Upstream T_1=35℃, p_1=229 kPa (abs)and V_1=704 m/s. Determine the temperature and stagnation pressure of the air stream leaving the shock.

$$M_1 = V_1 / \sqrt{kRT_1}$$

$$M_2{}^2 = \frac{1+[(k-1)/2]M_1{}^2}{kM_1{}^2-(k-1)/2}$$

$$\frac{T_2}{T_1} = \left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}$$

$$\frac{p_{0,2}}{p_{0,1}} = \left[1+\frac{2k}{k+1}(M_1^2-1)\right]^{\frac{-1}{k-1}}\left[\frac{(k+1)M_1^2}{2+(k-1)M_1^2}\right]^{\frac{k}{k-1}}$$

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Homework Review

Problem 12.68

If through a normal shock wave (in air), the absolute pressure rises from 275 to 410 kPa and the velocity diminishes from 460 to 346 m/s, what temperature are to be expected upstream and downstream from the wave?

$$\frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1) \qquad M_1 = V_1 / \sqrt{kRT_1} \qquad M_2{}^2 = \frac{1 + [(k-1)/2]M_1{}^2}{kM_1{}^2 - (k-1)/2} \qquad M_2 = V_2 / \sqrt{kRT_2}$$

Table D.2

Normal-Shock Flow Functions (one-dimensional flow, ideal gas, $k = 1.4$)

M_1	M_2	$p0_2/p0_1$	T_2/T_1	p_2/p_1	ρ_2/ρ_1
1.00	1.000	1.000	1.000	1.000	1.000
1.50	0.7011	0.9298	1.320	2.458	1.862
2.00	0.5774	0.7209	1.687	4.500	2.667
2.50	0.5130	0.4990	2.137	7.125	3.333
3.00	0.4752	0.3283	2.679	10.33	3.857
3.50	0.4512	0.2130	3.315	14.13	4.261
4.00	0.4350	0.1388	4.047	18.50	4.571
4.50	0.4236	0.09170	4.875	23.46	4.812
5.00	0.4152	0.06172	5.800	29.00	5.000

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Homework Review

Problem 12.71

The Concorde supersonic transport flew at M = 2.2 at 20 km altitude. Air is decelerated isentropically by the engine inlet system to a local Mach number of 1.3. The air passed through a normal shock and was decelerated further to M = 0.4 at the engine compressor section. Determine the temperature, pressure and stagnation pressure of the air entering the engine compressor.

$$M_1 \rightarrow P_{0,1}, P_1, T_1$$

$$M_2 \rightarrow P_{0,2}, P_2, T_2$$

$$M_3 \rightarrow P_{0,3}, P_3, T_3$$

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