Sichuan University - Pittsburgh Institute

ME 1071: Applied Fluids

Lecture 11 Introduction to Compressible Flow

Spring 2021

Weekly Study Plan

Weeks	Dates	Lectures
1	Mar. 9	Course Introduction, Fluids Review
2	Mar. 16	Chapter 8: Internal Incompressible Viscous Flow
3	Mar. 23	Chapter 8: Internal Incompressible Viscous Flow
4	Mar. 30	Chapter 8/Exam I Review
5	**Apr. 6**	**Exam I**
6	Apr. 13	Chapter 9: External Incompressible Viscous Flow
7	Apr. 20	Chapter 9: External Incompressible Viscous Flow
8	Apr. 25	Chapter 11: Flow in Open Channels
9	Apr. 27	Chapter 11: Flow in Open Channels
10	May. 11	Exam II Review
11	**May. 18**	**Exam II**
12	May. 25	Chapter 12: Introduction to Compressible Flow
13	Jun. 1	Chapter 12: Introduction to Compressible Flow
14	Jun. 8	Chapter 12: Introduction to Compressible Flow
15	Jun. 15	Chapter 5: CFD Related Topics
16	Jun. 22	Final Exam Review **(Final Exam at Jun. 26)**

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Outlines

- **Isentropic Flow with Area Variation**
- **Calculation of Normal Shock-Wave Properties**
- **Measurement of Velocity in Compressible Flows**

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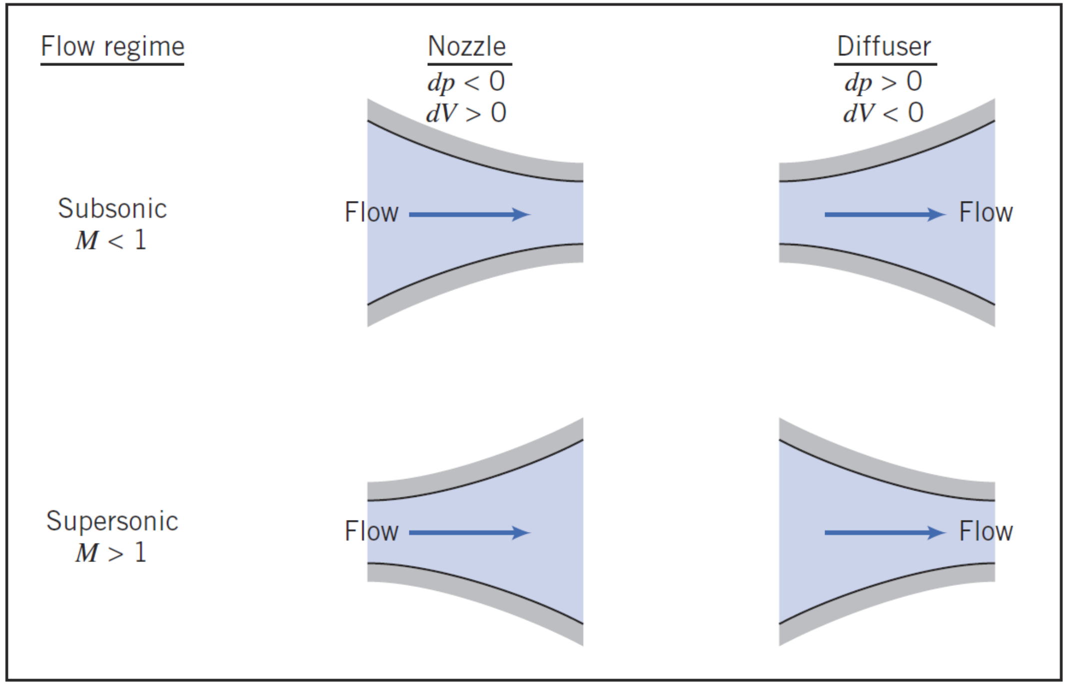
Isentropic Flow with Area Variation

- **Flow variation induced by area change**

Area-velocity relation

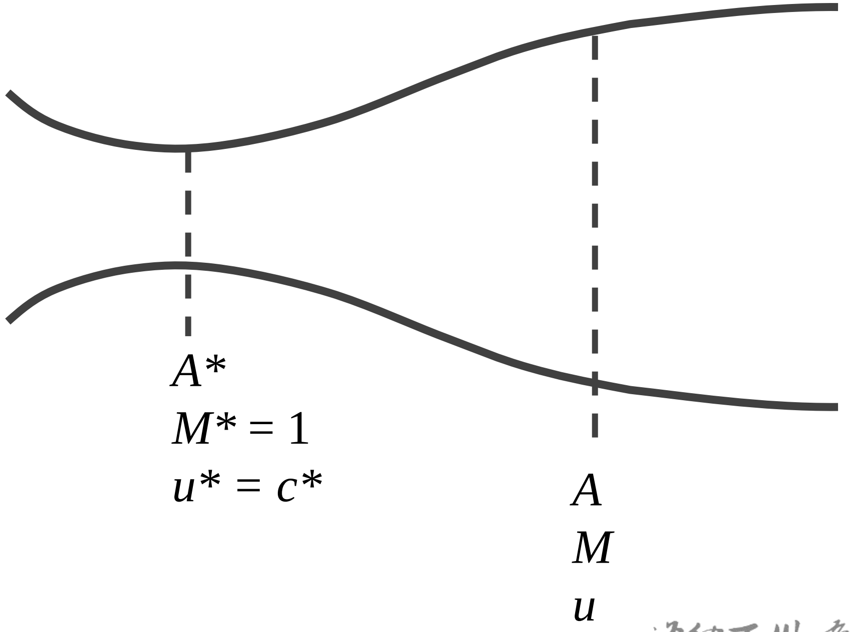
$$\boxed{\frac{dA}{A} = (M^2 - 1)\frac{du}{u}}$$



Nozzle and diffuser shapes as a function of initial Mach number.

Area-Mach number relation

$$\boxed{\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{k+1}\left(1+\frac{k-1}{2}M^2\right)\right]^{(k+1)/(k-1)}}$$



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Compressible Flow with Area Variation

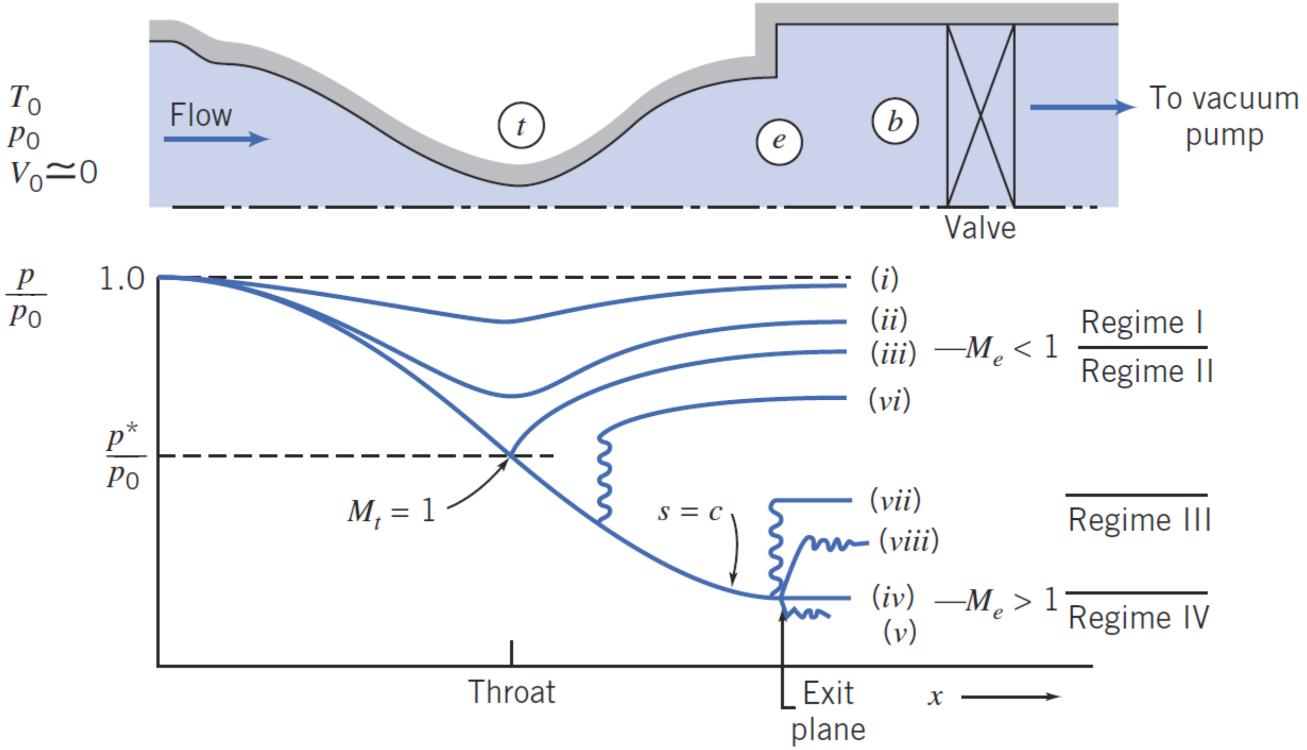
- **Compressible Flow in a Convergent-Divergent Nozzle**
  - Once there is shock wave, the flow is not isentropic.

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)}$$

How to calculate the properties of shock waves?



Pressure distributions for flow in a converging-diverging nozzle for different back pressures.

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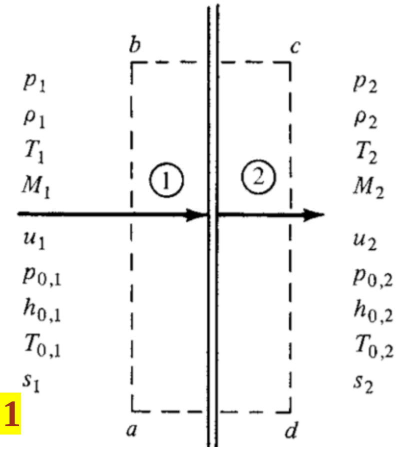
Outlines

- Isentropic Flow with Area Variation
- Calculation of Normal Shock-Wave Properties
- Measurement of Velocity in Compressible Flows

Calculation of Normal Shock-Wave Properties

- **Assumptions**
  - Steady flow, $\partial/\partial t = 0$.
  - Adiabatic, $\dot{q} = 0$.
  - No viscous effects.
  - No body forces, $\mathbf{f} = 0$.

  + One-dimensional flow, $M_1 >= 1$



Continuity	$\rho_1 u_1 = \rho_2 u_2$
Momentum	$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$
Energy	$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$
Enthalpy	$h_2 = c_p T_2$
Equation of state	$p_2 = \rho_2 R T_2$

$$\frac{p_1}{\rho_1 u_1} + u_1 = \frac{p_2}{\rho_2 u_2} + u_2 \xrightarrow{c=\sqrt{kp/\rho}} \frac{c_1^2}{ku_1} - \frac{c_2^2}{ku_2} = u_2 - u_1$$

$$c^2 = \frac{k+1}{2} c^{*2} - \frac{(k-1)u^2}{2}$$

Dividing by u_2-u_1

The Prandtl Relation $\boxed{c^{*2} = u_1 u_2}$

$$M_2^* = \frac{1}{M_1^*}$$

$$M^{*2} = \frac{(k+1)M^2}{2+(k-1)M^2}$$

$$\boxed{M_2^2 = \frac{1+[(k-1)/2]M_1^2}{kM_1^2-(k-1)/2}}$$

The relation between the Mach numbers ahead and behind the normal shock wave.

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Calculation of Normal Shock-Wave Properties

- The **Mach numbers** across a normal shock wave

$$\boxed{M_2^{\,2}=\frac{1+[(k-1)/2]M_1^{\,2}}{kM_1^{\,2}-(k-1)/2}}\quad\begin{cases}\text{Invalid} & \text{if } M_1<1\\ M_2=1 & \text{if } M_1=1 \quad \text{infinitely weak shock wave, Mach wave}\\ M_2<1 & \text{if } M_1>1\\ M_2\to\sqrt{\dfrac{k-1}{2k}}=0.378 & \text{if } M_1\to\infty\end{cases}$$

- The **ratios of thermodynamics properties** across a normal shock wave

Continuity $\rho_1 u_1=\rho_2 u_2 \longrightarrow \dfrac{\rho_2}{\rho_1}=\dfrac{u_1}{u_2} \longrightarrow \dfrac{\rho_2}{\rho_1}=\dfrac{u_1^{\,2}}{u_1u_2}=\dfrac{u_1^{\,2}}{c^{*2}}=M_1^{*2} \longrightarrow \boxed{\dfrac{\rho_2}{\rho_1}=\dfrac{u_1}{u_2}=\dfrac{(k+1)M_1^{\,2}}{2+(k-1)M_1^{\,2}}}$

Momentum $p_1+\rho_1u_1^2=p_2+\rho_2u_2^2 \longrightarrow p_2-p_1=\rho_1u_1^2-\rho_2u_2^2=\rho_1u_1^2(1-\dfrac{u_2}{u_1})$

$$\frac{p_2-p_1}{p_1}=\frac{\rho_1}{p_1}u_1^2(1-\frac{u_2}{u_1})=\frac{k\rho_1}{kp_1}u_1^2(1-\frac{u_2}{u_1})=\frac{ku_1^2}{a_1^2}(1-\frac{u_2}{u_1}) \longrightarrow \boxed{\frac{p_2}{p_1}=1+\frac{2k}{k+1}(M_1^2-1)}$$

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Calculation of Normal Shock-Wave Properties

- **The ratios of thermodynamics properties across a normal shock wave**

Equation of state $p = \rho RT$

Enthalpy $h_2 = c_p T_2$

$$\left.\begin{array}{l} p = \rho RT \\ h_2 = c_p T_2 \end{array}\right\} \longrightarrow \frac{h_2}{h_1} = \frac{T_2}{T_1} = \frac{p_2}{p_1}\frac{\rho_1}{\rho_2} \longrightarrow \boxed{\frac{h_2}{h_1} = \frac{T_2}{T_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1)\right]\frac{2 + (k-1)M_1^2}{(k+1)M_1^2}}$$

The five unknowns p_2, u_2, ρ_2, h_2, and T_2 are explicitly solved.

- **The ratios of thermodynamics properties in limiting case ($M_1 \to \infty$)**

$$\lim_{M_1 \to \infty} M_2 = \sqrt{\frac{1 + [(k-1)/2]M_1^2}{kM_1^2 - (k-1)/2}} = \sqrt{\frac{k-1}{2k}} = 0.378$$

$$\lim_{M_1 \to \infty} \frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(k+1)M_1^2}{2 + (k-1)M_1^2} = \frac{k+1}{k-1} = 6$$

$$\lim_{M_1 \to \infty} \frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1) = \infty$$

$$\lim_{M_1 \to \infty} \frac{h_2}{h_1} = \frac{T_2}{T_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1)\right]\frac{2 + (k-1)M_1^2}{(k+1)M_1^2} = \infty$$

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Calculation of Normal Shock-Wave Properties

- The **entropy change** across the normal shock wave

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \xrightarrow{M_1 >= 1} \boxed{s_2 - s_1 = c_p \ln\left\{\left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}\right\} - R\ln\left[1+\frac{2k}{k+1}(M_1^2-1)\right] \geq 0}$$

The second law of thermodynamics requires $M_1 \geq 1$.

- Within the shock wave large gradients in velocity and temperature occur.
- Friction and thermal conduction are significant.
- Therefore, the entropy increases across the shock wave.

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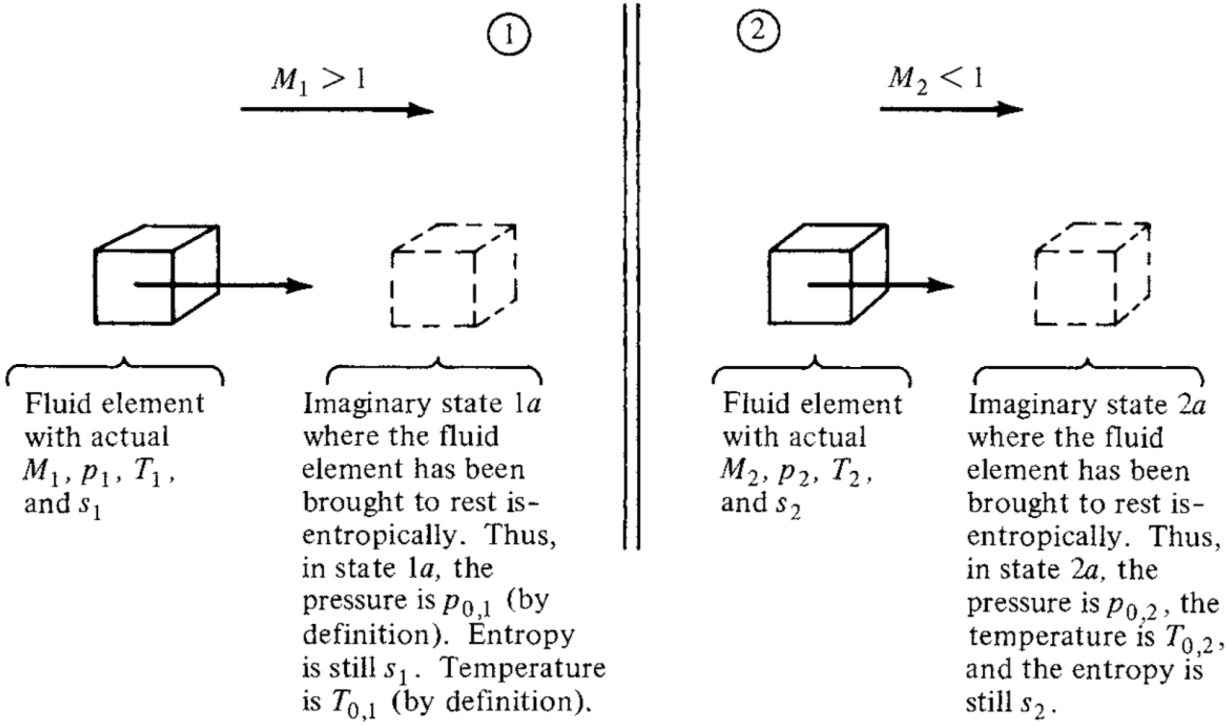
Calculation of Normal Shock-Wave Properties

- **Total conditions** **across the normal shock wave: total temperature**



$$\boxed{h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}}$$

$$c_p T_1 + \frac{u_1^2}{2} = c_p T_{0,1} \qquad c_p T_2 + \frac{u_2^2}{2} = c_p T_{0,2}$$

$$\boxed{T_{0,1} = T_{0,2}}$$

- Total temperature is constant across a stationary normal shock wave.
- Consistent with the conclusion derived for the adiabatic calorically perfect gas.

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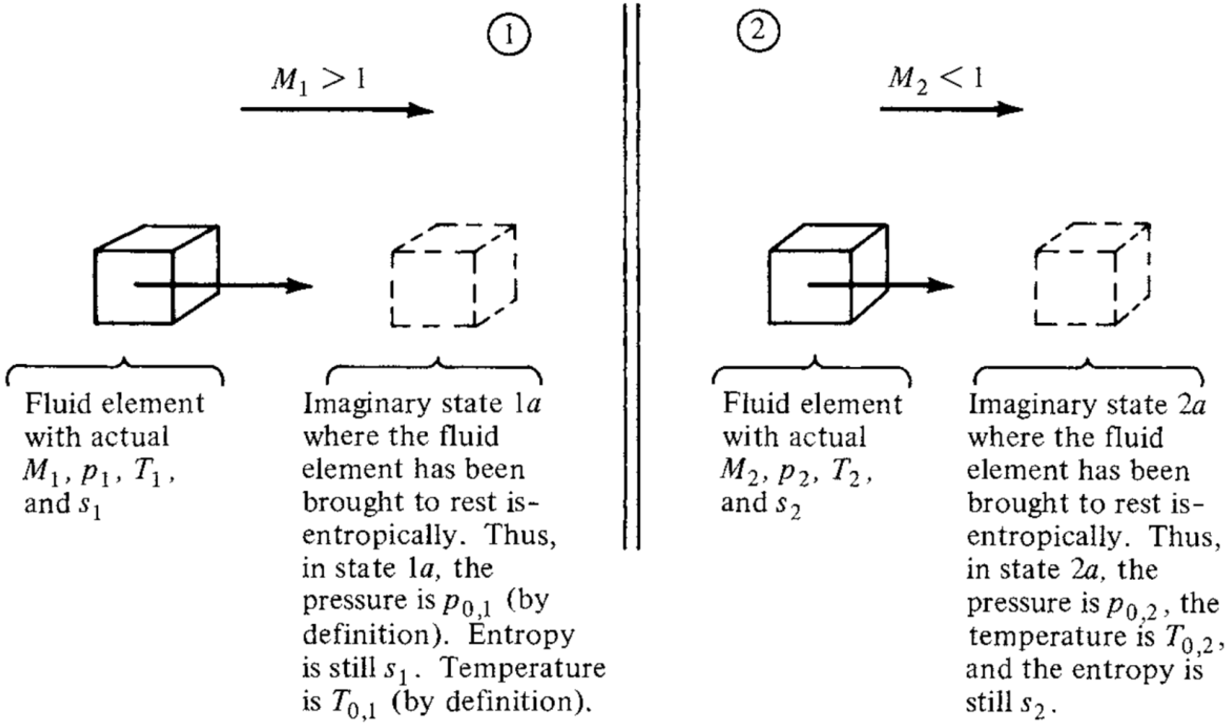
Calculation of Normal Shock-Wave Properties

- **Total conditions across the normal shock wave: total pressure**



$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

Isentropic processes for 1 and 2

$$s_2 - s_1 = c_p \ln\frac{T_{0,2}}{T_{0,1}} - R\ln\frac{p_{0,2}}{p_{0,1}}$$

$T_{1,0} = T_{2,0}$

$$s_2 - s_1 = -R\ln\frac{p_{0,2}}{p_{0,1}}$$

$$\boxed{\frac{p_{0,2}}{p_{0,1}} = e^{-(s_2 - s_1)/R}}$$

- Total pressure decreases across a normal shock wave.

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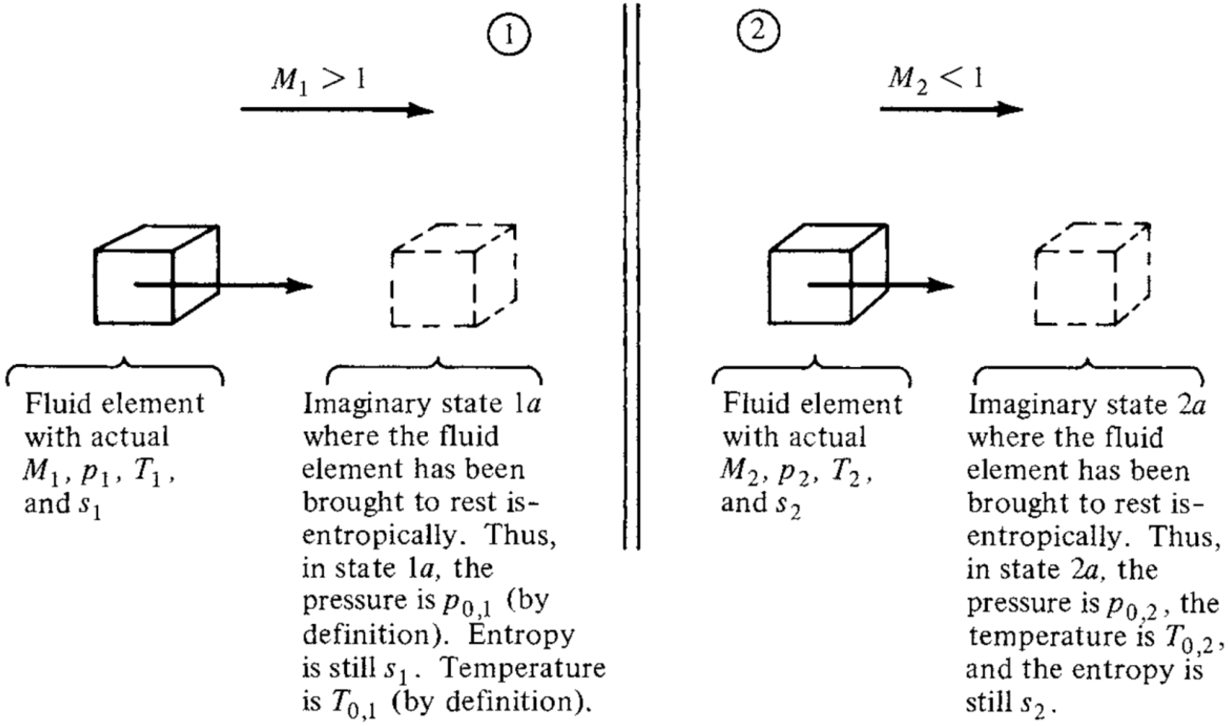
Calculation of Normal Shock-Wave Properties

- **Total conditions** **across the normal shock wave: total pressure**



$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1} \qquad s_2 - s_1 = -R\ln\frac{p_{0,2}}{p_{0,1}}$$

$$\downarrow$$

$$-R\ln\frac{p_{0,2}}{p_{0,1}} = c_p \ln\frac{T_2}{T_1} - R\ln\frac{p_2}{p_1}$$

$$\downarrow$$

$$\frac{p_{2,0}}{p_{1,0}} = \left(\frac{T_2}{T_1}\right)^{\frac{-k}{k-1}} \frac{p_2}{p_1}$$

$$\downarrow$$

$$\boxed{\frac{p_{0,2}}{p_{0,1}} = \left[1 + \frac{2k}{k+1}\left(M_1^2 - 1\right)\right]^{\frac{-1}{k-1}} \left[\frac{(k+1)M_1^2}{2+(k-1)M_1^2}\right]^{\frac{k}{k-1}}}$$

- Total pressure decreases across a normal shock wave.

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Calculation of Normal Shock-Wave Properties

- **Assumptions**
  - Steady flow, $\partial/\partial t = 0$.
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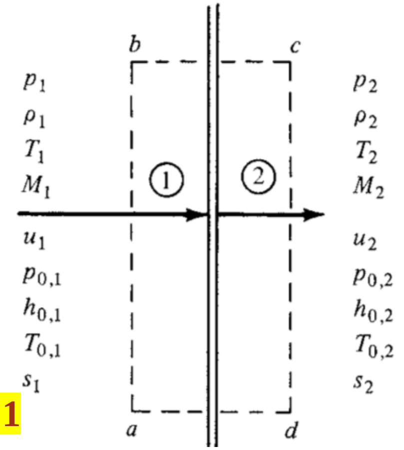
+ One-dimensional flow, $M_1 >= 1$



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$$\boxed{M_2^{\,2} = \frac{1+[(k-1)/2]M_1^{\,2}}{kM_1^{\,2}-(k-1)/2}}$$

$$\boxed{T_{0,1} = T_{0,2}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(k+1)M_1^2}{2+(k-1)M_1^2}}$$

$$\boxed{\frac{p_{0,2}}{p_{0,1}} = e^{-(s_2-s_1)/R}}$$

$$\boxed{\frac{p_2}{p_1} = 1+\frac{2k}{k+1}(M_1^2-1)}$$

$$\boxed{\frac{h_2}{h_1} = \frac{T_2}{T_1} = \left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}}$$

$$\boxed{s_2 - s_1 = c_p \ln\left\{\left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}\right\} - R\ln\left[1+\frac{2k}{k+1}(M_1^2-1)\right] \geq 0}$$

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Calculation of Normal Shock-Wave Properties

- **The variations of properties across a normal shock wave**

$$M_2{}^2=\frac{1+[(k-1)/2]M_1{}^2}{kM_1{}^2-(k-1)/2}$$

$$T_{0,1}=T_{0,2}$$

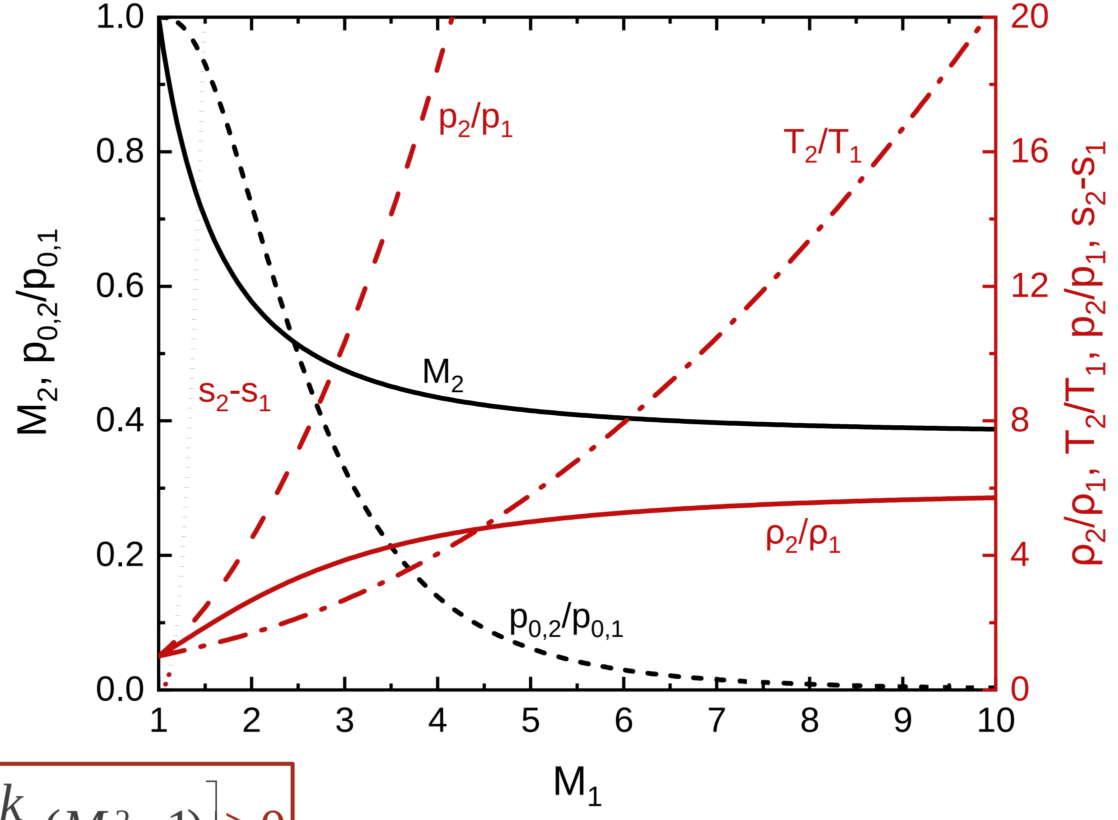
$$\frac{\rho_2}{\rho_1}=\frac{u_1}{u_2}=\frac{(k+1)M_1^2}{2+(k-1)M_1^2}$$

$$\frac{p_{0,2}}{p_{0,1}}=e^{-(s_2-s_1)/R}$$

$$\frac{p_2}{p_1}=1+\frac{2k}{k+1}(M_1^2-1)$$

$$\frac{h_2}{h_1}=\frac{T_2}{T_1}=\left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}$$

$$s_2-s_1=c_p\ln\left\{\left[1+\frac{2k}{k+1}(M_1^2-1)\right]\frac{2+(k-1)M_1^2}{(k+1)M_1^2}\right\}-R\ln\left[1+\frac{2k}{k+1}(M_1^2-1)\right]\geq 0$$



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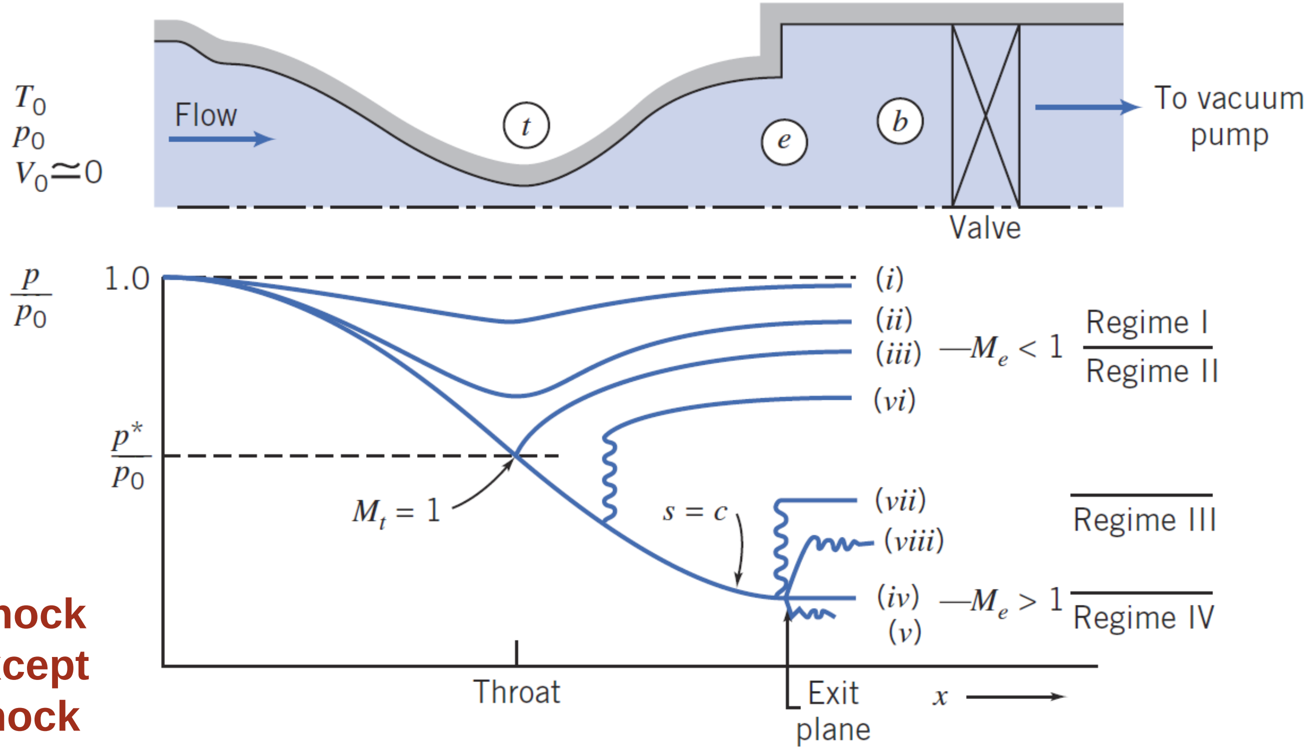
Compressible Flow with Area Variation

- **Compressible Flow in a Convergent-Divergent Nozzle**
  - Once there is shock wave, the flow is not isentropic.

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{1/(k-1)}$$



The flows before and after the normal shock wave are still respectively isentropic, except that all the properties will jump at the shock wave.

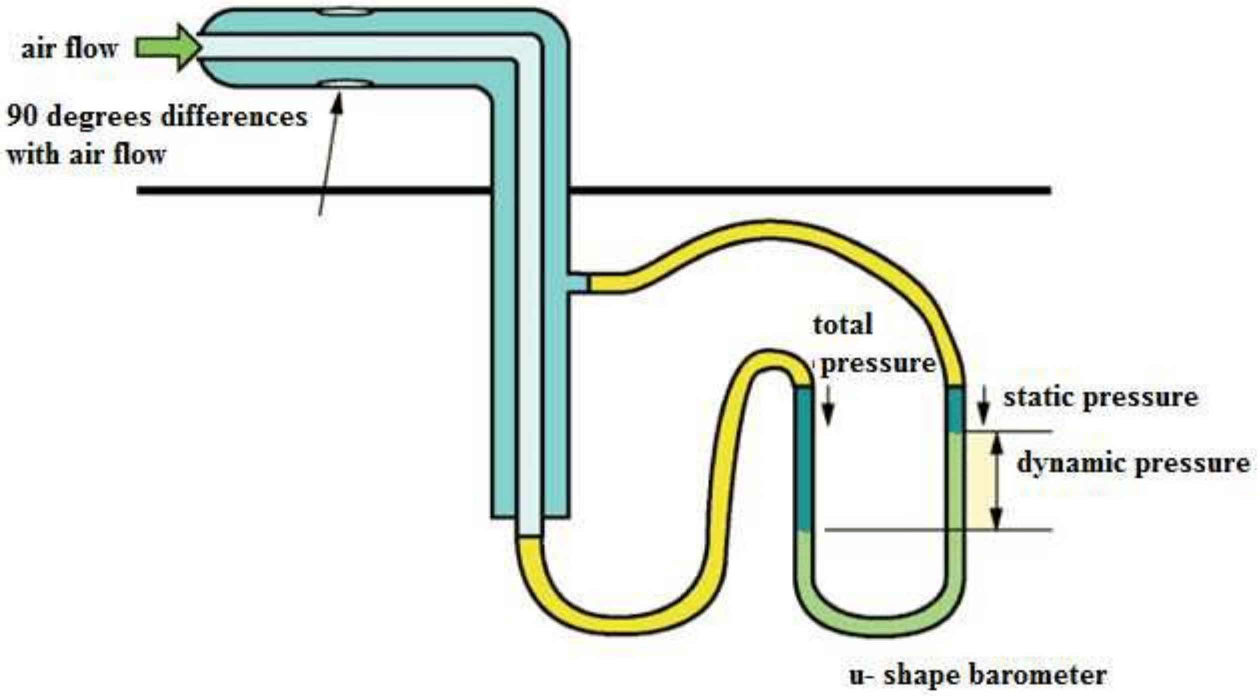
Pressure distributions for flow in a converging-diverging nozzle for different back pressures.

Outlines

- Introduction
- The Basic Normal Shock Equations
- Speed of Sound
- Special Forms of the Energy Equation
- When Is a Flow Compressible?
- Calculation of Normal Shock-Wave Properties
- **Measurement of Velocity in Compressible Flows**

Measurement of Velocity in Compressible Flows

- Velocity measurement using a pitot tube



- For **incompressible flow**

Bernoulli's Equation

$$\boxed{p_1+\frac{1}{2}\rho_1 u_1^2=p_2+\frac{1}{2}\rho_2 u_2^2}$$

Total pressure

$$p_0-p_1=\frac{1}{2}\rho u_1^2$$

Static pressure

Dynamic pressure

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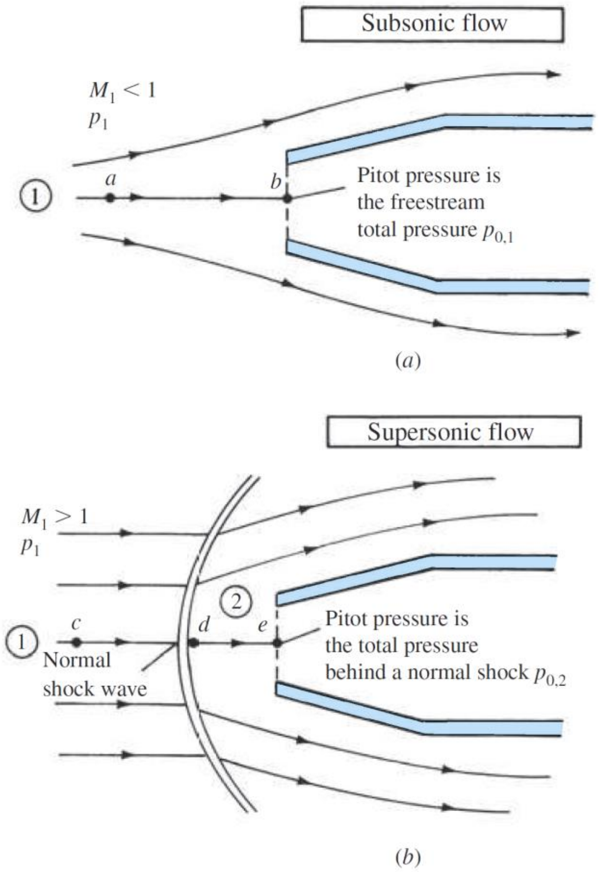
Measurement of Velocity in Compressible Flows

- Velocity measurement using a pitot tube



- For **subsonic compressible flow**

a → b is isentropic process.

$$\frac{p_{0,1}}{p_1} = \left(1 + \frac{k-1}{2} M_1^2\right)^{k/(k-1)}$$

$$\downarrow$$

$$M_1^2 = \frac{2}{k-1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(k-1)/k} - 1\right]$$

$$\downarrow$$

$$u_1^2 = \frac{2c_1^2}{k-1}\left[\left(\frac{p_{0,1}}{p_1}\right)^{(k-1)/k} - 1\right]$$

Comparing to incompressible flow, additional knowledge of c_1 is needed for subsonic compressible flow.

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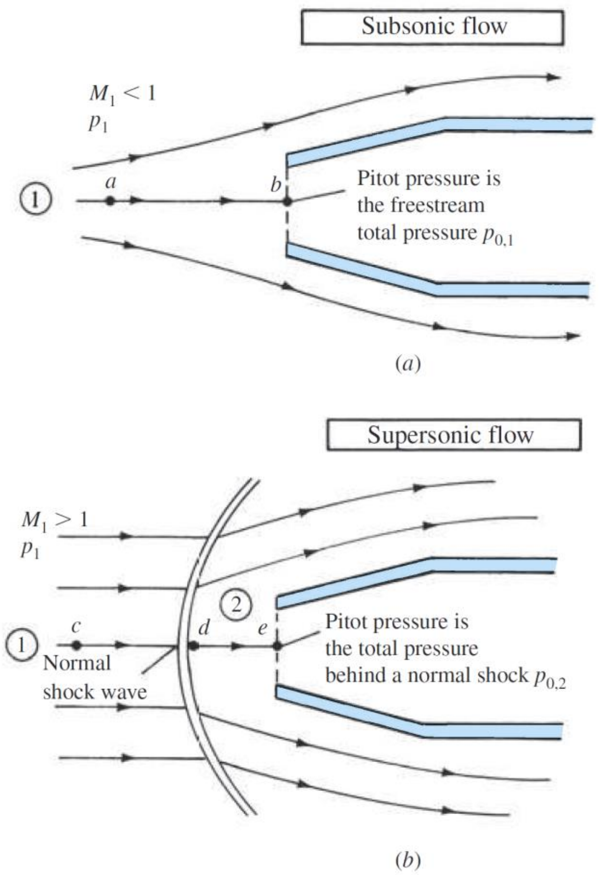
Measurement of Velocity in Compressible Flows

- Velocity measurement using a pitot tube



- For **supersonic compressible flow (method 1)**

c $\rightarrow$ the point before the shock wave is isentropic process.
d $\rightarrow$ e is also isentropic process.
However, the point before the shock wave $\rightarrow$ d is nonisentropic.

$$\frac{p_{0,2}}{p_1} = \frac{p_{0,2}}{p_2}\frac{p_2}{p_1}$$

$$\frac{p_{0,2}}{p_2} = \left(1 + \frac{k-1}{2} M_2^2\right)^{k/(k-1)} \qquad M_2{}^2 = \frac{1 + [(k-1)/2] M_1{}^2}{k M_1{}^2 - (k-1)/2} \qquad \frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{p_{0,2}}{p_1} = \left(\frac{(k+1)^2 M_1{}^2}{4kM_1{}^2 - 2(k-1)}\right)^{k/(k-1)} \left(1 + \frac{2k}{k+1}(M_1^2 - 1)\right)$$

The Rayleigh Pitot tube fomula

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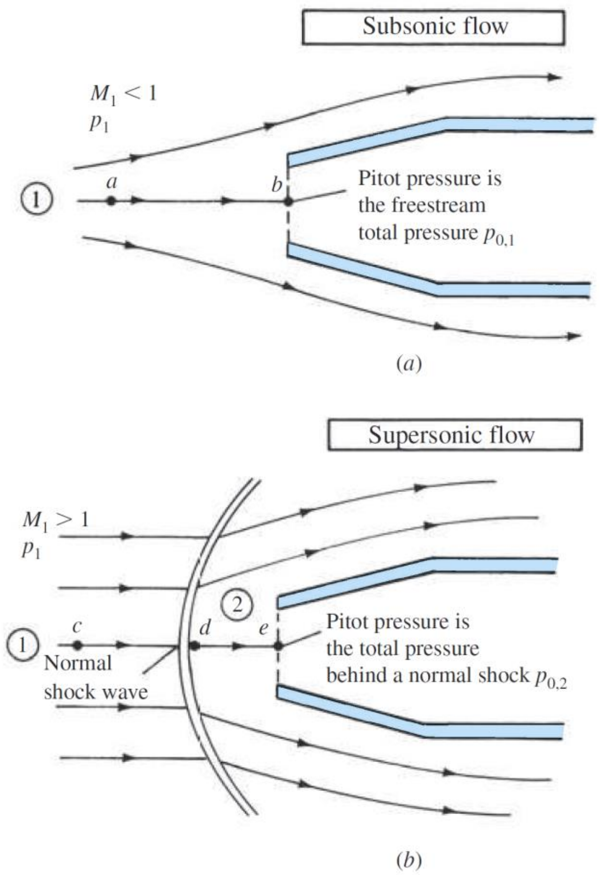
Measurement of Velocity in Compressible Flows

- Velocity measurement using a pitot tube



- For **supersonic compressible flow (method 2)**

d $\rightarrow$ e is isentropic process.
The point before the shock wave $\rightarrow$ e can be calculated using the results of the properties across the shock wave.

$$\frac{p_{0,2}}{p_1} = \frac{p_{0,2}}{p_{0,1}}\frac{p_{0,1}}{p_1}$$

$$\frac{p_{0,2}}{p_{0,1}} = \left[1+\frac{2k}{k+1}(M_1^2-1)\right]^{\frac{-1}{k-1}}\left[\frac{(k+1)M_1^2}{2+(k-1)M_1^2}\right]^{\frac{k}{k-1}} \qquad \frac{p_{0,1}}{p_1} = \left(1+\frac{k-1}{2}M_1^2\right)^{k/(k-1)}$$

$$\frac{p_{0,2}}{p_1} = \left(\frac{(k+1)^2 M_1{}^2}{4kM_1{}^2-2(k-1)}\right)^{k/(k-1)}\left(1+\frac{2k}{k+1}(M_1^2-1)\right)$$

The Rayleigh Pitot tube formula

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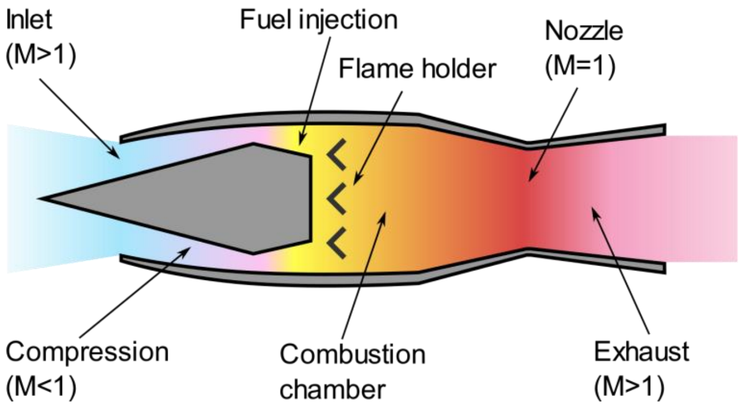
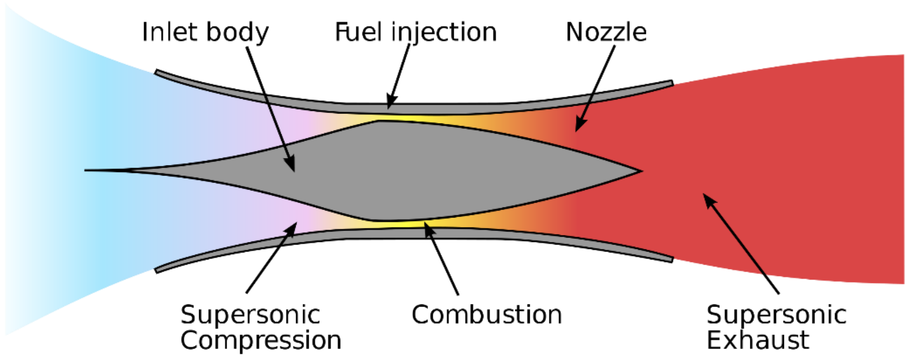
Calculation of Normal Shock-Wave Properties

- Examples 1 & 2



Ramjet (冲压发动机，ram撞击)



Scramjet (超燃冲压发动机)

- **Ramjet** is a form of airbreathing jet engine that uses the engine's forward motion to compress incoming air without a compressor.
- **Scramjet (supersonic combustion ramjet)** is a variant of a ramjet airbreathing jet engine in which combustion takes place in supersonic airflow.

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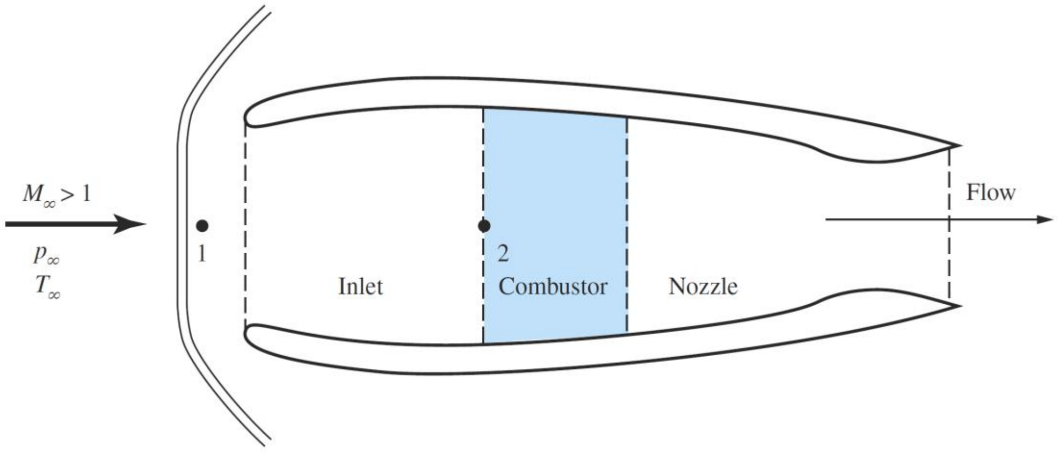
Calculation of Normal Shock-Wave Properties

- Examples 1 & 2



The ramjet is flying at Mach 2 at a standard altitude of 10 km, where the air pressure and temperature are 2.65 × 10^4 N/m^2 and 223.3 K, respectively. Calculate the air temperature and pressure at point 2 when the Mach number at that point is 0.2.

399 K, 1.42 atm

Repeat Example 1, except for a freestream Mach number M_∞ = 10. Assume that the ramjet has been redesigned so that the Mach number at point 2 remains equal to 0.2.

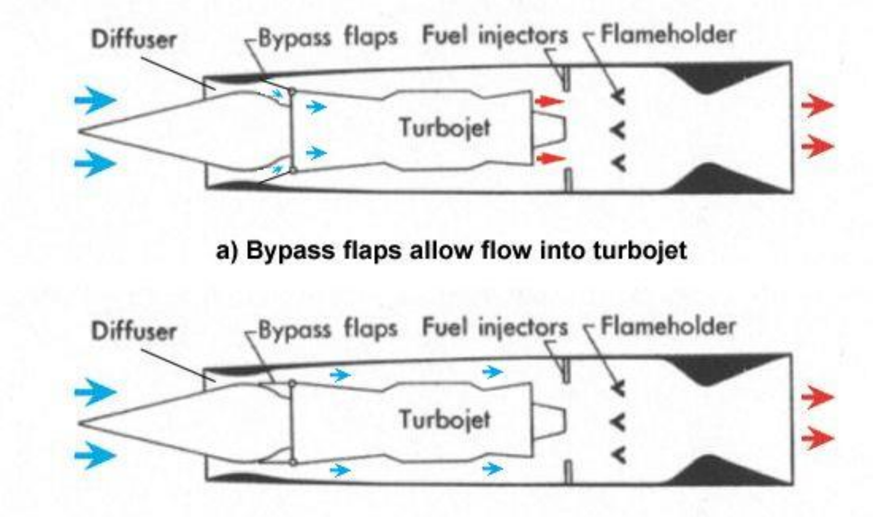
4690 K, 32.7 atm

$$\frac{T_{0,1}}{T_\infty} = 1 + \frac{k-1}{2} M_\infty^2 \qquad \frac{p_{0,1}}{p_\infty} = \left(1 + \frac{k-1}{2} M_\infty^2\right)^{k/(k-1)} \qquad \frac{p_{0,1}}{p_{0,\infty}} = \left[1 + \frac{2k}{k+1}(M_\infty^2 - 1)\right]^{\frac{-1}{k-1}} \left[\frac{(k+1)M_\infty^2}{2+(k-1)M_\infty^2}\right]^{\frac{k}{k-1}}$$

$$T_{1,0} = T_{2,0} \qquad \frac{T_{0,2}}{T_2} = 1 + \frac{k-1}{2} M_2^2 \qquad \frac{p_{0,2}}{p_2} = \left(1 + \frac{k-1}{2} M_2^2\right)^{k/(k-1)}$$

Calculation of Normal Shock-Wave Properties

- Example 3



a) Bypass flaps allow flow into turbojet

b) Bypass flaps block flow into turbojet during ramjet mode

Consider the Lockheed SR-71 Blackbird flying at a standard altitude of 25 km. The pressure measured by a Pitot tube on this airplane is 3.88 × 10^4 N/m^2. Calculate the velocity of the airplane. At an altitude of 25 km, p = 2.5273 × 10^3 N/m^2 and T = 216.66 K.

$$\frac{p_{0,1}}{p_{0,\infty}} = \left[1+\frac{2k}{k+1}(M_\infty^2-1)\right]^{\frac{-1}{k-1}}\left[\frac{(k+1)M_\infty^2}{2+(k-1)M_\infty^2}\right]^{\frac{k}{k-1}}$$

$$a=\sqrt{kRT}$$

$$V = Ma = 1003 \text{ m/s}$$

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Homework

Problem 12.54

Air flows from a large tank ($p = 650\ kPa,\ T = 550℃$) through a converging nozzle, with a throat area of 600 mm^2, and dischargers to the atmosphere. Determine the mass flow rate of the flow for isentropic flow through the nozzle.

$$\frac{p_t}{p_0} = \frac{101}{650} = 0.155 < 0.528 \rightarrow choked\ flow, M_t = 1.0$$

$$\frac{T_0}{T_t} = [1 + \frac{k-1}{2} M_t^2]$$

$$\frac{p_0}{p_t} = \left[1 + \frac{k-1}{2} M_t^2\right]^{\frac{\mathrm{k}}{\mathrm{k}-1}}$$

$$\dot{m} = \rho_t V_t A_t$$

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Homework

Problem 12.58

Air flows from a large tank ($p = 650\ kPa,\ T = 550℃$) through a converging nozzle, with a throat area of 600 mm^2, and dischargers to the atmosphere. Determine the mass flow rate of the flow for isentropic flow through the nozzle.

$$\sum F_x = 0 = p_{1,g}A_1 - p_{2,g}A_2 - R_x = -V_1(\dot{m}) + V_2(\dot{m})$$

$$R_x = p_{1,g}A_1 - p_{2,g}A_2 + \dot{m}(V_1 - V_2)$$

$$\frac{p_t}{p_0} = \frac{101}{171} = 0.6257 > 0.528 \rightarrow not\ choked\ flow$$

$$\frac{p_0}{p_t} = \left[1 + \frac{k-1}{2}M_t^2\right]^{\frac{\mathrm{k}}{\mathrm{k}-1}} \qquad \frac{p_0}{p_1} = \left[1 + \frac{k-1}{2}M_1^2\right]^{\frac{\mathrm{k}}{\mathrm{k}-1}} \qquad \frac{p_0}{p_2} = \left[1 + \frac{k-1}{2}M_2^2\right]^{\frac{\mathrm{k}}{\mathrm{k}-1}}$$

$$\frac{T_0}{T_t} = \left[1 + \frac{k-1}{2}M_t^2\right] \qquad \frac{T_0}{T_1} = \left[1 + \frac{k-1}{2}M_1^2\right] \qquad \frac{T_0}{T_2} = \left[1 + \frac{k-1}{2}M_2^2\right]$$

Homework

Problem 12.59

Air enters a converging-diverging nozzle at 2MPa and 313 K. At the exit of the nozzle, the pressure is 200 kPa. Assume adiabatic, frictionless flow through the nozzle. The throat area is 20 cm^2. What is the area at the nozzle exit? What is the mass flow rate of the air?

$$\frac{p_0}{p_e} = \left[1 + \frac{k-1}{2} M_e^2\right]^{\frac{k}{k-1}}$$

$$\frac{A}{A^*} = \frac{1}{M_e}\left[\frac{1 + \frac{k-1}{2} M_e^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

$$\dot{m} = A_e p_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

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Homework

Problem 12.64

Air discharges through a convergent-divergent nozzle which is attached to a large reservoir. At a point in the nozzle in a normal shock wave is detected across which the absolute pressure jumps from 69 to 207 kPa. Calculate the pressures in the throat of the nozzle and in the reservoir.

$$\frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{p_0}{p_1} = \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$

$$\frac{p_t}{p_0} = 0.528$$

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Homework

This assignment is due by **6pm on June 10th**.

- Upload your solution to BB.
- You can either type your solution out using a word editor like Microsoft Word or clearly hand write the information and then copy/scan your solution to a digital file.

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