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Applied Fluid Mechanics
Homework 11

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2018141521058

Applied Fluid Mechanics

Class Section 01

06/16/2021

Problem 12.63

12.63 Testing of a demolition explosion is to be evaluated. Sensors indicate that the shock wave generated at the instant of explosion is 30 MPa absolute. If the explosion occurs in air at 20°C and 101 kPa, find the speed of the shock wave, and the temperature and speed of the air just after the shock passes. As an approximation assume $k = 1.4$. Why is this an approximation?

Solution:

$$\frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\Rightarrow M_1 = 15.9606$$

$$V_S = V_1 = M_1\sqrt{kRT_1} = 5476.7 \ \ \mathrm{m/s}$$

$$\frac{T_2}{T_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1)\right]\frac{2 + (k-1)M_1^2}{(k+1)M_1^2}$$

$$\Rightarrow T_2 = 14797 \ \ \mathrm{K}$$

$$M_2^2 = \frac{1 + \left[\frac{k-1}{2}\right]M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

$$\Rightarrow M_2 = 0.3818$$

$$V_S - V = V_2 = M_2\sqrt{kRT_2} = 930.7071 \ \ \mathrm{m/s}$$

$$V = V_s - V_2 = 5476.7 \ \ \mathrm{m/s} - 930.7071 \ \ \mathrm{m/s} = 4546.0 \ \ \mathrm{m/s}$$

Problem 12.67

12.67 Air undergoes a normal shock. Upstream, $T_1 = 35°\mathrm{C}$, $p_1 = 229$ kPa absolute, and $V_1 = 704$ m/s. Determine the temperature and stagnation pressure of the air stream leaving the shock.

Solution:

$$M_1 = \frac{V_1}{\sqrt{kRT_1}} = 2.0011$$

$$M_2^2 = \frac{1 + \left[\frac{k-1}{2}\right]M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

$$\Rightarrow M_2 = 0.5772$$

$$\frac{T_2}{T_1} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1)\right]\frac{2 + (k-1)M_1^2}{(k+1)M_1^2}$$

$$\Rightarrow T_2 = 520.2711 \ \ \mathrm{K}$$

$$\mathrm{Appendix\ \ D.1{:}}\ \frac{p_1}{p_{0,1}} = 0.1278$$

$$\mathrm{Appendix\ \ D.2{:}}\ \frac{p_{0,2}}{p_{0,1}} = 0.7209$$

$$\frac{p_{0,2}}{p_{0,1}} = \left[1 + \frac{2k}{k+1}(M_1^2 - 1)\right]^{-\frac{1}{k-1}}\left[\frac{(k+1)M_1^2}{2 + (k-1)M_1^2}\right]^{\frac{k}{k-1}} = 0.7204$$

$$\Rightarrow p_{0,2} = \frac{\frac{p_{0,2}}{p_{0,1}}}{\frac{p_1}{p_{0,1}}} \cdot p_1 = \frac{0.7209}{0.1278} \times (229\ \text{kPa})$$

$$= 1.2918\ \text{MPa}$$

Problem 12.68

12.68 If, through a normal shock wave in air, the absolute pressure rises from 275 to 410 kPa and the velocity diminishes from 460 to 346 m/s, what temperatures are to be expected upstream and downstream from the wave?

Solution:

$$\frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\Rightarrow M_1 = 1.1920$$

$$M_1 = \frac{V_1}{\sqrt{kRT_1}}$$

$$\Rightarrow T_1 = 370.7921\ \text{K}$$

$$M_2^2 = \frac{1 + \left[\frac{k-1}{2}\right] M_1^2}{kM_1^2 - \frac{k-1}{2}}$$

$$\Rightarrow M_2 = 0.8472$$

$$M_2 = \frac{V_2}{\sqrt{kRT_2}}$$

$$\Rightarrow T_2 = 415.2487\ \text{K}$$

Problem 12.71

12.71 The Concorde supersonic transport flew at $M = 2.2$ at 20 km altitude. Air is decelerated isentropically by the engine inlet system to a local Mach number of 1.3. The air passed through a normal shock and was decelerated further to $M = 0.4$ at the engine compressor section. Assume, as a first approximation, that this subsonic diffusion process was isentropic and use standard atmosphere data for free-stream conditions. Determine the temperature, pressure, and stagnation pressure of the air entering the engine compressor.

Solution:

$$\text{Appendix A.3:} \begin{cases} T_\infty = 216.7\ \text{K} \\ p_\infty = 5.5293\ \text{kPa} \end{cases}$$

For $M_\infty = 2.2$,

$$\text{Appendix D.1:} \begin{cases} \frac{T_\infty}{T_0} = 0.5111 \\ \frac{p_\infty}{p_0} = 0.1001 \end{cases}$$

$$\Rightarrow \begin{cases} T_0 = 423.9709\ \text{K} \\ p_0 = 55.242\ \text{kPa} \end{cases}$$

For $M_1 = 1.3$,

$$\text{Appendix D.1:} \begin{cases} \frac{T_1}{T_0} = 0.7471 \\ \frac{p_1}{p_0} = 0.3748 \end{cases}$$

$$\Rightarrow \begin{cases} T_1 = 316.7656\ \text{K} \\ p_1 = 20.703\ \text{kPa} \end{cases}$$

$$\text{Appendix D.2:} \begin{cases} \frac{T_{0,2}}{T_{0,1}} = 1.1920 \\ \frac{p_{0,2}}{p_{0,1}} = 0.9579 \end{cases}$$

$$\Rightarrow \begin{cases} M_2 = 0.7860 \\ T_1 = 380.5846\ \text{K} \\ p_{0,3} = p_{0,2} = 57.0\ \text{kPa} \end{cases}$$

For $M_3 = 0.4$,

$$\text{Appendix D.1:} \begin{cases} \frac{T_3}{T_{0,3}} = 0.9690 \\ \frac{p_3}{p_{0,3}} = 0.8956 \end{cases}$$

$$\Rightarrow \begin{cases} T_3 = 414\ \text{K} \\ p_3 = 51.9\ \text{kPa} \end{cases}$$

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