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Applied Fluid Mechanics
Homework 10

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Applied Fluid Mechanics

Class Section 01

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Problem 12.54

12.54 Air flows from a large tank at $p = 650$ kPa absolute, $T = 550°C$ through a converging nozzle, with a throat area of 600 mm^2, and discharges to the atmosphere. Determine the mass rate of flow for isentropic flow through the nozzle.

Solution:

$$\frac{p_t}{p_0} = \frac{101}{650} = 0.155 < 0.528$$

Therefore, the air is choked flow, for which $M_t = 1.0$.

$$\frac{T_0}{T_t} = \left[1 + \frac{k-1}{2}M_t^2\right]$$

$$T_t = \frac{T_0}{\left[1 + \frac{k-1}{2}M_t^2\right]} = \frac{(550 + 273.15)\ \text{K}}{1 + \frac{1.4-1}{2} \times 1^2}$$
$$= 685.96\ \text{K}$$

$$\frac{p_0}{p_t} = \left[1 + \frac{k-1}{2}M_t^2\right]^{\frac{k}{k-1}}$$

$$p_t = \frac{p_0}{\left[1 + \frac{k-1}{2}M_t^2\right]^{\frac{k}{k-1}}}$$
$$= \frac{650\ \text{kPa}}{\left(1 + \frac{1.4-1}{2} \times 1^2\right)^{\frac{1.4}{1.4-1}}}$$
$$= 343.38\ \text{kPa}$$

$$\rho_t = \frac{p_t}{RT_t}$$
$$= \frac{343.38\ \text{kPa}}{(287\ \text{N}\cdot\text{m/kg}\cdot\text{K}) \times (685.96\ \text{K})}$$
$$= 1.74\ \text{kg/m}^3$$

$$c_t = \sqrt{kRT_t}$$
$$= \sqrt{1.4 \times (287\ \text{N}\cdot\text{m/kg}\cdot\text{K}) \times (685.96\ \text{K})}$$
$$= 524.99\ \text{m/s}$$

$$V_t = M_t c_t = 1 \times 524.99\ \text{m/s}$$
$$= 524.99\ \text{m/s}$$

$$\dot{m} = \rho_t V_t A_t = (1.74\ \text{kg/m}^3) \times (524.99\ \text{m/s}) \times (600 \times 10^{-6}\ \text{m}^2) = 0.5494\ \text{kg/s}$$

Problem 12.58

12.58 Air flows isentropically through a converging nozzle attached to a large tank, where the absolute pressure is 171 kPa and the temperature is 27°C. At the inlet section, the Mach number is 0.2. The nozzle discharges to the atmosphere; the discharge area is 0.015 m^2. Determine the magnitude and direction of the force that must be applied to hold the nozzle in place.

Solution:

$$\frac{p_t}{p_0} = \frac{101}{171} = 0.6257 > 0.528$$

Therefore, the air is not choked flow.

$$\frac{p_0}{p_t} = \left[1 + \frac{k-1}{2}M_t^2\right]^{\frac{k}{k-1}}$$

$$\frac{171\ \text{kPa}}{101\ \text{kPa}} = \left[1 + \frac{1.4-1}{2}M_t^2\right]^{\frac{1.4}{1.4-1}}$$

$$\Rightarrow M_t = 0.9010$$

$$\frac{T_0}{T_t} = \left[1 + \frac{k-1}{2}M_t^2\right]$$

$$T_t = \frac{T_0}{\left[1 + \frac{k-1}{2} M_t^2\right]}$$

$$= \frac{(27 + 273.15)\ \text{K}}{1 + \frac{1.4 - 1}{2} \times 0.9010^2}$$

$$= 258.2276\ \text{K}$$

$$c_t = \sqrt{kRT_t}$$

$$= \sqrt{1.4 \times (287\ \text{N} \cdot \text{m/kg} \cdot \text{K}) \times (258.2276\ \text{K})}$$

$$= 322.1116\ \text{m/s}$$

$$V_t = M_t c_t = 0.9010 \times 322.1116\ \text{m/s}$$
$$= 290.2103\ \text{m/s}$$

$$\rho_t = \frac{p_t}{RT_t}$$

$$= \frac{101\ \text{kPa}}{(287\ \text{N} \cdot \text{m/kg} \cdot \text{K}) \times (258.2276\ \text{K})}$$

$$= 1.3628\ \text{kg/m}^3$$

$$\dot{m} = \rho_t V_t A_t = (1.3628\ \text{kg/m}^3)$$
$$\times (290.2103\ \text{m/s})$$
$$\times (0.015\ \text{m}^2)$$
$$= 5.9325\ \text{kg/s}$$

$$\frac{p_0}{p_1} = \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$

$$p_1 = \frac{p_0}{\left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}}$$

$$= \frac{171\ \text{kPa}}{\left(1 + \frac{1.4 - 1}{2} \times 0.2^2\right)^{\frac{1.4}{1.4-1}}}$$

$$= 166.2969\ \text{kPa}$$

$$\frac{T_0}{T_1} = \left[1 + \frac{k-1}{2} M_1^2\right]$$

$$T_1 = \frac{T_0}{\left[1 + \frac{k-1}{2} M_1^2\right]}$$

$$= \frac{(27 + 273.15)\ \text{K}}{\left(1 + \frac{1.4 - 1}{2} \times 0.2^2\right)}$$

$$= 297.7679\ \text{K}$$

$$c_1 = \sqrt{kRT_1}$$

$$= \sqrt{1.4 \times (287\ \text{N} \cdot \text{m/kg} \cdot \text{K}) \times (297.7679\ \text{K})}$$

$$= 345.8947\ \text{m/s}$$

$$V_1 = M_1 c_1 = 0.2 \times 345.8947\ \text{m/s}$$
$$= 69.1789\ \text{m/s}$$

$$\rho_1 = \frac{p_1}{RT_1}$$

$$= \frac{166.2969\ \text{kPa}}{(287\ \text{N} \cdot \text{m/kg} \cdot \text{K}) \times (297.7679\ \text{K})}$$

$$= 1.9459\ \text{kg/m}^3$$

$$A_1 = \frac{\dot{m}}{\rho_t V_t}$$

$$= \frac{(5.9325\ \text{kg/s})}{(1.9459\ \text{kg/m}^3) \times (69.1789\ \text{m/s})}$$

$$= 0.0441\ \text{m}^2$$

$$\sum F_x = 0 = p_{1,g} A_1 - p_{2,g} A_2 - R_x$$
$$= -V_1(\dot{m}) + V_2(\dot{m})$$

$$R_x = p_{1,g} A_1 - p_{2,g} A_2 + \dot{m}(V_1 - V_2)$$
$$= [(166.2969\ \text{kPa})$$
$$- (101.3\ \text{kPa})]$$
$$\times (0.0441\ \text{m}^2)$$
$$- [(101.3\ \text{kPa})$$
$$- (101.3\ \text{kPa})]$$
$$\times (0.015\ \text{m}^2)$$
$$+ (5.9325\ \text{kg/s})$$
$$\times [(69.1789\ \text{m/s})$$
$$- (290.2103\ \text{m/s})]$$
$$= 1553\ \text{N}$$

Problem 12.59

12.59 Air enters a converging-diverging nozzle at 2 MPa absolute and 313 K. At the exit of the nozzle, the pressure is 200 kPa absolute. Assume adiabatic, frictionless flow through the nozzle. The throat area is 20 cm^2. What is the area at the nozzle exit? What is the mass flow rate of the air?

Solution:

$$\frac{p_0}{p_a} = \left[1 + \frac{k-1}{2} M_a^2\right]^{\frac{k}{k-1}}$$

$$\frac{2\ \text{MPa}}{200\ \text{kPa}} = \left[1 + \frac{1.4-1}{2} M_a^2\right]^{\frac{1.4}{1.4-1}}$$

$$\Rightarrow M_a = 2.1572$$

$$\frac{A}{A^*} = \frac{1}{M_a}\left[\frac{1 + \frac{k-1}{2} M_a^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

$$A_e = A_t \frac{1}{M_a}\left[\frac{1 + \frac{k-1}{2} M_a^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

$$= (20\ \text{cm}^2) \times \frac{1}{2.1572}$$

$$\times \left[\frac{1 + \frac{1.4-1}{2} \times 2.1572^2}{\frac{1.4+1}{2}}\right]^{\frac{1.4+1}{2\times(1.4-1)}}$$

$$= 38.6136\ \text{cm}^2$$

$$\dot{m} = A_e p_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

$$= (38.6136\ \text{cm}^2) \times (2\ \text{MPa})$$

$$\times \sqrt{\frac{1.4}{(287\ \text{N}\cdot\text{m/kg}\cdot\text{K}) \times (313\ \text{K})}}$$

$$\times \left(\frac{2}{1.4+1}\right)^{\frac{1.4+1}{2\times(1.4-1)}} = 17.6432\ \text{kg/s}$$

Problem 12.64

12.64 Air discharges through a convergent-divergent nozzle which is attached to a large reservoir. At a point in the nozzle a normal shock wave is detected across which the absolute pressure jumps from 69 to 207 kPa. Calculate the pressures in the throat of the nozzle and in the reservoir.

Solution:

$$\frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1)$$

$$\frac{207\ \text{kPa}}{69\ \text{kPa}} = 1 + \frac{2 \times 1.4}{1.4+1}(M_1^2 - 1)$$

$$\Rightarrow M_1 = 1.6475$$

$$\frac{p_0}{p_1} = \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$

$$p_0 = p_1\left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$

$$= (69\ \text{kPa})$$

$$\times \left(1 + \frac{1.4-1}{2}\right.$$

$$\left.\times 1.6475^2\right)^{\frac{1.4}{1.4-1}}$$

$$= 314.7669\ \text{kPa}$$

$$\frac{p_t}{p_0} = 0.528$$

$$p_t = 0.528 p_0 = 0.528 \times (314.7669\ \text{kPa})$$
$$= 166.1969\ \text{kPa}$$

— Christopher King —