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Applied Fluid Mechanics
Homework 09

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Applied Fluid Mechanics

Class Section 01

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Problem 12.34

12.34 Consider steady, adiabatic flow of air through a long straight pipe with $A = 0.05\ \text{m}^2$. At the inlet section ①, the air is at 200 kPa absolute, 60°C, and 146 m/s. Downstream at section ②, the air is at 95.6 kPa absolute and 280 m/s. Determine p_{0_1}, p_{0_2}, T_{0_1}, T_{0_2}, and the entropy change for the flow. Show static and stagnation state points on a Ts diagram.

Solution:

$$c_1 = \sqrt{kRT_1}$$

$$= \sqrt{(1.4) \times (286.9\ \ \text{J/kg} \cdot \text{K}) \times (333.15\ \ \text{K})}$$

$$= 365.8046\ \ \text{m/s}$$

$$M_1 = \frac{V_1}{c_1} = \frac{(146\ \ \text{m/s})}{(365.8046\ \ \text{m/s})} = 0.3991$$

$$p_{0_1} = p_1 \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$

$$= (200\ \ \text{kPa}) \times \left[1 + \frac{1.4-1}{2} \times 0.3991^2\right]^{\frac{1.4}{1.4-1}}$$

$$= 223.2039\ \ \text{kPa}$$

$$T_{0_1} = T_1 \left[1 + \frac{k-1}{2} M_1^2\right]$$

$$= (333.15\ \ \text{K}) \times \left[1 + \frac{1.4-1}{2} \times 0.3991^2\right]$$

$$= 343.7640\ \ \text{K}$$

$$T_{0_2} = T_{0_1} = 343.7640\ \ \text{K}$$

$$\rho_1 V_1 = \rho_2 V_2$$

$$\rho_2 = \rho_1 \cdot \frac{V_1}{V_2} = \frac{p_1}{RT_1} \cdot \frac{V_1}{V_2}$$

$$= \frac{(200\ \ \text{kPa})}{(286.9\ \ \text{J/kg} \cdot \text{K}) \times (333.15\ \ \text{K})} \times \frac{(146\ \ \text{m/s})}{(280\ \ \text{m/s})} = 1.0911\ \ \text{kg/m}^3$$

$$T_2 = \frac{p_2}{\rho_2 R}$$

$$= \frac{(95.6\ \ \text{kPa})}{(1.0911\ \ \text{kg/m}^3) \times (286.9\ \ \text{J/kg} \cdot \text{K})}$$

$$= 305.4027\ \ \text{K}$$

$$c_2 = \sqrt{kRT_2}$$

$$= \sqrt{(1.4) \times (286.9\ \ \text{J/kg} \cdot \text{K}) \times (305.4027\ \ \text{K})}$$

$$= 350.2400\ \ \text{m/s}$$

$$M_2 = \frac{V_2}{c_2} = \frac{(280\ \ \text{m/s})}{(350.2400\ \ \text{m/s})} = 0.7995$$

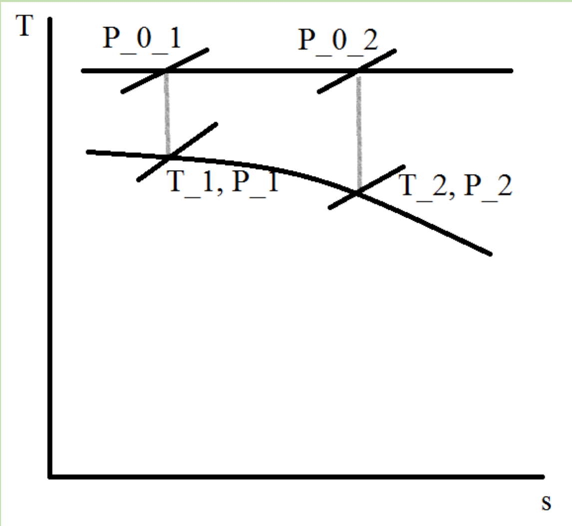
$$p_{0_2} = p_2 \left[1 + \frac{k-1}{2} M_2^2\right]^{\frac{k}{k-1}}$$

$$= (95.6\ \ \text{kPa}) \times \left[1 + \frac{1.4-1}{2} \times 0.7995^2\right]^{\frac{1.4}{1.4-1}}$$

$$= 145.6476\ \ \text{kPa}$$

$$s_2 - s_1 = -R\ln\frac{p_{0_2}}{p_{0_1}}$$
$$= -(286.9\ \mathrm{J/kg\cdot K})\times\ln\frac{(145.6476\ \mathrm{kPa})}{(223.2039\ \mathrm{kPa})}$$
$$= 122.4763\ \mathrm{J/kg\cdot K}$$

The T-s diagram is shown below:



Problem 12.38

12.38 Air flows from the atmosphere into an evacuated tank through a convergent nozzle of 38-mm tip diameter. If atmospheric pressure and temperature are 101.3 kPa and 15°C, respectively, what vacuum must be maintained in the tank to produce sonic velocity in the jet? What is the flow rate? What is the flow rate when the vacuum is 254 mm of mercury?

Solution:

$$p_1 = \frac{p_{0_1}}{\left[1+\frac{k-1}{2}M_1^2\right]^{\frac{k}{k-1}}}$$
$$= \frac{(101.3\ \mathrm{kPa})}{\left[1+\frac{1.4-1}{2}\times 1^2\right]^{\frac{1.4}{1.4-1}}}$$
$$= 53.5149\ \mathrm{kPa}$$

$$V_{ac1} = (P_0 - P_1)\times\frac{760\ \mathrm{mm}}{101.3\ \mathrm{kPa}}$$
$$= [(101.3\ \mathrm{kPa}) - (53.5149\ \mathrm{kPa})]\times\frac{760\ \mathrm{mm}}{101.3\ \mathrm{kPa}}$$
$$= 358.5058\ \mathrm{mm}$$

$$T_1 = \frac{T_0}{\left[1+\frac{k-1}{2}M_1^2\right]} = \frac{288.15\ \mathrm{K}}{\left[1+\frac{1.4-1}{2}\times 1^2\right]}$$
$$= 240.1250\ \mathrm{K}$$

$$V = c = \sqrt{kRT}$$
$$= \sqrt{(1.4)\times(286.9\ \mathrm{J/kg\cdot K})\times(240.1250\ \mathrm{K})}$$
$$= 310.5618\ \mathrm{m/s}$$

$$\dot{m}_1 = \rho VA = \frac{p}{RT}\cdot V\cdot\frac{\pi D^2}{4}$$
$$= \frac{53.5149\ \mathrm{kPa}}{(286.9\ \mathrm{J/kg\cdot K})\times(240.1250\ \mathrm{K})}\times(340.2034\ \mathrm{m/s})\times\frac{\pi\times(38\ \mathrm{mm})^2}{4}$$
$$= 0.2736\ \mathrm{kg/s}$$

$$h_0 = h + \frac{1}{2}V^2$$

For $V_{ac} = 254\ \mathrm{mm}$

$$V_{ac} = (P_0 - P)\times\frac{760\ \mathrm{mm}}{101.3\ \mathrm{kPa}}$$
$$254\ \mathrm{mm} = (101.3\ \mathrm{kPa} - P)\times\frac{760\ \mathrm{mm}}{101.3\ \mathrm{kPa}}$$
$$\Rightarrow p_2 = 67.4445\ \mathrm{kPa}$$

$$\frac{p_{0_2}}{p_2} = \left[1+\frac{k-1}{2}M_2^2\right]^{\frac{k}{k-1}}$$
$$\Rightarrow M_2 = 0.7850$$

$$T_2 = \frac{T_0}{\left[1 + \frac{k-1}{2} M_2^2\right]}$$

$$= \frac{288.15 \ \text{K}}{\left[1 + \frac{1.4-1}{2} \times 0.7850^2\right]}$$

$$= 256.5331 \ \text{K}$$

$$\frac{V_2}{c_2} = M_2$$

$$V_2 = M_2 c_2$$
$$= 0.7850$$
$$\times \sqrt{(1.4) \times (286.9 \ \text{J/kg} \cdot \text{K}) \times (256.5331 \ \text{K})}$$
$$= 251.9844 \ \text{m/s}$$

$$\dot{m}_2 = \rho V A = \frac{p}{RT} \cdot V \cdot \frac{\pi D^2}{4}$$
$$= \frac{67.4445 \ \text{kPa}}{(286.9 \ \text{J/kg} \cdot \text{K}) \times (256.5331 \ \text{K})}$$
$$\times (251.9844 \ \text{m/s}) \times \frac{\pi \times (38 \ \text{mm})^2}{4}$$
$$= 0.2619 \ \text{kg/s}$$

Problem 12.43

12.43 Nitrogen flows through a diverging section of duct with $A_1 = 0.15 \text{ m}^2$ and $A_2 = 0.45 \text{ m}^2$. If $M_1 = 0.7$ and $p_1 = 450$ kPa, find M_2 and p_2.

Solution:

$$p_{0_1} = p_1 \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$
$$= (450 \ \text{kPa})$$
$$\times \left[1 + \frac{1.4-1}{2} \times 0.7^2\right]^{\frac{1.4}{1.4-1}}$$
$$= 624.1956 \ \text{kPa}$$

$$\frac{A}{A^*} = \frac{1}{M} \left[\frac{1 + \frac{k-1}{2} M^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}$$

$$A_1^* = \frac{A_1}{\frac{1}{M_1} \left[\frac{1 + \frac{k-1}{2} M_1^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}}$$

$$= \frac{(0.15 \ \text{m}^2)}{\frac{1}{0.7} \times \left[\frac{1 + \frac{1.4-1}{2} \times 0.7^2}{\frac{1.4+1}{2}}\right]^{\frac{1.4+1}{2\times(1.4-1)}}}$$

$$= 0.1371 \ \text{m}^2$$

$$A_1^* = A_2^*$$

$$\frac{A_2}{\frac{1}{M_2} \left[\frac{1 + \frac{k-1}{2} M_2^2}{\frac{k+1}{2}}\right]^{\frac{k+1}{2(k-1)}}} = 0.1371 \ \text{m}^2$$

$$\Rightarrow M_2 = 0.1797$$

$$p_{0_1} = p_{0_2}$$

$$\Rightarrow p_2 = \frac{p_{0_2}}{\left[1 + \frac{k-1}{2} M_2^2\right]^{\frac{k}{k-1}}}$$

$$= \frac{(624.1956 \ \text{kPa})}{\left[1 + \frac{1.4-1}{2} \times 0.1797^2\right]^{\frac{1.4}{1.4-1}}}$$

$$= 610 \ \text{kPa}$$

Problem 12.46

12.46 Air, at an absolute pressure of 60.0 kPa and 27°C, enters a passage at 486 m/s, where $A = 0.02 \text{ m}^2$. At section ① downstream, $p = 78.8$ kPa absolute. Assuming isentropic flow, calculate the Mach number at section ②. Sketch the flow passage.

Solution:

$$c_1 = \sqrt{kRT_1}$$

$$= \sqrt{(1.4) \times (286.9 \ \text{J/kg} \cdot \text{K}) \times (300 \ \text{K})}$$

$$= 347.1282 \ \text{m/s}$$

$$M_1 = \frac{V_1}{c_1} = \frac{(486\ \ \text{m/s})}{(347.1282\ \ \text{m/s})} = 1.4001$$

$$p_{0_1} = p_1\left[1 + \frac{k-1}{2}M_1^2\right]^{\frac{k}{k-1}}$$

$$= (60\ \ \text{kPa}) \times \left[1 + \frac{1.4-1}{2} \times 1.4001^2\right]^{\frac{1.4}{1.4-1}}$$

$$= 190.9522\ \ \text{kPa}$$

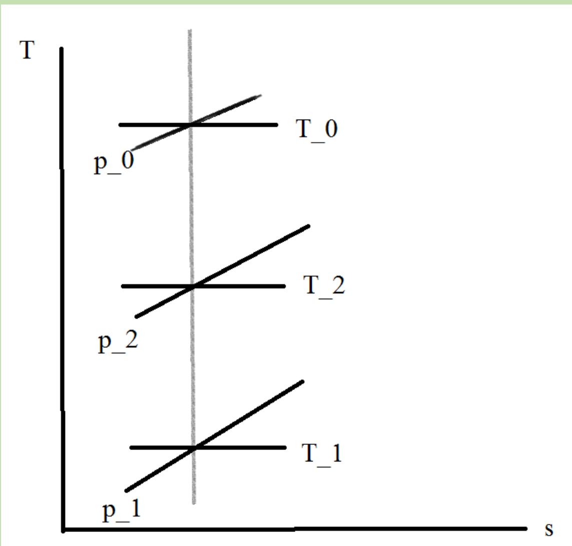
$$p_{0_1} = p_{0_2}$$

$$\frac{p_{0_2}}{p_2} = \left[1 + \frac{k-1}{2}M_2^2\right]^{\frac{k}{k-1}}$$

$$\frac{190.9522\ \ \text{kPa}}{78.8\ \ \text{kPa}} = \left[1 + \frac{1.4-1}{2} \times M_2^2\right]^{\frac{1.4}{1.4-1}}$$

$$\Rightarrow M_2 = 1.20$$

The T-s diagram is shown below:



— Christopher King —