Christopher King

Applied Fluid Mechanics Homework 08

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Applied Fluid Mechanics

Class Section 01

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Problem 12.9

12.9 Carbon dioxide flows at a speed of 10 m/s in a pipe and then through a nozzle where the velocity is 50 m/s, What is the change in gas temperature between pipe and nozzle? Assume that this is an adiabatic flow of a perfect gas.

Solution:

$$0 = \left(h_1 + \frac{V_1^2}{2}\right) - \left(h_2 + \frac{V_2^2}{2}\right)$$

$$h_2 - h_1 = \frac{1}{2}(V_1^2 - V_2^2)$$

$$= \frac{1}{2}[(10\ \text{m/s})^2 - (50\ \text{m/s})^2]$$

$$= -1.2\ \text{kJ/kg}$$

Assume a constant specific heat

$$h_2 - h_1 = c_p(T_2 - T_1)$$

$$\Delta T = \frac{h_2 - h_1}{c_p} = \frac{(-1.2\ \text{kJ/kg})}{(0.8404\ \text{kJ/kg}\cdot\text{K})}$$

$$= -1.4279\ \text{K}$$

Problem 12.21

12.21 A supersonic aircraft flies at 3 km altitude at a speed of 1000 m/s on a standard day. How long after passing directly above a ground observer is the sound of the aircraft heard by the ground observer?

Solution:

$$c_{3000} = \sqrt{kRT_{3000}}$$

$$= \sqrt{1.4 \times (286.9\ \text{J/kg}\cdot\text{K}) \times (268.7\ \text{K})}$$

$$= 328.5210\ \text{m/s}$$

$$M_{3000} = \frac{V}{c_{3000}} = \frac{1000\ \text{m/s}}{328.5210\ \text{m/s}} = 3.0439$$

$$\alpha_{3000} = \sin^{-1}\frac{1}{M_{3000}} = \sin^{-1}\frac{1}{3.0439}$$

$$= 19.1790°$$

$$x_{3000} = \frac{h}{\tan\alpha_{3000}} = \frac{3\ \text{km}}{\tan 19.1790°}$$

$$= 8.6250\ \text{km}$$

$$\Delta t_{3000} = \frac{x_{3000}}{V} = \frac{8.6250\ \text{km}}{1000\ \text{m/s}} = 8.6250\ \text{s}$$

$$c_0 = \sqrt{kRT_0}$$

$$= \sqrt{1.4 \times (286.9\ \text{J/kg}\cdot\text{K}) \times (288.2\ \text{K})}$$

$$= 340.2329\ \text{m/s}$$

$$M_0 = \frac{V}{c_0} = \frac{1000\ \text{m/s}}{340.2329\ \text{m/s}} = 2.9392$$

$$\alpha_0 = \sin^{-1}\frac{1}{M_0} = \sin^{-1}\frac{1}{2.9392} = 19.8911°$$

$$x_0 = \frac{h}{\tan\alpha_0} = \frac{3\ \text{km}}{\tan 19.8911°} = 8.2914\ \text{km}$$

$$\Delta t_0 = \frac{x_0}{V} = \frac{8.2914\ \text{km}}{1000\ \text{m/s}} = 8.2914\ \text{s}$$

Take average:

$$\Delta t = \frac{8.6250\ \text{s} + 8.2914\ \text{s}}{2} = 8.4582\ \text{s}$$

Problem 12.25

12.25 Compute the air density in the undisturbed air and at the stagnation point of an aircraft flying at 250 m/s in air at 28 kPa and 250°C. What is the percentage increase in density? Can we approximate this as an incompressible flow?

Solution:

$$c = \sqrt{kRT}$$

$$= \sqrt{1.4 \times (286.9\ \ \mathrm{J/kg \cdot K}) \times [(250 + 273.15)\ \ \mathrm{K}]}$$

$$= 458.3977\ \ \mathrm{m/s}$$

$$M = \frac{V}{c} = \frac{250\ \ \mathrm{m/s}}{458.3977\ \ \mathrm{m/s}} = 0.5454$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}}$$

$$= \left(1 + \frac{1.4-1}{2} \times 0.5454^2\right)^{\frac{1}{1.4-1}} = 1.1554$$

$$\frac{\rho_0 - \rho}{\rho} \times 100\% = \left(\frac{\rho_0}{\rho} - 1\right) \times 100\%$$

$$= (1.1554 - 1) \times 100\%$$

$$= 15.54\% > 5\%$$

Therefore, we cannot approximate this as an incompressible flow.

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