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Applied Fluid Mechanics
Homework 06

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2018141521058

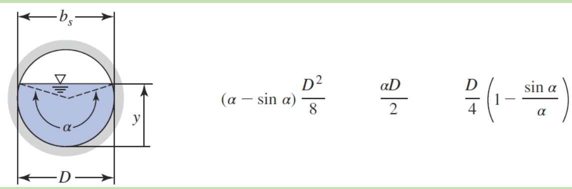
Applied Fluid Mechanics

Class Section 01

04/25/2021

Problem 11.1

Find out the flow area A, wetted perimeter P and hydraulic radius R_h for the open open-channel flow in a circular conduit for $y < \frac{D}{2}$.



Solution:

Wetted perimeter:

$$P = \alpha \cdot \frac{D}{2} = \frac{\alpha D}{2}$$

Flow area:

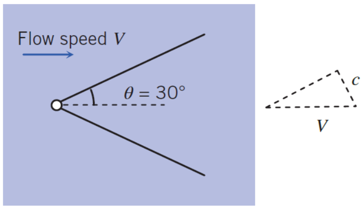
$$A = A_1 + A_2 = \frac{\alpha}{2\pi} \cdot \frac{\pi D^2}{4} + \frac{1}{2} \times \left(D \cdot \sin\frac{\alpha}{2}\right) \cdot \left(\frac{D}{2} \cdot \cos\frac{\alpha}{2}\right)$$
$$= \frac{\alpha D^2}{8} + \frac{1}{8} D^2 \sin\alpha$$
$$= (\alpha - \sin\alpha)\frac{D^2}{8}$$

Flow area:

$$R_h = \frac{A}{P} = \frac{(\alpha - \sin\alpha)\frac{D^2}{8}}{\frac{\alpha D}{2}} = \frac{D}{4}\left(1 - \frac{\sin\alpha}{\alpha}\right)$$

Problem 11.7

11.7 Surface waves are caused by a sharp object that just touches the free surface of a stream of flowing water, forming the wave pattern shown. The stream depth is 150 mm. Determine the flow speed and Froude number. Note that the wave travels at speed c (Eq. 11.6) normal to the wave front, as shown in the velocity diagram.



P11.7

Solution:

$$c = \sqrt{gy} = \sqrt{(9.81\ \mathrm{m/s^2}) \times (150\ \mathrm{mm})}$$
$$= 1.21\ \mathrm{m/s}$$

$$V = \frac{c}{\sin\theta} = \frac{1.21\ \mathrm{m/s}}{\sin 30°} = 2.43\ \mathrm{m/s}$$

$$Fr = \frac{V}{c} = \frac{1}{\sin\theta} = \frac{1}{\sin 30°} = 2$$

Problem 11.19

11.19 A rectangular channel 3 m wide carries a discharge of 0.57 $\mathrm{m^3/s}$ at 0.27 m depth. A smooth bump 0.06 m high is placed on the floor of the channel. Estimate the local change in flow depth caused by the bump.

Solution:

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_1$$

$$Q = V \cdot A = V \cdot b \cdot y$$

$$\Rightarrow V = \frac{Q}{by}$$

$$V_1 = \frac{Q}{by_1} = \frac{0.57\ \mathrm{m^3/s}}{(3\ \mathrm{m}) \times (0.27\ \mathrm{m})}$$
$$= 0.7037\ \mathrm{m/s}$$

$$\frac{V_1^2}{2g} + y_1 + z_1 = \frac{V_2^2}{2g} + y_2 + z_2 + H_1$$

$$\frac{(0.7037\ \mathrm{m/s})^2}{2 \times (9.81\ \mathrm{m/s^2})} + (0.27\ \mathrm{m})$$
$$= \frac{(0.57\ \mathrm{m^3/s})^2}{2 \times (9.81\ \mathrm{m/s^2}) \times (3\ \mathrm{m})^2 y_2^2} + y_2$$
$$+ (0.06\ \mathrm{m})$$

Therefore,

$$y_2 = 0.1755\ \mathrm{m}$$

Therefore, the local change in flow depth caused by the bump is equal to

$$\frac{y_2 - y_1}{y_1} = \frac{0.1755\ \mathrm{m} - 0.27\ \mathrm{m}}{0.27\ \mathrm{m}} = 35.0\%$$

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