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Applied Fluid Mechanics Homework 04

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Applied Fluid Mechanics

Class Section 01

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Problem 9.4

9.4 For flow around a sphere the boundary layer becomes turbulent around $Re_D \approx 2.5 \times 10^5$. Find the speeds at which (a) an American golf ball ($D = 43$ mm), (b) a British golf ball ($D = 41.1$ mm), and (c) a soccer ball ($D = 222$ mm) develop turbulent boundary layers. Assume standard atmospheric conditions.

Solution:

$$Re_D = \frac{VD}{v}$$

$$\Rightarrow V = \frac{Re_D v}{D} = \frac{(2.5 \times 10^5) \times (1.50 \times 10^{-5} \quad \mathrm{m^2/s})}{D} = \frac{3.75}{D}$$

(a)

$$V = \frac{3.75}{D} = \frac{3.75}{0.043} \quad \mathrm{m/s} = 87.21 \quad \mathrm{m/s}$$

(b)

$$V = \frac{3.75}{D} = \frac{3.75}{0.0411} \quad \mathrm{m/s} = 91.24 \quad \mathrm{m/s}$$

(c)

$$V = \frac{3.75}{D} = \frac{3.75}{0.222} \quad \mathrm{m/s} = 16.89 \quad \mathrm{m/s}$$

Problem 9.10

9.10 A simplistic laminar boundary-layer model is

$$\frac{u}{U} = \sqrt{2}\frac{y}{\delta} \qquad 0 < y \le \frac{\delta}{2}$$

$$\frac{u}{U} = (2-\sqrt{2})\frac{y}{\delta} + (\sqrt{2}-1) \qquad \frac{\delta}{2} < y \le \delta$$

Does this expression satisfy boundary conditions applicable to the laminar boundary-layer velocity profile? Evaluate δ^*/δ and θ/δ.

Solution:

The boundary conditions for the laminar boundary-layer velocity profile is

$$u(0) = 0, \left.\frac{du}{dy}\right|_{y=\delta} = 0$$

When $y = 0$,

$$\frac{u}{U} = \sqrt{2} \times 0 = 0$$

which satisfies the boundary condition.

When $y = \delta$,

$$\frac{du}{dy} = U(2-\sqrt{2})\frac{1}{\delta} \neq 0$$

which does not satisfy the boundary condition.

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy \approx \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

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$$\frac{\delta^*}{\delta}=\frac{1}{\delta}\int_0^{\delta}\left(1-\frac{u}{U}\right)dy$$

$$=\frac{1}{\delta}\left\{\int_0^{\frac{\delta}{2}}\left(1-\sqrt{2}\frac{y}{\delta}\right)dy+\int_{\frac{\delta}{2}}^{\delta}\left[1-\left[(2-\sqrt{2})\frac{y}{\delta}+(\sqrt{2}-1)\right]\right]dy\right\}$$

$$=\frac{1}{\delta}\left\{\left[\frac{\delta}{2}-\frac{\sqrt{2}}{2}\frac{\left(\frac{\delta}{2}\right)^2}{\delta}\right]+\left[\frac{\delta}{2}-\left[\frac{(2-\sqrt{2})}{2}\frac{\delta^2-\left(\frac{\delta}{2}\right)^2}{\delta}+(\sqrt{2}-1)\frac{\delta}{2}\right]\right]\right\}$$

$$=\frac{1}{2}-\frac{\sqrt{2}}{8}+\frac{1}{2}-\left[\frac{(2-\sqrt{2})}{2}\times\frac{3}{4}+\frac{(\sqrt{2}-1)}{2}\right]=0.3964$$

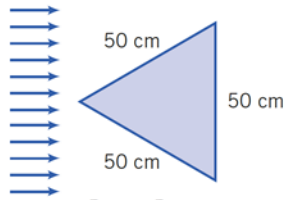
$$\theta=\int_0^{\infty}\frac{u}{U}\left(1-\frac{u}{U}\right)dy\approx\int_0^{\delta}\frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

$$\frac{\theta}{\delta}=\frac{1}{\delta}\int_0^{\delta}\frac{u}{U}\left(1-\frac{u}{U}\right)dy$$

$$=\frac{1}{\delta}\left\{\int_0^{\frac{\delta}{2}}\sqrt{2}\frac{y}{\delta}\left(1-\sqrt{2}\frac{y}{\delta}\right)dy+\int_{\frac{\delta}{2}}^{\delta}\left[(2-\sqrt{2})\frac{y}{\delta}+(\sqrt{2}-1)\right]\left[1-\left[(2-\sqrt{2})\frac{y}{\delta}+(\sqrt{2}-1)\right]\right]dy\right\}=0.1524$$

Problem 9.32

9.32 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5 m/s air flow. The air is 20°C and 101.3 kPa.



P9.32, P9.33

Solution:

$$Re_L=\frac{UL}{\nu}=\frac{(5\ \ \mathrm{m/s})\times(50\ \ \mathrm{cm})\times\frac{\sqrt{3}}{2}}{(1.50\times10^{-5}\quad\mathrm{m^2/s})}$$

$$=144338<5\times10^5$$

Therefore, the flow is laminar.

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho U^2}=\frac{0.730}{\sqrt{Re_x}}$$

$$\Rightarrow\tau_w=\frac{0.730\times\frac{1}{2}\rho U^2}{\sqrt{Re_x}}=\frac{0.365\rho U^2}{\sqrt{Re_x}}$$

$$F_D = \int_0^L \tau_w dA = \int_0^L \tau_w W\left(\frac{x}{L}\right) dx$$

$$= \int_0^L \frac{0.365\rho U^2}{\sqrt{Re_x}} W\left(\frac{x}{L}\right) dx$$

$$= \int_0^{(50\ \text{cm})\times\frac{\sqrt{3}}{2}} \left\{ \frac{0.365 \times (1.21\ \text{kg/m}^3) \times (5\ \text{m/s})^2}{\sqrt{\frac{(5\ \text{m/s}) \times x}{(1.50 \times 10^{-5}\ \ \text{m}^2/\text{s})}}} \times (50\ \text{cm}) \times \left[\frac{x}{(50\ \text{cm}) \times \frac{\sqrt{3}}{2}}\right] \right\} dx$$

$= 4.195 \times 10^{-3}$ N

Problem 9.33

9.33 Assume laminar boundary-layer flow to estimate the drag on the plate shown when it is placed parallel to a 5 m/s air flow, except that the base rather than the tip faces the flow. Would you expect this to be larger than, the same as, or lower than the drag for Problem 9.32?

Solution:

$$F_D = \int_0^L \tau_w dA = \int_0^L \tau_w W\left(\frac{x}{L}\right) dx$$

$$= \int_0^L \frac{0.365\rho U^2}{\sqrt{Re_x}} W\left(\frac{x}{L}\right) dx$$

$$= \int_0^{(50\ \text{cm})\times\frac{\sqrt{3}}{2}} \left\{ \frac{0.365 \times (1.21\ \text{kg/m}^3) \times (5\ \text{m/s})^2}{\sqrt{\frac{(5\ \text{m/s}) \times x}{(1.50 \times 10^{-5}\ \ \text{m}^2/\text{s})}}} \times (50\ \text{cm}) \times \left(1 - \left[\frac{x}{(50\ \text{cm}) \times \frac{\sqrt{3}}{2}}\right]\right) \right\} dx$$

$= 8.383 \times 10^{-3}$ N

Therefore, the drag is larger than that of Problem 9.32.

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