Christopher King

Applied Fluid Mechanics
Homework 02

Christopher King

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Applied Fluid Mechanics

Class Section 01

03/23/2021

Problem 8.78

Water flows from a tank with a very short outlet pipe. Estimate the exit flow rate. How could the flow rate be increased?

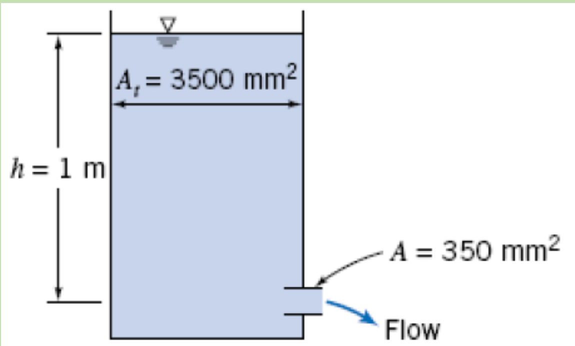
$$h_{l_T} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$h_{l_T} = \left(\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{\bar{V}_2^2}{2}\right)$$

$$h_{l_T} = h_l + h_{lm} = h_{lm} = \frac{K\bar{V}_2^2}{2}$$

$$\left(\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{\bar{V}_2^2}{2}\right) = \frac{K\bar{V}_2^2}{2}$$

$$V_1 = V_2\frac{A_1}{A_2}$$



Solution:

$$h_{l_T} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$h_{l_T} = \left(\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{\bar{V}_2^2}{2}\right)$$

$$h_{l_T} = h_l + h_{lm} = h_{lm} = \frac{K\bar{V}_2^2}{2}$$

$$\left(\frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{\bar{V}_2^2}{2}\right) = \frac{K\bar{V}_2^2}{2}$$

$$V_1 = V_2 \frac{A_1}{A_2}$$

$$\Rightarrow V_2 = \sqrt{\frac{2gh}{\left[1 + K - \left(\frac{A_2}{A_1}\right)^2\right]}} = \sqrt{\frac{2 \times (9.81\ \mathrm{m/s^2}) \times (1\ \mathrm{m})}{\left[1 + (0.78) - \left[\frac{(350\ \mathrm{mm^2})}{(3500\ \mathrm{mm^2})}\right]^2\right]}} = 3.33\ \mathrm{m/s}$$

$$Q = V_2 A_2 = (3.33\ \mathrm{m/s}) \times (350\ \mathrm{mm^2}) = 1.17 \times 10^{-3}\ \mathrm{m^3/s}$$

Improvement:

(1) Add a diffuser.
(2) Round the entrance.

Problem 8.105

A pool is to be filled that has a 1.5 m diameter and is 0.76 m deep. The pool is located 5.5 m above the water source which travels through a 15 m long, 1.6 cm diameter hose that is very smooth. Neglecting minor losses, how long will it take to fill if the water pressure at the source is 414 kPa?

$$h_{l_T} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$h_{l_T} = \left(\frac{p_1}{\rho}\right) - (gz_2)$$

$$h_{l_T} = h_l + h_{lm} = h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$\left(\frac{p_1}{\rho}\right) - (gz_2) = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$\bar{V} = \sqrt{\frac{2D\left(\frac{p_1}{\rho} - gz_2\right)}{f\,\mathrm{L}}} = \sqrt{\frac{2(0.016\,m)\left(\frac{414000}{1000} - (9.81)(5.5 + .76)\right)}{f(15\text{ m})}} = \frac{0.87}{\sqrt{f}}$$

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{1000\bar{V}(0.016\,m)}{0.00101} = 15841.6\bar{V}$$

$$\frac{1}{\sqrt{f}} = -2.0\,log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) = -2.0\,log\left(\frac{2.51}{Re\sqrt{f}}\right)$$

Guess f=0.015:

$$\bar{V} = \frac{0.87}{\sqrt{0.015}} = 7.10\frac{m}{s} \quad \rightarrow \quad Re = 15841.6(7.10) = 1.1\times 10^5 \quad \rightarrow \quad f = 0.0177$$

Solution:

$$h_{l_T} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$h_{l_T} = \left(\frac{p_1}{\rho}\right) - (gz_2)$$

$$h_{l_T} = h_l + h_{lm} = h_l = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$\left(\frac{p_1}{\rho}\right) - (gz_2) = f\frac{L}{D}\frac{\bar{V}^2}{2}$$

$$\bar{V} = \sqrt{\frac{2D\left(\frac{p_1}{\rho} - gz_2\right)}{f\,\mathrm{L}}} = \sqrt{\frac{2(0.016\,m)\left(\frac{414000}{1000} - (9.81)(5.5 + .76)\right)}{f(15\text{ m})}} = \frac{0.87}{\sqrt{f}}$$

$$Re = \frac{\rho\bar{V}D}{\mu} = \frac{1000\bar{V}(0.016\,m)}{0.00101} = 15841.6\bar{V} = 15841.6\frac{0.87}{\sqrt{f}}$$

$$\frac{1}{\sqrt{f}} = -2.0\,log\left(\frac{\frac{e}{D}}{3.7} + \frac{2.51}{Re\sqrt{f}}\right) = -2.0\,log\left(\frac{2.51}{Re\sqrt{f}}\right) = -2.0\,log\left(\frac{2.51}{15841.6\frac{0.87}{\sqrt{f}}\sqrt{f}}\right)$$

$$= -2.0\,log\left(\frac{2.51}{15841.6 \times 0.87}\right)$$

$$\Rightarrow f = 0.0179$$

$$V = 6.51\ \mathrm{m/s}$$

$$Q = \frac{\pi \times (1.6\ \mathrm{cm})^2}{4} \times (6.51\ \mathrm{m/s}) = 1.31 \times 10^{-3}\ \mathrm{m^3/s}$$

$$\forall = (0.76\ \mathrm{m}) \times \frac{\pi \times (1.5\ \mathrm{m})^2}{4} = 1.343\ \mathrm{m^3}$$

$$t = \frac{\forall}{Q} = \frac{(1.343\ \mathrm{m^3})}{(1.31 \times 10^{-3}\ \mathrm{m^3/s})} = 1026.5\ \mathrm{s}$$

Problem 8.120

Determine the smallest standard commercial steel pipe that will allow for a static pressure to be greater than -6m H_2O gage.

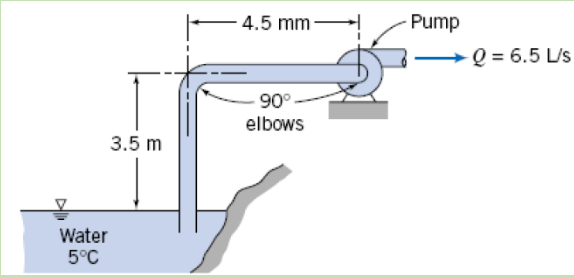
$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right)$$

$$\sum h_l + \sum h_{l_m} = -\left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right) = \frac{V_2^2}{2}\left[f\frac{L}{D} + K_{ent} + 2K_{elb}\right]$$

$$\frac{p_2}{\rho g} = -z_2 - \frac{V_2^2}{2g}\left[1 + f\frac{L}{D} + K_{ent} + 2K_{elb}\right] = -3.5 - \frac{V_2^2}{19.62}\left[3.15 + 8\frac{f}{D}\right]$$

$$D = 0.0254\ \mathrm{m} \rightarrow V = 12.83\frac{m}{s} \rightarrow Re = 2.96 \times 10^5, \frac{e}{D} = .00181 \rightarrow f = .024$$

$$\frac{p_2}{\rho g} = -93.35 \neq -6$$



Solution:

$$h_{l_T}=\sum h_l+\sum h_{l_m}=\left(\frac{p_1}{\rho}+\frac{\bar{V}_1^2}{2}+gz_1\right)-\left(\frac{p_2}{\rho}+\frac{\bar{V}_2^2}{2}+gz_2\right)$$

$$\sum h_l+\sum h_{l_m}=-\left(\frac{p_2}{\rho}+\frac{\bar{V}_2^2}{2}+gz_2\right)=\frac{V_2^2}{2}\left[f\frac{L}{D}+K_{ent}+2K_{elb}\right]$$

$$\frac{p_2}{\rho g}=-z_2-\frac{V_2^2}{2g}\left[1+f\frac{L}{D}+K_{ent}+2K_{elb}\right]=-3.5-\frac{V_2^2}{19.62}\left[3.15+8\frac{f}{D}\right]$$

$$\frac{1}{\sqrt{f}}=-1.8\log\left[\left(\frac{\frac{e}{D}}{3.7}\right)^{1.11}+\frac{2.51}{Re}\right]\Rightarrow f=\left\{\frac{1}{-1.8\log\left[\left(\frac{\frac{e}{D}}{3.7}\right)^{1.11}+\frac{2.51}{Re}\right]}\right\}^2$$

$$=\left\{\frac{1}{-1.8\log\left[\left(\frac{\frac{0.000046}{D}}{3.7}\right)^{1.11}+\frac{2.51}{\frac{5480.832477}{D}}\right]}\right\}^2$$

$$
\begin{cases}
h_2 = -3.5 - \dfrac{V_2^2}{19.62}\left[3.15 + 8\dfrac{f}{D}\right] \\
V = \dfrac{Q}{\frac{\pi D^2}{4}} = \dfrac{(6.5\ \mathrm{L/s})}{\frac{\pi D^2}{4}} = \dfrac{8.276 \times 10^{-3}}{D^2} \\
Re = \dfrac{4Q}{\pi v D} = \dfrac{4 \times (6.5\ \mathrm{L/s})}{\pi \times (1.51 \times 10^{-6}\ \mathrm{m^2/s})D} = \dfrac{5480.832477}{D} \\
\dfrac{e}{D} = \dfrac{0.000046}{D} \\
f = \left\{\dfrac{1}{-1.8\log\left[\left(\dfrac{\frac{0.000046}{D}}{3.7}\right)^{1.11} + \dfrac{2.51}{\frac{5480.832477}{D}}\right]}\right\}^2
\end{cases}
$$

$$\Rightarrow -6 = -3.5 - \frac{\left(\frac{8.276\times 10^{-3}}{D^2}\right)^2}{19.62}\left[3.15 + 8\frac{\left\{\frac{1}{-1.8\log\left[\left(\frac{\frac{0.000046}{D}}{3.7}\right)^{1.11} + \frac{2.51}{\frac{5480.832477}{D}}\right]}\right\}^2}{D}\right]$$

$$\Rightarrow D = 0.054\ \text{m}$$

From Table 8.5, I can know that the inside diameter for suitable commercial steel pipe is 52.501 mm.

Problem 8.129

Calculate the minimum pressure needed at the pump outlet for a 38 L/s flow rate and the input power required if the pumping efficiency is 70%.

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_3}{\rho} + \frac{\bar{V}_3^2}{2} + gz_3\right) + \Delta h_{pump}$$

$$\Delta h_{pump} = gz_3 + \frac{\bar{V}_3^2}{2} + \frac{V^2}{2}\left[f\frac{L}{D} + K_{ent} + K_{90^\circ} + 2K_{45^\circ} + 15 + K_v\right]$$

$$Q \rightarrow V_3 = 4.84\frac{m}{s} \rightarrow Re = 4.25 \times 10^5, \frac{e}{D} = .000015 \rightarrow f$$

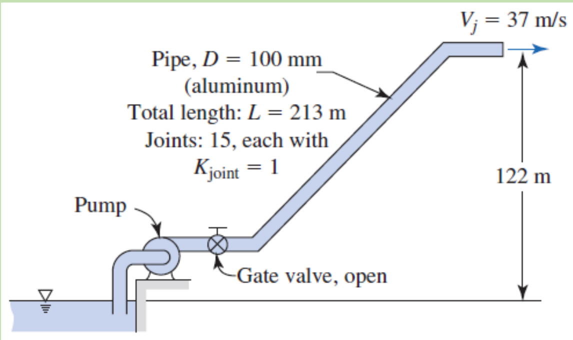
$$\Delta h_{pump} = (9.81)(122) + \frac{37^2}{2} + \frac{4.84^2}{2}\left[f\frac{213}{.1} + .75 + .7 + 2(.2) + 15 + .2\right]$$

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right) + \Delta h_{pump}$$

$$p_2 = \rho \Delta h_{pump}$$

$$\dot{W}_{p,th} = \dot{m}\Delta h_{pump} = \rho Q \Delta h_{pump}$$

$$\dot{W}_p = \frac{\dot{W}_{p,th}}{\eta}$$



Solution:

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_3}{\rho} + \frac{\bar{V}_3^2}{2} + gz_3\right) + \Delta h_{pump}$$

$$\Delta h_{pump} = gz_3 + \frac{\bar{V}_3^2}{2} + \frac{V^2}{2}\left[f\frac{L}{D} + K_{ent} + K_{90^\circ} + 2K_{45^\circ} + 15 + K_v\right]$$

$$Q \to V_3 = 4.84\frac{m}{s} \to Re = 4.25\times 10^5, \frac{e}{D} = .000015 \to f$$

$$= \left\{\frac{1}{-1.8\log\left[\left(\frac{\frac{e}{D}}{3.7}\right)^{1.11} + \frac{2.51}{Re}\right]}\right\}^2$$

$$= \left\{\frac{1}{-1.8\log\left[\left(\frac{0.000015}{3.7}\right)^{1.11} + \frac{2.51}{4.25\times 10^5}\right]}\right\}^2 = 0.0116$$

$$\Delta h_{pump} = (9.81)(122) + \frac{37^2}{2} + \frac{4.84^2}{2}\left[f\frac{213}{.1} + .75 + .7 + 2(.2) + 15 + .2\right]$$

$$= (9.81)(122) + \frac{37^2}{2}$$
$$+ \frac{4.84^2}{2}\left[0.0116\times\frac{213}{0.1} + 0.75 + 0.7 + 2(0.2) + 15 + 0.2\right] = 2370\ \mathrm{m^2/s^2}$$

$$h_{l_T} = \sum h_l + \sum h_{l_m} = \left(\frac{p_1}{\rho} + \frac{\bar{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \frac{\bar{V}_2^2}{2} + gz_2\right) + \Delta h_{pump}$$

$$p_2 = \rho\Delta h_{pump} = (1000\ \mathrm{kg/m^3})\times(2370\ \mathrm{m^2/s^2}) = 2370\ \mathrm{kPa}$$

$$\dot{W}_{p,th} = \dot{m}\Delta h_{pump} = \rho Q\Delta h_{pump} = (1000\ \mathrm{kg/m^3})\times(38\ \mathrm{L/s})\times(2370\ \mathrm{m^2/s^2}) = 90.1\ \mathrm{kW}$$

$$\dot{W}_p = \frac{\dot{W}_{p,th}}{\eta} = \frac{(90.1\ \mathrm{kW})}{(70\%)} = 128.68\ \mathrm{kW}$$

Christopher King