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Applied Fluid Mechanics
Homework 01

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Applied Fluid Mechanics

Class Section 01

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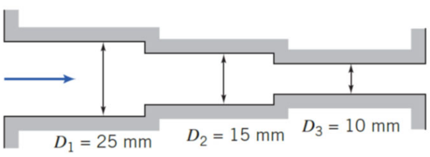
Problem 8.3

8.3 Air at 40°C flows in a pipe system in which diameter is decreased in two stages from 25 mm to 15 mm to 10 mm. Each section is 2 m long. As the flow rate is increased, which section will become turbulent first? Determine the flow rates at which one, two, and then all three sections first become turbulent. At each of these flow rates, determine which sections, if any, attain fully developed flow.



P8.3

$$Q_1 = \frac{Re_{crit}\pi v D_1}{4}$$

$$Q_2 = \frac{Re_{crit}\pi v D_2}{4}$$

$$Q_3 = \frac{Re_{crit}\pi v D_3}{4}$$

$$L_{laminer} = 0.06ReD,\ \ Turbulent: L_{min} = 25D \qquad L_{max} = 40D$$

$\boldsymbol{Q_3}$:

$$L_{min,3} = 25D_3 \qquad L_{max,3} = 40D_3$$

$$L_{laminer,3} = 0.06\left(\frac{4Q_3}{\pi v D_3}\right)D_3,$$

$$L_{laminer,2} = 0.06\left(\frac{4Q_3}{\pi v D_2}\right)D_2$$

$$L_{laminer,1} = 0.06\left(\frac{4Q_3}{\pi v D_1}\right)D_1$$

$\boldsymbol{Q_2}$:

$$L_{min,3} = 25D_3 \qquad L_{max,3} = 40D_3$$

$$L_{min,2} = 25D_2 \qquad L_{max,2} = 40D_2$$

$$L_{laminer,2} = 0.06\left(\frac{4Q_2}{\pi v D_2}\right)D_2$$

$$L_{laminer,1} = 0.06\left(\frac{4Q_2}{\pi v D_1}\right)D_1$$

$\boldsymbol{Q_1}$:

$$L_{min,3} = 25D_3 \qquad L_{max,3} = 40D_3$$

$$L_{min,2} = 25D_2 \qquad L_{max,2} = 40D_2$$

$$L_{min,1} = 25D_1 \qquad L_{max,1} = 40D_1$$

$$L_{laminer,1} = 0.06\left(\frac{4Q_1}{\pi v D_1}\right)D_1$$

Solution:

$$Q_1 = \frac{Re_{crit}\pi v D_1}{4} = \frac{(2300)\times\pi\times(1.69\times10^{-5}\ \ \mathrm{m^2/s})\times(25\ \ \mathrm{mm})}{4} = 7.63\times10^{-4}\ \ \mathrm{m^3/s}$$

$$Q_2 = \frac{Re_{crit}\pi v D_2}{4} = \frac{(2300)\times\pi\times(1.69\times10^{-5}\ \ \mathrm{m^2/s})\times(15\ \ \mathrm{mm})}{4} = 4.58\times10^{-4}\ \ \mathrm{m^3/s}$$

$$Q_3 = \frac{Re_{crit}\pi v D_3}{4} = \frac{(2300)\times\pi\times(1.69\times10^{-5}\ \ \mathrm{m^2/s})\times(10\ \ \mathrm{mm})}{4} = 3.05\times10^{-4}\ \ \mathrm{m^3/s}$$

Therefore, the third section will become turbulent first.

$\boldsymbol{Q_3}$:

For turbulent,

$$L_{min,3} = 25D_3 = 0.25 \ \text{m} < L$$

$$L_{max,3} = 40D_3 = 0.40 \ \text{m} < L$$

Therefore, it is fully developed.

For laminar,

$$L_{laminer,3} = 0.06\left(\frac{4Q_3}{\pi v D_3}\right)D_3 = 0.06 \times \left[\frac{4\times(3.05\times10^{-4} \ \text{m}^3/\text{s})}{\pi\times(1.69\times10^{-5} \ \text{m}^2/\text{s})\times(10 \ \text{mm})}\right] \times (10 \ \text{mm}) = 1.38 \ \text{m} < L$$

Therefore, it is fully developed.

$$L_{laminer,2} = 0.06\left(\frac{4Q_3}{\pi v D_2}\right)D_2 = 0.06 \times \left[\frac{4\times(3.05\times10^{-4} \ \text{m}^3/\text{s})}{\pi\times(1.69\times10^{-5} \ \text{m}^2/\text{s})\times(15 \ \text{mm})}\right] \times (15 \ \text{mm}) = 1.38 \ \text{m} < L$$

Therefore, it is fully developed.

$$L_{laminer,1} = 0.06\left(\frac{4Q_3}{\pi v D_1}\right)D_1 = 0.06 \times \left[\frac{4\times(3.05\times10^{-4} \ \text{m}^3/\text{s})}{\pi\times(1.69\times10^{-5} \ \text{m}^2/\text{s})\times(25 \ \text{mm})}\right] \times (25 \ \text{mm}) = 1.38 \ \text{m} < L$$

Therefore, it is fully developed.

$\boldsymbol{Q_2}$:

For turbulent,

$$L_{min,3} = 25D_3 = 0.25 \ \text{m} < L$$

$$L_{max,3} = 40D_3 = 0.40 \ \text{m} < L$$

Therefore, it is fully developed.

$$L_{min,2} = 25D_2 = 0.375 \ \text{m} < L$$

$$L_{max,2} = 40D_2 = 0.6 \ \text{m} < L$$

Therefore, it is fully developed.

For laminar,

$$L_{laminer,2} = 0.06\left(\frac{4Q_2}{\pi v D_2}\right)D_2 0.06 \times \left[\frac{4\times(4.58\times10^{-4} \ \text{m}^3/\text{s})}{\pi\times(1.69\times10^{-5} \ \text{m}^2/\text{s})\times(15 \ \text{mm})}\right] \times (15 \ \text{mm}) = 2.07 \ \text{m} > L$$

Therefore, it is not fully developed.

$$L_{laminer,1} = 0.06\left(\frac{4Q_2}{\pi v D_1}\right)D_1 = 0.06 \times \left[\frac{4\times(4.58\times10^{-4} \ \text{m}^3/\text{s})}{\pi\times(1.69\times10^{-5} \ \text{m}^2/\text{s})\times(25 \ \text{mm})}\right] \times (25 \ \text{mm}) = 2.07 \ \text{m} > L$$

Therefore, it is not fully developed.

$\boldsymbol{Q_1}$:

For turbulent,

$$L_{min,3} = 25D_3 = 0.25 \ \text{m} < L$$

$$L_{max,3} = 40D_3 = 0.40 \ \text{m} < L$$

Therefore, it is fully developed.

$$L_{min,2} = 25D_2 = 0.375 \ \text{m} < L$$

$$L_{max,2} = 40D_2 = 0.6 \ \text{m} < L$$

Therefore, it is fully developed.

$$L_{min,1} = 25D_1 = 0.625 \ \text{m} < L$$

$$L_{max,1} = 40D_1 = 1 \ \text{m} < \text{L}$$

Therefore, it is fully developed.

$$L_{laminer,1} = 0.06\left(\frac{4Q_1}{\pi v D_1}\right)D_1$$

$$= 0.06$$

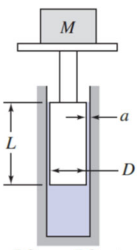
$$\times\left[\frac{4\times(7.63\times10^{-4}\ \ \mathrm{m^3/s})}{\pi\times(1.69\times10^{-5}\ \ \mathrm{m^2/s})\times(25\ \ \mathrm{mm})}\right]$$

$$\times(15\ \ \mathrm{mm}) = 3.45\ \ \mathrm{m} > L$$

Therefore, it is not fully developed.

Problem 8.11

8.11 A large mass is supported by a piston of diameter $D = 100$ mm and length $L = 100$ mm. The piston sits in a cylinder closed at the bottom, and the gap $a = 0.025$ mm between the cylinder wall and piston is filled with SAE 10 oil at 20°C. The piston slowly sinks due to the mass, and oil is forced out at a rate of 6×10^{-6} $\mathrm{m^3/s}$. What is the mass (kg)?



P8.11, P8.14

Flow between stationary plates:

$$V_{ave} = \frac{Q}{a\pi D} \quad \rightarrow \quad Re = \frac{aV_{ave}}{v}$$

$$\frac{Q}{l} = \frac{a^3\Delta p}{12\mu L}$$

$$\Delta p \rightarrow F \rightarrow m$$

Solution:

$$Re = \frac{aV_{ave}}{v} = \frac{a\dfrac{Q}{a\pi D}}{v} = \frac{Q}{v\pi D}$$

$$= \frac{(6\times10^{-6}\ \ \mathrm{m^3/s})}{(1.1\times10^{-4}\ \ \mathrm{m^2/s})\times\pi\times(100\ \ \mathrm{mm})}$$

$$= 0.174$$

$$\frac{Q}{l} = \frac{a^3\Delta p}{12\mu L} \Rightarrow \Delta p = \frac{12\mu LQ}{la^3}$$

$$= \frac{12\times(1\times10^{-1}\ \ \mathrm{N\cdot s/m^2})\times(100\ \ \mathrm{mm})\times(6\times10^{-6}\ \ \mathrm{m^3/s})}{(100\ \ \mathrm{mm})\times\pi\times(0.025\ \ \mathrm{mm})^3}$$

$$= 146.7\ \ \mathrm{MPa}$$

$$\frac{mg}{\dfrac{\pi D^2}{4}} = \Delta p$$

$$m = \frac{\Delta p\dfrac{\pi D^2}{4}}{g}$$

$$= \frac{(146.7\ \ \mathrm{MPa})\times\dfrac{\pi\times(100\ \ \mathrm{mm})^2}{4}}{(9.8\ \ \mathrm{m/s^2})}$$

$$= 1.18\times10^5\ \ \mathrm{kg}$$

Problem 8.18

8.18 Consider fully developed laminar flow between infinite parallel plates separated by gap width $d = 10$ mm. The upper plate moves to the right with speed $U_2 = 0.5$ m/s; the lower plate moves to the left with speed $U_1 = 0.25$ m/s. The pressure gradient in the direction of flow is zero. Develop an expression for the velocity distribution in the gap. Find the volume flow rate per unit depth ($\mathrm{m^3/s/m}$) passing a given cross section.

$$\frac{dP}{dx} = 0 \quad \rightarrow \quad \frac{d\tau}{dy} = \mu\frac{d^2u}{dy^2} = 0$$

$$\frac{d^2u}{dy^2} = 0 \quad \rightarrow \quad u(y),\ u(0) = -U_1,\ u(d) = U_2$$

$$Q = \int u\,dA = b\int_0^d u\,dy \quad \rightarrow \quad \frac{Q}{b} = ???$$

Solution:

$$\frac{dP}{dx} = 0 \quad \rightarrow \quad \frac{d\tau}{dy} = \mu\frac{d^2u}{dy^2} = 0$$

$$\frac{d^2u}{dy^2} = 0 \quad \rightarrow \quad u(y) = C_1y + C_2,\ u(0) = -0.25\ \ \mathrm{m/s},\ u(0.01\ \ \mathrm{m}) = 0.5\ \ \mathrm{m/s}$$

$$\Rightarrow \begin{cases} C_2 = -0.25 \ \ \mathrm{m/s} \\ C_1 = 75 \ \ \mathrm{s^{-1}} \end{cases}$$

$$\Rightarrow u(y) = 75y - 0.25$$

$$Q = \int u\, dA = b\int_0^d u\, dy$$

$$\rightarrow \frac{Q}{b} = \int_0^d u\, dy = \int_0^{0.01} (75y - 0.25)\, dy$$

$$= \left(\frac{75}{2}y^2 - 0.25y\right)\Big|_0^{0.01}$$

$$= 0.00125 \ \ \mathrm{m^3/s/m}$$

Problem 8-39

8.39 Consider first water and then SAE 10W lubricating oil flowing at 40°C in a 6-mm diameter tube. Determine the maximum flow rate (and the corresponding pressure gradient, $\partial p/\partial x$) for each fluid at which laminar flow would be expected.

Solution:

Water:

$$Re = \frac{\rho \bar{V} D}{\mu}$$

$$Q = \frac{Re_{crit}\pi v D_1}{4}$$

$$= \frac{(2300) \times \pi \times (6.59 \times 10^{-7} \ \ \mathrm{m^2/s}) \times (6 \ \ \mathrm{mm})}{4}$$

$$= 7.14 \times 10^{-6} \ \ \mathrm{m^3/s}$$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} = -\frac{8\mu Q}{\pi R^4} = -\frac{128\mu Q}{\pi D^4}$$

$$= -\frac{128 \times (6.53 \times 10^{-4} \ \ \mathrm{N \cdot s/m^2}) \times (7.14 \times 10^{-6} \ \ \mathrm{m^3/s})}{\pi \times (6 \ \ \mathrm{mm})^4}$$

$$= -146.63 \ \ \mathrm{Pa/m}$$

SAE:

$$Re = \frac{\rho \bar{V} D}{\mu}$$

$$Q = \frac{Re_{crit}\pi v D_1}{4}$$

$$= \frac{(2300) \times \pi \times (3.8 \times 10^{-5} \ \ \mathrm{m^2/s}) \times (6 \ \ \mathrm{mm})}{4}$$

$$= 4.12 \times 10^{-4} \ \ \mathrm{m^3/s}$$

$$Q = -\frac{\pi R^4}{8\mu}\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial x} = -\frac{8\mu Q}{\pi R^4} = -\frac{128\mu Q}{\pi D^4}$$

$$= -\frac{128 \times (3.4 \times 10^{-2} \ \ \mathrm{N \cdot s/m^2}) \times (4.12 \times 10^{-4} \ \ \mathrm{m^3/s})}{\pi \times (6 \ \ \mathrm{mm})^4}$$

$$= -4.40 \times 10^5 \ \ \mathrm{Pa/m}$$

Problem 8.40

8.40 For fully developed laminar flow in a pipe, determine the radial distance from the pipe axis at which the velocity equals the average velocity.

$$\bar{V} = \frac{Q}{A} = -\frac{R^2}{8\mu}\frac{\delta p}{\delta x}$$

$$u = -\frac{R^2}{4\mu}\frac{\delta p}{\delta x}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Solution:

$$\bar{V} = \frac{Q}{A} = -\frac{R^2}{8\mu}\frac{\delta p}{\delta x}$$

$$u = -\frac{R^2}{4\mu}\frac{\delta p}{\delta x}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$-\frac{R^2}{4\mu}\frac{\delta p}{\delta x}\left[1 - \left(\frac{r}{R}\right)^2\right] = -\frac{R^2}{8\mu}\frac{\delta p}{\delta x}$$

$$\left[1 - \left(\frac{r}{R}\right)^2\right] = \frac{1}{2}$$

$$\left(\frac{r}{R}\right)^2 = \frac{1}{2}$$

$$r = \frac{R}{\sqrt{2}}$$

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