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1. $\rho = 888\ kg/m^3$ $\mu = 0.800\ kg/m\cdot s$ $D = 0.05\ m$ $L = 40\ m$

$P_1 = 745\ kPa$ $P_2 = 97\ kPa$

(a) $$\dot{Q}_a = \frac{(\Delta P - \rho g L \sin\theta_a)\pi D^4}{128 \mu L} = \frac{(745-97)k\ \pi \times 0.05^4}{128 \times 0.800 \times 40}$$

$$= \boxed{3.11 \times 10^{-3}\ m^3/s}$$

$$V_a = \frac{\dot{Q}_a}{\frac{\pi D^2}{4}}$$

$$Re = \frac{\rho V D}{\mu} = \frac{4 Q \rho}{\pi D \mu} = \frac{4 \times 3.11 \times 10^{-3} \times 888}{\pi \times 0.05 \times 0.8} = 87.8 < 2300$$

Therefore, it is laminar

(b) $$\dot{Q}_b = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128 \mu L} = \frac{[(745-97)k - 888 \times 9.81 \times 40 \times \sin 15°]\ \pi \times 0.05^4}{128 \times 0.800 \times 40}$$

$$= \boxed{2.67 \times 10^{-3}\ m^3/s}$$

$$V_b = \frac{\dot{Q}_b}{\frac{\pi D^2}{4}}$$

$$Re = \frac{\rho V D}{\mu} = \frac{4 Q \rho}{\pi D \mu} = \frac{4 \times 2.67 \times 10^{-3} \times 888}{\pi \times 0.05 \times 0.8} = 75.58 < 2300$$

Therefore, it is laminar

(c) $$\dot{Q}_c = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128 \mu L} = \frac{[(745-97)k - 888 \times 9.81 \times 40 \times \sin -15°]\ \pi \times 0.05^4}{128 \times 0.800 \times 40}$$

$$= \boxed{3.54 \times 10^{-3}\ m^3/s}$$

$$V_c = \frac{\dot{Q}_c}{\frac{\pi D^2}{4}}$$

$$Re = \frac{\rho V D}{\mu} = \frac{4 Q \rho}{\pi D \mu} = \frac{4 \times 3.54 \times 10^{-3} \times 888}{\pi \times 0.05 \times 0.8} = 100.02 < 2300$$

Therefore, it is laminar.

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Solution 1

2. $$\alpha = \frac{\int_A \rho v^3 dA}{\dot{m} \bar{V}^2} = \frac{\int_A \rho \{2\bar{V}[1-(\frac{r}{R})^2]\}^3 dA}{\rho \bar{V} A \cdot \bar{V}^2}$$

$$= \frac{\int_A 8 \rho \bar{V}^3 [1-(\frac{r}{R})^2]^3 dA}{\rho A \bar{V}^3}$$

$$= \frac{8}{A} \int_A [1-(\frac{r}{R})^2]^3 dA$$

$$= \frac{8}{A} \int_0^{2\pi} \int_0^R [1-(\frac{r}{R})^2]^3 r dr d\theta$$

$$= \frac{8}{\pi R^2} \int_0^{2\pi} \int_0^R -\frac{1}{R^6}(r^2-R^2)^3 r dr d\theta$$

$$= -\frac{8}{\pi R^8} \int_0^{2\pi} \int_0^R (r^2-R^2)^3 r dr d\theta$$

$$= -\frac{8}{\pi R^8} \times 2\pi \times \frac{1}{8}(r^2-R^2)^4 \Big|_0^R$$

$$= -\frac{8}{\pi R^8} 2\pi \times -\frac{1}{8} R^8$$

$$= \boxed{2}$$

Solution 2. Because α is dimensionless, we can simplify the calculation of α in radical situation, which means we let $R=1$

$$\alpha = \frac{\int_A \rho v^3 dA}{\dot{m} \bar{V}^2} = \frac{\int_A \rho \{2V[1-r^2]\}^3 dA}{\rho \bar{V}^3 A} = \frac{8}{\pi R^2} \int_A [1-r^2]^3 dA$$

$$= \frac{8}{\pi} \int_0^{2\pi} \int_0^1 (1-r^2)^3 r dr d\theta = 16 \int_0^1 (1-r^2)^3 r dr = 16 \times \frac{1}{8} = \boxed{2}$$

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Air @ 35°C

$\rho = 1.145\ kg/m^3$ $\mu = 1.895\times10^{-5}\ kg/m\cdot s$ $\nu = 1.655\times10^{-5}\ m^2/s$

initial guesses for D and f are 0.267 , 0.018

$Q = 0.35\ m^3/s$

$$Re = \frac{\rho V D}{\mu} = \frac{\rho\cdot\frac{4Q}{\pi D^2}\cdot D}{\mu} = \frac{4\rho Q}{\pi\mu D} = \frac{4\times1.145\times0.35}{\pi\times1.895\times10^{-5}\times D}$$

$$f\frac{L}{D}\frac{V^2}{2} = g\cdot 20\,m$$

$$f\cdot\frac{150}{D}\times\frac{\left(\frac{4Q}{\pi D^2}\right)^2}{2} = 196.2$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{2.51}{Re\sqrt{f}}\right)$$

$$\Rightarrow \frac{1}{\sqrt{f}} = -2.0\log\frac{2.51}{\frac{45093}{\sqrt[5]{f}}\cdot\sqrt{f}}$$

$\Rightarrow f = 0.01796$

$\Rightarrow D = \boxed{0.26726\ m}$

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4. Water @ 10°C

$\rho = 999.7\ kg/m^3 \quad \mu = 1.307\times10^{-3}\ kg/m\cdot s$

$$\left(f\frac{L}{D} + \Sigma K\right)\frac{V^2}{2} = g\Delta h \quad -\frac{V^2}{2}$$

$$V = \frac{4Q}{\pi D^2} = \frac{4\times0.006}{\pi\times0.05^2} = 3.0558\ m/s$$

$$9.81\Delta h = \left(f\frac{89}{0.05} + 0.5 + 0.3\times2 + 0.2 + 1.06\right)\times\frac{3.0558^2}{2}$$

$$9.81\Delta h = (1780f + 3.36)\times\frac{3.0558^2}{2}$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right)$$

$$Re = \frac{4\rho Q}{\pi\mu D} = \frac{4\times999.7\times0.006}{\pi\times1.307\times10^{-3}\times0.05} = 116865.2707$$

$$\Rightarrow f = 0.031519$$

$$\Rightarrow \Delta h = \frac{(1780f + 3.36)\times\frac{3.0558^2}{2}}{9.81} = 28.30\ m$$

$$\Rightarrow Z_1 = \Delta h + Z_2$$

$$= 28.30 + 4$$

$$= \boxed{32.30\ m}$$