SCUPI

Sichuan University - Pittsburgh Institute

ME 1049

Mechatronics Lab

DC Motor Speed Control

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Mechanical Engineering Department

Lab 5: DC Motor Speed Control

Figure 0: Vehicle cruise control is only one of the many applications of DC motor speed control

One of the most common tasks that automation, robotics, and industrial engineers are called upon to perform when creating industrial systems is to control the speed of a DC motor. From automation in manufacturing, to automotive systems, autonomous systems and even aerospace, DC motors are used to actuate critical systems, and their speed needs to be controlled to perform within specific design criteria. As an introduction to control systems, the control of a DC motor serves as an excellent starting point because DC motors are relatively easy to model and control. The DC motor system on the Quanser Controls Board was designed to make that experience even easier by: A) tuning the dynamics of the motor to match a theoretical linear model accurately, and B) creating an interface to the hardware that makes sending commands to the amplifier system and reading the sensors quick and easy. Despite the ease of use, the skills gained in these exercises are directly applicable to a multitude of exciting emerging application areas in engineering.

Learning Objectives

After completing this lab, you should be able to complete the following activities.

1. Tune PI control parameters for speed control of a DC motor.
2. Design PI control parameters to meet speed control specifications.
3. Use lead compensator design to control the speed of a DC motor.

Required Tools and Technology

Platform: NI ELVIS III	✓ View the NI ELVIS III User Manual http://www.ni.com/en-us/support/model.ni-elvis-iii.html
Hardware: Quanser Controls Board	✓ View the Controls Board User Manual http://www.ni.com/en-us/support/model.quanser-controls-board-for-ni-elvis-iii.html
Software: LabVIEW Version 18.0 or Later Toolkits and Modules: • LabVIEW Real-Time Module • NI ELVIS III Toolkit • LabVIEW Control Design & Simulation	• Before downloading and installing software, refer to your professor or lab manager for information on your lab's software licenses and infrastructure • Download & Install for NI ELVIS III http://www.ni.com/academic/download • View Tutorials http://www.ni.com/academic/students/learn-labview/

Expected Deliverables

In this lab, you will collect the following deliverables:

- ✓ Evidence of the behavior of the DC motor with various control gains
- ✓ Step responses of the system to various sets of control gains
- ✓ Peak time and percent overshoot specifications
- ✓ Calculated control gains to meet the specifications
- ✓ Measured speed response to the designed control gains
- ✓ Measured peak time and percent overshoot
- ✓ Evidence of the effect of the design parameters on the control response
- ✓ Transfer function of the DC motor using nominal values of gain and time constant
- ✓ Expression for the magnitude response in terms of frequency
- ✓ Calculated crossover frequency
- ✓ Bode plot of $P_i(s)$ and corresponding phase margin and crossover frequency
- ✓ Calculated proportional gain K_c
- ✓ Bode plot of $K_cP_i(s)$
- ✓ Calculated values of phase lead (ϕ_m), phase margin frequency (ω_m), and α
- ✓ Transfer function of the lead compensator
- ✓ Calculated locations of the poles and zero of the lead compensator
- ✓ Bode plot of the lead compensator
- ✓ Screenshot of the actual and desired response of the system using the designed lead compensator

1 Qualitative PI Control Design

1.1 Theory and Background

The speed of the Quanser Controls Board DC motor is controlled using a proportional-integral (PI) control system. PI control combines a measure of the current instantaneous error of the system with the accumulated error over time. The block diagram of the closed-loop system is shown in Figure 1-1:

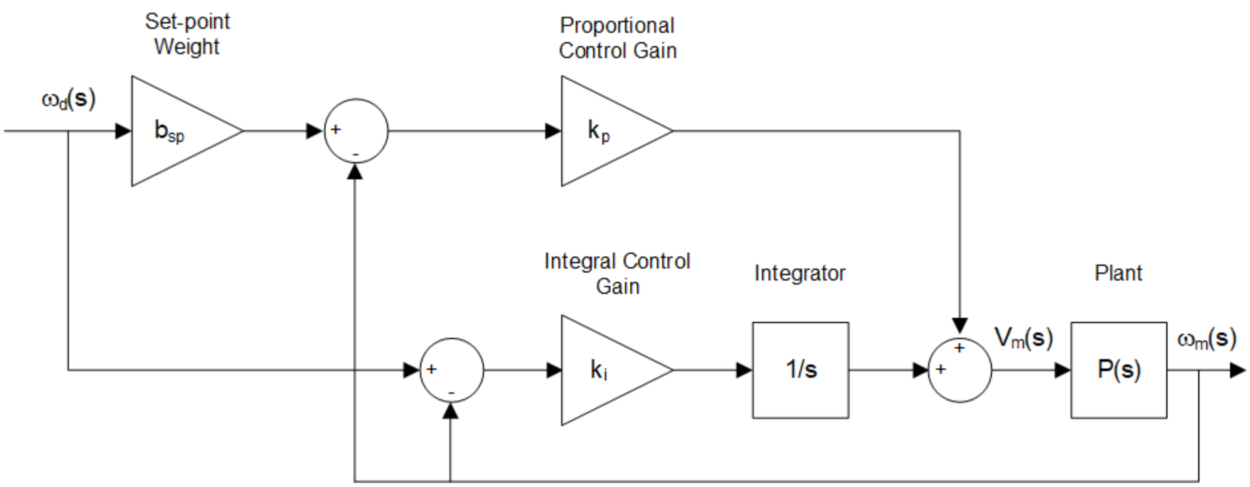

Figure 1-1: Closed loop PI control system block diagram

The transfer function representing the DC motor speed-voltage relation with steady-state gain *K* and time constant *τ* is

Equation 1-1

$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1}$$

and will be used to design the PI controller. The input-output relation in the time-domain for a PI controller with set-point weighting is

Equation 1-2

$$u = k_p(b_{sp}\omega_d - \omega_m) + \frac{k_i(\omega_d - \omega_m)}{s},$$

where k_p is the proportional gain, k_i is the integral gain, and b_{sp} is the set-point weight. The closed loop transfer function from the speed reference ω_d to the angular motor speed output ω_m is

Equation 1-3

$$G_{\omega_m,\omega_d}(s) = \frac{K(k_p b_{sp} s + k_i)}{\tau s^2 + (K k_p + 1)s + K k_i}.$$

1.2 Implement

1. Open the project **Quanser Controls Board.lvproj**, and then open **DC Motor Speed Control.vi** listed under the NI ELVIS III.

2. Run the VI. The DC motor should begin rotating and the scopes should look similar to Figure 1-2

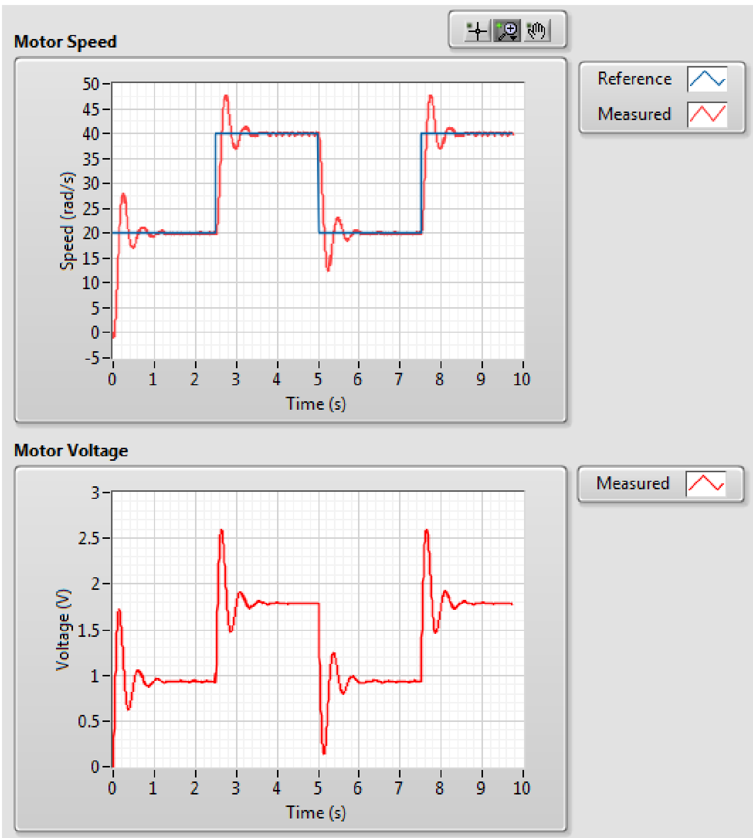

Figure 1-2: Default response of the PI control application

3. In the Signal Generator section of the front panel set:
    - Amplitude (rad/s) to **10**
    - Frequency (Hz) to **0.2**
    - Offset (rad/s) to **80**
4. In the Control Parameters section set:
    - kp (V s/rad) to **0.050**
    - ki (V/rad) to **1.00**
    - bsp to **0.00**
5. Examine the behavior of the measured speed, shown in red, with respect to the reference speed, shown in blue, in the Speed (rad/s) scope.
6. Increment and decrement k_p in steps of 0.005 V s/rad.
7. Observe the changes in the measured signal with respect to the reference signal in response to the updated proportional gains.
8. Set k_p to 0 V s/rad and k_i to 0 V/rad. The motor should stop spinning.
9. Increment the integral gain k_i in steps of 0.10 V/rad, between 0.1 V/rad and 2.00 V/rad.
10. Examine the response of the measured speed in the Speed (rad/s) scope.
11. Click on the **Stop** button to stop the VI.

2 Quantitative PI Control Design

2.1 Theory and Background

The standard second-order transfer function has the form

Equation 2-1

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ω_n is the natural frequency and ζ is the damping ratio. This leads to the standard desired closed-loop characteristic polynomial of

Equation 2-2

$$s^2 + 2\zeta\omega_0 s + \omega_0^2,$$

where ω_0 is the undamped closed loop frequency and ζ is the damping ratio. The denominator of the transfer function shown in Equation 1-3 in Section 1.1 is the characteristic equation of the Quanser Controls Board DC motor system, and matches the desired characteristic equation in Equation 2-2 with the following gains:

Equation 2-3

$$k_p = \frac{-1 + 2\zeta\omega_0\tau}{K}$$

and

Equation 2-4

$$k_i = \frac{\omega_0^2\tau}{K}.$$

Large values of ω_0 give large values of controller gain. The damping ratio, ζ, and the set-point weight parameter, b_{sp}, can be used to adjust the speed and overshoot of the response to reference values.
There is no tachometer sensor present on the Quanser Controls Board to measure the speed. Instead, the derivative of the encoder signal is calculated using a first-order differentiating filter.

Second-order Step Response

The properties of the response of a second-order system as defined by Equation 2-1 depend on the values of the parameters ω_n and ζ.

Consider a second-order system as shown in Equation 2-1 subjected to a step input given by

Equation 2-5

$$R(s) = \frac{R_0}{s}$$

with a step amplitude of $R_0 = 1.5$. The system response to this input is shown in Figure 2-1, where the red trace is the output response *y(t)* and the blue trace is the step input *r(t).*

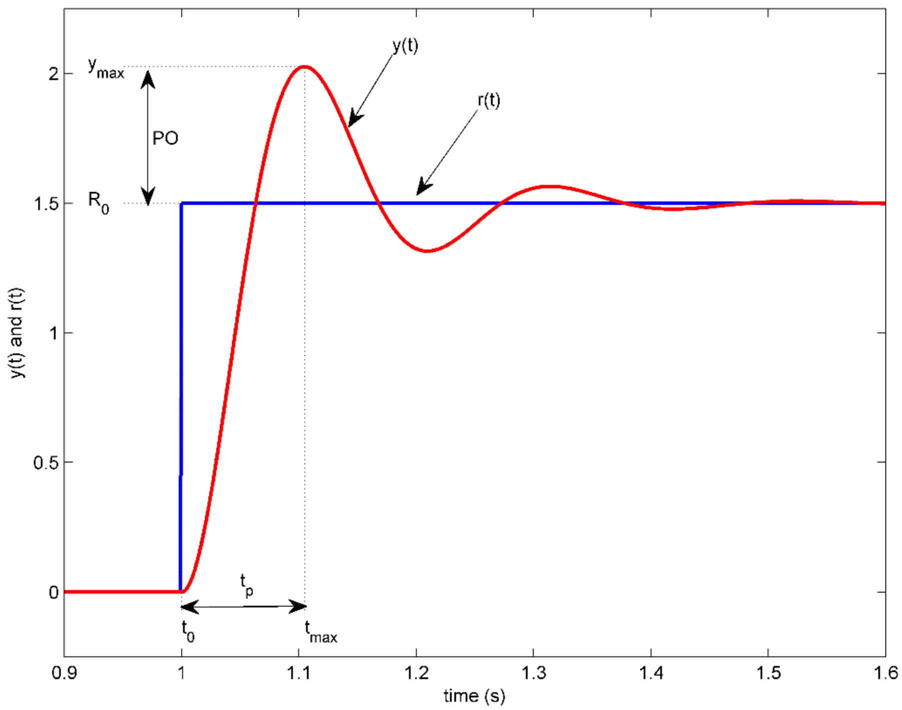

Figure 2-1: Typical step response of second-order system

Peak Time and Overshoot

The maximum value of the response is denoted by the variable y_{max} and it occurs at a time t_{max}. For a response similar to Figure 2-1, the percent overshoot is found using

Equation 2-6

$$PO = \frac{100(y_{\max} - R_0)}{R_0}.$$

In a second-order system, the amount of overshoot depends solely on the damping ratio parameter and it can be calculated using the equation

Equation 2-7

$$PO = 100e^{\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)}.$$

From the initial step time t_0 the time it takes for the response to reach its maximum value is

Equation 2-8

$$t_p = t_{\max} - t_0.$$

This is called the peak time of the system. It depends on both the damping ratio and natural frequency of the system. It can be derived analytically as

Equation 2-9

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}.$$

Generally speaking, the damping ratio affects the shape of the response while the natural frequency affects the speed of the response.

2.2 Implement

Quantitative PI Control Design

1. Calculate the expected peak time, t_p, and percent overshoot, *PO*, given the following design specifications:
   - ζ = **0.75**,
   - ω_0 = **16** rad/s.

   Optional: You can also design a VI that simulates the Quanser Controls Board first-order model with PI control and have it calculate the peak time and overshoot.
2. Calculate the proportional and integral control gains k_p and k_i, respectively, according to the design specifications for the model parameters
   - K = 22.6 rad/(V s)
   - τ = 0.12 s.

   If desired, you can conduct an experiment to find more precise model parameters as outlined in the experimental modeling laboratory.
3. Open the project **Quanser Controls Board.lvproj**, open **DC Motor Speed Control.vi**.
4. Run the VI. The DC motor should begin rotating and the scopes should look similar to Figure 1-2.
5. In the *Signal Generator* section of the front panel set:

- Amplitude (rad/s) to **10**
- Frequency (Hz) to **0.2**
- Offset (V) to **80**

6. In the *Control Parameters* section, enter the control gains that you found in Step 2, and ensure that the b_{sp} is set to zero.
7. When you have collected the rising and falling transient response, stop the application by clicking on the **Stop** button.
8. Capture the measured speed response. Make sure you include both the Speed (rad/s) and the control signal Voltage (V) scopes.
9. Measure the peak time and percentage overshoot of the observed response. If the specifications have not been satisfied, adjust the proportional gain k_p and integral gain k_i to meet the specifications and capture your system response plots.
10. Click on the **Stop** button to stop the VI.

3 Lead Compensator Design

3.1 Theory and Background

PID control uses integration and derivative operations to design a compensator such that the overall system response follows a desired trajectory. More generally, the design of a controller for a system can be regarded as a filter design problem. In this context, a PD controller is a high-pass filter that introduces a positive phase and is also referred to as a lead controller. Contrary, a PI controller is a low-pass filter, and because it introduces a negative phase over some frequency range, therefore it is also called a lag controller. In this lab you will design and implement a lead controller to control the speed of a DC motor.

The generic transfer function of a simple lead or lag compensator can be expressed as:

Equation 3-1

$$C_{\text{lead, lag}}(s) = K_1 \frac{s+z}{s+p} = K_c \frac{1+\alpha T s}{1+Ts}$$

which is a low-pass or phase lag controller for $\alpha < 1$ (or $p > z$), and a high-pass or phase lead controller for $\alpha > 1$ (or $p < z$).

The proportional gain of the lead compensator is used to attain a certain crossover frequency. In general, increasing the gain, and respectively the crossover frequency, essentially increases the bandwidth of the system, thus decreasing the system's peak time (speeding up the response). A gain of $K_c > 1$ decreases the system's phase margin and, if K_c is chosen too large, will lead to large overshoots in the system response. For design purposes, K_c is often chosen such that it increases the bandwidth of the system to about half the desired bandwidth. The lead compensator will add additional gain such that the combination of K_c and lead compensator result in the desired system bandwidth.

Even though lag compensators work well in theory, they often struggle with the saturation limits of actual hardware, and may not be able to achieve a zero steady-state error specification. In this lab, we will design a lead compensator in series with an integrator as in Figure 3-1 to achieve zero steady-state error. The resulting controller has the form:

Equation 3-2

$$C(s) = K_c \frac{1 + \alpha T s}{(1 + T s) s}$$

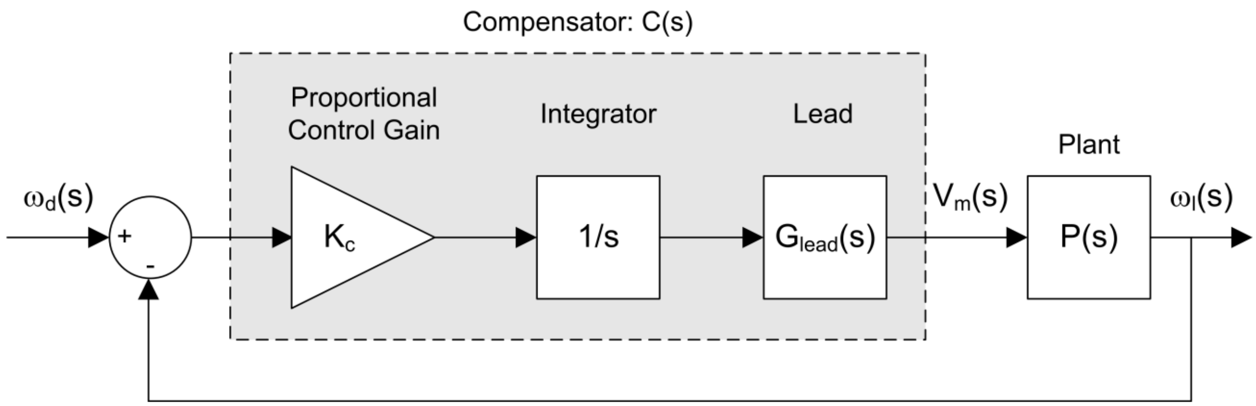

Figure 3-1: Closed-loop speed control with lead compensator

Lead Compensator Design Procedure

The two main design parameters for a lead compensator are the desired phase margin and the desired crossover frequency. The phase margin mainly affects the shape of the response, and a higher phase margin implies a more stable response with less overshoot. As a rule of thumb, the overshoot percentage (PO) will not go beyond 5% for a phase margin of at least 75 deg.

The crossover frequency is defined as the frequency where the gain of the system is 1 (or 0 dB in a Bode plot). This parameter mainly affects the speed of the response and a larger ω_m implies a decrease in the peak time. As a rule of thumb, the peak time t_p will not be more than 0.05 s with a crossover frequency of at least 75 rad.

The design process for a lead compensator can be summarized as follows:

1. Generate the Bode plot of the open-loop uncompensated system.
2. The lead compensator itself will add some gain to the closed-loop system response. To make sure that the bandwidth requirement of the design can be met, a proportional gain K_c needs to be added such that the open-loop crossover frequency is about a factor of two below the desired system bandwidth.
3. Determine the necessary additional phase lead ϕ_m for the plant with open-loop gain K_c. To do so, compute:

Equation 3-3

$$\varphi_m = PM_{\text{des}} - PM_{\text{meas}} + 5$$

   i.e. add 5 degrees to the desired phase margin and subtract the open-loop measured phase margin.
4. Compute α. To attain the maximum phase ϕ_m at the frequency ω_m as shown in Figure 3-2, the compensator is required to add 20 log10 (α) of gain. Here, ω_m is the geometric mean of the two corner frequencies from the zero and pole of the lead compensator, respectively, i.e:

Equation 3-4

$$log_{10}\,\omega_m = \frac{1}{2}\left(log_{10}\left(\frac{1}{\alpha T}\right) + log_{10}\left(\frac{1}{T}\right)\right)$$

   Solving for ω_m reveals:

Equation 3-5

$$\omega_m = \frac{1}{\sqrt{\alpha}T}$$

The proportional gain of the lead compensator is used to attain a certain crossover frequency. In general, increasing the gain, and respectively the crossover frequency, essentially increases the bandwidth of the system, thus decreasing the system's peak time (speeding up the response). A gain of $K_c > 1$ decreases the system's phase margin and, if K_c is chosen too large, will lead to large overshoots in the system response. The lead compensator is used to dampen the overshoot and increase the overall stability of the system by increasing the phase margin. The frequency response of the lead compensator in (Equation 3-1) is given by substituting s = jω as:

Equation 3-6

$$G_{\mathrm{lead}}(j\omega) = \frac{1 + j\omega\alpha T}{1 + j\omega T}$$

with the corresponding magnitude and phase:

Equation 3-7

$$|G_{\mathrm{lead}}(j\omega)| = \sqrt{\frac{1 + \omega^2\alpha^2 T^2}{1 + \omega^2 T^2}}$$

$$\varphi_m = atan(\omega\alpha T) - atan\,(\omega T)$$

Using the trigonometric identity

$$tan(\alpha - \beta) = \frac{tan(\alpha) + tan\,(\beta)}{1 + tan(\alpha)\,tan\,(\beta)}$$

On Equation 3-7 yields:

Equation 3-8

$$tan(\varphi_m(j\omega)) = \frac{\omega\alpha T - \omega T}{1 + (\omega\alpha T)(\omega T)}$$

Noting:

$$tan(\alpha) = \pm\frac{\sin\,(\alpha)}{\sqrt{1 - sin^2(\alpha)}}$$

and using Equation 3-5, one can find:

Equation 3-9

$$sin(\varphi_m) = \frac{\alpha - 1}{\alpha + 1}$$

Thus, if ϕ_m is known, α can be determined by solving:

Equation 3-10

$$\alpha = \frac{1 + sin(\varphi_m)}{1 - \ sin(\varphi_m)}$$

5. Determine the value of T using Equation 3-5. To do so, place the corner frequencies of the lead compensator such that ϕ_m is located at ω_m, i.e. the new gain crossover frequency (the geometric mean of $1/\alpha T$ and $1/T$) where the compensator has a gain of 10 dB. By design, ω_m is the frequency at which the system with compensator has 0 dB gain. Therefore, ω_m has to be placed at the frequency where the magnitude of the uncompensated system is $G(j\omega) = -10 \log10 \alpha$ dB. Then, ω_m is obtained by finding the corresponding frequency in the uncompensated Bode plot.
6. Determine the pole and zero of the lead compensator.
7. Check whether or not the compensator fulfills the design requirements. To do so, draw the Bode plot of the compensated system and check the resulting phase margin and check whether or not the system response meet the desired characteristics. Repeat the design steps for a different ϕ_m if necessary.

A typical Bode plot of a lead compensator is shown in Figure 3-2.

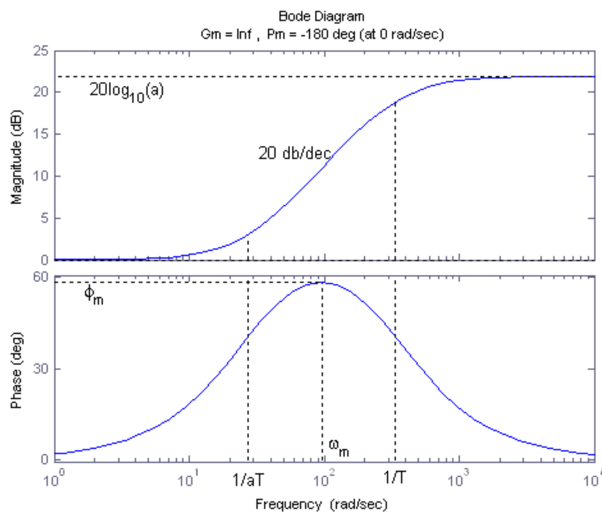

Figure 3-2: Bode plot of a typical lead compensator

3.2 Implement

In this lab, you will design a lead compensator for the speed control of the DC motor. Recall that the input voltage to output speed transfer function for the Controls board is given by:

Equation 3-11

$$P(s) = \frac{K}{\tau s + 1}$$

As stated in the background section, we want to design a controller that is in series with an integrator to guarantee zero steady-state. For the design purpose of the lead compensator, we assume that the integrator is part of the plant model, i.e.:

Equation 3-12

$$P_i(s) = P(s)\frac{1}{s}$$

The control design should fulfill the following design requirements for steady-state error (e_{ss}), peak time (t_p), percentage overshoot (PO), phase margin (PM) and system bandwidth (ω_m):

Equation 3-13

$$\begin{aligned} e_{ss} &= 0 \\ t_p &= 0.05\text{s} \\ PO &\leq 5\% \\ PM &\geq 75\text{deg} \\ \omega_m &\geq 75\text{rad/s} \end{aligned}$$

1. What is the transfer function representation of $P_i(s)$?
2. Find the magnitude of the frequency response of the system transfer function Equation 3-12 that is in series with an integrator ($|P_i(s)|$) in terms of the frequency ω.
3. The system has a gain of 1 (or 0 dB) at the crossover frequency ω_c. Find an expression for the crossover frequency in terms of the model parameters K and τ

for $P_i(s)$. Use this expression to determine the crossover frequency for the DC motor using the following nominal parameters K = 22.6 rad/s/V and τ = 0.12 s.

4. Open the project **Quanser Controls Board.lvproj**, and open **Bode Plot.vi**. Use this VI to generate the Bode plot of $P_i(s)$. First, using the **Transfer Function** numeric control, enter the coefficients of $P_i(s)$ that you determined in Step 1.
5. Run the VI.
6. The VI will automatically generate the magnitude and phase plots. The VI also displays the system's phase margin (in deg), and phase margin/crossover frequency (in rad/s). Compare your derived crossover frequency form Step 3 with the phase margin/crossover frequency you obtained using the VI. Take a screenshot of your results.
7. Find the proportional gain K_c that is necessary such that $K_cP_i(s)$ has a crossover frequency of 35 rad/s (about half the desired closed-loop bandwidth). To confirm your finding, use **Bode Plot.vi** to generate a Bode plot of $K_cP_i(s)$. Take a screenshot of your results.
8. Determine the necessary phase lead ϕ_m that the lead compensator needs to add for the system $K_cP_i(s)$.
9. Compute α.
10. Determine ω_m.
11. Does ω_m meet the design requirement of $\omega_m \geq 75$ rad/s? Comment on what you could do to ensure you meet this requirement.
12. Determine the transfer function of the lead compensator $G_{lead}(s)$. Start by evaluating T.
13. Determine the pole and zero location of the lead compensator. Generate the Bode plot of your lead compensator and verify that you have the desired phase margin at the desired frequency.
14. Validate your result by obtaining the closed-loop bode plot with proportional gain K_c and lead compensator $G_{lead}(s)$. Do you have the desired phase margin at the desired frequency? Take a screenshot of your results.
15. Open **Lead Compensator.vi** and implement your lead compensator $G_{lead}(s)$ with proportional gain K_c. To do so, navigate to the block diagram and set the parameters of the **G_lead** block with the coefficients of your lead compensator. Navigate to the front panel set **K_c** to the proportional gain you calculated earlier.
16. Set the **Signal Generator** parameters as follows:
    a. **Amplitude (rad/s):** 10
    b. **Frequency (Hz):** 0.2
    c. **Offset (rad/s):** 30
17. Run the VI.
18. Does the system response match the desired characteristics? Try varying the value of K_c and see if you can improve the overall system response. Take screenshots of your results. A sample system response is shown in Figure 3-3.
19. If no further experiments are required, stop your VI and power down the unit.

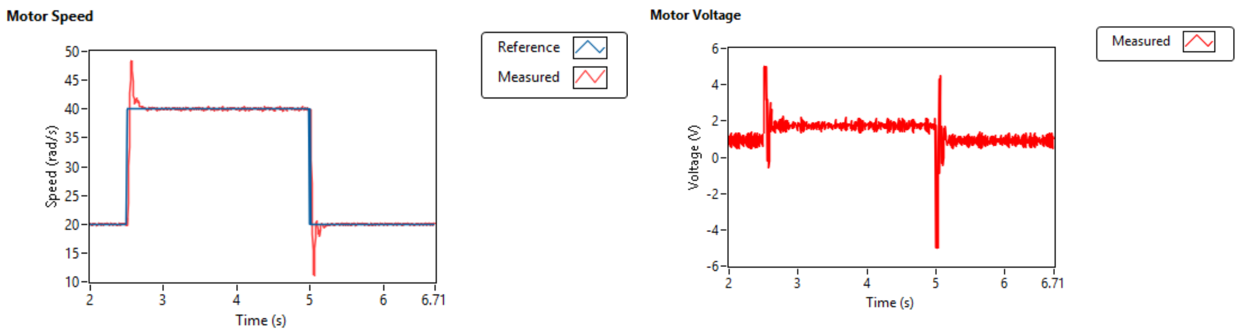

Figure 3-3: Sample system response using a lead controller

4 For the Report

The report format will be a MEMO. Be sure to include:

<u>Qualitative PI Control Design</u>

1. Explain the behavior of the measured speed with respect to the reference speed in Step 5.
2. Explain the observed performance differences in Step 7.
3. Describe and explain the response of the measured speed in Step 10 when k_i is set low compared to when k_i is high.

<u>Quantitative PI Control Design</u>

1. Attach the measured speed response that you captured in Step 8.
2. What effect does increasing the specification ζ have on the measured speed response? How about on the control gains? Hint: Start by examining Equation 2-7.
3. What effect does increasing the specification ω_0 have on the measured speed response and the generated control gains? Hint: Start by examining Equation 2-9.

<u>Lead Compensator Design</u>

1. What is the expression for the magnitude response in terms of frequency that you derived in Step 2?
2. What are the phase lead ϕ_m, α, and ω_m that you determined in Steps 8 through 10?
3. What is the transfer function representation of $G_{lead}(s)$?