To: Professor Qi Lu

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Subject: ME1042 Lab 05 PD Control of Unstable Systems

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On 21$^{\text{th}}$ October, Gordon Lou, Owen Chen, Frederic Liu, Yanjun He, and I conducted our fifth experiment in the course Mechanical Measurements 2, from which we have studied the control of the ball and beam system with proportion controller and phase advanced controller.

Proportional control is to multiply the error signal of the system by a constant gain k_p (Jia et al., 2020). This type of controller can be used to improve the steady-state error, increase the speed of the system's response, but can also cause the system to go unstable. Also, it cannot manipulate the ball's position because it has no anticipatory action (Zhou, 2018).

Phase advance control is to add an anticipatory action, which is accounted for with adding positive phase to the system. This type of controlled can speed up the system response. Because it has anticipatory actions, it is also able to manipulate the ball's position as well (Kong et al., 2010).

In the first part of this experiment, we will understand how the closed-loop proportional control affects the steady-state beam angle's measurement. The block diagram of using proportional control to manipulate the beam's angle is shown in Figure 3 and the resulting closed-loop transfer function $T_1(s)$ is shown in Equation 1. We understand how proportional control affects the beam angle's measurement is done by implementing the block diagram in Figure 3. The Simulink results are shown in Figure 4, 5, and 6 with $k_{p1} = 1$, $k_{p1} = 5$, and $k_{p1} = 8$, respectively.

In the next part of this experiment, we will show how closed-loop proportional control affects the steady state ball position's measurement. The block diagram of using proportional control to manipulate the ball's position is shown in Figure 7 and the resulting closed-loop transfer function $T_2(s)$ is shown in Equation 5. We understand how proportional control affects the ball's position is done by implementing the block diagram in Figure 7. The Simulink results are shown in Figure 8, 9, and 10 with $k_{p2} = 0.1$, $k_{p2} = 0.3$, and $k_{p2} = 0.5$, respectively.

The previous test shows why incorporating closed-loop proportional control within the apparatus's design will not manipulate the ball's position. The reason is proportional control has no anticipatory action and therefore another control structure is needed. One such control structure is a phase advance controller (lead controller), which can anticipate the ball's position. First, showing how the phase advance controller can manipulate the ball's position is done in Simulink before it's implemented on the equipment. The block diagram of implementing the phase advance controller to manipulate ball's position is shown in Figure 11 and the resulting closed-loop transfer function T3(s) is expressed in Equation 9. We understand how phase advanced control affects the ball's position is done by implementing the block diagram in Figure 11. The Simulink results are shown in Figure 12, 13, and 14.

Then, we will discuss the effects of varying k_{p1} and k_{p2}. k_{p1} is related to the angular response speed of beam. Since it is in the closed loop that affects the beam angle's measurement and the transfer function is shown in Equation 1., the time constant, as shown in Equation 16, decreases as k_{p1} increasing. Therefore, the increase of k_{p1} makes the whole system sensitive to the angle of beam. For k_{p2}, since it is in the master loop, it relatives to position of the ball. A larger k_{p2} will make the system sensitive to small changes in the position of the ball, resulting in a larger deflection of the beam. At the same time, according to the results of our Simulink, the increase of k_{p1} will make the system tend to be stable while the increase of k_{p2} will make the system tend to be unstable.

In this experiment, we find that $G_m = 1$ will make the simulation of gain set "a" unstable, but the real system is stable. Therefore, we assume $G_m = 8$, which will make the simulation match the real system well. And under this assumption, the system behavior of simulation under all gain sets is the same as the system behavior of real system. We think gain set "c" is the best set in our experiment, because of fast response with zero steady state error and minimum overshoot. I think the reasons are that bigger k_{p1} will make the system more stable, and smaller k_{p2} will make the system faster to converge. But in the experiment, we only do the "a", "c", and "d" gain sets. If we do others and use same logic in the previous analysis, we think "f" will be the best, because of the biggest k_{p1} and smallest k_{p2}.

During this experiment, we also notice that the defects of the experimental instrument would interfere with the theoretical results. The experimental instrument consists of three parts: a balance beam, a cam, and a motor. The Angle of the beam can be precisely controlled by changing the Angle of the cam by a programmed motor. By constantly adjusting the Angle of the beam, the ball can be controlled in the specified position on the beam. However, in actual experiments, we found that the program does not match the cam sometimes. In the program, the angle of the beam will be greater than the actual angle of the beam. At this time, the cam will rotate excessively, causing the angle of the beam to change in the opposite direction. The ball becomes unstable and moves out of orbit directly. This is more likely to happen when k_{p1} and k_{p2} are larger. In order to avoid this situation, we can only choose a smaller initial distance in the experiment.

After this experiment, with observing and recording the response of the ball and beam system with different controllers, we have learned a lot about the proportional control and the phase advance control, and have a better glimpse of the characteristics as well as the application condition for both of the controllers, which promotes the theories we have learned in class. In the future, if we have the opportunity to research from automation in manufacturing, to automotive systems, autonomous systems and even aerospace (Adib Murad et al., 2020), today's experiment will provide us with tremendous help regarding to this usefulness and significance.

Figure 1: CE106 Ball and Beam Apparatus.

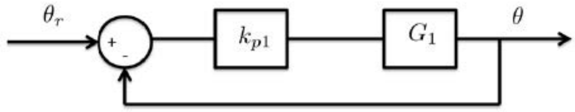

Figure 3: Block diagram of incorporating proportional control to manipulate the beam's angle.

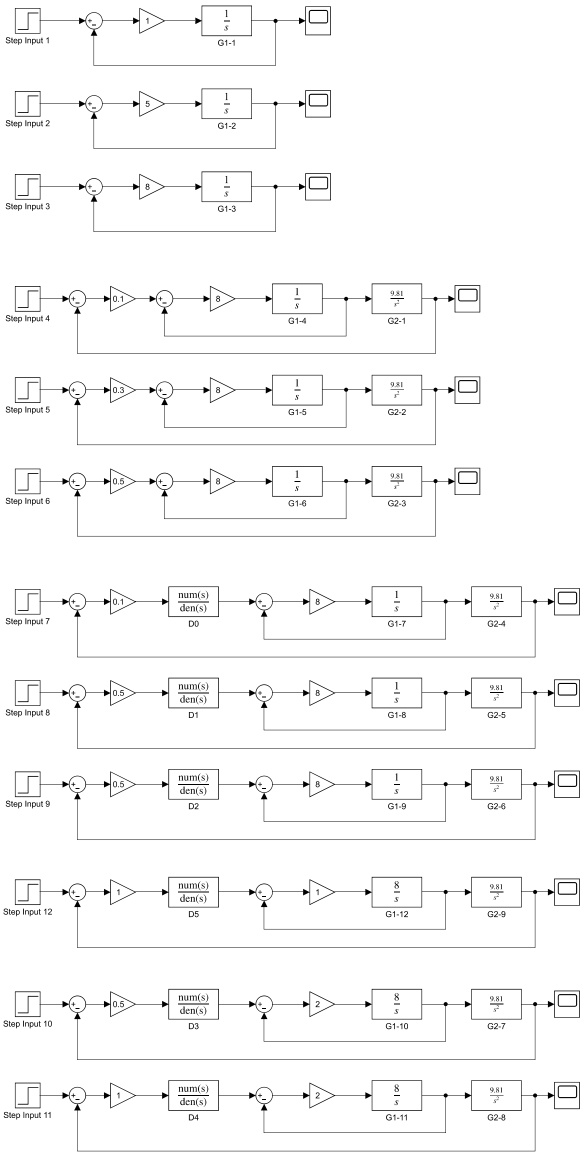

Figure 2: Block diagram in the Simulink.

Equation for transfer function for the slave loop:

$$T_1(s) = \frac{k_{p1} G_m}{s + k_{p1} G_m} \tag{1}$$

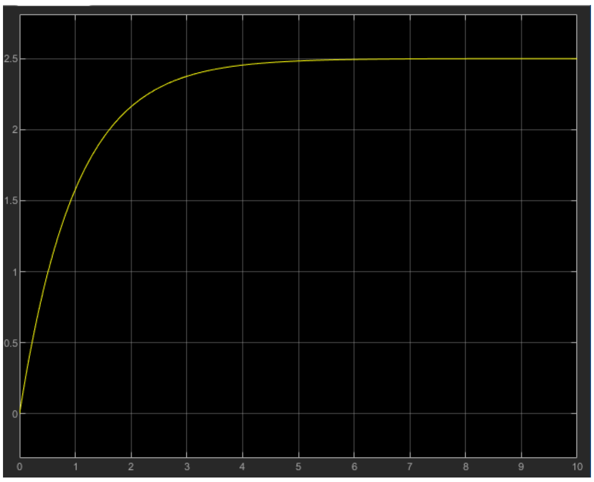

Figure 4: Output for incorporating proportional control to manipulate the beam's angle with $k_{p1} = 1$.

Equation for poles of the system in Figure 4:

$$s = -1 \tag{2}$$

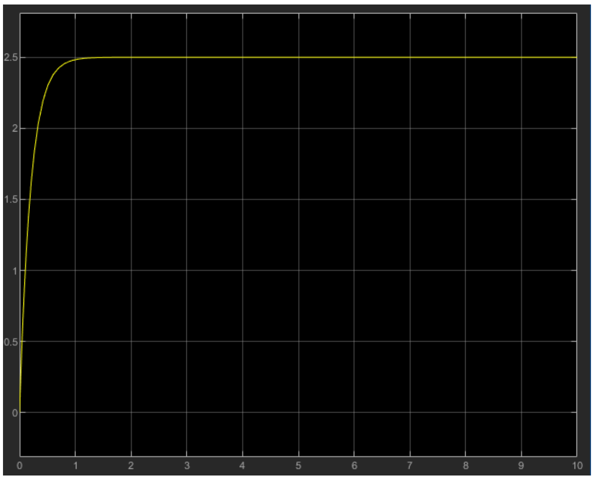

Figure 5: Output for incorporating proportional control to manipulate the beam's angle with $k_{p1} = 5$.

Equation for poles of the system in Figure 5:

$$s = -5 \tag{3}$$

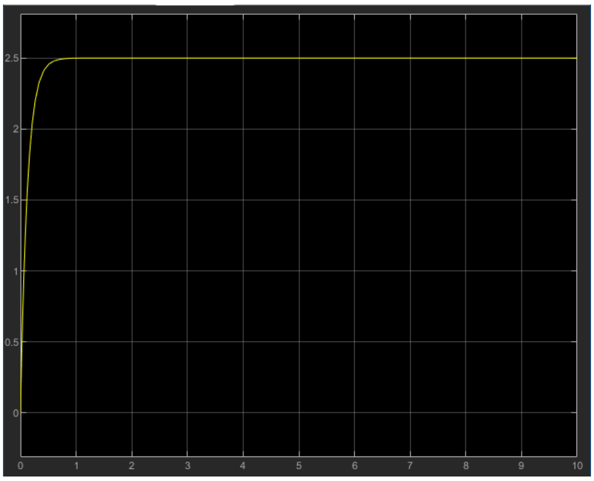

Figure 6: Output for incorporating proportional control to manipulate the beam's angle with $k_{p1} = 8$.

Equation for poles of the system in Figure 6:

$$s = -8 \tag{4}$$

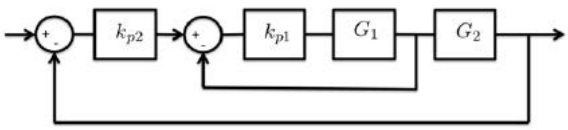

Figure 7: Block diagram of incorporating proportional control to manipulate the balls position.

Equation for transfer function for the whole loop:

$$T_2(s) = \frac{g k_{p1} k_{p2} G_m}{s^3 + k_{p1} G_m s^2 + g k_{p1} k_{p2} G_m} \tag{5}$$

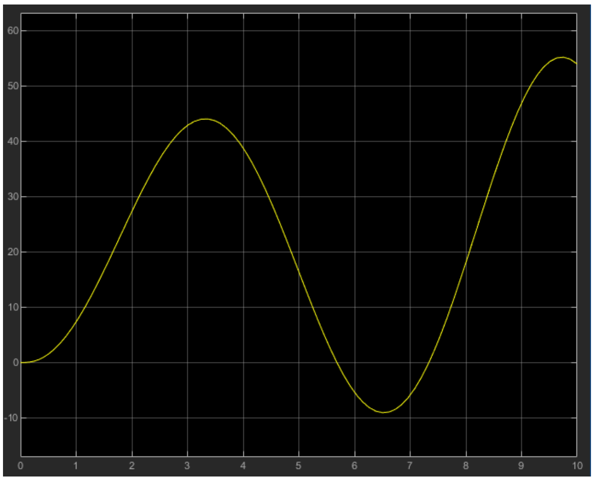

Figure 8: Output for incorporating proportional control to manipulate the balls position with $k_{p1} = 8$ and $k_{p2} = 0.1$.

Equation for poles of the system in Figure 8:

$$\begin{cases} s_1 = -8.11906 \\ s_2 = 0.0595276 - 0.981362i \\ s_3 = 0.0595276 + 0.981362i \end{cases} \tag{6}$$

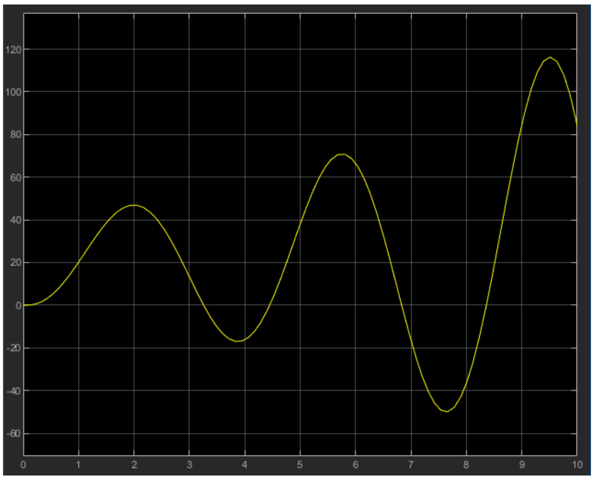

Figure 9: Output for incorporating proportional control to manipulate the balls position with $k_{p1} = 8$ and $k_{p2} = 0.3$.

Equation for poles of the system in Figure 9:

$$\begin{cases} s_1 = -8.33861 \\ s_2 = 0.169303 - 1.67177i \\ s_3 = 0.169303 + 1.67177i \end{cases} \tag{7}$$

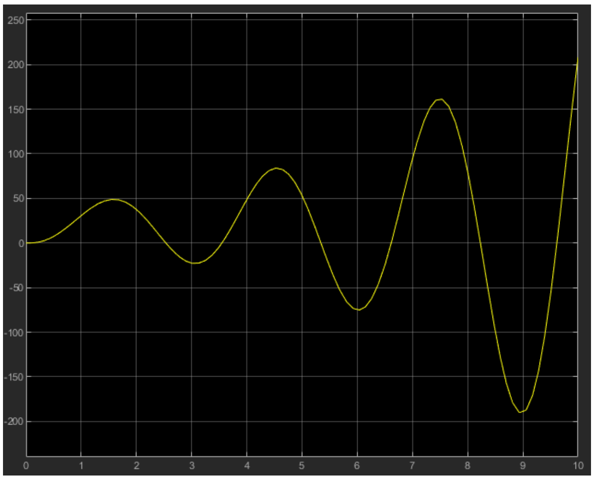

Figure 10: Output for incorporating proportional control to manipulate the balls position with $k_{p1} = 8$ and $k_{p2} = 0.5$.

Equation for poles of the system in Figure 10:

$$\begin{cases} s_1 = -8.53826 \\ s_2 = 0.269129 - 2.12682i \\ s_3 = 0.269129 + 2.12682i \end{cases} \tag{8}$$

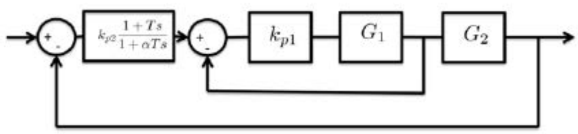

Figure 11: Block diagram of incorporating the phase advanced control to manipulate the ball's position.

Equation for transfer function for the phase advanced control loop:

$$T_3(s) = \frac{g k_{p1} k_{p2} G_m (Ts + 1)}{\alpha T s^4 + (k_{p1} G_m \alpha T + 1) s^3 + k_{p1} G_m s^2 + g k_{p1} k_{p2} G_m (Ts + 1)} \tag{9}$$

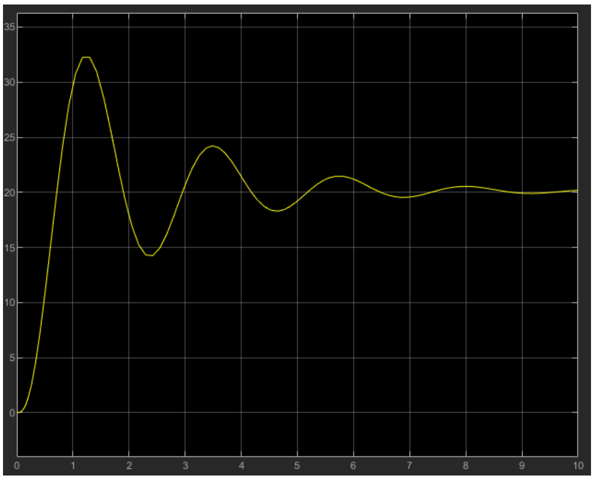

Figure 12: Output for incorporating the phase advanced control to manipulate the ball's position with $k_{p1} = 8$, $k_{p2} = 0.1$, $T = 4$, and $\alpha = 0.1$.

Equation for poles of the system in Figure 12:

$$\begin{cases} s_1 = -9.22938 \\ s_2 = -0.265526 \\ s_3 = -0.5002548 - 2.28451i \\ s_4 = -0.5002548 + 2.28451i \end{cases} \tag{10}$$

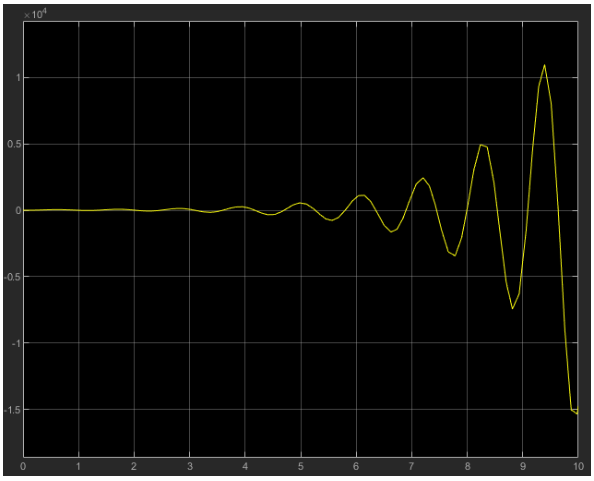

Figure 13: Output for incorporating the phase advanced control to manipulate the ball's position with $k_{p1} = 8$, $k_{p2} = 0.5$, $T = 4$, and $\alpha = 0.1$.

Equation for poles of the system in Figure 13:

$$\begin{cases} s_1 = -11.6218 \\ s_2 = -0.252836 \\ s_3 = -0.687334 - 5.73697i \\ s_4 = -0.687334 + 5.73697i \end{cases} \tag{11}$$

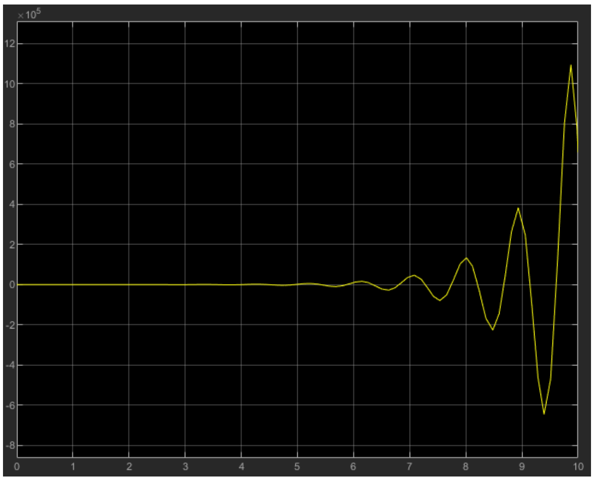

Figure 14: Output for incorporating the phase advanced control to manipulate the ball's position with $k_{p1} = 8$, $k_{p2} = 0.5$, $T = 6$, and $\alpha = 0.05$.

Equation for poles of the system in Figure 14:

$$\begin{cases} s_1 = -13.5753 \\ s_2 = -0.167554 \\ s_3 = 1.20474 - 7.4869i \\ s_4 = 1.20474 + 7.4869i \end{cases} \tag{12}$$

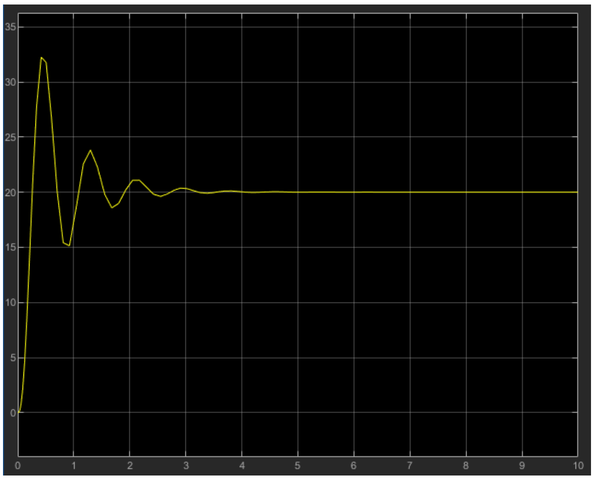

Figure 15: Output for incorporating the phase advanced control to manipulate the ball's position with $G_m = 8$, $k_{p1} = 1$, $k_{p2} = 1$, $T = 1$, and $\alpha = 0.05$.

Equation for poles of the system in Figure 15:

$$\begin{cases} s_1 = -16.7098 \\ s_2 = -1.10788 \\ s_3 = -1.59116 - 9.0694i \\ s_4 = -1.59116 + 9.0694i \end{cases} \tag{13}$$

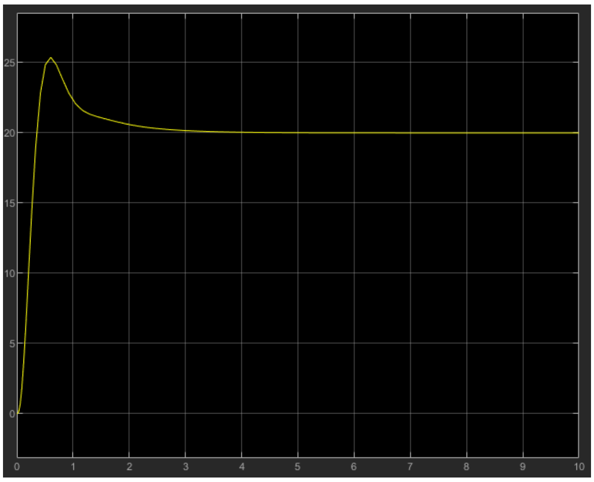

Figure 16: Output for incorporating the phase advanced control to manipulate the ball's position with $G_m = 8$, $k_{p1} = 2$, $k_{p2} = 0.5$, $T = 1$, and $\alpha = 0.05$.

Equation for poles of the system in Figure 16:

$$\begin{cases} s_1 = -5.69847 \\ s_2 = -1.32934 \\ s_3 = -7.4861 - 12.2948i \\ s_4 = -1.59116 + 12.2948i \end{cases} \tag{14}$$

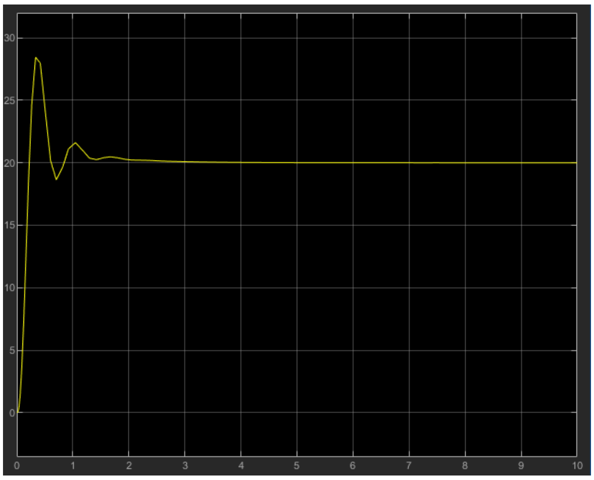

Figure 17: Output for incorporating the phase advanced control to manipulate the ball's position with $G_m = 8$, $k_{p1} = 2$, $k_{p2} = 1$, $T = 1$, and $\alpha = 0.05$.

Equation for poles of the system in Figure 17:

$$\begin{cases} s_1 = -14.0108 \\ s_2 = -1.11815 \\ s_3 = -3.43555 - 13.7324i \\ s_4 = -3.43555 + 13.7324i \end{cases} \tag{15}$$

Equation for time constant in the slave loop:

$$\tau = \frac{1}{k_{p1} G_m} \tag{16}$$

References

Adib Murad, M. A., Tzounas, G., & Milano, F. (2020). Modeling and simulation of fractional order pi control limiters for power systems**this work is supported by the science foundation ireland, by funding mohammed ahsan adib murad, georgios tzounas, and federico milano, under investigator programme grant no. sfi/15/ia/3074. *IFAC-PapersOnLine*, 53(2), 13107–13112. 21th IFAC World Congress.

Jia, Y., Chai, T., Wang, H., & Su, C.-Y. (2020). A signal compensation based cascaded pi control for an industrial heat exchange system. *Control Engineering Practice*, 98, 104372.

Kong, H., Liu, J., & Cui, G. (2010). Study on field-weakening theory of brushless dc motor based on phase advance method. In *2010 International Conference on Measuring Technology and Mechatronics Automation*, volume 3 (pp. 583–586).: IEEE.

Zhou, J. (2018). Adaptive pi control of bottom hole pressure during oil well drilling. *IFAC-PapersOnLine*, 51(4), 166–171. 3rd IFAC Conference on Advances in Proportional-Integral-Derivative Control PID 2018.