To: Professor Qi Lu

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Subject: ME1042 Lab 03 Programmable Logic Controllers

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On 30$^{\text{th}}$ September, Gordon Lou, Owen Chen, Frederic Liu, Yanjun He, and I conducted our third experiment in the course Mechanical Measurements 2, from which we have investigated the fundamentals of programmable logic controllers (PLCs) and their operation as it relates to fluid transport and liquid level.

First, we will show how to derive the differential equation of the system by applying the law of conservation of mass, presented in Equation 1 and obtain the function of water level with respect to time. Consider a tank with a valve in the experiment, as shown in Figure 1, where p_a is the atmospheric pressure and q_i and q_o are the volume flow rates into and out of the tank, respectively. The cross-sectional area of the tank is A, and the liquid height is h. The liquid leaves the tank through the valve, for which the hydraulic resistance is R. The density of the liquid is constant, ρ. Note that the total fluid mass in the tank is $\rho A h$. For constant cross-sectional area and constant density, the left-hand side of Equation 1 can be rewritten as Equation 2. The right-hand side of Equation 1 can be rewritten as Equation 3. Labeling point 1 at the upstream side of the valve and point 2 at the downstream side of the valve, the hydraulic resistance of the valve can be expressed as Equation 4. Assume that the pressure p_1 can be approximated as the hydrostatic pressure, $p_1 = p_a + \rho g h$, and the pressure p_2 can be approximated as the atmospheric pressure, $p_2 = p_a$. Substituting p_1 and p_2 into Equation 4 gives Equation 5. Hence, we can get Equation 6 or Equation 7. Substituting Equations 2 and 7 into Equation 1 results in Equation 8. Rearranging the equation gives Equation 9, which is a first-order ordinary differential equation relating the liquid height h and the inlet volume flow rate q_i. Introducing the expression of the hydraulic capacitance C, given by Equation 10, we can rewrite Equation 9 as Equation 11. Equation 11 describes the dynamic behavior of a single-tank liquid-level system with capacitance C and resistance R, as shown schematically in Figure 1. Solving differential equation of Equation 11 yields Equation 12, where A is a constant, which can be determined by initial condition. Because volume flow rates into the tank is zero when we open the valve to drain the tank 1. q_i is equal to zero. Hence, Equation 12 can be rewritten as Equation 13. And in the experiment, we record that when the time is about $14.14\ \mathrm{s}$, the volume of water in the tank is about half of its initial volume, through which we get $A = 130\ \mathrm{mm}$ and $RC = 20.3997\ \mathrm{s}$. Therefore, we can obtain the function of water level with respect to time as shown in Equation 14.

In fact, the relationship between the time obtained above and the water level in the container is the actual situation. If it were ideal, R would not exist. So here, let us derive the relationship between the time and the water level in the container under the ideal case. According to Torricelli's theorem, the velocity of a fluid, with height, h, draining from a hole in a tank is governed by Equation 15 (Malcherek, 2016). Using the water in the tank as a control volume mass in Equation 2 and applying conservation of mass in Equation 1 yields Equation 16. Substituting Equation 15 into Equation 16 obtains Equation 17. Then, through Equation 18, Equation 19, and Equation 20, we get the relationship between height and time as shown in Equation 21. According to the initial condition, which is that the initial height in the tank is equal to maximum height-$130\ \mathrm{mm}$, we can get the actual relationship between height and time as shown in Equation 24. Substituting the parameters of the water tank, including tank diameter and valve diameter gets Equation 25. The plot for time (x-axis) vs. height of fluid in cylinder (y-axis) and indication on my plot of the time to reach the draining level is shown in Figure 2.

Next, we will explore why the use of PLCs is an effective way to handle the tasks of updating an old relay system for automated fluid level control. PLC has a variety of functions including relay switch, timing, counting, signal processing, and so on, which ensures the efficient completion of the above tasks. In addition, PLC has many advantages over the relay logic, like more reliability, more flexibility, lower cost, more communications capability, faster response time, easier to troubleshoot, easier to test field devices. These benefits let PLC not only can efficiently complete the above tasks, but also be competent for most of the tasks which makes our company benefit from it and have more competitive. The logic of the program in PLC is shown in Figure 4. The simplified logic is also shown in Figure 5.

Then, we will discuss how close the obtained value matches the solved value. We plot the ideal model shown in Equation 25 and the experimental data into one graph as shown in Figure 3. The timer constant that obtained through trial and error in the lab is not very close to the solved time. When solving the drain problem, assumptions like Torricelli's theorem and conservation of mass are used. Conservation of mass is valid, since the loss of mass is negligible. Then, the problem is Torricelli's theorem. This theorem has much to do with conservation of energy, but the energy loss caused by friction is large in the experiment. When the liquid level is high, the flow rate is high, which leads to greater friction. This also explains the large slope difference at the beginning.

After this experiment, we have learnt how to analyze PLC logic and programming, which promotes the theories we have learned in class. In the future, if we have the opportunity to research from machine operation to control (Frey & Litz, 2000), today's experiment will provide us with tremendous help regarding to this usefulness and significance.

Equation for law of conservation of mass:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = q_{mi} - q_{mo} \tag{1}$$

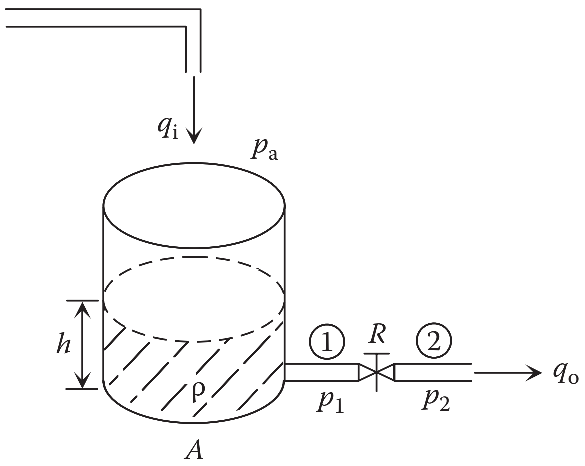

Figure 1: A single-tank liquid-level system with a valve.

Equation for derivation the differential equation of the tank system:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\rho A h\right) = \rho A \frac{\mathrm{d}h}{\mathrm{d}t} \tag{2}$$

$$q_{mi} - q_{mo} = \rho q_i - \rho q_o \tag{3}$$

$$R = \frac{\Delta p}{\Delta q_m} = \frac{p_1 - p_2}{\rho q_o} \tag{4}$$

$$R = \frac{\rho g h}{\rho q_o} \tag{5}$$

$$\rho q_i - \rho q_o = \rho q_i - \frac{\rho g h}{R} \tag{6}$$

$$q_{mi} - q_{mo} = \rho q_i - \frac{\rho g h}{R} \tag{7}$$

$$\rho A \frac{\mathrm{d}h}{\mathrm{d}t} = \rho q_i - \frac{\rho g h}{R} \tag{8}$$

$$\frac{RA}{g}\frac{\mathrm{d}h}{\mathrm{d}t} + h = \frac{R}{g} q_i \tag{9}$$

$$C = \rho A\left(h\right)\frac{1}{\rho g} = \frac{A\left(h\right)}{g} \tag{10}$$

$$RC\frac{\mathrm{d}h}{\mathrm{d}t} + h = \frac{R}{g} q_i \tag{11}$$

Equation for obtaining the function of water level with respect to time :

$$h\left(t\right) = Ae^{-\frac{t}{RC}} + \frac{R}{g} q_i \tag{12}$$

$$h\left(t\right) = Ae^{-\frac{t}{RC}} \tag{13}$$

$$h\left(t\right) = 130e^{-\frac{t}{20.3997}} \tag{14}$$

The governing equation for the velocity of a fluid with height h:

$$v = \sqrt{2gh} \tag{15}$$

Equation for derivation the differential equation of the tank system:

$$\rho A_c \frac{\mathrm{d}h}{\mathrm{d}t} + \rho v A_v = 0 \tag{16}$$

$$A_c \frac{\mathrm{d}h}{\mathrm{d}t} + \sqrt{2gh} A_v = 0 \tag{17}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{A_v}{A_c}\sqrt{2gh} \tag{18}$$

$$\frac{\mathrm{d}h}{\sqrt{h}} = -\frac{A_v}{A_c}\sqrt{2g}\mathrm{d}t \tag{19}$$

$$\int \frac{\mathrm{d}h}{\sqrt{h}} = \int -\frac{A_v}{A_c}\sqrt{2g}\mathrm{d}t \tag{20}$$

$$2\sqrt{h} = -\frac{A_v}{A_c}\sqrt{2g}t + C \tag{21}$$

$$\sqrt{h} = \frac{-\frac{A_v}{A_c}\sqrt{2g}t + C}{2} \tag{22}$$

$$\sqrt{h} = -\frac{A_v}{2A_c}\sqrt{2g}t + \sqrt{0.13} \tag{23}$$

$$h = \left(-\frac{A_v}{2A_c}\sqrt{2gt} + \sqrt{0.13} \right)^2 \tag{24}$$

$$h = (-0.0134t + 0.3606)^2 \times 1000 \text{ mm} \tag{25}$$

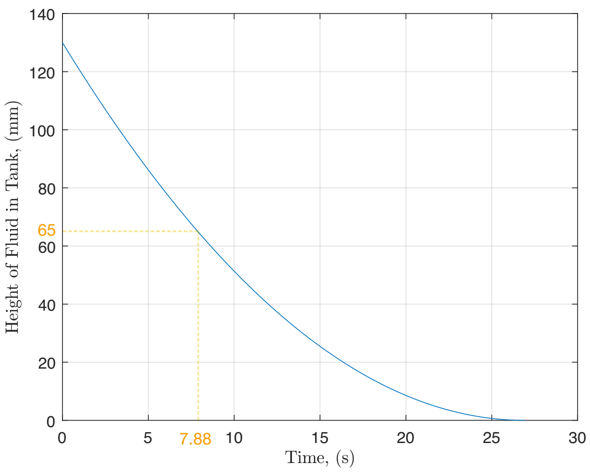

Figure 2: Time vs. height of fluid in cylinder.

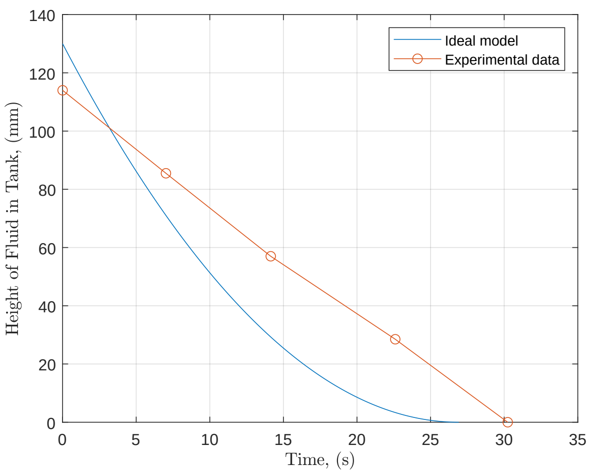

Figure 3: Comparison between ideal model and experimental data.

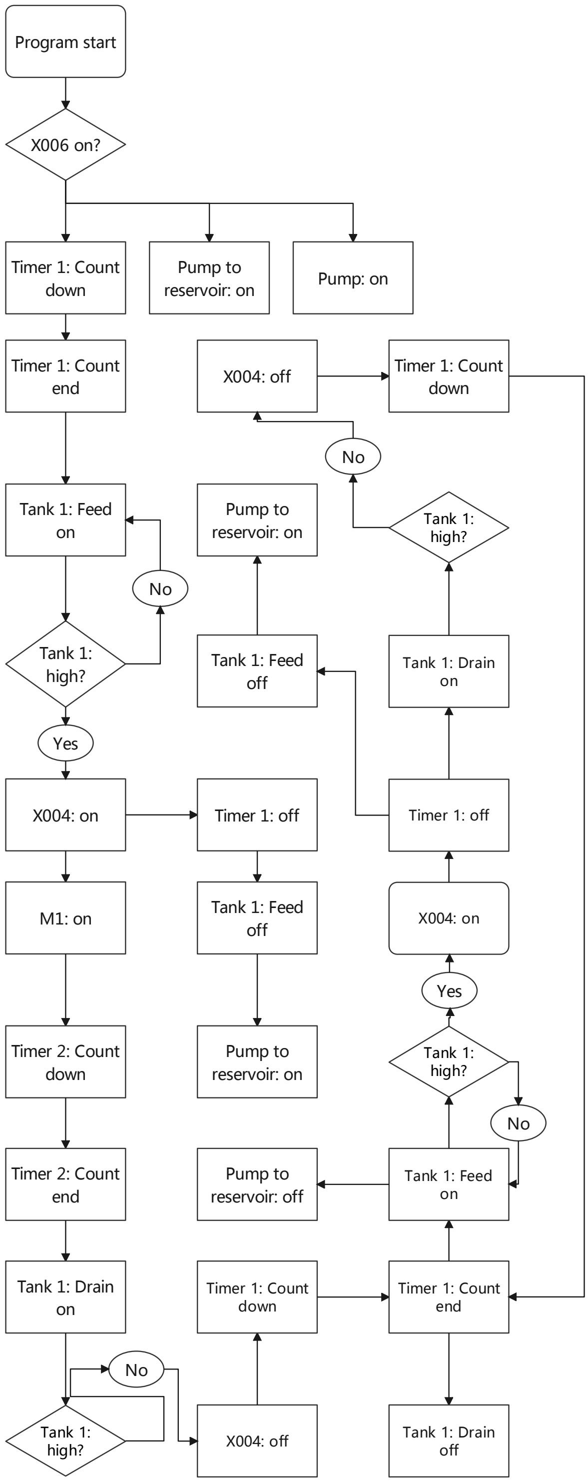

Figure 4: Flow chart of using the inputs and outputs available on the CE123 and CE111 to simulate water tank.

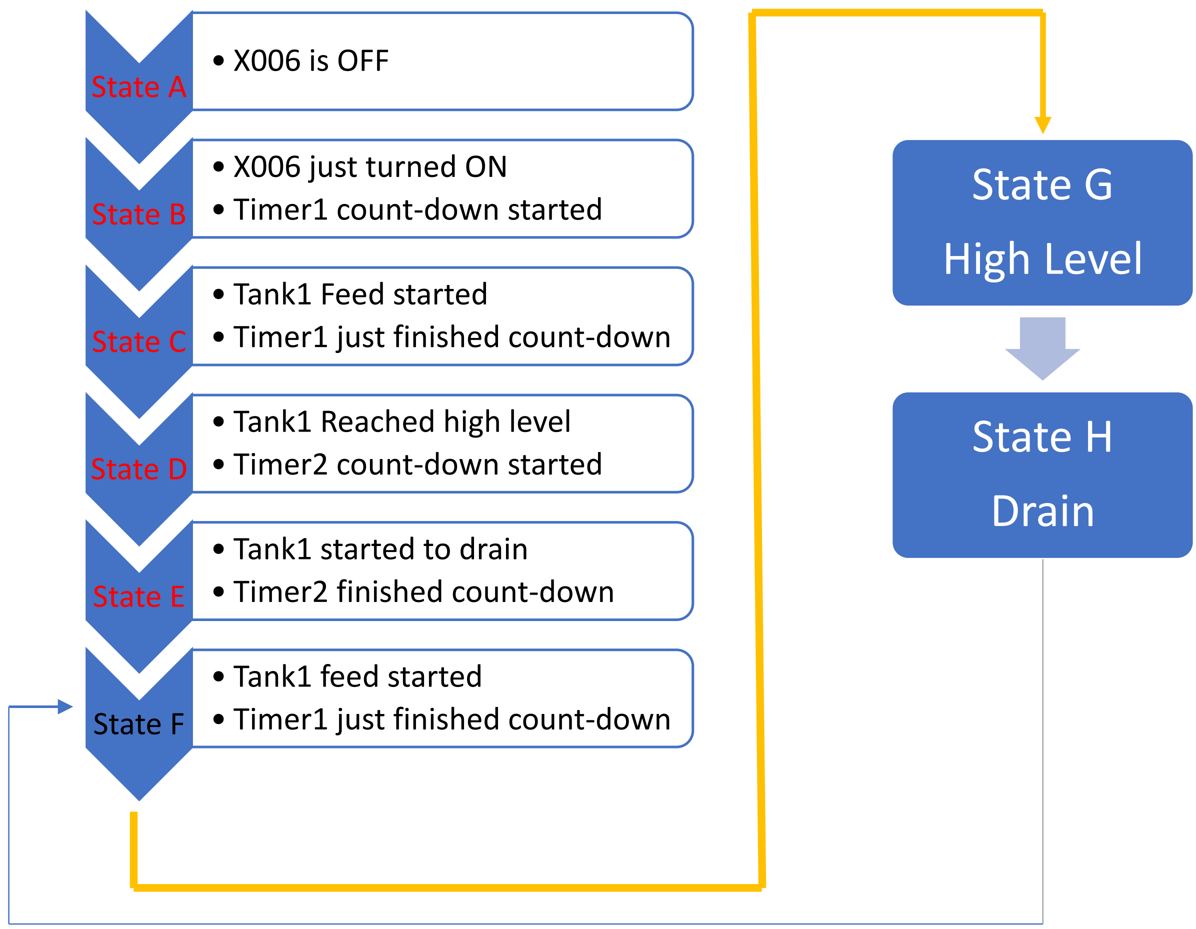

Figure 5: Simplified logic of program.

References

Frey, G. & Litz, L. (2000). Formal methods in plc programming. In *Smc 2000 conference proceedings. 2000 ieee international conference on systems, man and cybernetics.'cybernetics evolving to systems, humans, organizations, and their complex interactions'(cat. no. 0*, volume 4 (pp. 2431–2436).: IEEE.

Malcherek, A. (2016). History of the torricelli principle and a new outflow theory. *Journal of Hydraulic Engineering*, 142(11), 02516004.