To: Professor Lu

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Subject: ME1042 Lab 02 Geared Systems

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On 23$^{\text{rd}}$ September, Gordon Lou, Owen Chen, Frederic Liu, Yanjun He, and I conducted our second experiment in the course Mechanical Measurements 2, from which we have determined the mechanical efficiency of a simple and compound geared system, considered the advantages/disadvantages of a compound system, and experimentally determined the rotational inertia of a shaft with friction and compare to a value found from geometry. Our results are shown in Table 1 and Table 2.

At the beginning, we plot three figures in order to accurately analyze the data: the Motor Torque vs. Brake Torque for both systems (on same axes) along with a linear fit for both as shown in Figure 1, Efficiency vs. Motor Torque for a simple and compound system (on same axes) as shown in Figure 2, and $(mg–ma)$ vs. a for the Experiment 2 along with a linear fit as shown in Figure 3.

Next, we discuss the results, using Section 2.3 and Section 3.2 as a guide along with the plots.

First, in Figure 1 which is about the actual motor torque vs. the brake torque for the simple and the compound systems, we can find that the developed appropriate curve for each case is linear, and it coincides well with each data point. Also, we can obviously find that the slopes of the fitting curves corresponding to each case are different, among which the slope corresponding to the compound system is much larger.

By analyzing the practical meaning and the physical principle of this figure, we find that **the slope of the curve represents mechanical advantage (MA)** while **the intercept of the curve with the horizontal axis represents the inbuilt friction**. The slope corresponding to the compound system is larger than the slope corresponding to the simple system shows that the mechanical advantage of the compound system is larger than that of the simple system. One similarity is that the intercepts of both curves with the horizontal axis is close which indicates that the inbuilt frictions of two cases are almost the same.

Second, in Figure 2 which is about the efficiency (%) vs. the actual motor torque for both systems, we can find that the efficiency of the simple system is larger that that of the compound system and the slope of the compound system increases significantly when the actual motor torque is small.

Because the calculation of efficiency of the gear system is shown in Equation 1, we can know that when comparing the overall efficiency, although T_{brake}/T_{motor} always looks larger for compound system (Figure 1), $\omega_{brake}/\omega_{motor}$ is shrinking the overall value even more. From a physical point of view, this result is also reasonable, because the compound system means more gear parts, more inbuilt frictions, and more dissipation of power. At the same time, for the compound system, the power dissipation accounts for a large proportion of the total input power when the total input power is low, which leads to low efficiency and explains why the slope of the compound system increases significantly when T_{motor} is around $10 \sim 50\ \mathrm{mN \cdot m}$.

The above phenomena and analysis have great enlightenment for engineering application. When using a compound system over a simple system, the advantage is that the output torque is significantly amplified while the disadvantage is that the mechanical efficiency is relatively low, especially at a small input power.

Third, the Figure 3 is $(mg–ma)$ vs. a for the Experiment 2. In the figure, the developed appropriate curve is linear, and it coincides well with the data points. According to Equation 2, the slope of the curve in this figure is the constant, I/r^2, where I is the inertia of the mass rotating about axis and r is the radius of this rotation mass.

Then, we calculate the moment of inertia of our system. The critical formula of the experiment 2 is shown in Equation 2, in which the acceleration can be calculated by Equation 3. Therefore, combining Equation 2 and 3, we can get the equation for calculate the moment of inertia of our system as shown in Equation 4. The results are shown in Table 3. And based on this, we can calculate the mean and standard derivation of our dataset as shown in Equation 5 and 6. Then, combined with Equation 7 and 8, we can know that the uncertainty of our dataset is shown in Equation 9. Therefore, the moment of inertia measured in this experiment is shown in Equation 10.

Finally, we conduct the t-test for the data from Experiment 2 and use a 95% confidence.

According to Figure 4 and 5, the theoretical inertia value of the total body is shown in Equation 11, in which the calculation of theoretical inertia value of the flywheel is shown in Equation 12 and the calculation of theoretical inertia value of the pulley is shown in Equation 14. And because the inertia of A and G are too small, we only need to consider the inertia of B, C and F. So, Equation 12 can be simplified and the theoretical inertia value of the flywheel is shown in Equation 13. Also, because the inertia of brass collect and missing hole are too small, we only need to consider the inertia of the solid body. So, Equation 14 can be simplified and the theoretical inertia value of the pulley is shown in Equation 15. Based on the previous analysis, we can know that the theoretical inertia value of the total body is shown in Equation 16. Then, we conduct the t test with 95% confidence using Equation 17, Because $t_0 > t_{24,95}$, we should reject the null hypothesis. This situation may be due to the fact that the experiment we did was too stable, resulting in a very small standard deviation, which resulted in a large number of calculations when we performed the t test (Kim, 2015).

After this experiment, we have learnt how to analyze the gear system including gear set, rolling cylinders, types of gears, simple gear trains, compound gear trains and calculated mechanical advantages, velocity ratio, and efficiency, which promotes the theories we have learned in class. In the future, if we have the opportunity to research from vehicles to watches (Lynwander, 2019) (Zanzi & Pedrero, 2005), today's experiment will provide us with tremendous help regarding to this usefulness and significance.

Table 1: Data Table for Simple System and Compound System.

	Nominal Motor Torque $(\mathrm{mN \cdot m})$	Actual Motor Torque $(\mathrm{mN \cdot m})$	Motor Speed (rpm)	Brake Torque $(\mathrm{mN \cdot m})$	Brake Speed (rpm)	Motor Power (W)	Dynamo-meter Power (W)
Simple System	10	10	602	14	401	0.61	0.58
	20	20	602	28	401	1.25	1.18
	30	30	603	42	403	1.88	1.78
	40	40	603	57	402	2.54	2.39
	50	50	603	70	402	3.13	2.94
	60	60	603	84	401	3.77	3.53
	70	70	603	98	402	4.41	4.13
Compound System	10	10	599	21	200	0.62	0.44
	20	20	603	48	201	1.25	1.01
	30	30	603	75	201	1.86	1.57
	40	40	602	103	201	2.52	2.17
	50	50	603	131	201	3.17	2.75
	60	60	603	159	201	3.80	3.32
	70	70	603	185	201	4.41	3.88

Table 2: Results for Experiment Inertia of a Shaft with Friction.

Mass (g)	Angular Acceleration $(\mathrm{rad/s^2})$				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
100	25.6	25.6	25.7	25.7	25.5
150	38.1	38.1	38.1	38.1	38.1
200	49.8	49.8	49.8	49.8	49.8
250	61.1	61.1	61.1	61.2	61.2
300	72.1	72.1	72.1	72.1	72.2

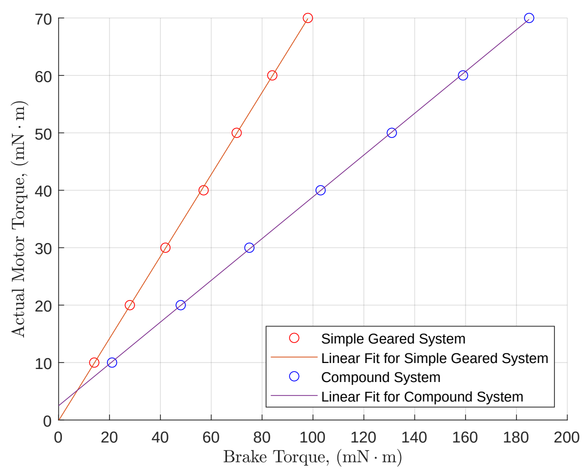

Figure 1: Motor Torque vs. Brake Torque for Both Systems along with a Linear Fit for Both.

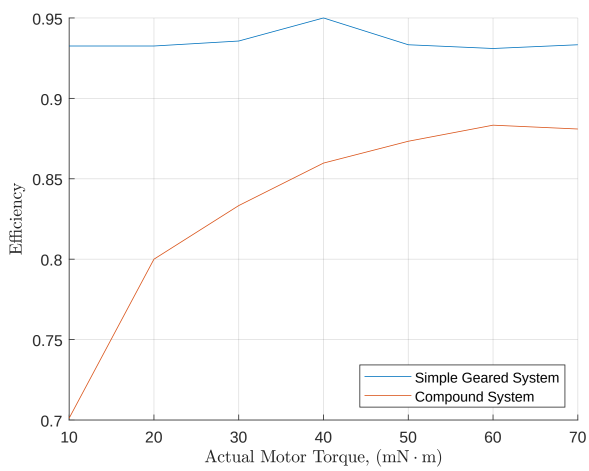

Figure 2: Efficiency vs. Motor Torque for a Simple and Compound System.

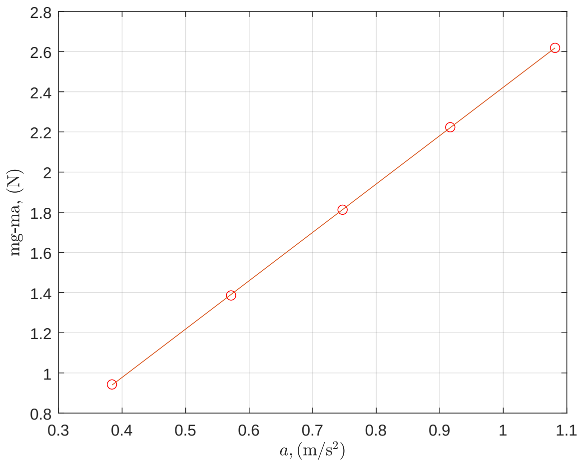

Figure 3: (mg – ma) vs. a. for the Experiment 2 along with a Linear Fit.

Equation for efficiency of gear system:

$$\varepsilon(\%) = \frac{MA}{VR} \times 100 = \frac{T_{brake}}{T_{motor}} \times \frac{\omega_{brake}}{\omega_{motor}} \times 100 \quad (1)$$

Equation for force analysis of gear system:

$$mg - ma = I\frac{a}{r^2} \quad (2)$$

Equation for conversion of linear and angular acceleration:

$$a = \alpha r \quad (3)$$

Equation for calculation of moment of inertia:

$$I = \frac{r^2}{a}(mg - ma) = \frac{mgr}{\alpha} - mr^2 \quad (4)$$

Table 3: Results for Experiment Inertia of a Shaft with Friction.

Mass (g)	Moment of inertia ($\times 10^{-3}$ kg $\cdot$ m^2)				
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
100	0.5523	0.5523	0.5501	0.5501	0.5546
150	0.5456	0.5456	0.5456	0.5456	0.5456
200	0.5460	0.5460	0.5460	0.5460	0.5460
250	0.5458	0.5458	0.5458	0.5459	0.5459
300	0.5448	0.5448	0.5448	0.5448	0.5439

Equation for calculation mean of dataset:

$$\bar{I} = \frac{\sum_{i=1}^{N} I_i}{N} = 5.467 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \quad (5)$$

Equation for calculation standard deviation of dataset:

$$s_I = \sqrt{\frac{\sum_{i=1}^{N}(I_i - \bar{I})}{N-1}} = 2.798 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \quad (6)$$

Equation for degree of freedom:

$$v = N - 1 = 5 \times 5 - 1 = 24 \quad (7)$$

Equation for critical value of 95% confidence t test:

$$t_{24,95} = 2.067 \quad (8)$$

Equation for calculation of uncertainty of dataset:

$$u_I = t_{24,95} s_{\bar{I}} = t_{24,95}\frac{s_I}{\sqrt{N}} = 1.157 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \quad (9)$$

Equation for moment of inertia:

$$I = \bar{I} \pm u_I = 5.467 \times 10^{-4} \pm 1.157 \times 10^{-6} \text{ kg} \cdot \text{m}^2 \quad (10)$$

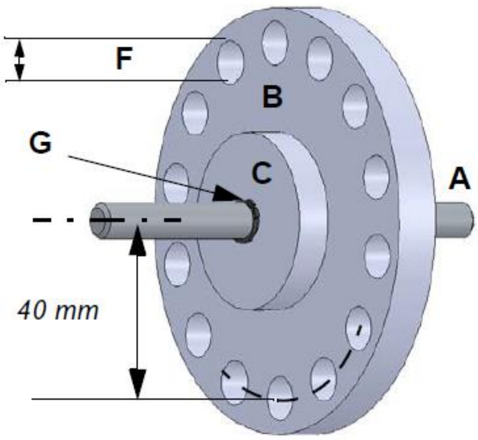

Figure 4: Flywheel dimensions.

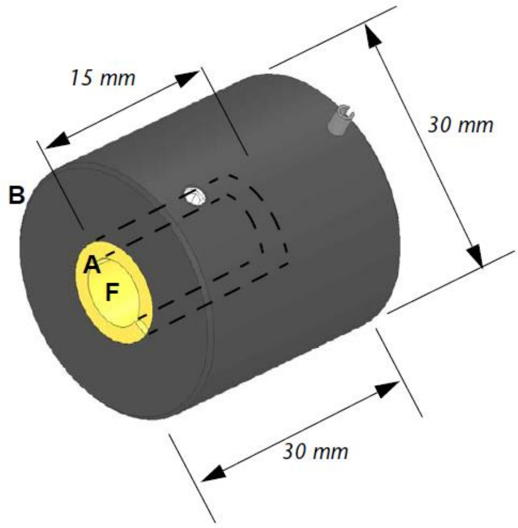

Figure 5: Load pulley dimensions.

Equation for calculation of theoretical inertia value of the total body:

$$I_{th} = I_{flywheel} + I_{pulley} \tag{11}$$

Equation for theoretical inertia value of the flywheel:

$$I_{flywheel} = I_A + I_B + I_C - I_F - I_G \tag{12}$$

$$\begin{aligned} I_{flywheel} &= I_B + I_C - I_F \\ &= \frac{1}{2} m_B r^2 + \frac{1}{2} m_c r^2 \\ &\quad - 14 \times \frac{1}{2} m_F \left(r^2 + d^2\right) \\ &= 5.998 \times 10^{-4}\ \mathrm{kg \cdot m^2} \end{aligned} \tag{13}$$

Equation for theoretical inertia value of the pulley:

$$I_{pulley} = I_{brass\ collect} + I_{solid\ body} - I_{missing\ hole} \tag{14}$$

$$I_{pulley} = \frac{1}{2} m r^2 = 1.909 \times 10^{-5}\ \mathrm{kg \cdot m^2} \tag{15}$$

Equation for theoretical inertia value of the total body:

$$I_{th} = I_{flywheel} + I_{pulley} = 6.189 \times 10^{-4}\ \mathrm{kg \cdot m^2} \tag{16}$$

Equation for t test:

$$t_0 = \frac{\bar{I} - I_{th}}{S_{\bar{x}}} = 129.1 > t_{24,95} \tag{17}$$

References

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Lynwander, P. (2019). *Gear drive systems: Design and application*. CRC Press.

Zanzi, C. & Pedrero, J. I. (2005). Application of modified geometry of face gear drive. *Computer Methods in Applied Mechanics and Engineering*, 194(27-29), 3047–3066.