To: Professor Lu

From: Christopher Jin, Frederic Liu, Gordon Lou, Owen Chen, Yanjun He

Subject: ME1042 Lab 01 Forced and Free Vibrations

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On 2nd and 16th September, Gordon Lou, Owen Chen, Frederic Liu, Yanjun He, and I conducted our first experiment in the course Mechanical Measurements 2, from which we have explored the fundamentals of mechanical vibrations, including natural frequency and damping on a rigid beam.

In the first part of experiment, we researched the influence of added mass on the natural frequency of the cantilever beam and also examine the impact of frequency ratio on the amplitude and phase lag of vibration. In order to achieve these two goals, we recorded the natural frequency with different masses added to the cantilever beam and also the amplitude and phase lag of vibration with different frequencies of force applied to cantilever beam into Table 4, whose results are shown in Figure 1, 2, and 3, respectively.

Firstly, we conduct the error analysis in this experiment. Through our measurements as shown in Figure 1, It is obvious that the measured value is less than the theoretical value. After calculation, there is about 5% error between the measured value and the theoretical value and the measured value is always less than the theoretical value. The theoretical frequency is given by Equation 6. From this, we can know that the first possibility of error is caused by that the actual I_A is larger than the theoretical I_A. If the mass of the connecting part is ignored when measuring the mass of each part of the system, or the reading is smaller than the actual value, I_A will be smaller. Another possibility is that the actual k is smaller than the measured k. For example, when measuring the spring k value, the reading is too large, which will also lead to theoretical frequency greater than measured frequency (Sorokin & Ershova, 2003). In addition, we regard beam as a rigid body in Equation 6. However, in fact, the beam also has a certain elasticity. If the elasticity of the beam is considered, the k of the system will be smaller than that of the spring.

Secondly, we compare the natural frequency and damping ratio for two conditions - undamped and fully shut. For the damping ratio, the damp of the fully shut case is mainly come from the viscous damper. For the undamped case, it comes from the air fraction. In theory, because the damping liquid has a damping constant much larger than the air, the fully shut case should have a damping ratio larger than that of the undamped case. From Table 1, the recorded data conforms our expectation from the theory. The damping ratio of the fully shut case is 0.029, which is 14.5 times larger than that of the undamped case. For the natural frequency, in theory, from Equation 1, the damped natural frequency becomes lower as the damping ratio increases. However, in our results, the two natural frequencies are nearly the same. We hold the opinion that contradict is mainly because the damping ratio is too small in the order of magnitudes, which has low influence on the damped natural frequency (So et al., 2000). In the experiment we use the same beam for both the undamped and the full shut case, which means the system would have the same natural frequency. Assuming the natural frequency of the undamped beam in ideal conditions is ω_n , and substitute the damping ratio into Equation 1, we can obtain the damped natural frequency for the undamped case is: $\omega_d = 0.999998\omega_n$, while for the fully shut case is $\omega_d = 0.999579\omega_n$. The two natural frequencies vary after four decimal places, so we can say that the effect of the damper has low effect on the damped natural frequency.

Thirdly, we find the magnification factor for a speed ratio of 1.0 for two sets of results and compare them. For forced vibration, the magnification factor is shown in Equation 2. When

r approaches to 0, the magnification factor β approaches to 1. Therefore, when speed ratio is small, $\beta \approx 1$. In other words, $X \approx X_{st}$. And from Figure 2, we can know that when r approaches to 0.75, the amplitude is converging to some vale and the value is already stable. Also, damper system ($\zeta > 0$) will converge first than the undamped system ($\zeta \approx 0$). Therefore, we can use $X_{r=0.75, fully\ shut}$ to replace X_{st}. According to the previous deduction, the magnification factors for the condition of undamped and fully shut in this experiment can be calculate using Equation 4, which are equal to 135.6 and 13.4, respectively. The difference between these two magnification factors are huge, which is because when r approaches to 1, the magnification factor is shown in Equation 5. Therefore, at $r = 1$, the magnification factor β only depends on the damping ratio. In undamped case, the damping ratio $\zeta_{undamped} \approx 0$, which will make the magnification factor extremely high. But in fully shut case, the damping ratio $\zeta_{fully\ shut}$ is much bigger than 0, which will decrease the magnification factor of this system dramatically. That's why $\beta_{undamped} \gg \beta_{fully\ shut}$ at $r = 1.0$. And we also use Equation 5 to calculate the theoretical magnification factors, which are shown in Table 1.

Finally, we give our own comments on the amplitude and phase lag values vs speed ratio plot. In undamped and forced vibration case, the amplitude as shown in Equation 8 should approach infinity when r is close to 1. The result is consistent with theoretical analysis. According to the Figure 2, the amplitude of undamped case approaches to 0 when the frequency ratio is further away from 1; The amplitude becomes very large and increases rapidly when the frequency ratio approaches 1. Since the recorded data points are discrete and the test equipment has more or less damping, there will not be infinite situation in real experiment. In damped and forced vibration case, the amplitude is shown in Equation 9. Since the denominator has minimum value when the value of r is the same as Equation 10, the maximum amplitude occurs at r slightly greater than 1. The result is consistent with theoretical analysis. The amplitude of damped case also approaches to 0 when the frequency ratio is further away from 1 and it has maximum value when the frequency ratio is about 1.01. The maximum value is 2.019 cm based on the experimental data.The phase lag as shown in Equation 11 should increase from $0°$ to $180°$ and it will increase rapidly as r approaches 1 (Çalım, 2009). Also, the lower the damping ratio, the higher the slope around $r = 1$. The figure drawn by the two groups of data as shown in Figure 3 is both consistent with the theoretical analysis. The slope of undamped curve is obviously larger than that of damped curve, and the ordinate scale ranges from $180°$ to $360°$, which is equivalent to the scale range from $0°$ to $180°$.

After this experiment, we have understood the fundamentals of natural frequency and damping on a rigid beam by observing oscillation affects, which promotes the theories we have learned in class. In the future, if we have the opportunity to research civil engineering and geotechnical monitoring, today's experiment will provide us with tremendous help regarding to its usefulness and significance.

In the second part of experiment, we researched the influence of added mass on the natural frequency of the cantilever beam of simply supported beam and also examine the impact of frequency ratio on the amplitude and phase lag of vibration.

Firstly, we find the effective mass using Rayleigh's improved solution as shown in Equation 12 and then calculate the theoretical oscillation frequency using Equation 13 whose results are shown in Table 3. Compared the calculation to the measured experimental value, we can find that the theoretical natural frequencies are always slightly smaller than the measured one, which may be because the blocks we added have a certain amount of wear, resulting in the actual mass of the block being less than 400 grams. Therefore, the frequency we measured is greater than the frequency calculated.

Secondly, we compare the natural frequency and damping ratio for two conditions - undamped and fully shut in the simply supported case. For the damping ratio of the simply supported beam experiment, comparing the result of the fully shut with the undamped case, the result shows the same tendency as the beam and spring experiment, which is because the damping constant of the damping liquid is larger than that of the air. However, when comparing two sets of the damping ratios between the two experiments, our group find that the simply supported case shows a larger damping ratio than that of the beam and spring case whether or not the system is damped. As discussed in the previous part, the effective mass of the simply supported beam is shown in Equation 12, which is lower than the total mass of the system. According of the definition of the in Equation 16, damping ratio is negatively correlated with the effective mass. Thus, with the same damping constant, the decreasing of the effective mass is the main cause of the increasing of the damping ratio. For the natural frequency, from Equation 1, the damped natural frequency becomes lower as the damping ratio increases. This time, as the damping ratio becomes larger, the decrease in the recorded value of the natural frequency becomes more prominent, the value of the fully shut case is $0.12\ \mathrm{rad/s}$ smaller than that of the undamped case.

Thirdly, we find the magnification factor for a speed ratio of 1.0 for two sets of results in the simply supported case and compare them. We use Equation 5 to calculate the theoretical magnification factors, which are shown in Table 2. At $r = 1$, the magnification factor β only depends on the damping ratio. In undamped case, the damping ratio $\zeta_{undamped} \approx 0$, which will make the magnification factor extremely high. But in fully shut case, the damping ratio $\zeta_{fully\ shut} > 0$, which will decrease the magnification factor of this system dramatically. That's why $\beta_{undamped} \gg \beta_{fully\ shut}$ at $r = 1.0$.

After this experiment, we have learnt how to analyze the beam model in the case of free vibration and simple support, calculate the theoretical natural frequency of the beam and compare it with the measured frequency, and find several reasons for the error and studied forced vibration and damping, which promotes the theories we have learned in class. In the future, if we have the opportunity to research from mechanical vibration to dynamic system (Liu & Li, 2020), today's experiment will provide us with tremendous help regarding to this usefulness and significance.

Table 1: Data Table for Damping Condition Calculations.

Case	Natural Frequency (Hz)	Damping Ratio	Magnification Ratio
Undamped	6.43	0.002	250
Fully Shut	6.43	0.029	17.2

Table 2: Data Table for Damping Condition Calculations in Simply Supported Case.

Case	Natural Frequency (Hz)	Damping Ratio	Magnification Ratio
Undamped	15.00	0.0035	143
Fully Shut	14.88	0.041	12.2

Equation for the relationship between the damped natural frequency and the damping ratio:

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} \tag{1}$$

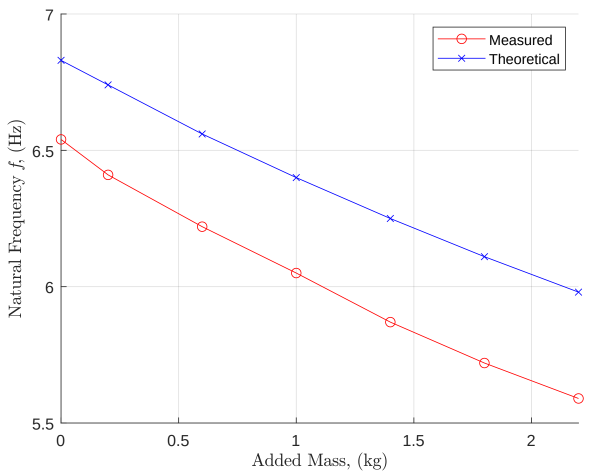

Figure 1: Frequency vs Added Mass for Both Theoretical and Measured Values.

Equation for magnification factor:

$$\beta = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \tag{2}$$

Equation for speed ratio:

$$r = \frac{\omega}{\omega_n} \tag{3}$$

Equation for magnification factor in this experiment:

$$\beta = \frac{X_{r=1.0}}{X_{st}} \approx \frac{X_{r=1.0}}{X_{r=0.75,fully\ shut}} \tag{4}$$

Equation for magnification factor at natural frequency:

$$\beta = \lim_{r \to 1} \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} = \frac{1}{2\zeta} \tag{5}$$

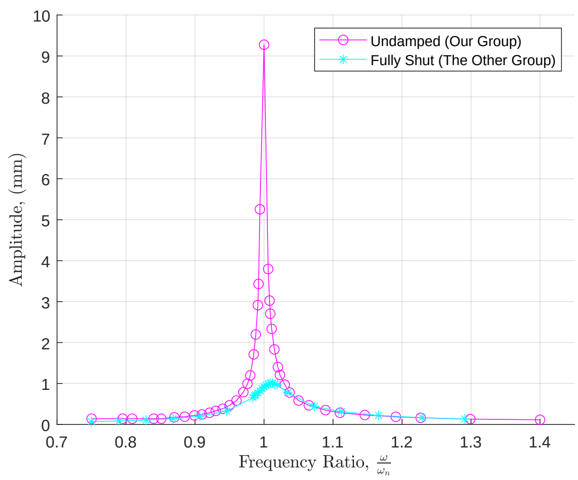

Figure 2: The Relationship between Amplitude and Frequency Ratio during excitation damping.

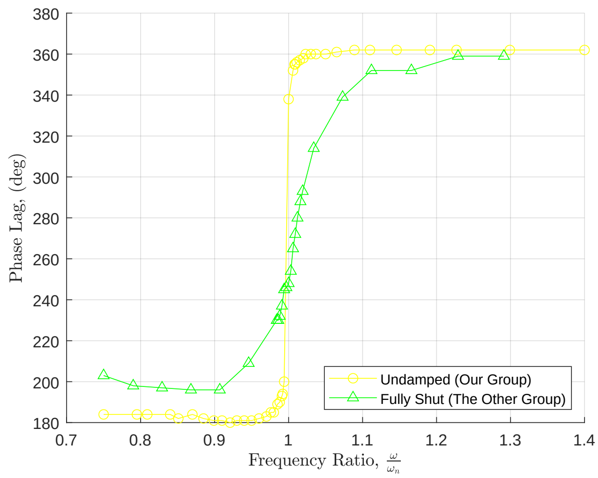

Figure 3: The Relationship between Phase Lag and Frequency Ratio during excitation damping.

Equation for theoretical frequency for cantilever beam:

$$\omega = \sqrt{\frac{kl_{spring}^2}{I_A}} \tag{6}$$

in which

$$\begin{aligned} I_A = &\left[\frac{1}{3} m_{beam} l_{beam}^2\right] \\ &+ \left[\left(\frac{m_{spring}}{3} + m_{fixing}\right) l_{spring}^2\right] \\ &+ \left[m_{exciter} l_{exciter}^2\right] \end{aligned} \tag{7}$$

Equation for amplitude for vibration of beam:

$$X = \left|\frac{\delta_{st}}{1 - r^2}\right| \tag{8}$$

Equation for amplitude for vibration of beam in damped and forced vibration case:

$$\begin{aligned} X &= \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \\ &= \frac{1}{\sqrt{(4\zeta^2 + 1) r^4 - 2r^2 + 1}} \end{aligned} \tag{9}$$

in which X reaches minimum when

$$r = \frac{1}{\sqrt{4\zeta^2 + 1}} \tag{10}$$

Equation for phase lag for vibration of beam:

$$\tan\phi = \frac{2\zeta r}{1 - r^2} \tag{11}$$

Equation for effective mass (Rayleigh):

$$m_{effective} = m_{exciter} + \frac{17}{35} m_{beam} \tag{12}$$

Equation for improved natural frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{6EI_{beam}}{m_{effective} l_3^3}} \tag{13}$$

Equation for forced vibration:

$$I_A\ddot{\theta} + kx_S l_{spring} + cl_{damper}\dot{x}_d = l_{exciter} Q \sin\Omega t \tag{14}$$

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega^2\theta = \frac{l_{exciter} Q}{I_A} Q \sin\Omega t \tag{15}$$

Equation for damping ratio calculation:

$$\zeta = \frac{c}{2\sqrt{m_{eff} k_{eff}}} \tag{16}$$

Equation for stiffness of simply supported beam:

$$k_{beam} = \frac{48EI_{beam}}{l_{beam}^3} \tag{17}$$

Equation for period and frequency of simply supported beam:

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m_{exciter} l_{beam}^3}{48EI_{beam}}} \tag{18}$$

Table 3: Simply supported beam data table for result calculations.

Added Mass (kg)	Total Exciter Mass (kg)	Effective Mass (kg)	Natural Frequency, f (Hz)		$1/f^2$
			Measured	Theoretical	
0	4.2	5.0	15.53	15.50	0.00415
0.2	4.4	5.2	15.35	15.20	0.00424
0.6	4.8	5.6	14.78	14.64	0.00458
1.0	5.2	6.0	14.24	14.15	0.00493
1.4	5.6	6.4	13.77	13.70	0.00527
1.8	6.0	6.8	13.38	13.29	0.00559
2.2	6.4	7.2	13.01	12.91	0.00591

Table 4: Recorded Data for Amplitude and Phase Angle with respect to Speed Ratio

Speed Ratio	Speed (Hz)	X_{pp}	Phase Angle
0.750	4.82	0.2804	184
0.795	5.11	0.2804	184
0.809	5.20	0.2804	184
0.840	5.40	0.2804	184
0.851	5.47	0.2804	182
0.870	5.60	0.3488	184
0.885	5.69	0.3774	182
0.899	5.78	0.4415	181
0.910	5.85	0.4878	181
0.921	5.92	0.5696	180
0.930	5.98	0.6622	181
0.940	6.04	0.7682	181
0.950	6.11	0.9366	181
0.960	6.17	1.166	182
0.970	6.24	1.576	183
0.976	6.28	1.987	185
0.980	6.30	2.395	185
0.985	6.34	3.422	189
0.988	6.36	4.393	190
0.991	6.37	5.828	193
0.992	6.38	6.865	194
0.994	6.39	10.51	200
1.000	6.43	18.55	338
1.006	6.47	7.594	352
1.008	6.48	6.049	355
1.009	6.49	6.049	355
1.011	6.50	6.669	356
1.015	6.53	3.665	357
1.020	6.56	2.803	358
1.023	6.58	2.417	360
1.030	6.62	1.943	360
1.037	6.67	1.559	360
1.050	6.75	1.170	360
1.065	6.85	0.9360	361
1.089	7.00	0.6998	362
1.110	7.14	0.5718	362
1.146	7.37	0.4614	362
1.191	7.67	0.3731	362
1.227	7.89	0.3245	362
1.299	8.35	0.2605	362
1.400	9.00	0.2274	362

References

Çalım, F. F. (2009). Free and forced vibrations of non-uniform composite beams. *Composite Structures*, 88(3), 413–423.

Liu, C.-S. & Li, B. (2020). Forced and free vibrations of composite beams solved by an energetic boundary functions collocation method. *Mathematics and Computers in Simulation*, 177, 152–168.

So, R., Zhou, Y., & Liu, M. (2000). Free vibrations of an elastic cylinder in a cross flow and their effects on the near wake. *Experiments in Fluids*, 29(2), 130–144.

Sorokin, S. & Ershova, O. (2003). Forced and free vibrations of rectangular sandwich plates with parametric stiffness modulation. *Journal of sound and vibration*, 259(1), 119–143.

Appendices

1 Code for plot in the lab

Input Matlab source for plotting three graphs in this lab:

```
clc; clf; clear all;
% Clear all the things that are remained in the before command.
%% Problem 3.1.1
hold on;
% Begin to plot the figure in one graph.
M = [0 0.2:0.4:2.2];    % Added mass.
fm = [6.54 6.41 6.22 6.05 5.87 5.72 5.59];  % Measured natural frequency.
ft = [6.83 6.74 6.56 6.40 6.25 6.11 5.98];  % Theoretical natural frequency.
plot(M, fm, 'ro-'); % Plot added mass with measured natural frequency.
plot(M, ft, 'bx-'); % Plot added mass with theoretical natural frequency.
grid on;    % Open the grid on the graph.
xlim([0 2.2]);  % Set the limit of x axis to [0 2.2].
xlabel('Added Mass, (kg)','interpreter','latex');
% Set the x label to the Added Mass, (kg).
ylabel('Natural Frequency\it \ f\rm, (Hz)','interpreter','latex');
% Set the y label to the Natural Frequency\it \ f\rm, (Hz).
legend('Measured','Theoretical');
% Set the Legend to 'Measured','Theoretical'.
hold off;
% Stop continuing plot the figure.
%% Problem 3.1.4.1
figure; %Open another figure.
hold on;
% Begin to plot the figure in one graph.
sr1 = [
    0.75
    0.795
    0.809
    0.84
    0.851
    0.87
    0.885
    0.899
    0.91
    0.921
    0.93
    0.94
    0.95
    0.96
    0.97
    0.976
    0.98
    0.985
    0.988
    0.991
    0.992
    0.994
    1
    1.006
    1.008
    1.009
    1.011
    1.015
    1.02
    1.023
    1.03
    1.037
```

```
    1.05
    1.065
    1.089
    1.11
    1.146
    1.191
    1.227
    1.299
    1.4
    ];  % Speed ratio for undamped.
amp1 = [
    0.2804
    0.2804
    0.2804
    0.2804
    0.2804
    0.3488
    0.3774
    0.4415
    0.4878
    0.5696
    0.6622
    0.7682
    0.9366
    1.166
    1.576
    1.987
    2.395
    3.422
    4.393
    5.828
    6.865
    10.51
    18.55
    7.594
    6.049
    5.408
    4.669
    3.665
    2.803
    2.417
    1.943
    1.559
    1.17
    0.936
    0.6998
    0.5718
    0.4614
    0.3731
    0.3245
    0.2605
    0.2274
    ]/2;    % Amplitude for undamped.
plot(sr1, amp1, 'mo-'); % Plot the speed ratio with amplitude for undamped.
sr2 = [
    0.75
    0.79
    0.829
    0.868
    0.907
    0.946
    0.984
    0.986
```

```
    0.988
    0.991
    0.994
    0.997
    1
    1.003
    1.006
    1.009
    1.012
    1.016
    1.019
    1.034
    1.073
    1.112
    1.166
    1.229
    1.291
    ];  % Speed ratio for fully shut.
amp2 = [
    0.1368
    0.17
    0.212
    0.2826
    0.4018
    0.6534
    1.316
    1.413
    1.446
    1.546
    1.634
    1.755
    1.833
    1.92
    1.964
    2.019
    2.019
    1.975
    1.943
    1.557
    0.8784
    0.6314
    0.4414
    0.331
    0.2694
    ]/2;    % Amplitude ratio for fully shut.
plot(sr2, amp2, 'c*-');  % Plot the speed ratio with amplitude for fully shut.
grid on;    % Open the grid on the graph.
xlim([0.7 1.45]);  % Set the limit of x axis to [0.7 1.45].
xlabel('Frequency Ratio, $\frac{\omega}{\omega_n}$','interpreter','latex');
% Set the x label to the Frequency Ratio, $\frac{\omega}{\omega_n}$.
ylabel('Amplitude, (cm)','interpreter','latex');
% Set the y label to the Amplitude, (cm).
legend('Undamped (Our Group)','Fully Shut (The Other Group)');
% Set the Legend to 'Undamped (Our Group)','Fully Shut (The Other Group)'.
hold off;
% Stop continuing plot the figure.
%% Problem 3.1.4.1
figure; %Open another figure.
hold on;
% Begin to plot the figure in one graph.
pha1 = [
    184
    184
```

```
     184
     184
     182
     184
     182
     181
     181
     180
     181
     181
     181
     182
     183
     185
     185
     189
     190
     193
     194
     200
     248
     262
     265
     265
     266
     267
     268
     270
     270
     270
     270
     271
     272
     272
     272
     272
     272
     272
     272
     ];  % Phase lag for undamped.
plot(sr1, pha1, 'yo-'); % Plot the speed ratio with phase lag for undamped.
pha2 = [
     203
     198
     197
     196
     196
     209
     230
     230
     232
     237
     245
     246
     248
     254
     265
     272
     280
     288
     293
     314
     339
```

```
    352
    352
    359
    359
    ];  % Phase lag for fully shut.
plot(sr2, pha2, 'g^-'); % Plot the speed ratio with phase lag for fully shut.
grid on;    % Open the grid on the graph.
% xlim([0.7 1.45]);
xlabel('Frequency Ratio, $\frac{\omega}{\omega_n}$','interpreter','latex');
% Set the x label to the Frequency Ratio, $\frac{\omega}{\omega_n}$.
ylabel('Phase Lag, ($\deg$)','interpreter','latex');
% Set the y label to the Phase Lag, ($\deg$).
legend('Undamped (Our Group)','Fully Shut (The Other Group)','Location','SouthEast'
    ↪ );
% Set the Legend to 'Undamped (Our Group)','Fully Shut (The Other Group)'.
hold off;
% Stop continuing plot the figure.
```