Sichuan University - Pittsburgh Institute

ME 1042

Dynamics/Controls Lab

PD Control of Unstable Systems

Revised October 2020

Mechanical Engineering Department

Goal: To show how feedback control can be used to stabalize an unstable system.

Equipment Needed:
CE106 Ball and Beam Apparatus
CE120 Controller Board
Computer with Matlab and Simulink

1 Introduction and Basic Theory

Controlling unstable systems is a vital aspect of control theory, especially in the aerospace industry. The purpose of this lab is to demonstrate how feedback control theory can stabilize an unstable system by incorporating it within the system's design. This experiment concentrates on controlling a ball's position as it rolls on top of a ball and beam apparatus. The remainder of this section discusses a basic overview of control theory and the dynamics describing the ball and beam apparatus.

1.1 *Overview of Control Systems*

A control system is an interconnection of components to have a system achieved a desired response. The two basic components of a control system are the plant (process), a mathematical model of the system (transfer function) and the controller. The objective is to have the plant output achieve certain goals (e.g. tracking a desired signal) and if the plant's output does not achieve the goals then one approach of achieving these goals is to use feedback control theory.
Feedback control achieves the desired goals by comparing the difference between the plant's output and the desired signal to create an error signal then the error signal is sent to the controller (another mathematical model accounting for the plant's deficiencies) to manipulate the plant's input to satisfy the goals. A graphical representation of a feedback system (closed-loop system) is called a block diagram as shown in Figure 1, where $R(s)$ is the desired signal, $E(s)$ is the error signal, $U(s)$ is the control signal, $Gc(S)$ is the controller, and $G(s)$ is the plant. In the block diagram, the error signal is $E(s) = R(s) - Y(s)$, the control signal is $U(s) = Gc(s)E(s)$, and the plant's output is $Y(s) = G(s)U(s)$.

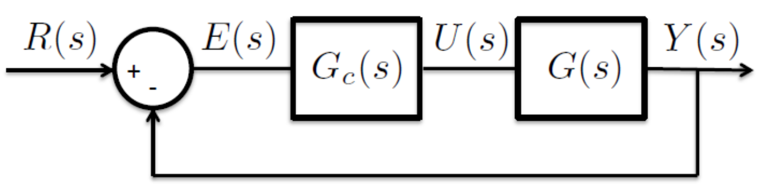

Figure 1: A block diagram of a feedback control system.

In tracking control problems, it is desired to have the plant's output track the desired signal, which is equivalent to having the error signal be zero. The plant's output and the error signal are expressed in the Laplace domain by

$$Y(s) = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)} R(s)$$

$$E(s) = \frac{1}{1 + G(s)G_c(s)} R(s)$$

One approach to examine if the controller design satisfied the goals is to apply the final-value theorem (FVT) to the first equation listed. For a constant desired signal, R(s) = r/s (where r is the constant value input), the final value of the plant's output is

$$y(\infty) = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s)$$

and can only be applied to a closed-loop system if the roots of the denominator of Y(s)/R(s) have negative real parts. Note the only way the plant's output will track a constant desired signal is if zero is one of the roots of the denominator of $G_c(S)G(S)$.

1.2 Phase Advance Control Structure

The ball and beam apparatus is an open-loop unstable system, which requires feedback control to stabilize the ball's position. When the ball's desired position is a constant, the ball accelerates at a uniform rate towards one end of the beam. Thus, stabilizing the ball's position requires some type of predictive control or phase advance control to predict the ball's movement and to make changes to the beam's angle. Phase advance control gives a positive phase over a range of frequencies by having a first order zero slower than a first order pole, which is expressed by the transfer function

$$H(s) = \frac{Ts + 1}{\alpha Ts + 1}$$

and whose bode plot is shown in Figure 2. Providing positive phase allows the phase advance controller to anticipate the ball's position by making necessary changes to the beam's angle.

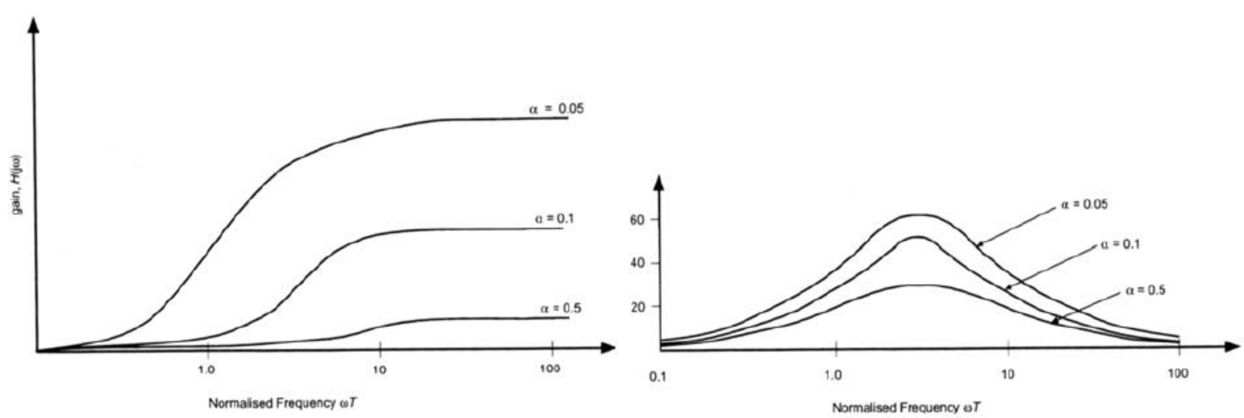

Figure 2: Frequency response of the phase advance controller for different values of α.

1.3 Ball and Beam Apparatus Dynamics

The ball and beam apparatus works by applying a voltage input *u(s)* to the profile cam that is attached to the motor shaft as shown in Figure 3.
The angular velocity $\dot{\theta}$ of the cam is proportional to the input voltage u. The beam angle θ is obtained by integrating the cam's angular velocity and is related to the input voltage with the transfer function

$$\frac{\theta(s)}{u(s)} = G_1(s) = \frac{G_m}{s}$$

where G_m is the proportionality constant. When combining feedback control and proportionality gain k_{p1} with $G_1(s)$ results in the beam's angle tracking the desired beam angle θ_r as shown in Figure 4. The closed-loop beam's angle is described by

$$\theta(s) = \frac{k_{p1}G_1(s)}{1 + k_{p1}G_1(s)}\theta_r(s)$$

Increasing the proportional gain, k_{p1}, will increase the speed in that the apparatus responds.

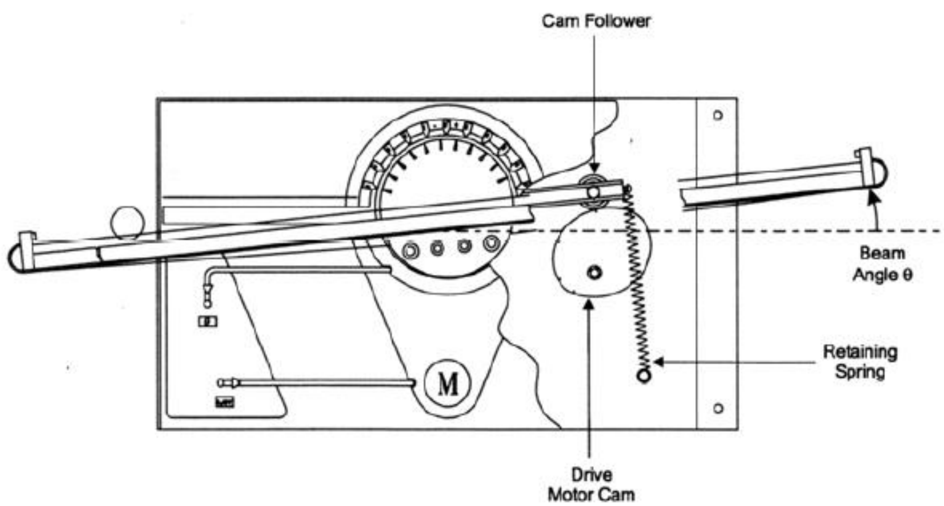

Figure 3: Schematic of how the drive motor and beam work.

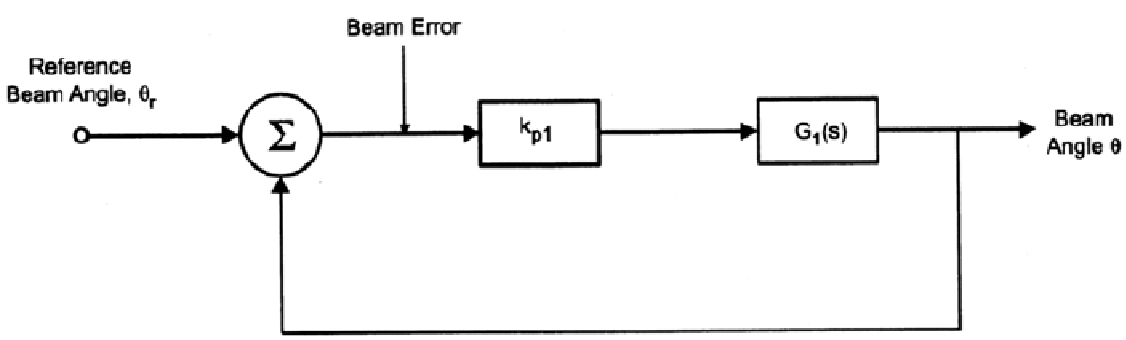

Figure 4: Block diagram of the feedback control loop to manipulate the beam's angle.

The ball's position on the beam is obtained by realizing the force acting on the ball is proportional to the gravity $\boldsymbol{g}$, acting along the beam. Let's assume there is no friction between the ball and the beam, then the beam's angle is related to the ball's position *x(s)* by the transfer function

$$\frac{x(s)}{\theta(s)} = G_2(s) = \frac{g}{s^2}$$

This double integrator is the reason why manipulating the ball's position along the beam is challenging. This challenge is alleviated by incorporating feedback control, especially phase advance control (lead controller), into the apparatus's design.

2 Proportional Control of Beam Angle and Ball Position

Before the phase advance controller is incorporated within the apparatus's design, another control structure called proportional control is incorporated within the apparatus's design. The purpose of using the proportional controller (no anticipatory control) is to show why using an anticipatory type controller is necessary to control the ball's position. This experiment is broken down into two parts and each part is to be completed in Simulink.

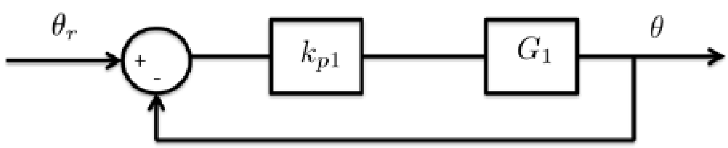

Figure 5: Block diagram of incorporating proportional control to manipulate the beam's angle.

The first part of this experiment shows how the closed-loop proportional control affects the steady-state beam angle's measurement. The block diagram of using proportional control to manipulate the beam's angle is shown in Figure 5 and the resulting closed-loop transfer function *T1(s)* is

$$T_1(s) = \frac{k_{p1}G_m}{s + k_{p1}G_m}$$

Understanding how proportional control affects the beam angle's measurement is done by implementing the block diagram in Figure 5 with the following parameters: the desired beam's angle is 2.5, $G_m = 1$ (*Gm* is found within $G_1(s)$), the sampling period is 0.001 s, and the simulation duration is 10 s. Since the value of the proportional control gain affects the system's speed of response, different values will be used for the proportional control gain. Initially let $k_{p1} = 1$ then run the simulation and plot the beam's angle as a function of time. Answer the following:

- Provide comments about the system's speed of response and its steady-state error.
- Using the given parameters, are the roots of the closed-loop characteristic equation in agreement with the plot characteristics of the beam's angle and why?
- Repeat the above for $k_{p1} = 5$ and $k_{p1} = 8$ and provide comments on the same equations.
- Comment on how the system responds when the proportional control gain is increased.

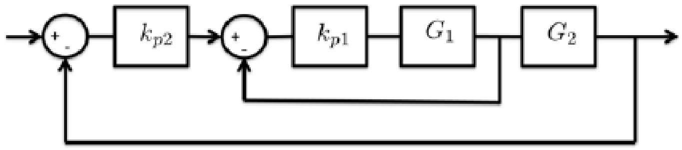

Figure 6: Block diagram of incorporating proportional control to manipulate the balls position.

The next part of this experiment shows how closed-loop proportional control affects the steady state ball position's measurement. The block diagram of using proportional control to manipulate the ball's position is shown in Figure 6 and the resulting closed-loop transfer function $T_2(s)$ is

$$T_2(s) = \frac{gk_{p1}k_{p2}G_m}{s^3 + k_{p1}G_m s^2 + gk_{p1}k_{p2}G_m}$$

Understanding how proportional control affects the ball's position is done by implementing the block diagram in Figure 6 with the following parameters: the proportional control gain for the beam angle is k_{p1} = 8, G_m = 1, gravity is 9.81 m/s^2, the beam's reference position is 20 cm, the sampling period is 0.001 s, and the simulation duration is 10 s. Since the proportional control gain for the ball's position will affect the system's response, different proportional control gains are used. Initially let the proportional gain for the ball's position be k_{p2} = 0.1 then run the simulation and plot the ball's position as a function of time. Answer the following:

- Provide comments about the system's speed of response and the system's behavior.
- Using the given parameters, are the roots of the closed-loop characteristic equation in agreement with the plot characteristics of the ball's position and why?
- Repeat the simulation when k_{p2} = 0.3 and k_{p2} = 0.5 and provide comments on the same questions.
- Comment on how the system responds when the proportional gain control is increased. Explain why proportional gain does not work.

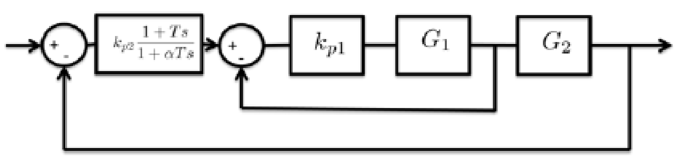

Figure 7: Block diagram of incorporating the phase advanced control to manipulate the ball's position.

The previous test shows why incorporating closed-loop proportional control within the apparatus's design will not manipulate the ball's position. The reason is proportional control has no anticipatory action and therefore another control structure is needed. One such control structure is a phase advance controller (lead controller), which can anticipate the ball's position. First, showing how the phase advance controller can manipulate the ball's position is done in Simulink before it's implemented on the equipment. The block diagram of implementing the phase advance controller to manipulate ball's position is shown in Figure 7 and the resulting closed-loop transfer function $T_3(s)$ is expressed

$$T_3(s) = \frac{gk_{p1}k_{p2}G_m(Ts+1)}{\alpha Ts^4 + (k_{p1}G_m\alpha T + 1)s^3 + k_{p1}G_m s^2 + gk_{p1}k_{p2}G_m(Ts+1)}$$

In Simulink, use the following parameters: k_{p1} = 8, G_m = 1, gravity is 9.81 m/s^2, the beam's reference position is 20 cm, the sampling period is 0.001 s, and the simulation duration is 10 s. Initially let k_{p2} = 0.1, α = 0.1, and T = 4, then run the simulation, plot the ball's position as a function of time, and **comment on the system's response**. Also answer the following:

- Using the given parameters, are the roots of the closed-loop characteristic equation in agreement with the plot characteristics of the ball's position and why?
- Now, let's analyze how increasing the value of k_{p2} affects the system's response. Let's change k_{p2} = 0.5 and use the same values for α and T, then repeat the above experiment.
- Finally, change α = 0.05 and T = 6 and keep the value k_{p2} = 0.5, then run the simulation and provide comments on the same equations. How does changing the parameters of the phase advance controller α and T affect the system's response?

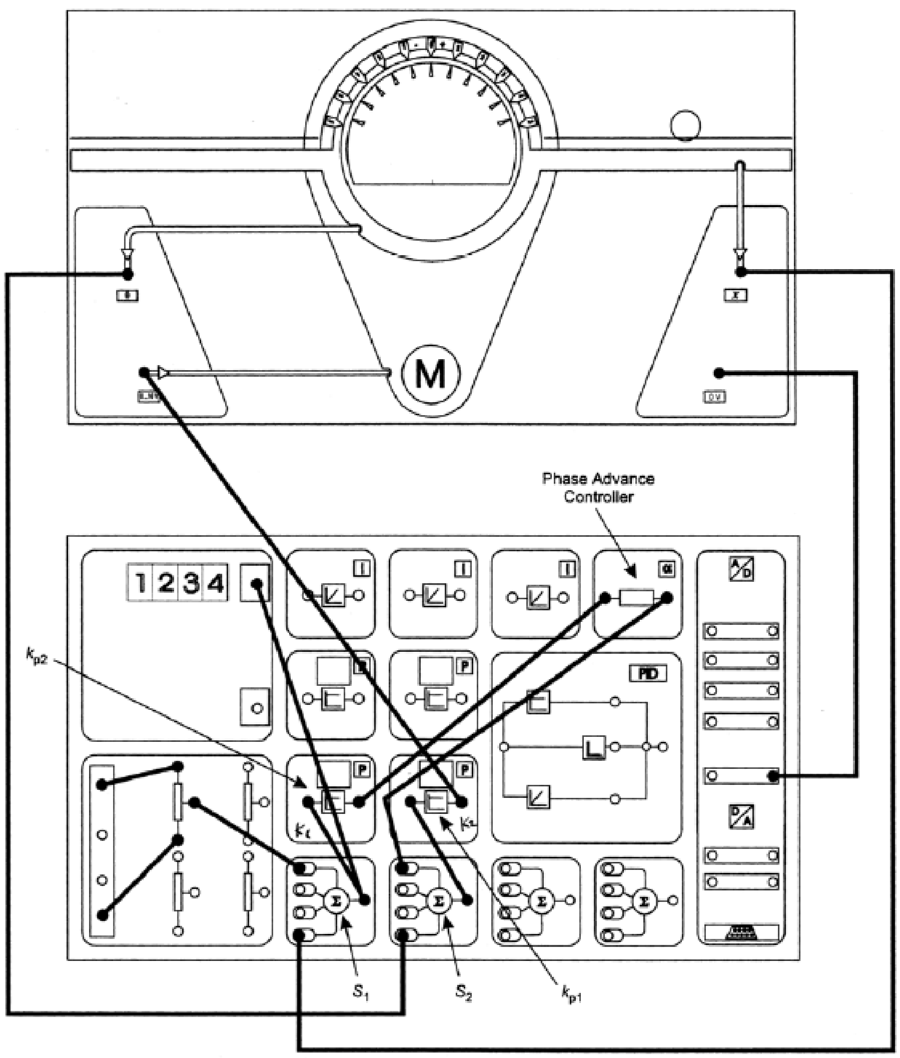

Figure 8: Schematic showing how to connect the CE106 to the CE120 to implement the phase advanced control.

3 Experiment: Phase Advanced Control of Ball Position

The phase advance controller must be incorporated into the apparatus to manipulate the ball's position. This will be done by using specific controller gains to control the ball's position. The following procedure is to be followed during laboratory:

1. Connecting the CE106 with the CE120 as shown in Figure 8.
2. Check all electrical and mechanical connections to make sure that they are secure.
3. Set the controller gains to $k_{p1} = 1$, $k_{p2} = 0.5$, $\tau = 1$, and $\alpha = 0.05$. Note: If during the course of any experiments the system appears to be going unstable, carefully remove the ball from the system so that it is not lost or damaged.
4. Place the ball on the beam at 0 cm. Check to see if the controller is working as expected by gently moving the ball to one side. You should notice the beam tilts to restore the ball's position back to 0 cm.
5. Use the potentiometer to change the desired ball's position to 20 cm. As you turn the potentiometer, the beam will tilt and automatically move the ball to the corresponding position on the beam.
6. **Note:**
    - The error signal once the system reaches steady-state.
    - Observations concerning how the system may or may not oscillate.
    - Observations concerning the time to reach steady-state.
    - Behavior of the ball during the trials.
    - Stability of the trial.
    - General behavior of the system (e.g. sights, sounds, etc.).
7. Repeat the above procedure with the following gain sets:
    a. $k_{p1} = 1$, $k_{p2} = 1$, $\tau = 1$, and $\alpha = 0.05$
    b. $k_{p1} = 1$, $k_{p2} = 3$, $\tau = 1$, and $\alpha = 0.05$
    c. $k_{p1} = 2$, $k_{p2} = 0.5$, $\tau = 1$, and $\alpha = 0.05$
    d. $k_{p1} = 2$, $k_{p2} = 1$, $\tau = 1$, and $\alpha = 0.05$
    e. $k_{p1} = 2$, $k_{p2} = 3$, $\tau = 1$, and $\alpha = 0.05$
    f. $k_{p1} = 3$, $k_{p2} = 0.5$, $\tau = 1$, and $\alpha = 0.05$
    g. $k_{p1} = 3$, $k_{p2} = 1$, $\tau = 1$, and $\alpha = 0.05$
    h. $k_{p1} = 3$, $k_{p2} = 3$, $\tau = 1$, and $\alpha = 0.05$

4 <u>For the Report</u>

A. For each simulation, provide a plot with the axes labeled and provide a title (3 types of controllers and 3 sets of parameters for each controller makes total 9 plots), give the roots of the characteristic equation. Also give answers to the questions that were asked.
B. Describe the effects of varying k_{p1}.
C. Describe the effects of varying k_{p2}.
D. Some of the gain sets used in this experiment are stable in simulation but may not be in the real system, why?
E. Which gain set do you think is best? Give your reasons.

The form of the report should be an extended memo. This will include a few paragraphs of introduction to state the purpose of the study (no theory is needed) and a description of the pre-experiments and the main lab experiment, followed by results and discussion.