SCUPI

Sichuan University - Pittsburgh Institute

ME 1042

Theory of Machines Lab

Forced and Free Vibrations

Revised
September 2020

Mechanical Engineering Department

Forced and Free Vibrations

Goal: The objective of this experiment is to understand the fundamentals of natural frequency and damping on a rigid beam by observing oscillation affects.

Equipment Needed:
TM 1016 Free and Forced Vibrations Device
Versatile Data Acquisition System (VDAS) – Hardware and Software
Desktop Computer
Five 400g Masses and mass holder
Dashpot Damper and Damper Fluid

1 Introduction and Basic Theory

1.1 *Free Vibrations of a Beam and Spring*

Free vibration occurs in a system when it given an initial input of energy. The system then vibrates at its natural frequency until the initial energy has been dissipated due to friction, damping or some other sort of absorber.

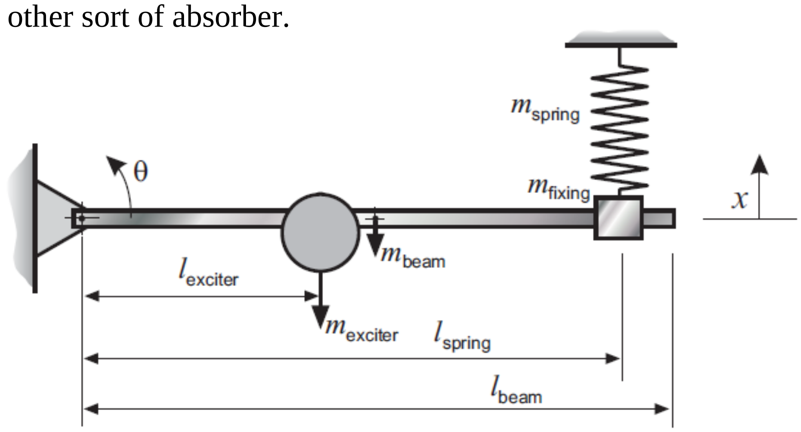

Figure 1: Beam and spring assembly.

Summing the moments about the pivot point gives the following equation of motion that can be used along with the total mass moment of inertia:

$$I_A\ddot{\theta} + kx_s l_{spring} = 0$$

$$I_A = I_{beam} + I_{spring} + I_{exciter}$$

$$I_A = \left[\frac{1}{3}m_{beam}l_{beam}^2\right] + \left[\left(\frac{m_{spring}}{3} + m_{fixing}\right)l_{spring}^2\right] + \left[m_{exciter}l_{exciter}^2\right]$$

The second order differential equation form for this system based on a small angle approximation can be written as:

$$\ddot{\theta} + \frac{kl_{spring}^2}{I_A}\theta = 0$$

A general solution to this equation can be seen below where A is the amplitude and α is the phase shift:

$$\theta = A\cos\left(\sqrt{\frac{kl_{spring}^2}{I_A}}t - \alpha\right)$$

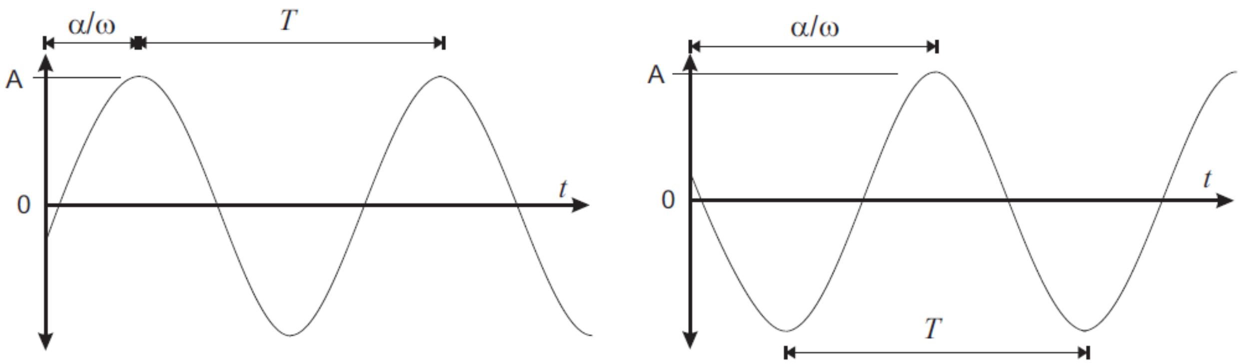

Figure 2: Periodic motion curves detailing amplitude and phase shift.

Figure 2 details examples of periodic motion curves and from the figures along with the oscillation equation, the period and angular velocity can be found.

$$T = 2\pi\sqrt{\frac{I_A}{kl_{spring}^2}}$$

$$\omega = \sqrt{\frac{kl_{spring}^2}{I_A}}$$

By substituting these angular relationship into the 2nd order differential equation above, the natural frequency for the oscillation of the beam can be found from the following form:

$$\ddot{\theta} + \omega^2\theta = 0$$

1.2 Viscous Damping

No vibrating system is ideal, in that without continued applied energy, vibrations will eventually reduce to zero, due to friction or damping the energy of the oscillations. If a constant external force influences an oscillating system such that is causes a decrease in the amplitude of the oscillation, then the system is damped. The most common form of damping is viscous damping where the damping force is proportional and in opposition to the velocity of the displacement. The quantification of viscous damping is shown as a damping coefficient, with the letter *c*.

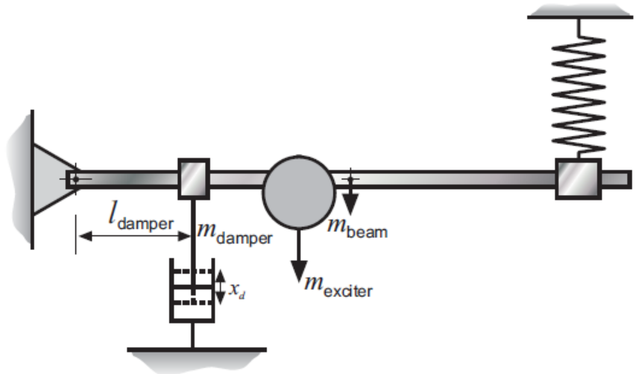

Figure 3 shows the damper in place on the beam and spring system which changes the equation of motion to:

$$I_A\ddot{\theta} = -kx_s l_{spring} - cl_{damper}\dot{x}_d$$
$$\ddot{\theta} + \frac{cl_{damper}^2}{I_A}\dot{\theta} + \frac{kl_{spring}^2}{I_A}\theta = 0$$

The natural frequency can be substituted into the equation along with the decay coefficient γ:

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega^2\theta = 0$$

To account for the inertia of the damper, assume the damper piston acts as a point mass. The following relationships define the total moment of inertia for the system:

$$I_{damper} = m_{damper}l_{damper}^2$$
$$I_A = \left[\frac{1}{3}m_{beam}l_{beam}^2\right] + \left[\left(\frac{m_{spring}}{3} + m_{fixing}\right)l_{spring}^2\right] + [m_{exciter}l_{exciter}^2] + m_{damper}l_{damper}^2$$

A solution to the second order differential equation of the displacement can be realized in the following standard from allowing for the subsequent relationships being found:

$$\theta = Ce^{rt}$$
$$r^2 + 2\gamma r + \omega^2 = 0$$
$$r = -\gamma \pm \sqrt{\gamma^2 - \omega^2}$$

Based on the relationship of γ to ω, three different damping scenarios will exist, which are highlighted in Figure 4.

1. Underdamped, where $\gamma < \omega$, oscillates with a gradually reducing amplitude until equilibrium is reached.
2. Critically damped, where $\gamma = \omega$, once displaced the system will return to equilibrium in the shortest possible time without oscillating
3. Overdamped, where $\gamma > \omega$, Oscillation will not occur and a gradual return to equilibrium will occur at a rate slower than the critically damped situation.

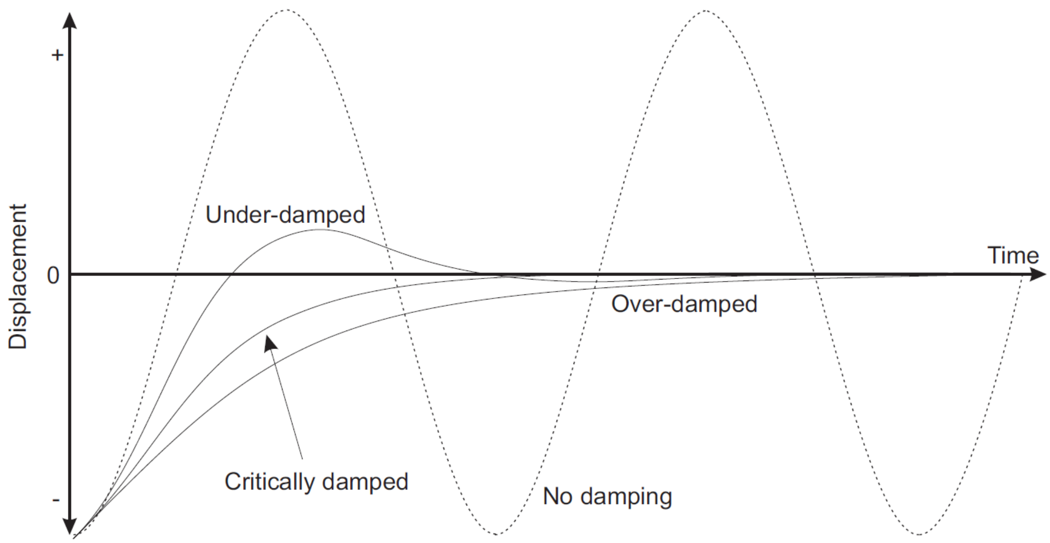

Figure 4: System oscillations relating the different modes of damping ranging from No damping to Overdamped.

A relationship between the damping coefficient (c) of a specific system to that of the critically damped coefficient (c_c) is called the damping ratio ζ as defined below. Table 1 gives a relationship between damping ratio to the range of damping situations, along with the solutions to the 2nd order differential displacement equations.

$$\zeta = \frac{c}{c_c}$$

Table 1: Damping ratio relationships based on damping condition.

Damping Condition	Damping coefficient	Damping Ratio	Displacement Equation
Undamped	$c = 0$	$\zeta = 0$	$\theta = A\cos(\omega t - \alpha)$
Underdamped	$c < c_c$	$0 < \zeta < 1$	$\theta = Ae^{-\gamma t}\cos\left(\sqrt{\omega^2 - \gamma^2}t - \alpha_d\right)$
Critically Damped	$c = c_c$	$\zeta = 1$	$\theta = C_1 e^{-\omega t} + C_2 e^{-\omega t}$
Overdamped	$c > c_c$	$\zeta > 1$	$\theta = C_1 e^{r_1 t} + C_2 e^{r_2 t}$

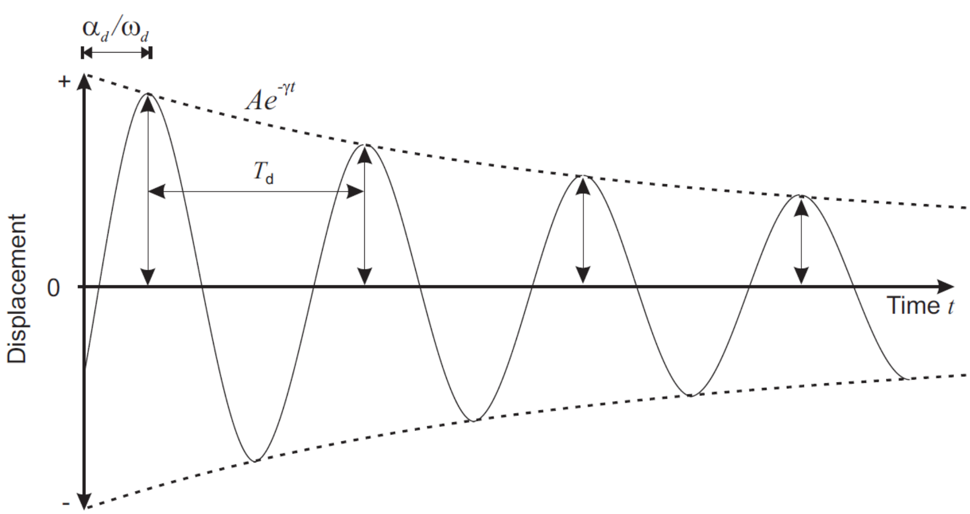

Figure 5: Underdamped oscillation curve.

Forced and Free Vibrations

The damping frequency can be determined based upon the natural frequency of the system and the damping coefficient. Likewise a relationship between damping ratio and the frequency ratio:

$$\omega_d = \sqrt{\omega^2 - \gamma^2}$$

$$\frac{\omega_d}{\omega} = \sqrt{1 - \zeta^2}$$

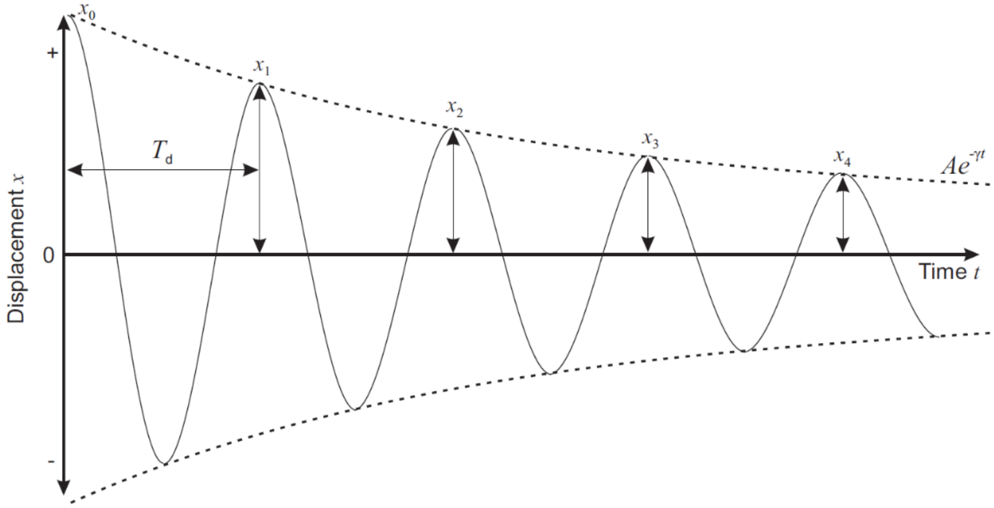

Figure 6: Logarithmic decrement approximation for calculating damping.

A common technique used to calculate the damping frequency and damping period T_d is the logarithmic decrement method, as depicted in Figure 6. This technique uses the natural log of the ratio of amplitude of successive peaks over a certain number of periods apart. Usually 3 or more peaks are used for a better accuracy of measurement. The relationships below are used to solve for these terms:

$$\frac{x_0}{x_{0+j}} = \frac{Ae^{-\gamma t_0}}{Ae^{-\gamma(t_0 + jt_d)}} = e^{\gamma j T_d} = e^{j\delta}$$

$$\delta = \frac{1}{j} \ln \frac{x_0}{x_{0+j}} = \gamma T_d = \frac{2\pi\gamma}{\omega_d}$$

1.3 Forced Vibrations of a Beam and Spring

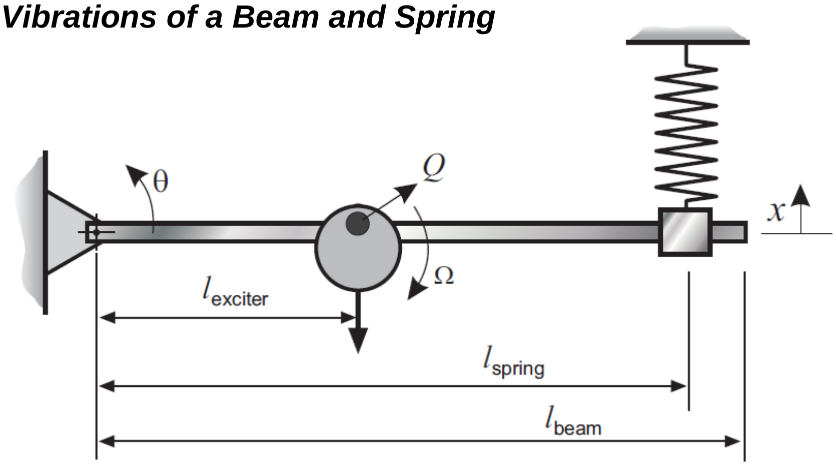

Figure 7: Beam and spring system under the influence of an oscillating force.

Forced and Free Vibrations
An oscillating force Q, as shown in Figure 7, that may be implemented on the beam spring system causes a forced vibration on the system that does not represent the natural frequency. Inserting this force into the 2nd order oscillation form can be seen below:

$$I_A\ddot{\theta} + kx_s l_{spring} + cl_{damper}\dot{x}_d = l_{exciter}\,Q\sin\Omega t$$

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega^2\theta = \frac{l_{exciter}}{I_A}Q\sin\Omega t$$

A solution to this equation based on a few parameters and differentiation can be seen below along with additional definitions for magnification factor and phase angle:

$$\theta = \frac{l_{exciter}}{\omega^2 I_A}Q\beta\sin(\Omega t - \phi)$$

$$\beta = \frac{1}{\sqrt{\left(1-\frac{\Omega^2}{\omega^2}\right)^2 + \left(\frac{2\zeta\Omega}{\omega}\right)^2}}$$

$$\tan\phi = \frac{\frac{2\zeta\Omega}{\omega}}{1-\frac{\Omega^2}{\omega^2}}$$

1.4 Simply Supported Beam

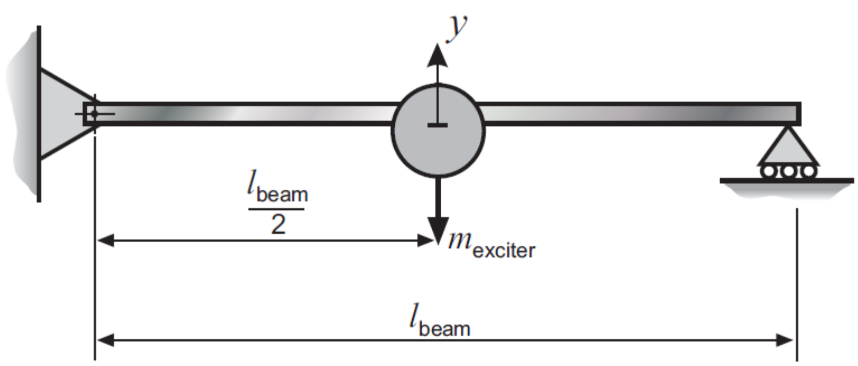

Figure 8: Simply supported beam arrangement under the influence of an excitation force.

For a simply supported beam with an applied load as in Figure 8 above, the flexural integrity of the beam is directly comparable to that of a spring.

$$k_{beam} = \frac{48EI_{beam}}{l_{beam}^3}$$

The motion of the beam can be considered as linear in the y-direction, so the following forms for the displacement and frequency can be used:

$$\ddot{y} + \omega^2 y = 0$$

$$\omega^2 = \frac{k_{beam}}{m_{exciter}}$$

The period and frequency can then be represented as before in the rigid beam system with the displacement equation in phase from shown below:

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m_{exciter} l_{beam}^3}{48EI_{beam}}}$$

$$y = A\cos(\omega t - \alpha)$$

Rayleigh's improved beam theory accounts for the both the mass of the excitation weight and the mass of the beam, deriving an effective mass in the following form:

$$m_{effective} = m_{exciter} + \frac{17}{35} m_{beam}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m_{effective} l_{beam}^3}{48EI_{beam}}}$$

Because the masses attach to the central location of the beam, the stiffness of the beam is actually increased resulting a modification to the period.

$$k_{beam} = \frac{6EI_{beam}}{\frac{l_{beam}}{2}^3}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m_{effective}\frac{l_{beam}}{2}^3}{6EI_{beam}}}$$

1.5 Simply Supported Beam with Damping

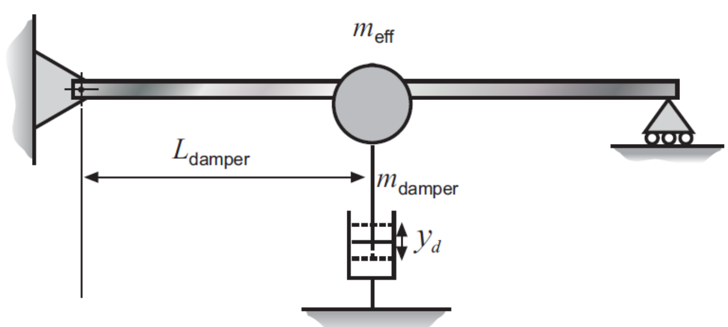

Figure 9: Simply supported beam under a damped excitation force.

To account for damping as shown in Figure 9 for the motion equation, the following relationships
apply:

$$m_{eff}\ddot{y} + c\dot{y} + k_{eff} = 0$$

$$m_{eff} = m_{mass} + \frac{17}{35}m_{beam} + m_{damper}$$

$$\ddot{y} + 2\gamma\dot{y} + \omega^2 y = 0$$

$$y = Ae^{-\gamma t}\cos(\omega_d t - \alpha_d)$$

1.6 Simply Forced Vibrations of a Simply Supported Beam

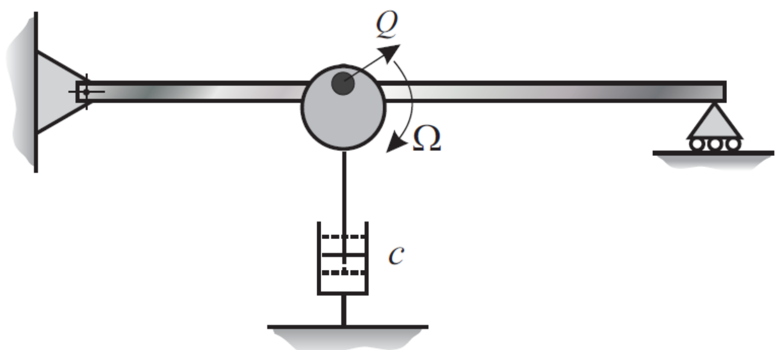

Figure 10: Forced vibration damped, simply supported beam arrangement.

The relations for forced damping for the simply supported beam are the same as before with the oscillation force included:

$$m_{eff}\ddot{y} + c\dot{y} + k_{eff}y = Q\sin\Omega t$$

$$m_{eff} = m_{mass} + \frac{17}{35}m_{beam} + m_{damper}$$

$$\ddot{y} + 2\gamma\dot{y} + \omega^2 y = \frac{Q}{m_{eff}}\sin\Omega t$$

$$y = \frac{Q}{k_{eff}}\beta\sin(\Omega t - \alpha)$$

2 Experimental Procedure

2.1 Week 1: Rigid Beam and Spring System

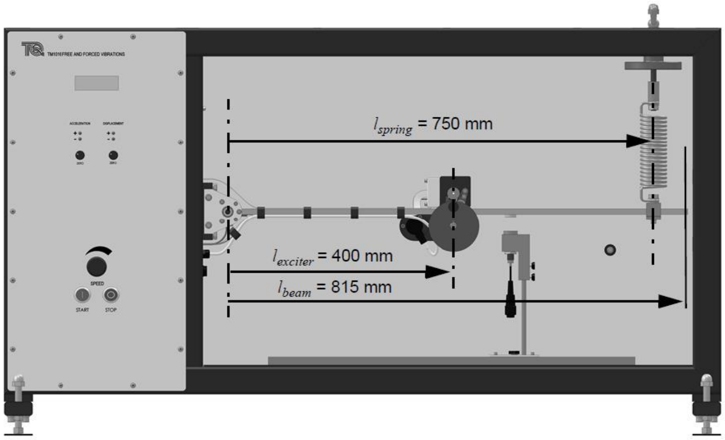

Figure 11: Rigid beam and spring system setup for the TM 1016.

1. Make sure the system is setup in the beam and spring arrangement as shown in Figure 11.
2. Open the VDAS software and make the settings as rigid under the beam option, free vibration under the mode, 2 mm for the displacement channel and a time displacement of 50 ms.
3. Make sure the VDAS trace begins at the center of the chart as a zero reference value.
4. Start the VDAS and press down on the beam and release. You should be able to monitor the oscillation curve after releasing the displacement.
5. Stop the VDAS program and use the curser tools to measure the distance between two peaks. This will allow you to approximate the natural frequency.
6. Add the mass holder to the system with no added mass and make sure the zero reference value is maintained. Also add the mass of the mass holder to the added mass location in VDAS.
7. Under these new conditions, find the natural frequency again.
8. Repeat the natural frequency measurement after adding each of the 400 g masses one at a time.
9. Use Table 2 as a means to track your data for each step condition.

Table 2: Data table for rigid beam and spring calculations.

Added Mass (kg)	Total Exciter Mass (kg)	I_{mass} (kg.m^2)	I_A (kg.m^2)	Natural Frequency f (Hz)	
				Measured	Theoretical
0	4.2				
Mass Holder = 0.2 kg	4.4				
400 g + 0.2 kg = 0.6 kg	4.8				
800 g + 0.2 kg = 1.0 kg	5.2				
1200 g + 0.2 kg = 1.4 kg	5.6				
1600 g + 0.2 kg = 1.8 kg	6.0				
2000 g + 0.2 kg = 2.2 kg	6.4				
m_{beam} = 1.65 kg l_{beam} = 0.815 m $l_{exciter}$ = 0.4 m		m_{spring} = 0.388 kg l_{spring} = 0.75 m m_{fixing} = 0.09 kg		I_{beam} = 0.365 kg.m^2 I_{spring} = 0.123 kg.m^2 k = 3800 N.m^{-1}	

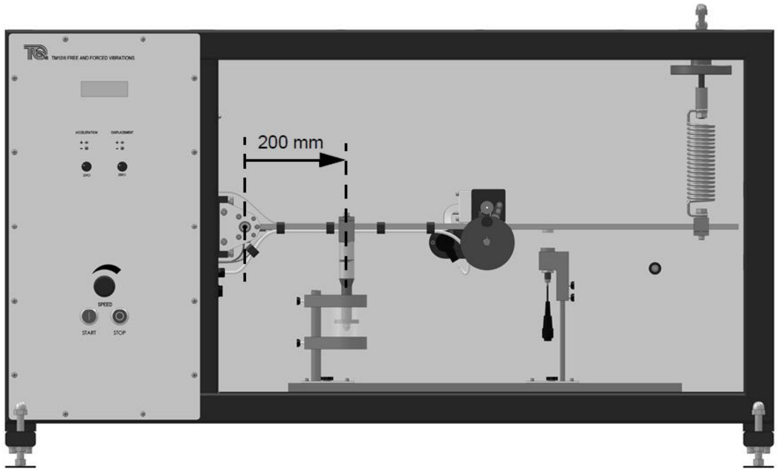

Figure 12: Rigid beam and spring setup with dashpot damper in place for the TM 1016.

10. Fit the dashpot damper to the rigid beam and spring system, as shown in Figure 12.
11. Do not add damping fluid yet to the damper cylinder. Include the added weight of the damper piston to the VDAS but keep the settings as no damping.
12. Find the natural frequency for the system as before.
13. Enter the natural frequency into the VDAS interface and set the timebase as 500 ms
14. Start the VDAS and press down on the beam and release. Wait for a good number of oscillations to occur before stopping the VDAS. Scroll across the x-axis to find the cleanest section of the waveform.
15. The view should show a non-decaying wave. Use the channel 1 tools to select the underdamped model to fit the data. VDAS should then update the damping parameter fields.
16. Change the VDAS setting to forced vibrations. Make sure the natural frequency value you measured is inserted so that speed ratio will be calculated. Also update the channel 1 displacement to 0.1 mm and the damper as attached.
17. Start the VDAS and press the exciter start button.
18. To duplicate the plot in Figure 13, adjust the speed slowly in steps of 0.25 Hz from a speed ratio of 0.75. As you approach a speed ration of 1.0, adjust the speed in steps of 0.1 Hz. At each step, wait about 1 minute for the oscillations to stabilize and then record the displacement amplitude and phase lag.
19. Add fluid to the dashpot damper and set the piston to the fully open location and repeat the test from step 12.
20. Do the same for the half open and fully shut conditions. Use Table 3 for data recording.

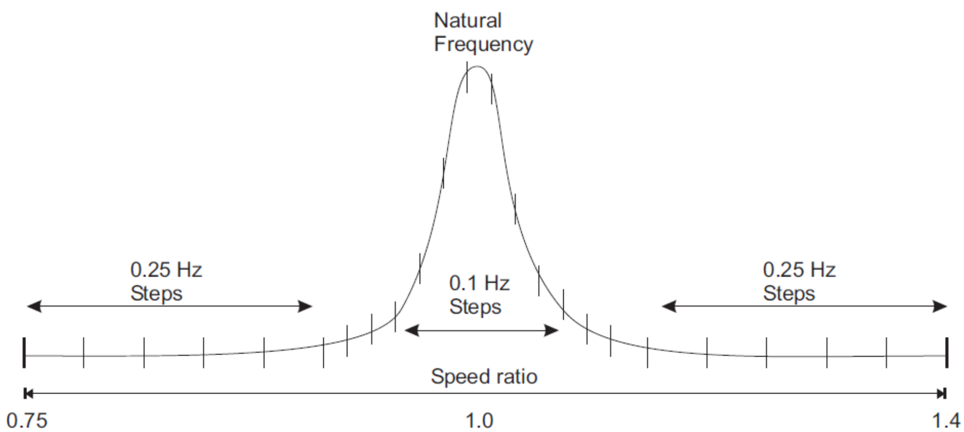

Figure 13: Frequency vs speed ratio during excitation damping.

Table 3: Data table for damping condition calculations.

Damping Condition	Nat. Frequency	Damping Ratio	Magnification Factor
Undamped			
Fully Opened			
Half Opened			
Fully Shut			

2.2 Week 2 Simply Supported Beam

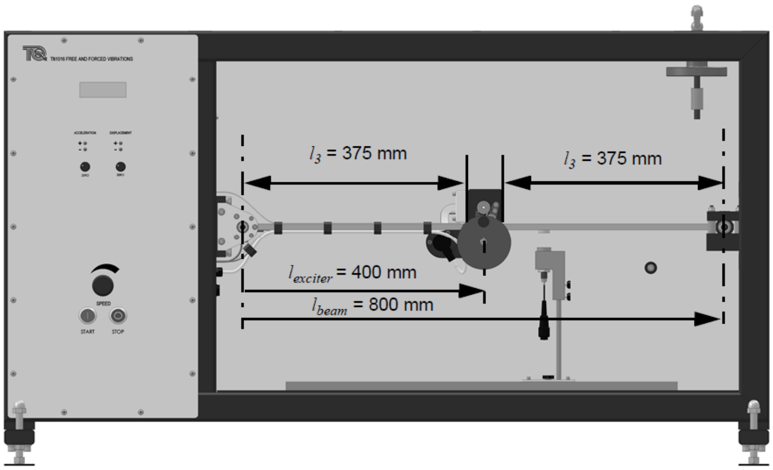

Figure 14: Simply supported beam with dashpot damper in place for the TM1016.

1. Make sure the system is set up for the simply supported beam as shown in Figure 14.
2. Find the natural frequency of the system with no added mass, as done in previous steps.
3. Add the mass holder to the system with no added mass and include the mass of the holder in the VDAS settings.
4. Find the natural frequency and repeat for each added 400 g masses. Remember to include the added mass input into the VDAS settings.
5. Use Table 4 as a data table for result calculations.

Table 4: Simply supported beam data table for result calculations.

Added Mass (kg)	Total Exciter Mass (kg)	Effective Mass (kg)	Natural Frequency f (Hz)		$1/f^2$
			Measured	Theoretical	
0	4.2				
Mass Holder = 0.2 kg	4.4				
400 g + 0.2 kg = 0.6 kg	4.8				
800 g + 0.2 kg = 1.0 kg	5.2				
1200 g + 0.2 kg = 1.4 kg	5.6				
1600 g + 0.2 kg = 1.8 kg	6.0				
2000 g + 0.2 kg = 2.2 kg	6.4				
m_{beam} = 1.65 kg 17/35 m_{beam} = 0.8 kg l_3 = 0.375 m			I_{beam} = 2.083 x 10^{-9} m^4 E = 2.00 x 10^{11} Pa $6EI_{beam}$ = 2.50 x 10^3		

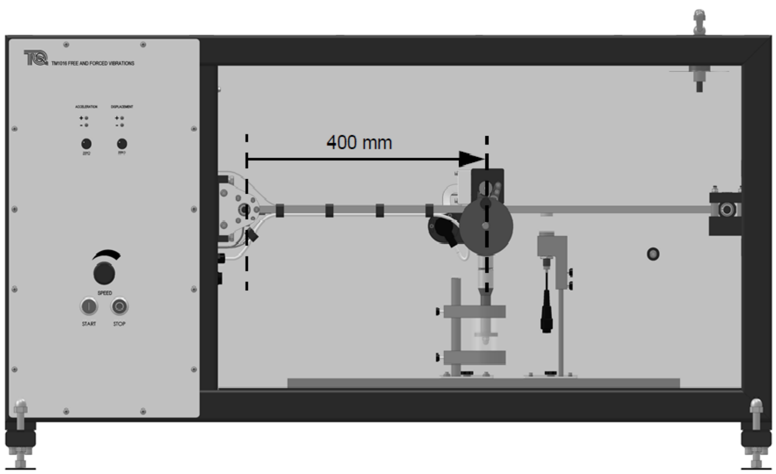

Figure 15: Simply supportive beam setup under damping conditions.

6. Make sure the system is set up for the simply supported beam with the dashpot damper, as shown in Figure 15.
7. Follow the steps used in the rigid beam and spring system under damping.
8. Use Table 3 again as a data table for result calculations.

3 For the Report

The extended memo report for this lab will be due 1 week after the second week of lab. The memo report that is due must include the information outlined below.

3.1 Week 1: Rigid Beam and Spring

1. Plot frequency vs added mass for both theoretical and measured values. Compare them and explain any associated errors or differences.
2. Compare the natural frequency and damping ratio for TWO conditions.
3. Find the magnification factor for a speed ratio of 1.0 for TWO sets of results and compare.
4. Plot your amplitude and phase lag values vs speed ratio.

3.2 Week 2: Simply Supported Beam

1. Find the effective mass using Rayleigh's improved solution and then calculate the theoretical oscillation frequency. Compare the calculation to the measured experimental value.
2. Compare the natural frequency and damping ratio for TWO conditions.
3. Find the magnification factor for a speed ratio of 1.0 for TWO sets of results and compare.
4. Do your results compare well with those shown in theory? Can you explain any differences?