UNSTEADY FEEDBACK CONTROL

LAB 4

STABILITY: AN EXAMPLE

A lifting body is a fixed-wing aircraft or spacecraft configuration in which the body itself produces lift.

M2-F2

STABILITY: AN EXAMPLE

Bruce Peterson

M2-F3

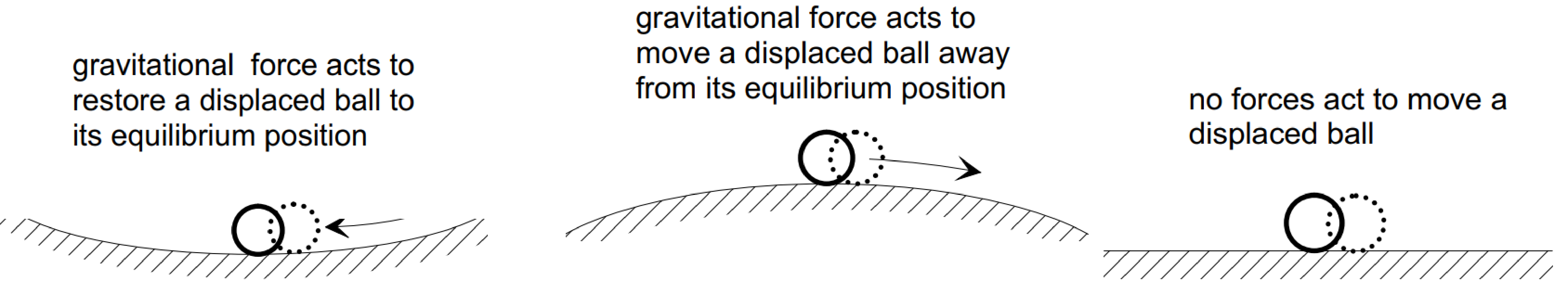

5

UNSTEADY FEEDBACK CONTROL

- Stability
- Control strategy: cascade control
- Ball and beam modelling
- Phase advanced control
- Goal: to show how feedback control can be used to stabilize an unstable system

STABILITY

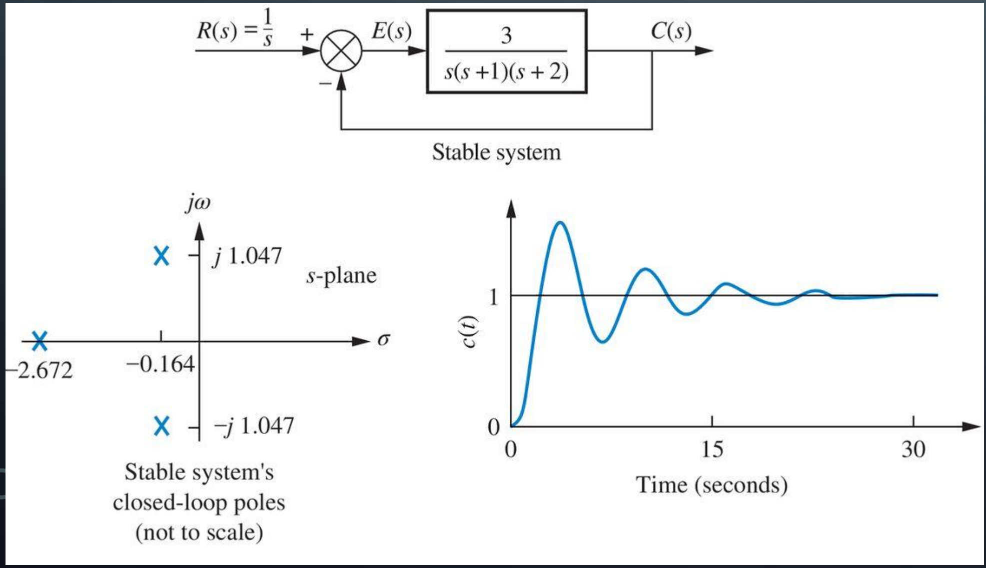
- Stability conditions

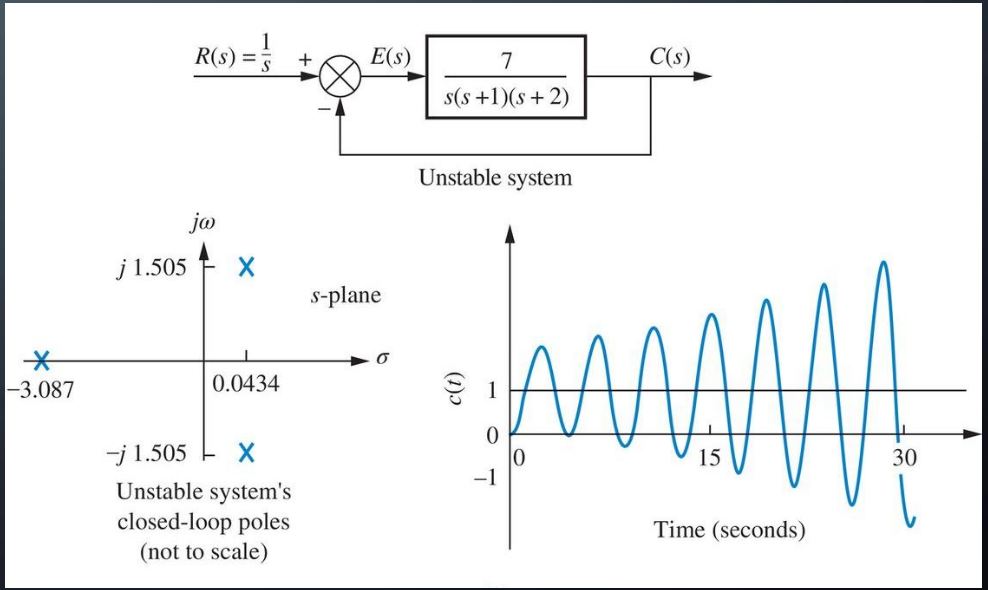


(a) a stable system (b) an unstable system (c) a neutrally stable system

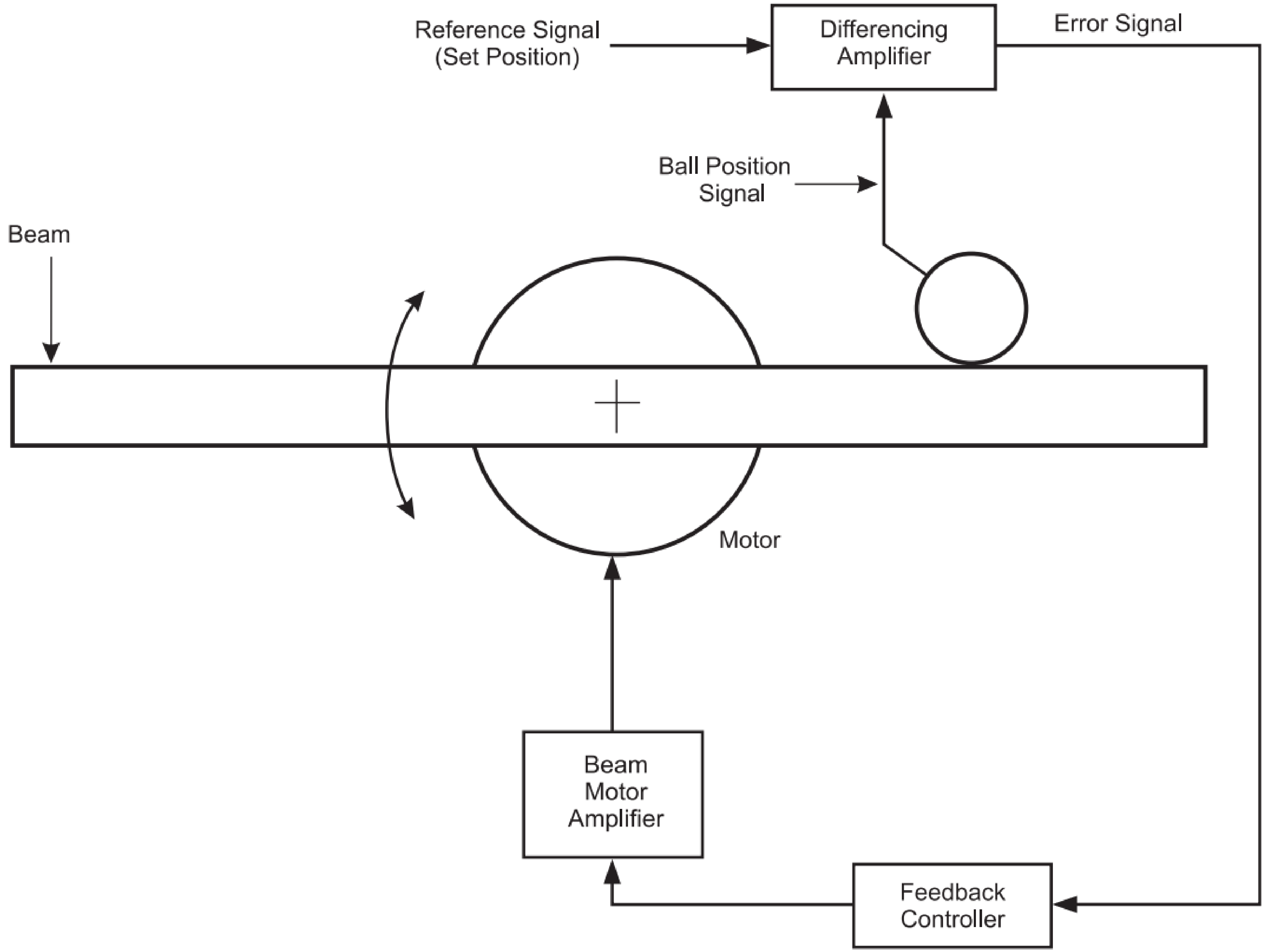
STABILITY

- Stable systems have closed-loop transfer functions with poles only in the left half-plane.
- Unstable systems have closed-loop transfer functions with at least one pole in the right half-plane.
- A linear system is defined to be stable only if all of its poles have negative real parts.



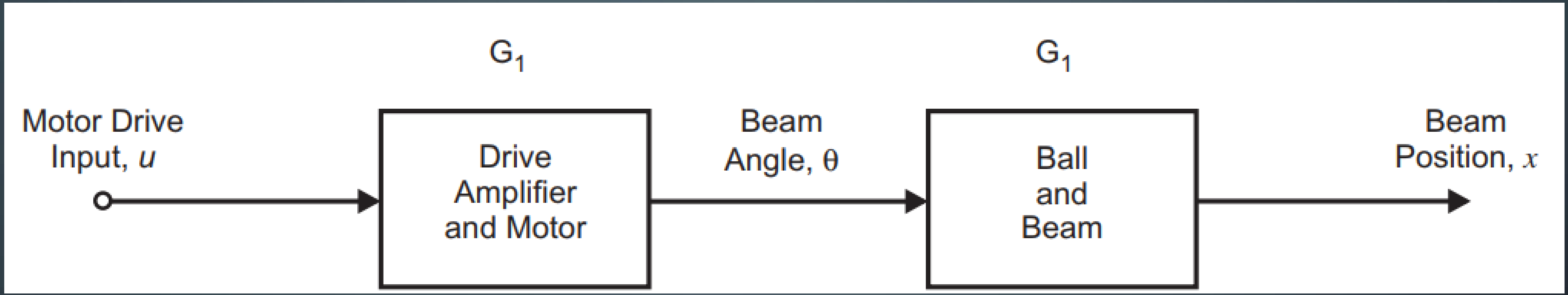


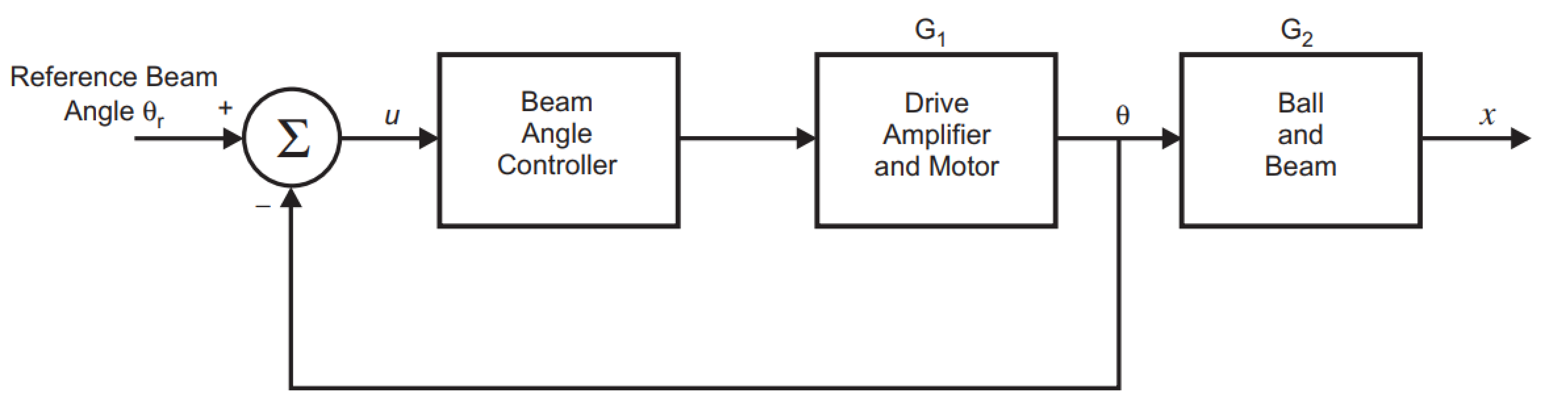
BALL AND BEAM



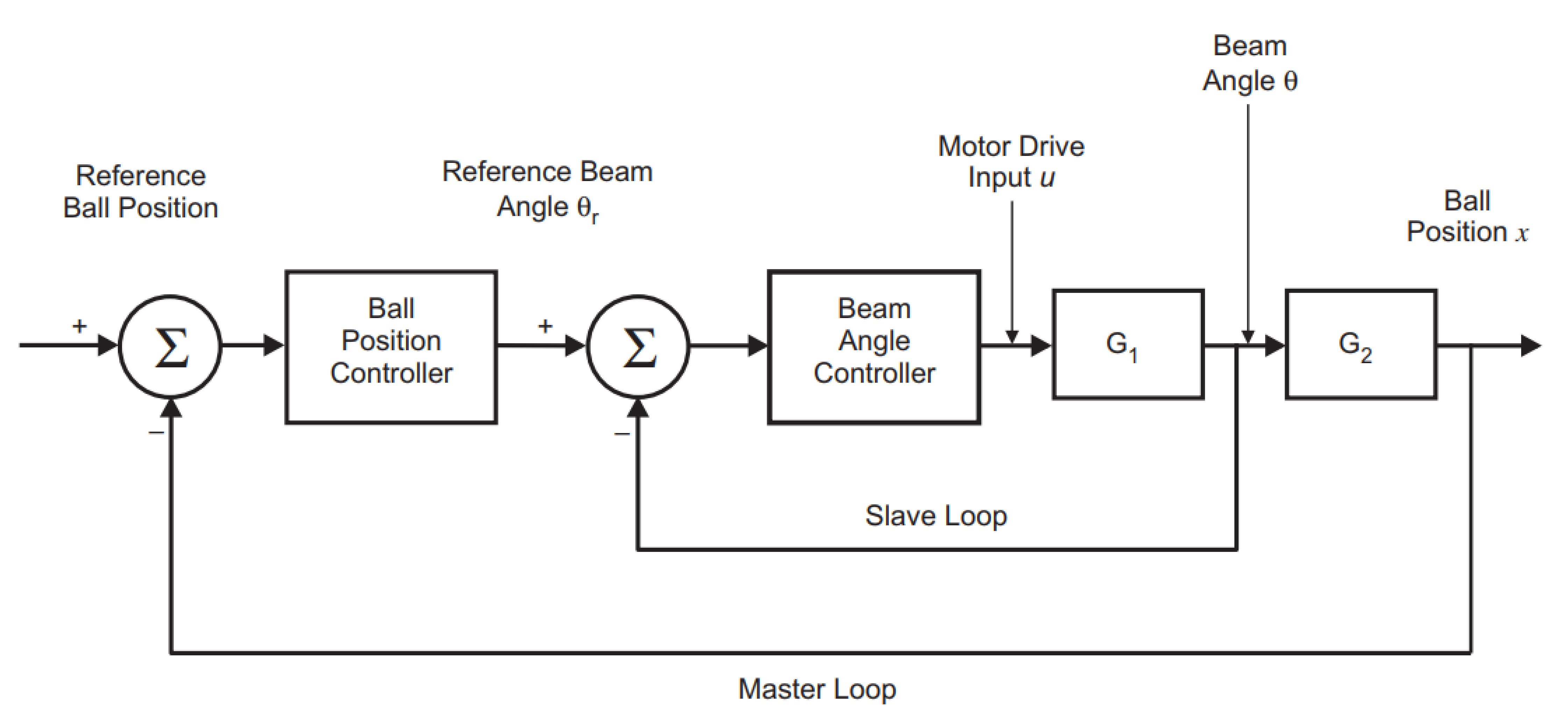
Reference Signal
(Set Position)
Differencing
Amplifier
Error Signal
Ball Position
Signal
Beam
Motor
Beam
Motor
Amplifier
Feedback
Controller

CONTROL STRATEGY: CASCADE CONTROL

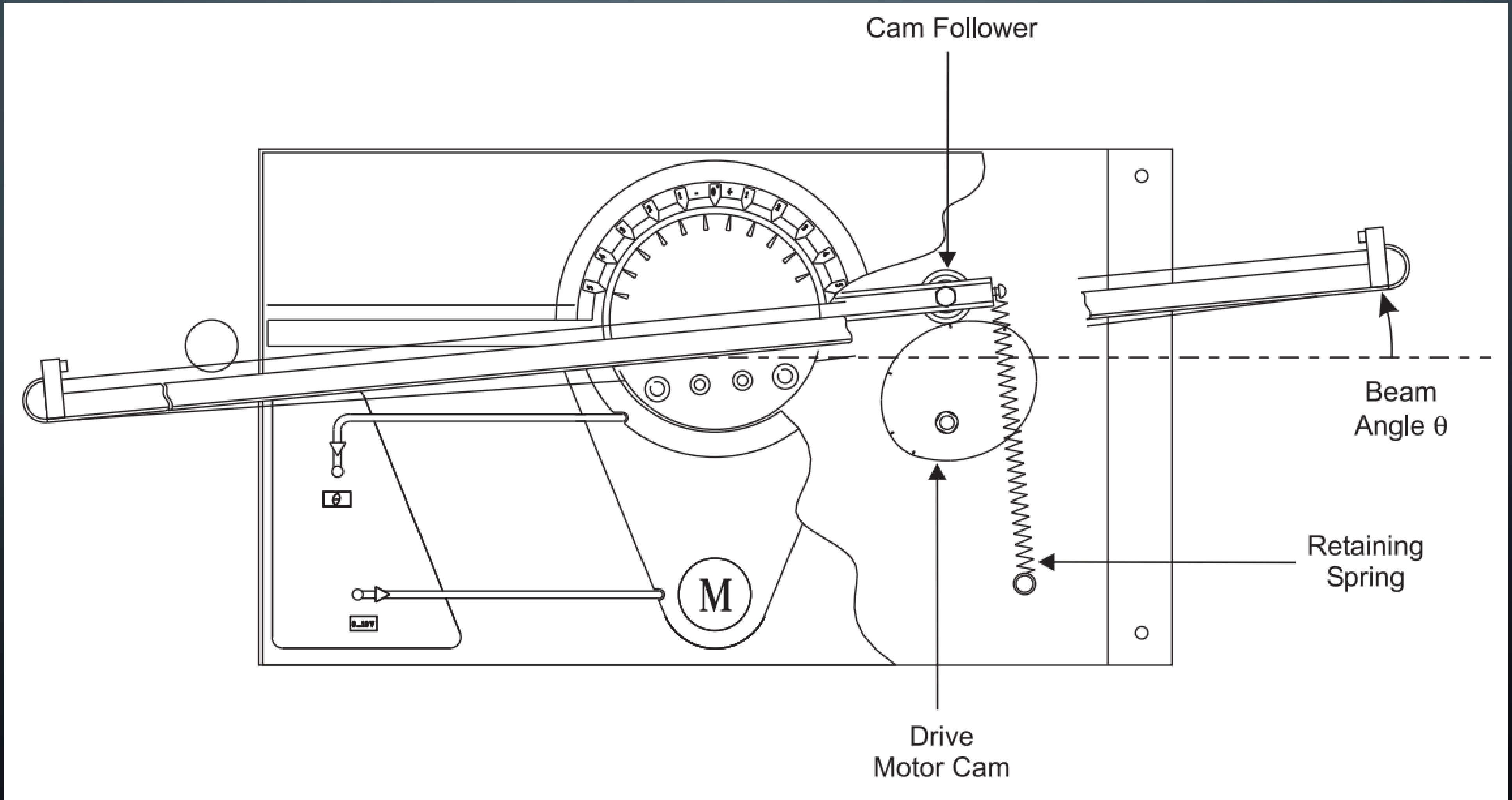




CONTROL STRATEGY: CASCADE CONTROL



MODELLING

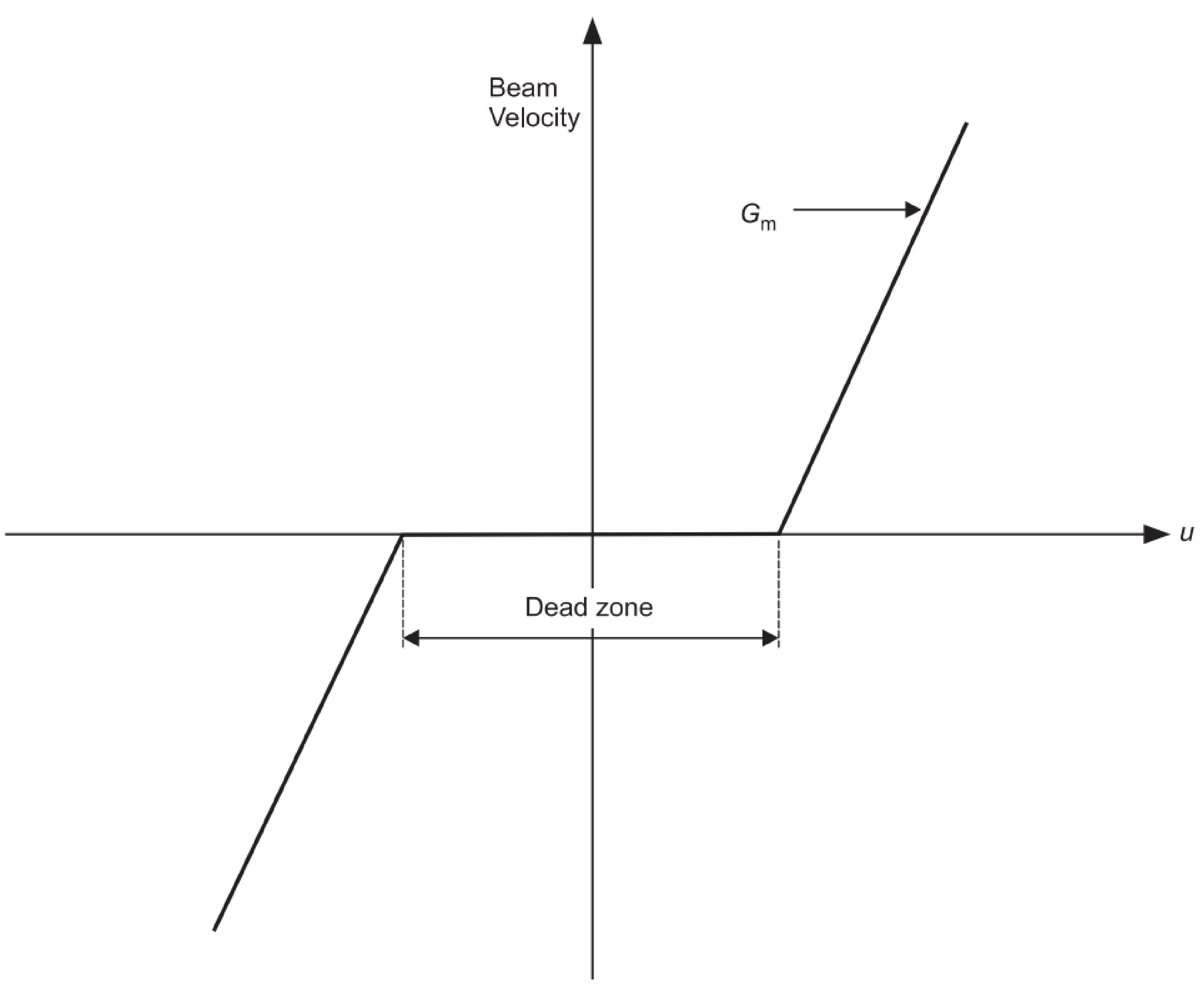


Cam Follower
Beam Angle θ
Retaining Spring
Drive Motor Cam
M

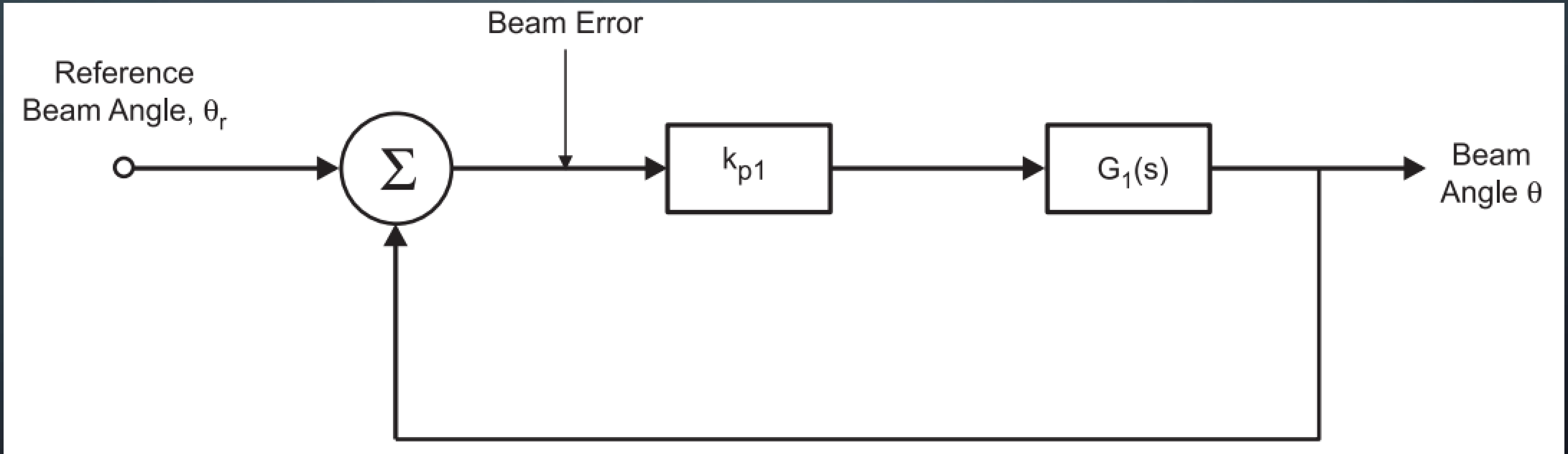
DRIVE MOTOR AND BEAM: MODELLING

$$\frac{d\theta}{dt} = G_m u$$

$$\theta(s) = \frac{G_m}{s} u(s)$$

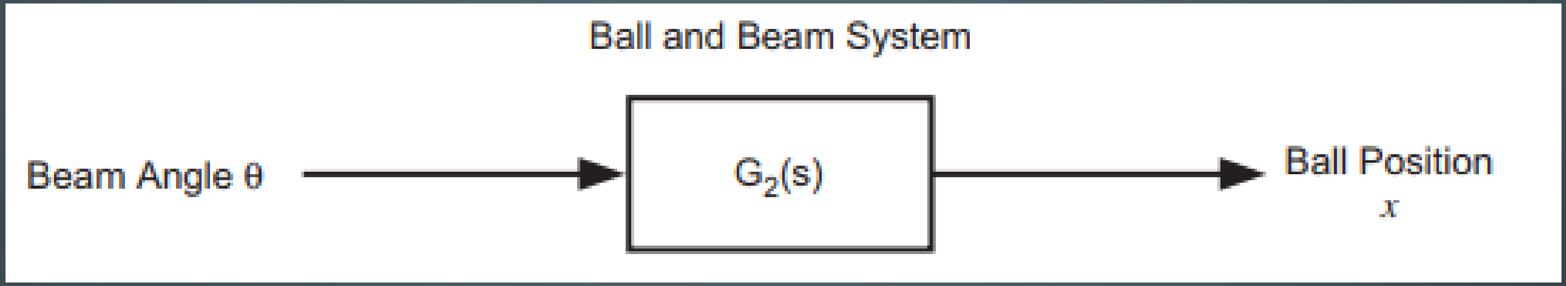


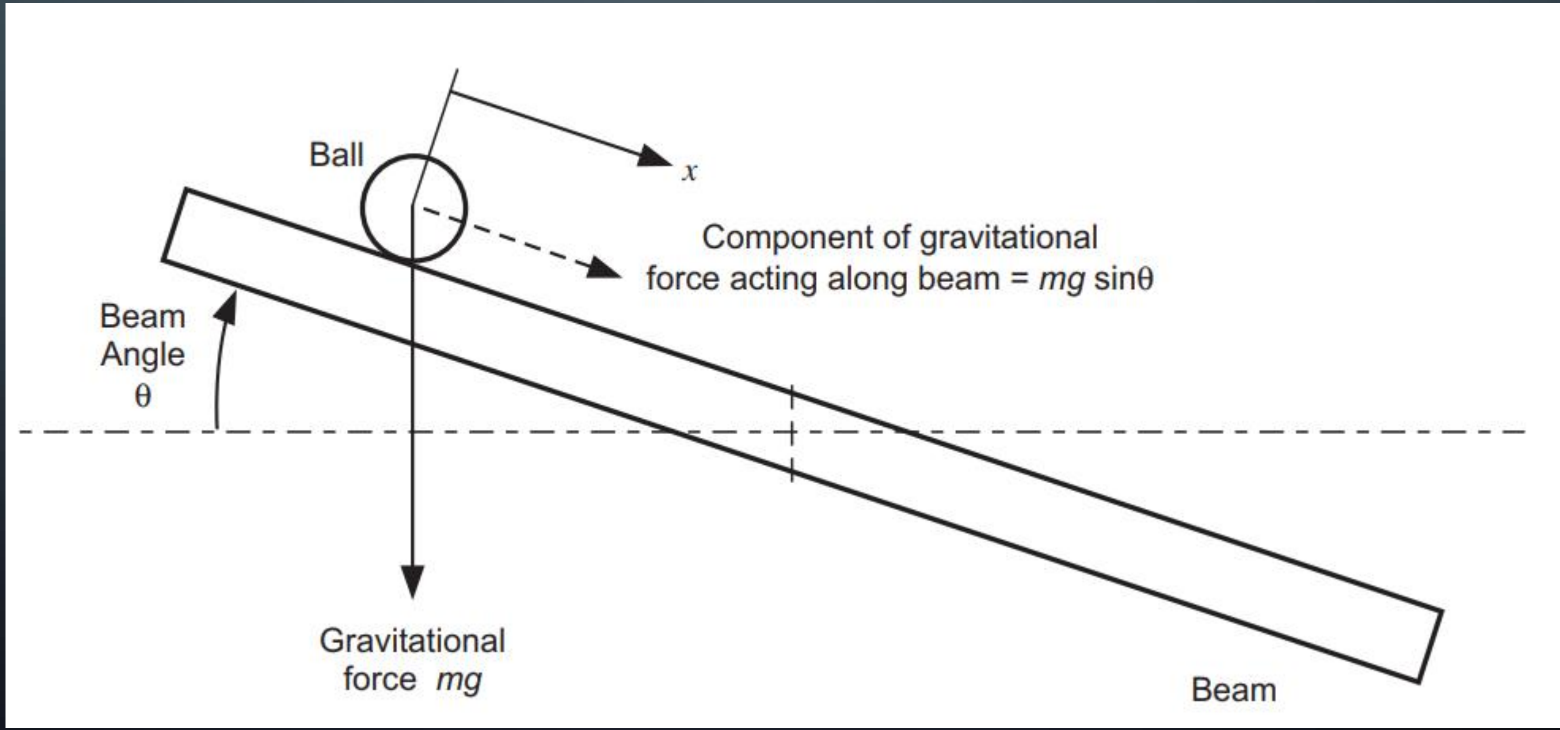
DRIVE MOTOR AND BEAM: CONTROL



$$\theta(s) = \frac{k_{p1}G_1}{1 + k_p G_1}\theta_r(s)$$

BALL AND BEAM SYSTEM: MODEL





BALL AND BEAM SYSTEM: MODEL

- Assume no friction

$$mg\sin(\theta) = m\ddot{x}$$

- Consider small angle

$$\sin(\theta) = \theta$$
$$\ddot{x} = g\theta$$

- Laplace transform

$$x(s) = \frac{g}{s^2}\theta(s)$$
$$G_2(s) = \frac{g}{s^2}$$

CONTROLLER TYPES

- Proportional control - multiplies the error signal by a constant gain k_p. This type of controller can be used to improve the steady-state error, increase the speed of the system's response, but can also cause the system to go unstable.

$$G_c(s) = k_p$$

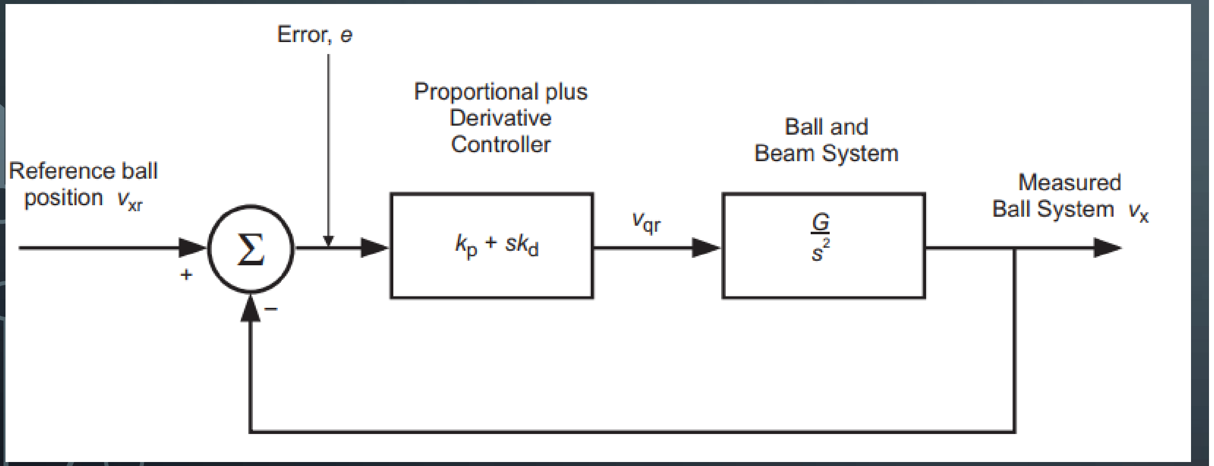
- Proportional and Integral control - multiplies the error signal by a constant gain k_p and multiplies the sum of all the past values of the error signal by a constant gain k_i. For constant desired signals, integral control makes the steady-state error zero.

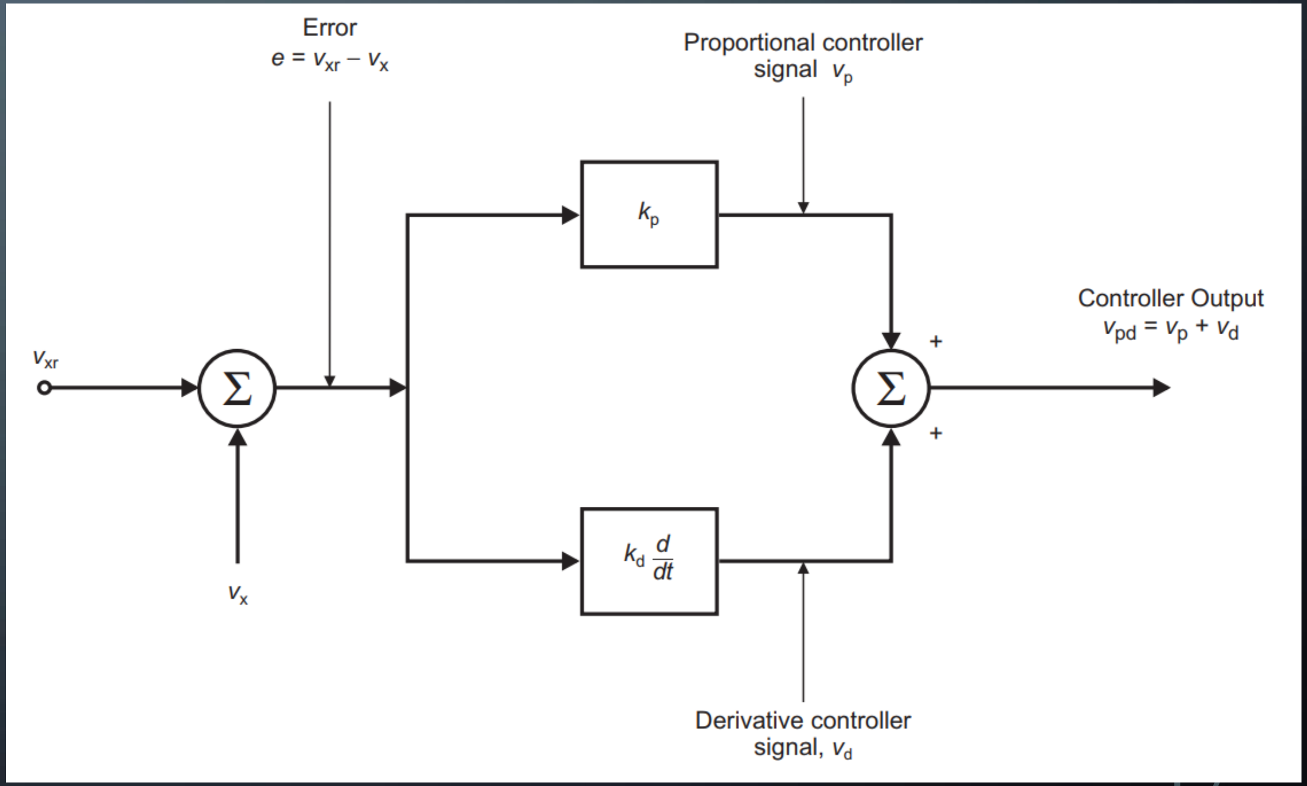
$$G_c(s) = k_p + \frac{k_i}{s}$$

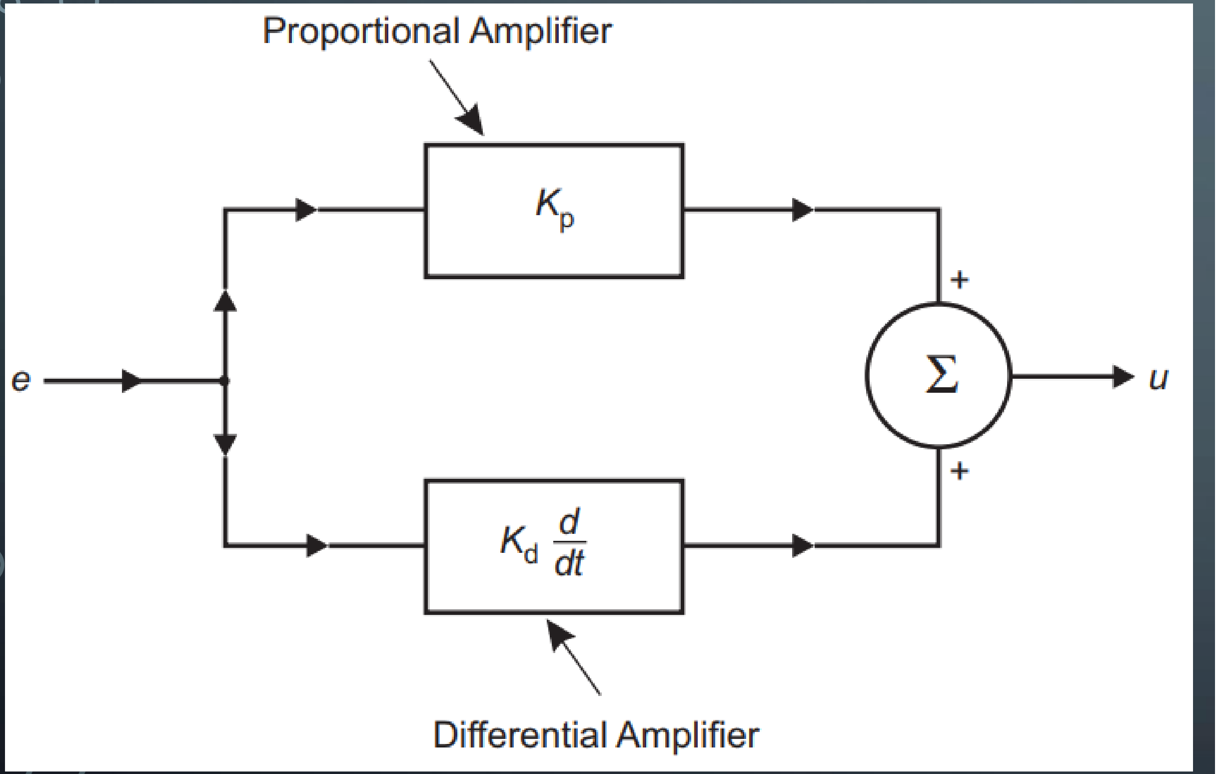
- Phase advance control (Lead compensation) - is used to speed up the system's response by adding anticipatory action to it. This anticipatory action is accounted for with adding positive phase to the system.

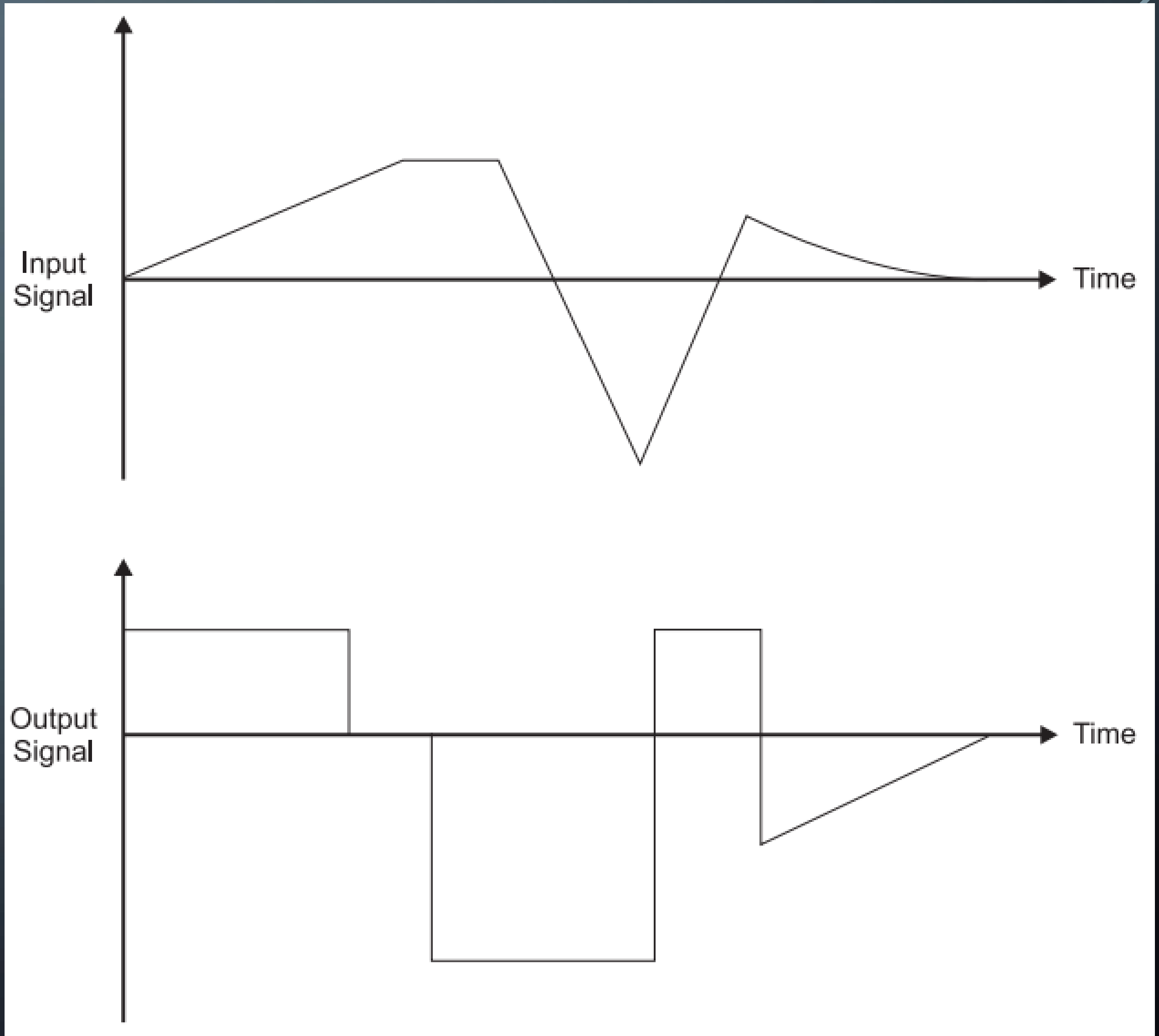
$$G_c(s) = \frac{Ts + 1}{\alpha Ts + 1} \quad \alpha < 1$$

BALL AND BEAM SYSTEM: PD CONTROL



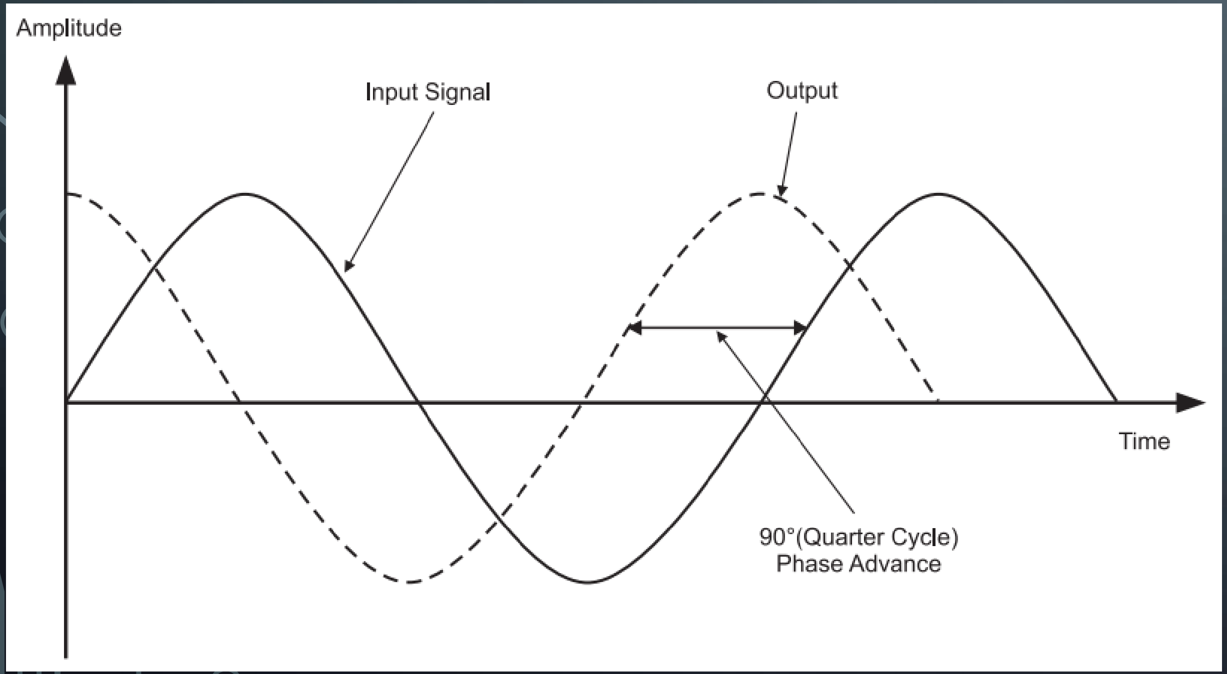


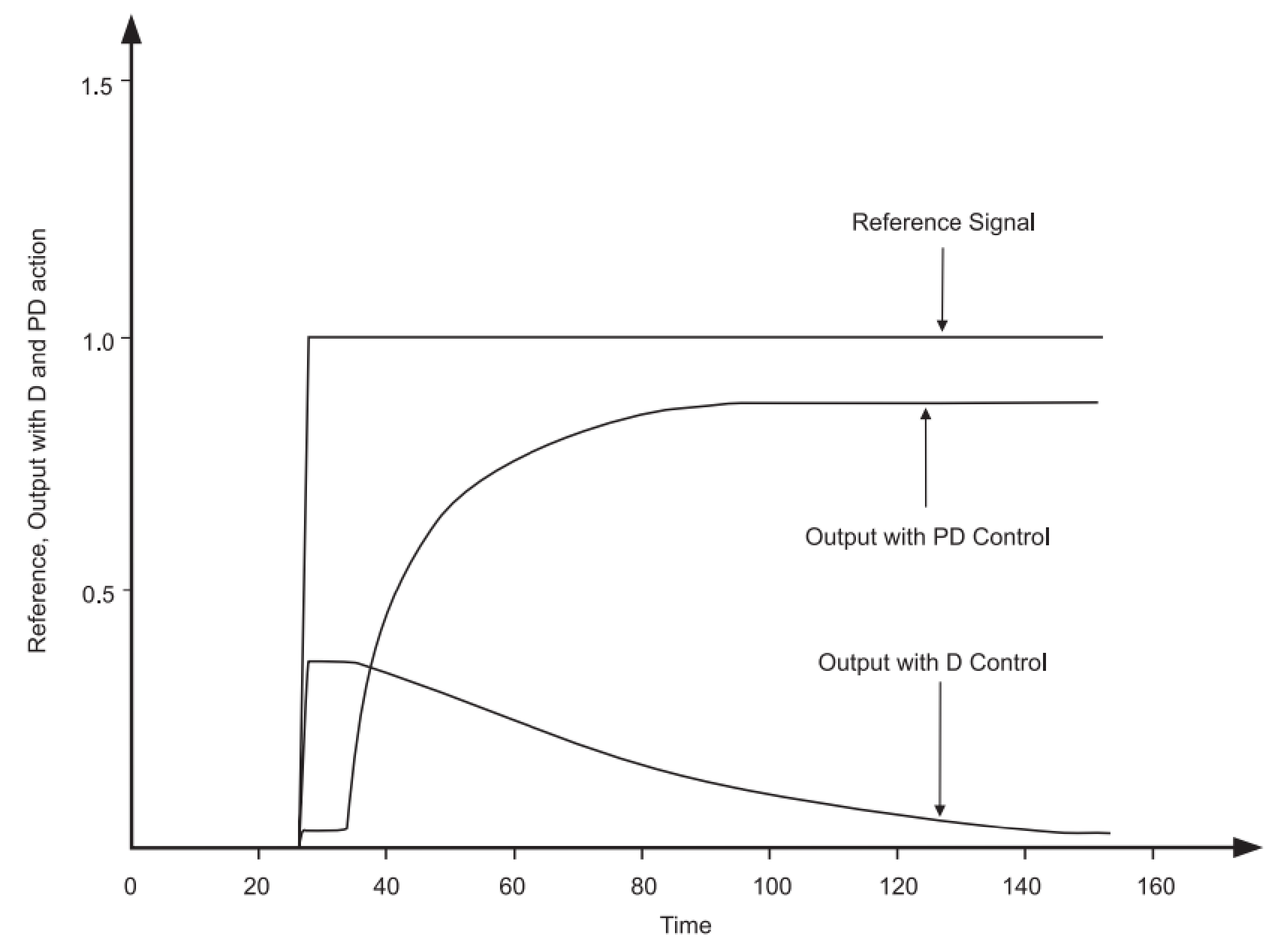

Proportional Amplifier
K_p
e
Σ
u
+
+
$K_d \frac{d}{dt}$
Differential Amplifier


Input Signal
Time
Output Signal
Time

PD CONTROLLER

- Predictive capability



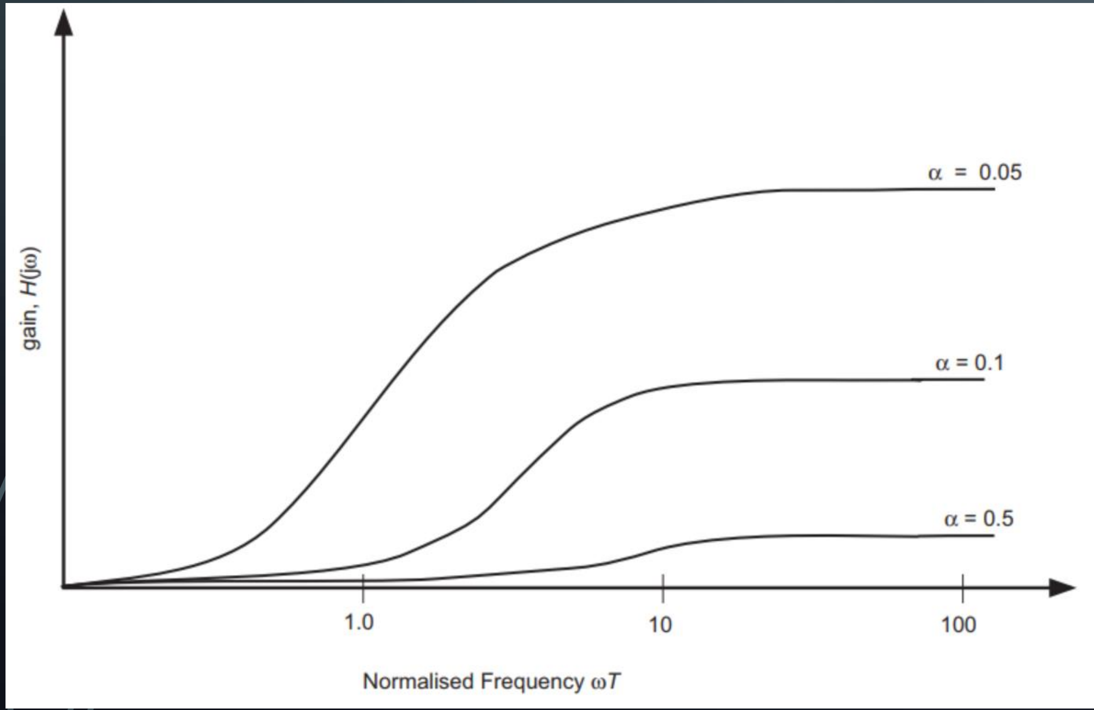


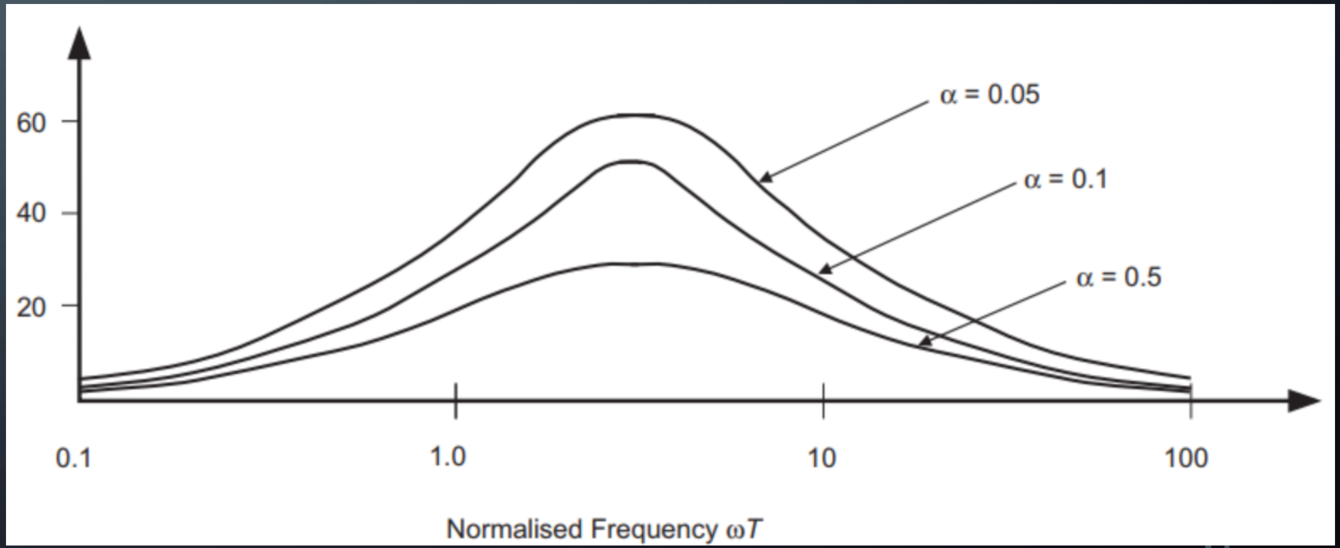
BALL AND BEAM SYSTEM: PHASE ADVANCE CONTROL

- Phase advance control (lead compensator)

$$H(s) = \frac{1 + sT}{1 + s\alpha T}\ \alpha < 1;\ \ H(j\omega) = \frac{1 + j\omega T}{1 + j\omega\alpha T}$$

$$\phi_{\max} = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right);\ \ \omega_{\max} = \frac{1}{T\sqrt{\alpha}}$$

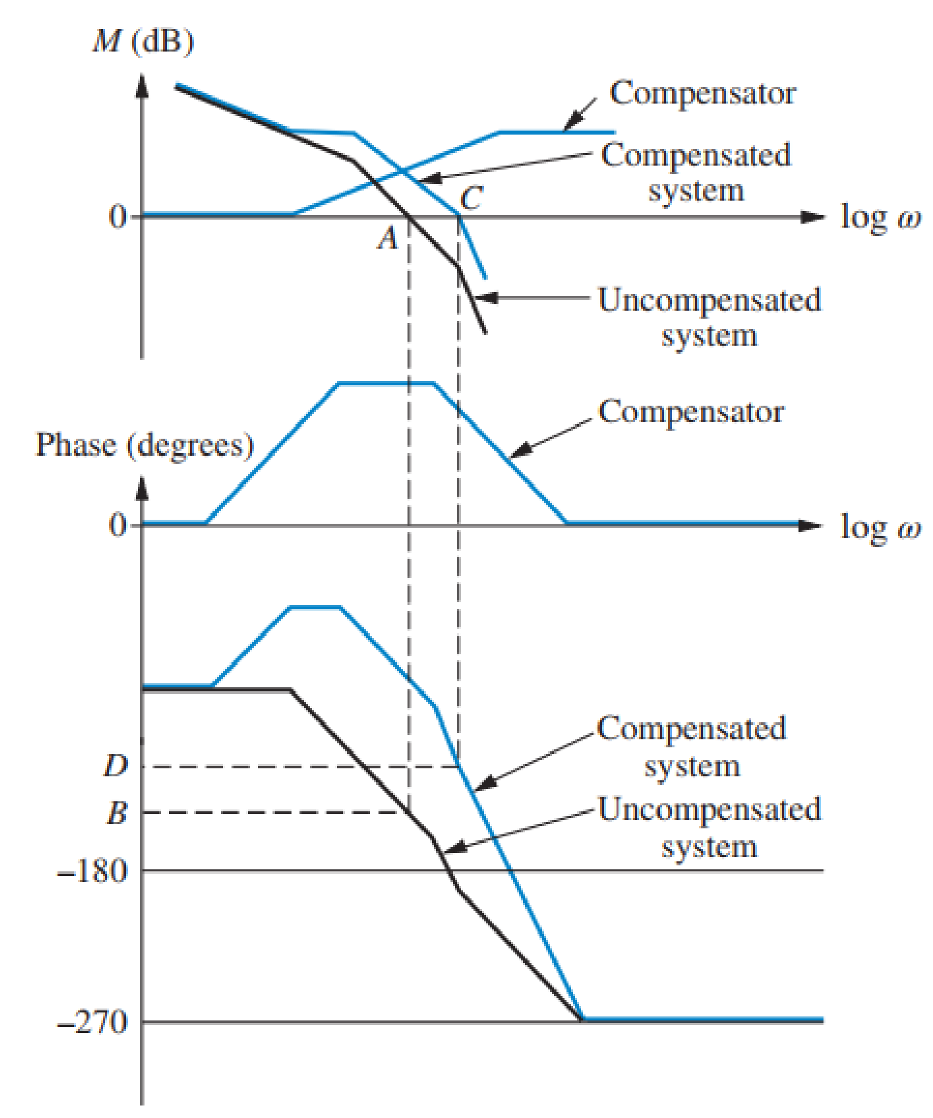




DESIGNING A LEAD COMPENSATOR WITH BODE PLOT

- Use Bode plots
- Common design criteria
  - Overshoot
  - Peak time

Design Criteria	Affecting Parameter
Overshoot	Phase margin
Peak time	Gain crossover frequency



DESIGNING A LEAD COMPENSATOR WITH ROOT LOCUS

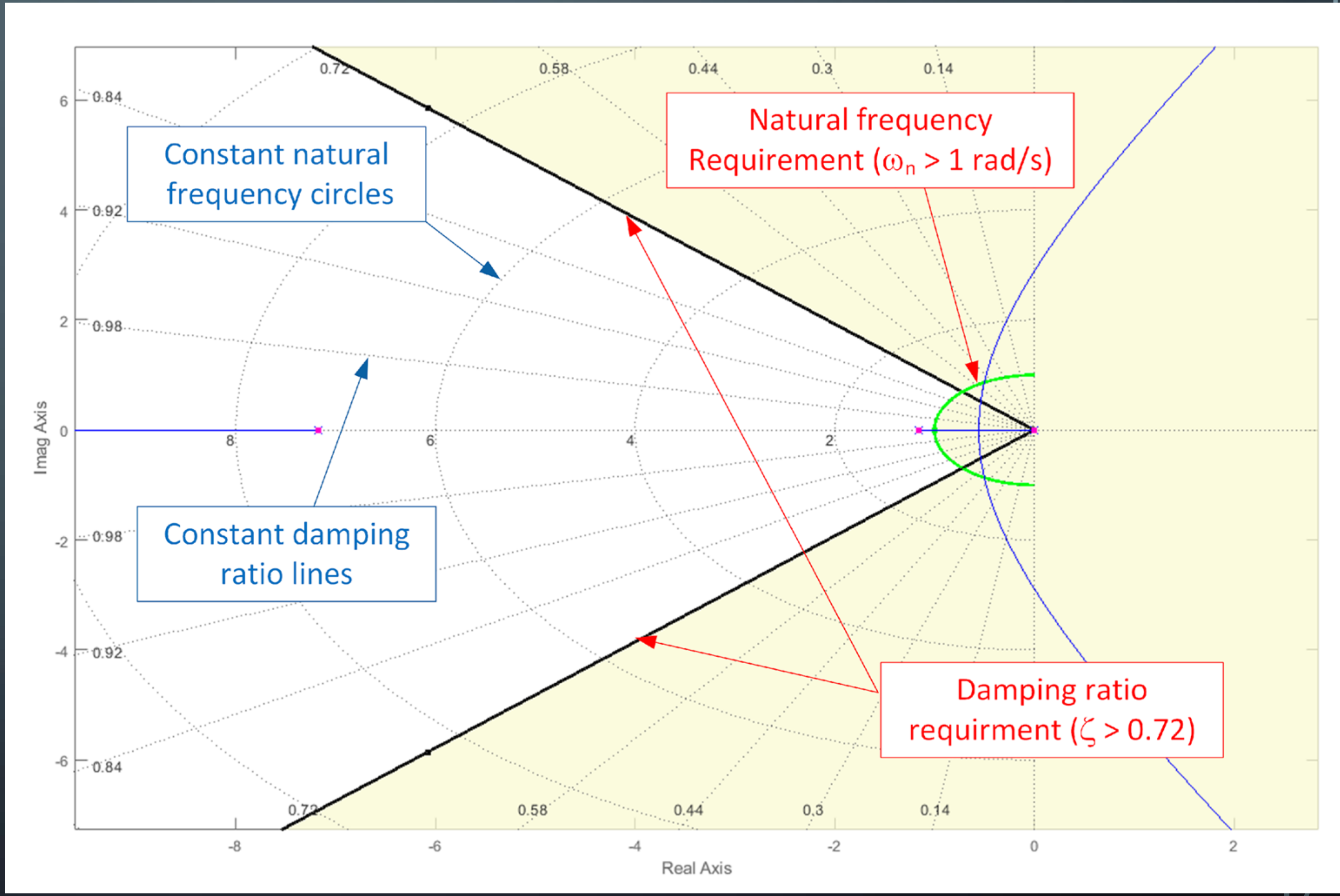
- Graphically superimpose the requirements on the root locus plot

$$P(s) = \frac{22.3}{0.12\,s^3 + s^2 + s}$$

- Requirements:

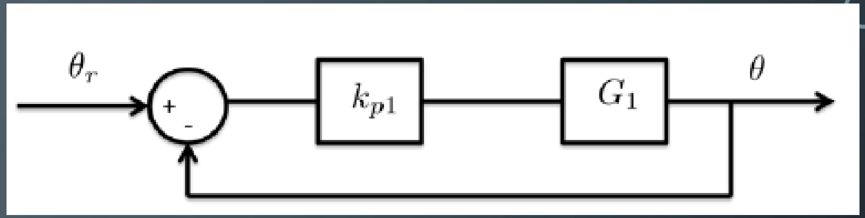
$$\omega_n > 1 \text{ rad/s}$$
$$\zeta > 0.72$$

- Poles must fall inside the white intersecting area!
- Add a lead compensator to shift the locus inside the intersecting region

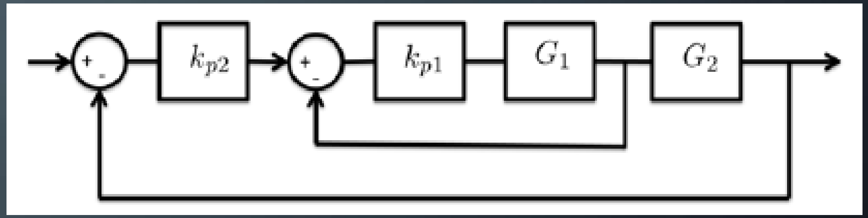


SIMULINK TESTING

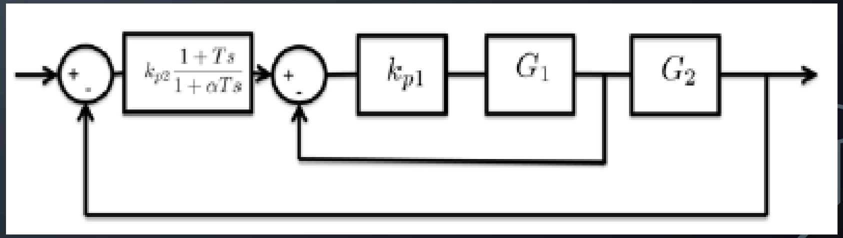
$$T_1(s) = \frac{k_{p1}G_m}{s + k_{p1}G_m}$$



$$T_2(s) = \frac{gk_{p1}k_{p2}G_m}{s^3 + k_{p1}G_m s^2 + gk_{p1}k_{p2}G_m}$$



$$T_3(s) = \frac{gk_{p1}k_{p2}G_m(Ts+1)}{\alpha Ts^4 + (k_{p1}G_m\alpha T + 1)s^3 + k_{p1}G_m s^2 + gk_{p1}k_{p2}G_m(Ts+1)}$$



PHASE ADVANCED CONTROL EXPERIMENT