GEARED SYSTEMS

LAB 2

OUTLINE

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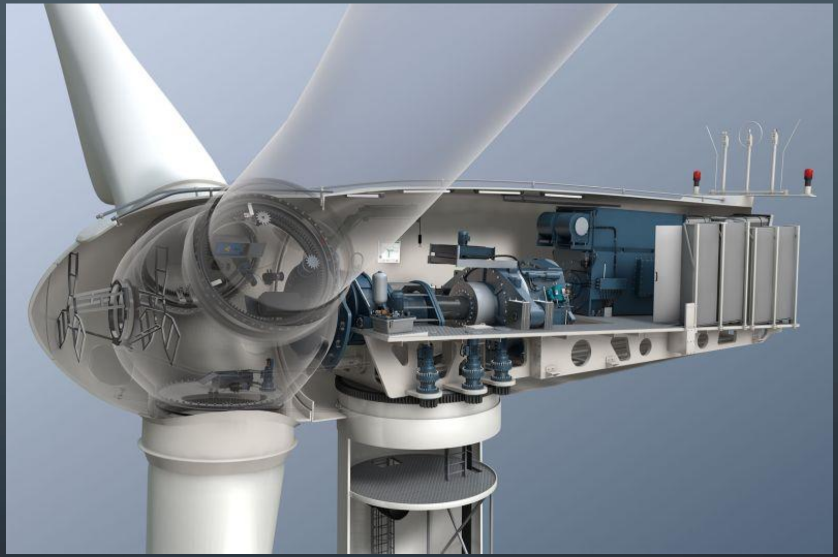
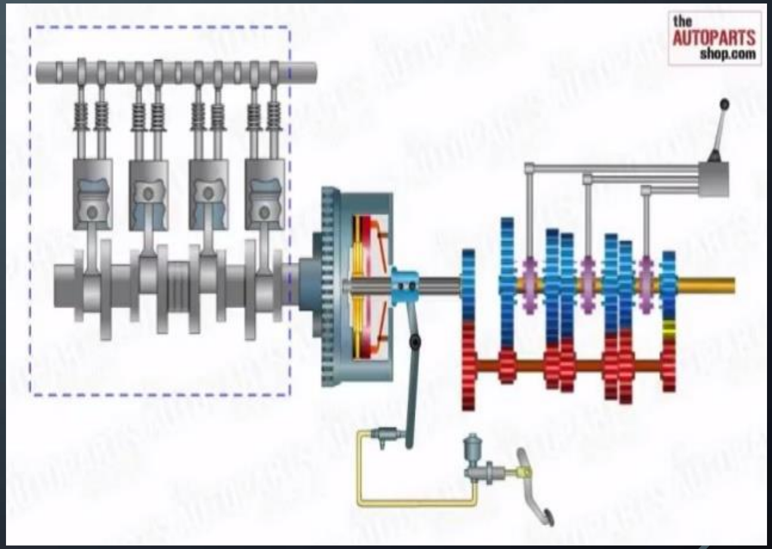
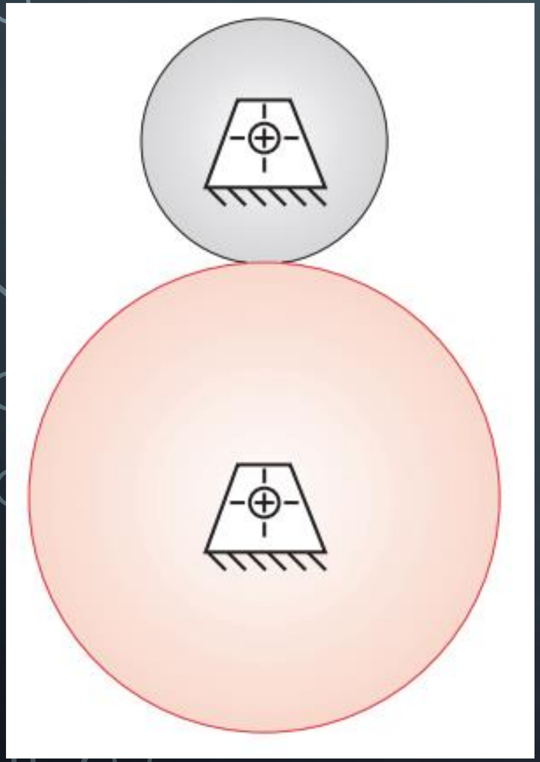
  - Rolling Cylinders & Belt
  - Gear Set
  - Equivalent to Four Bar Linkage
  - Involute Tooth Form
- Types of Gears
- Mechanical Advantage
- Simple Gear Trains
- Compound Gear Trains
- Efficiency
- Moments of Inertia
- Hypothesis Testing

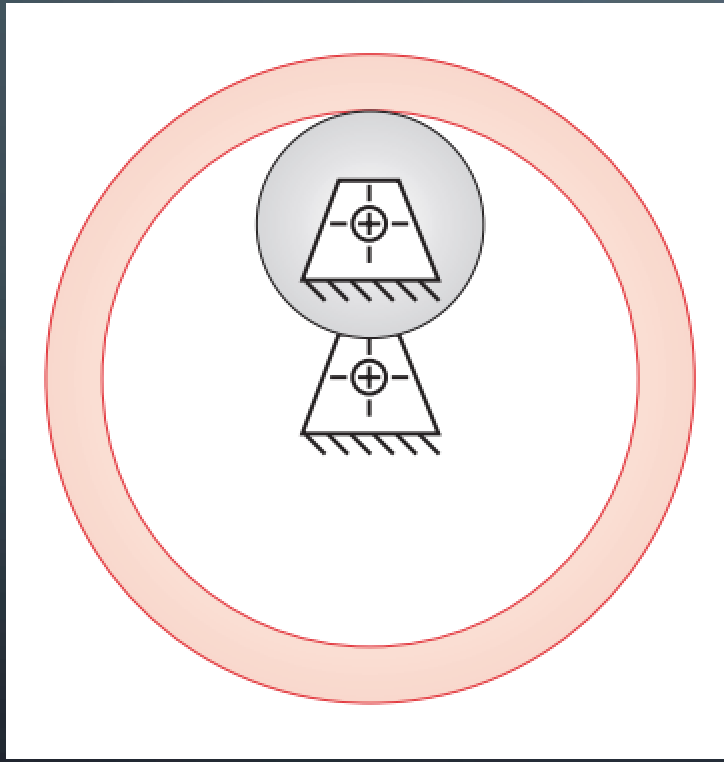
APPLICATIONS

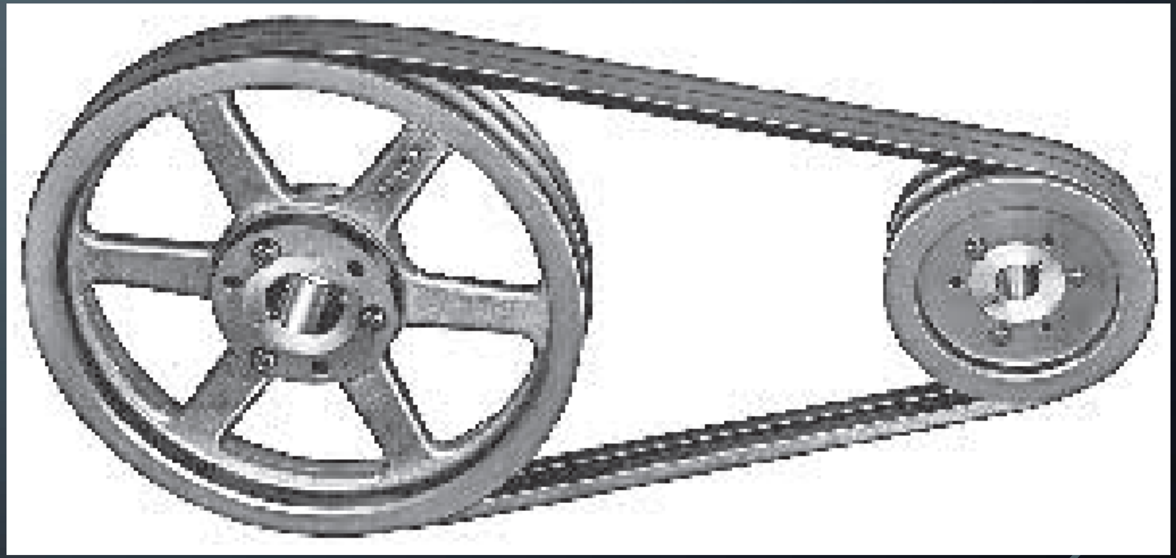




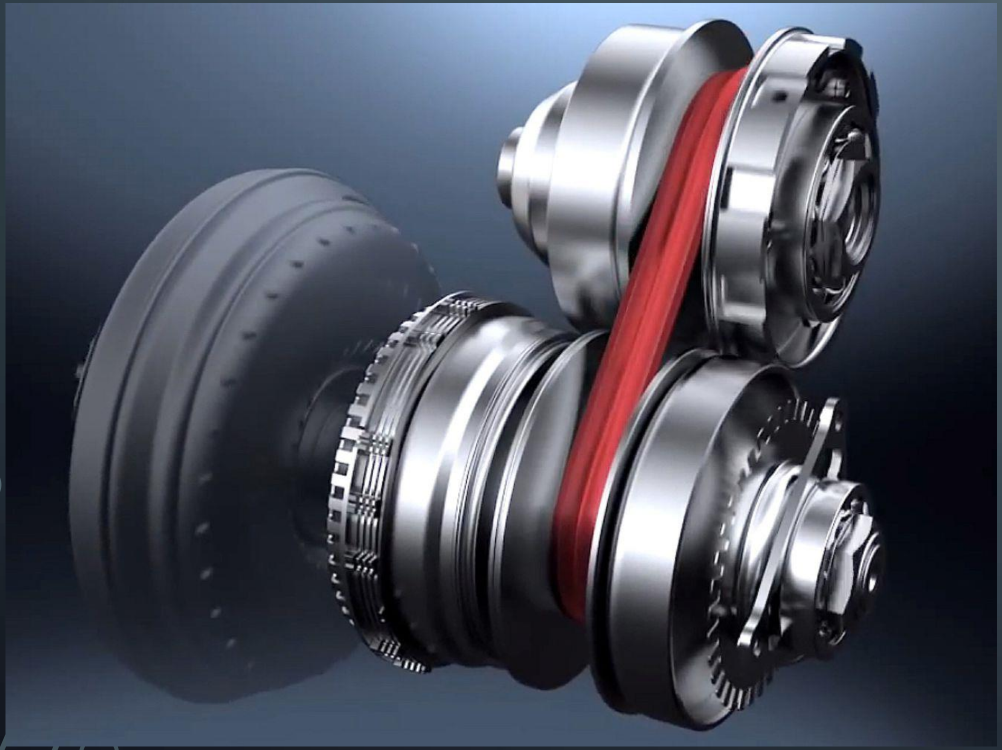
ROLLING CYLINDERS & BELT



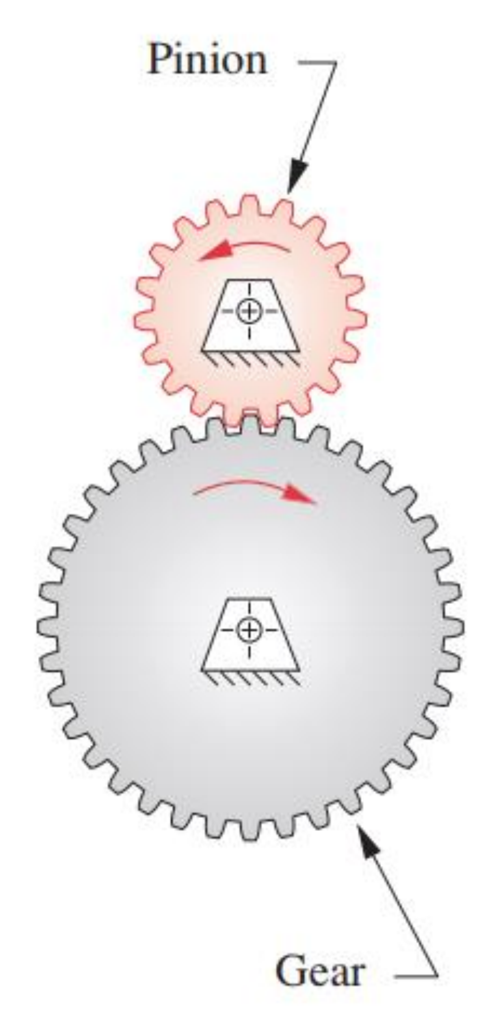




CONTINUOUSLY VARIABLE TRANSMISSION (CVT)



GEAR SET



Fundamental law of gearing:

The angular velocity ratio between the gears of a gearset remains constant throughout the mesh

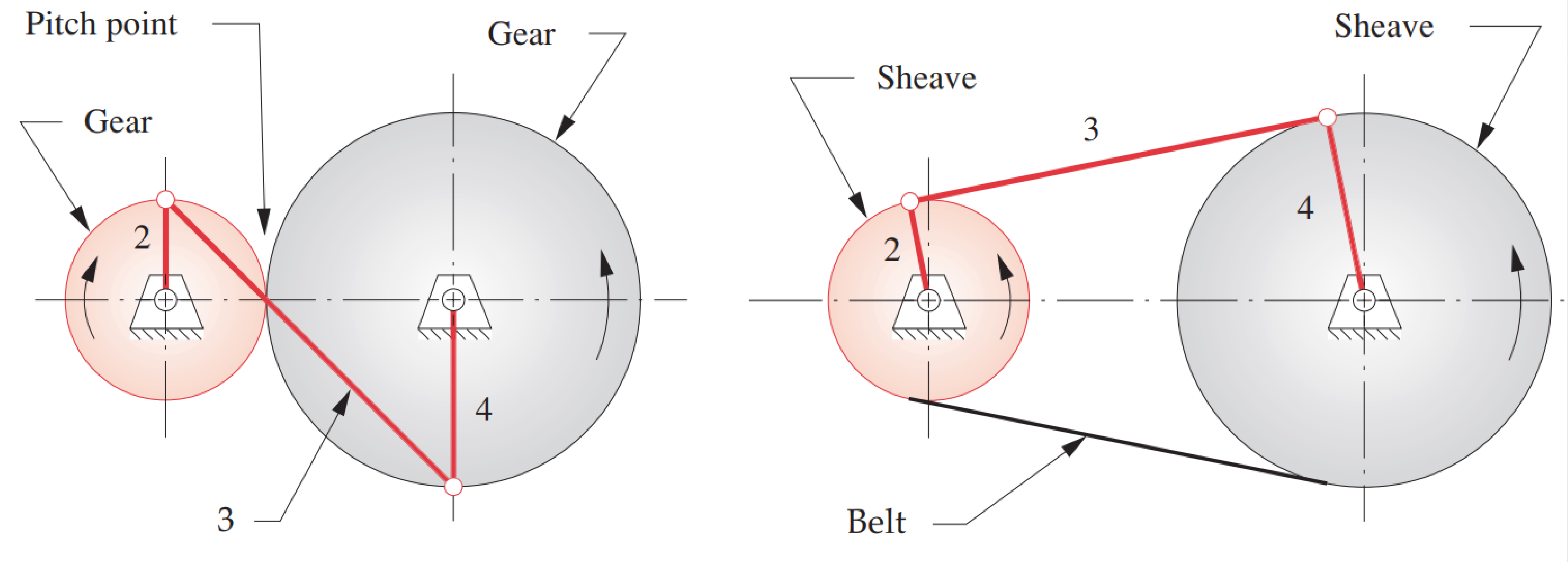
Speed ratio:

$$m_V = \frac{\omega_{out}}{\omega_{in}} = \pm\frac{r_{in}}{r_{out}} = \pm\frac{d_{in}}{d_{out}}$$

Torque ratio:

$$m_T = \frac{\omega_{in}}{\omega_{out}} = \pm\frac{r_{out}}{r_{in}} = \pm\frac{d_{out}}{d_{in}}$$

EQUIVALENT TO FOUR BAR LINKAGE

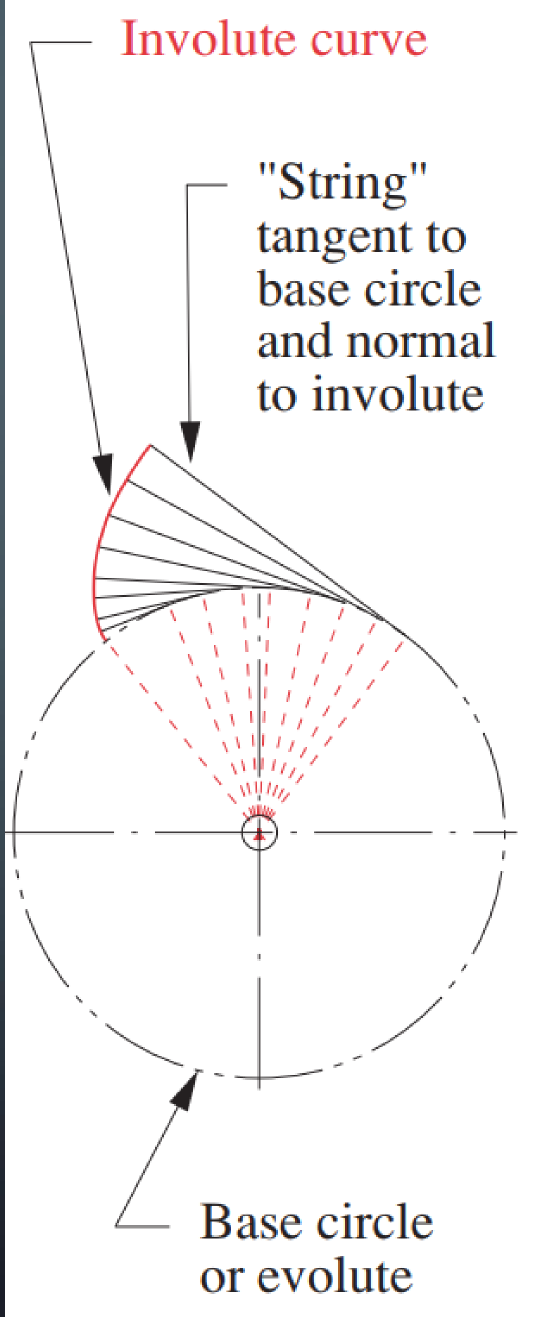


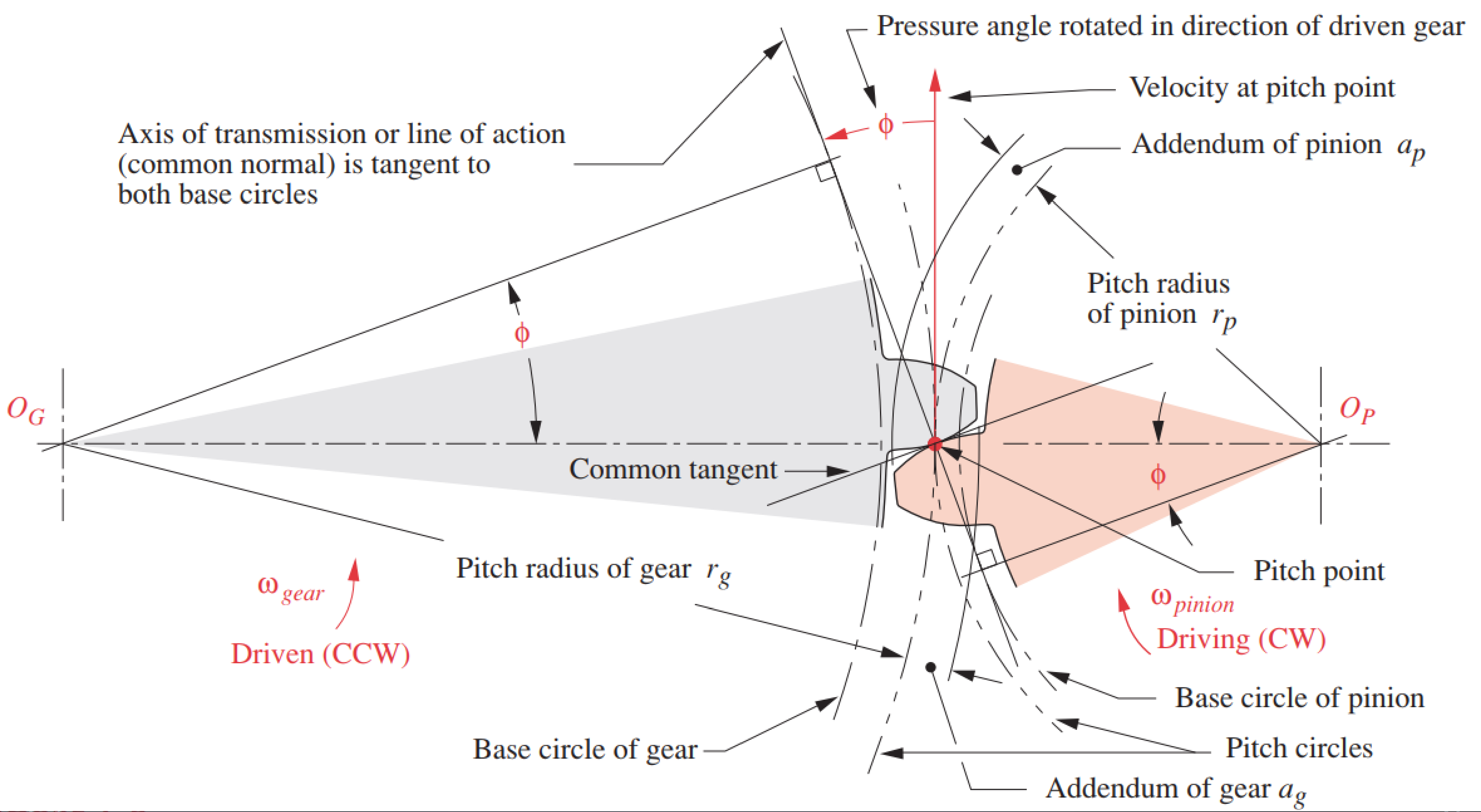
(a) Gear train

(b) Belt train

INVOLUTE TOOTH FORM

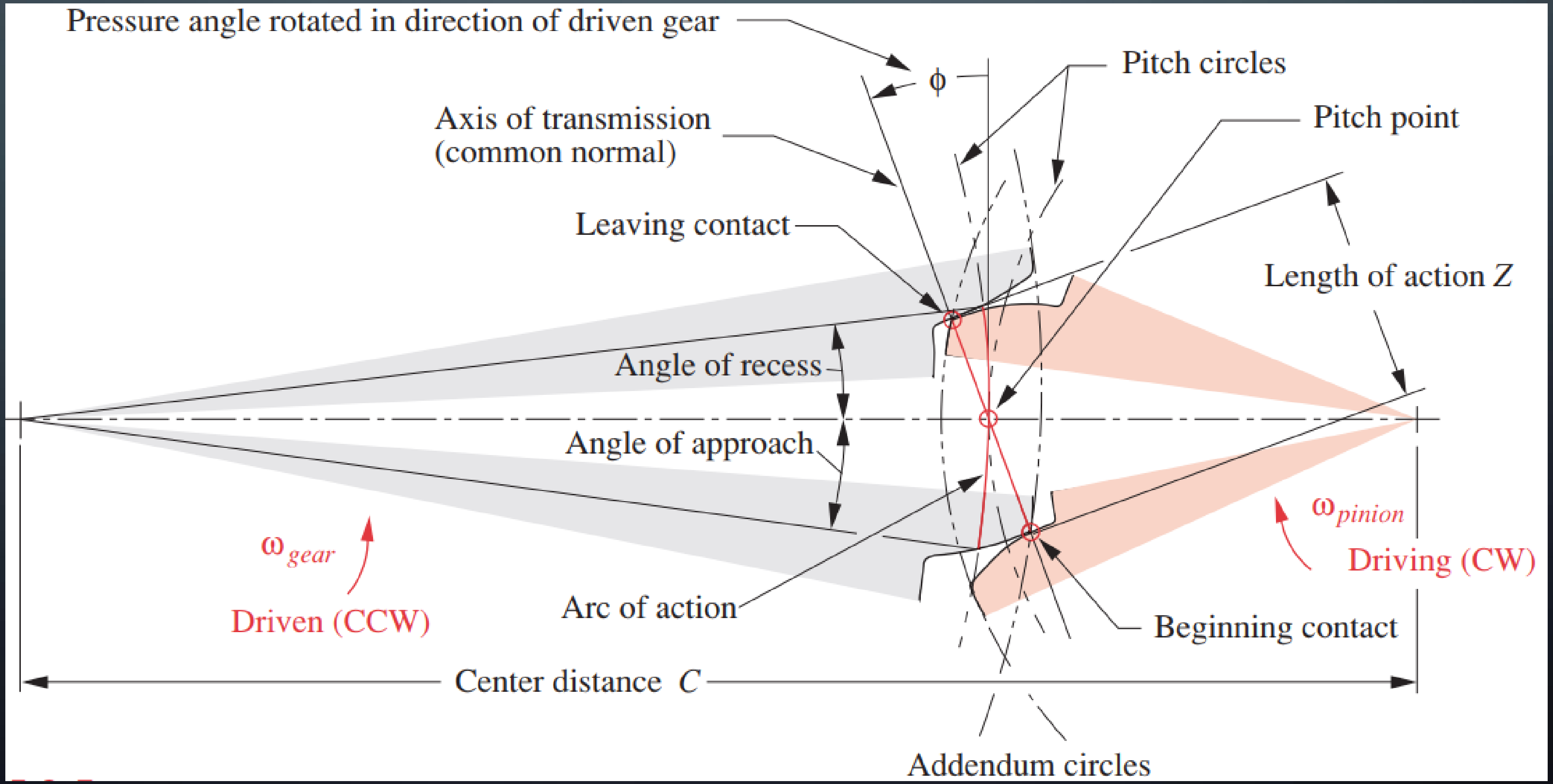
- The string is always tangent to the cylinder.
- The center of curvature of the involute is always at the point of tangency of the string with the cylinder.
- A tangent to the involute is then always normal to the string, the length of which is the instantaneous radius of curvature of the involute curve.



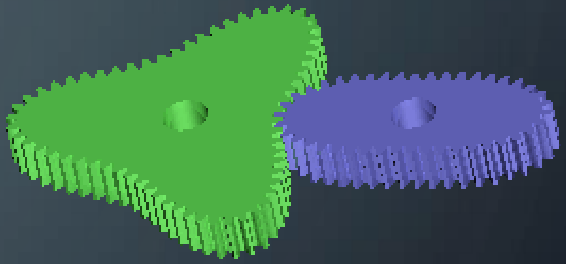

Pressure angle rotated in direction of driven gear
Velocity at pitch point
Axis of transmission or line of action (common normal) is tangent to both base circles
Addendum of pinion a_p
ϕ
Pitch radius of pinion r_p
O_G
O_P
Common tangent
Pitch point
Pitch radius of gear r_g
ω_{gear}
Driven (CCW)
ω_{pinion}
Driving (CW)
Base circle of pinion
Base circle of gear
Pitch circles
Addendum of gear a_g

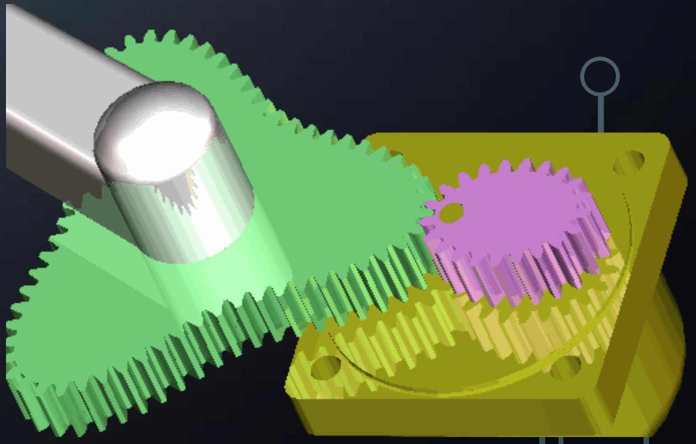
INVOLUTE TOOTH FORM

- Fundamental law of gearing in a more kinematically formal way as: the common normal of the tooth profiles, at all contact points within the mesh, must always pass through a fixed point on the line of centers, called the pitch point.

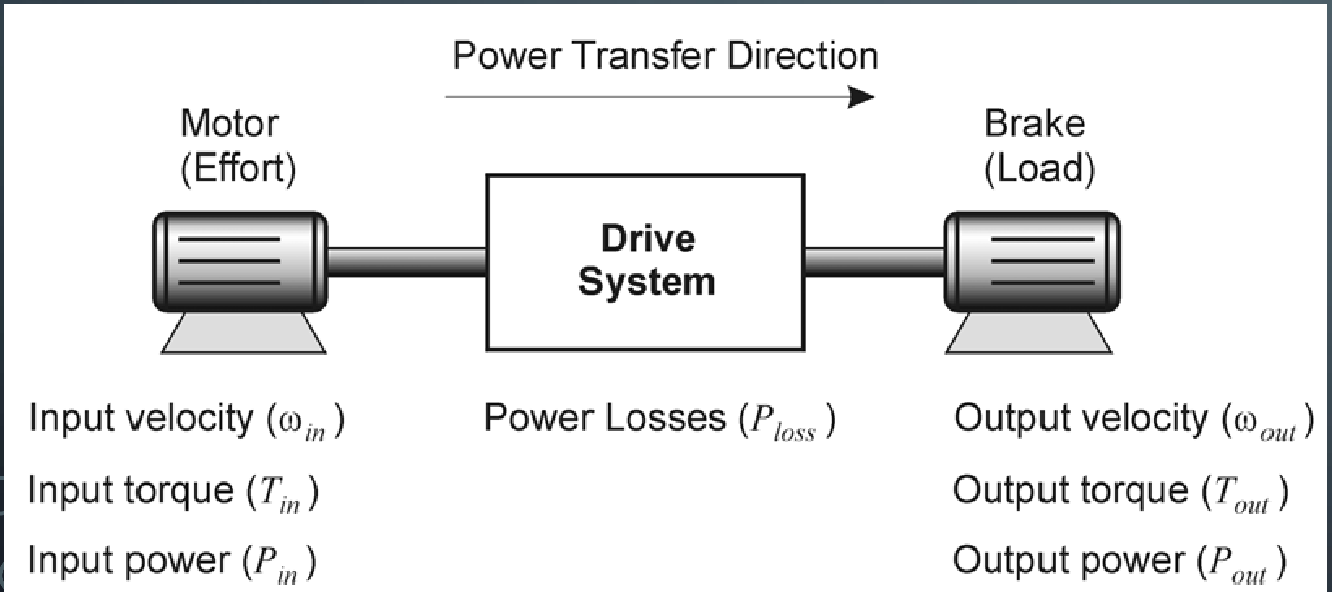


TYPES OF GEARS





MECHANICAL ADVANTAGE



$$\text{Mechanical Advantage (MA)} = \frac{Tq_{out}}{Tq_{in}}$$

$$\text{Velocity Ratio (VR)} = \frac{\omega_{in}}{\omega_{out}}$$

$$\text{Efficiency (in\%)} = \frac{MA}{VR} \times 100$$

EFFICIENCY

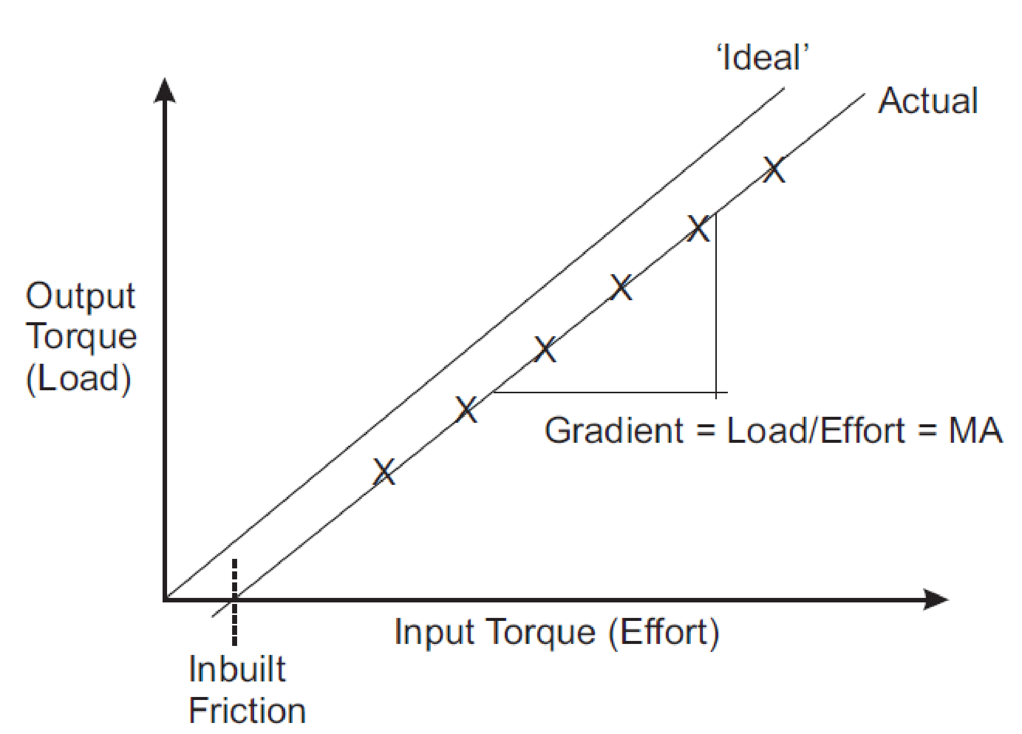
Shaft Power (in Watts) = Shaft Torque (in Nm) x Shaft Speed (in radians)

Motor Shaft Power is the input power (P_{in}) to a drive unit

$$P_{in} = \text{Motor torque x motor speed}$$

Dynamometer Shaft Power is the output power (P_{out}) from a drive unit.

$$P_{out} = \text{Dynamometer torque x dynamometer speed}$$



$$\eta = \frac{P_{out}}{P_{in}} \times 100$$

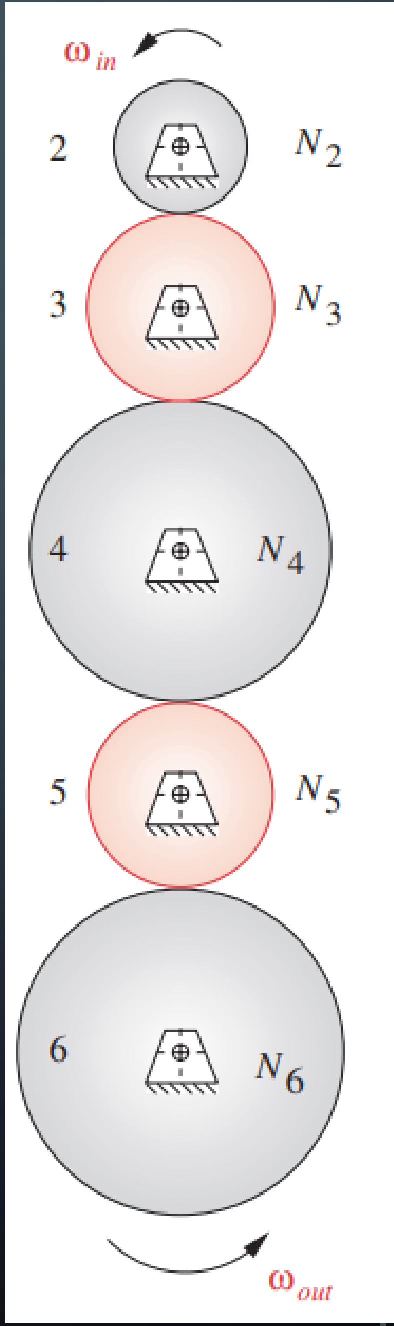
Efficiency (%) = (Gradient/VR) x 100

SIMPLE GEAR TRAINS

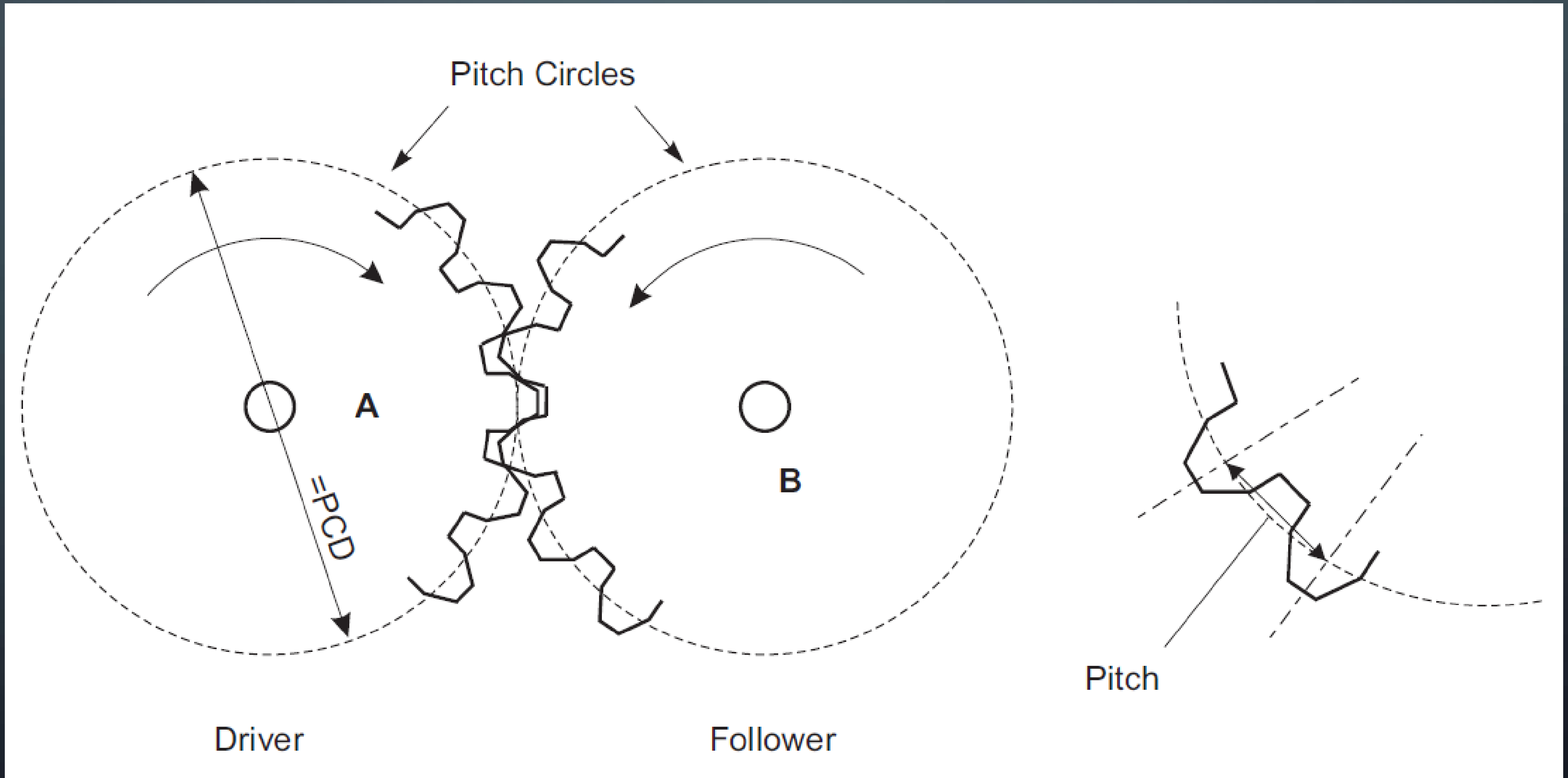
- A simple gear train is one in which each shaft carries only one gear, the most basic, two-gear example of which is shown in Figure

$$m_V = \left(-\frac{N_2}{N_3}\right)\left(-\frac{N_3}{N_4}\right)\left(-\frac{N_4}{N_5}\right)\left(-\frac{N_5}{N_6}\right) = +\frac{N_2}{N_6}$$

$$m_V = \pm\frac{N_{in}}{N_{out}}$$

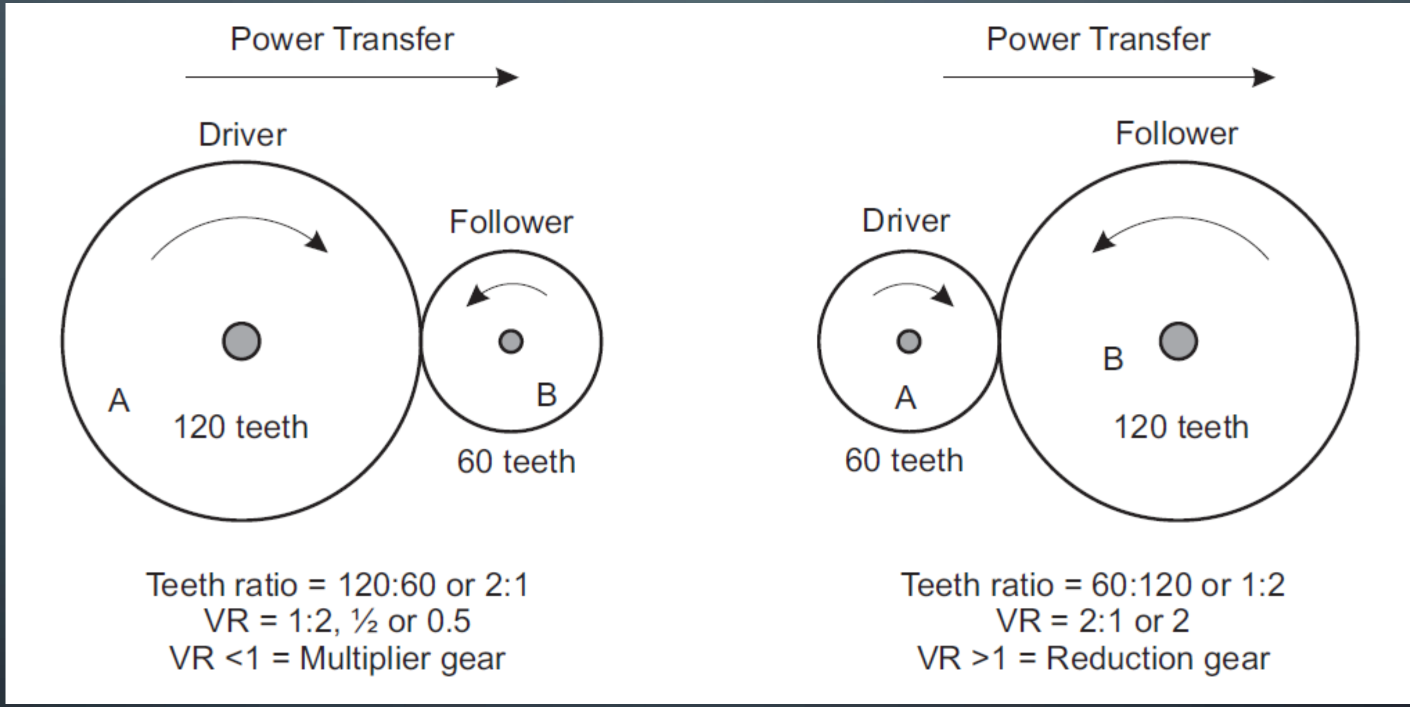


SIMPLE GEAR TRAINS



$$\text{Teeth ratio} = \frac{\text{Teeth on Gear A}}{\text{Teeth on Gear B}} = \frac{\text{Pitch Circle Diameter of Gear A}}{\text{Pitch Circle Diameter of Gear B}}$$

GEAR RATIO



$$\textit{Velocity (gear) ratio} = \frac{\text{Velocity of Driver}}{\text{Velocity of Follower}}$$

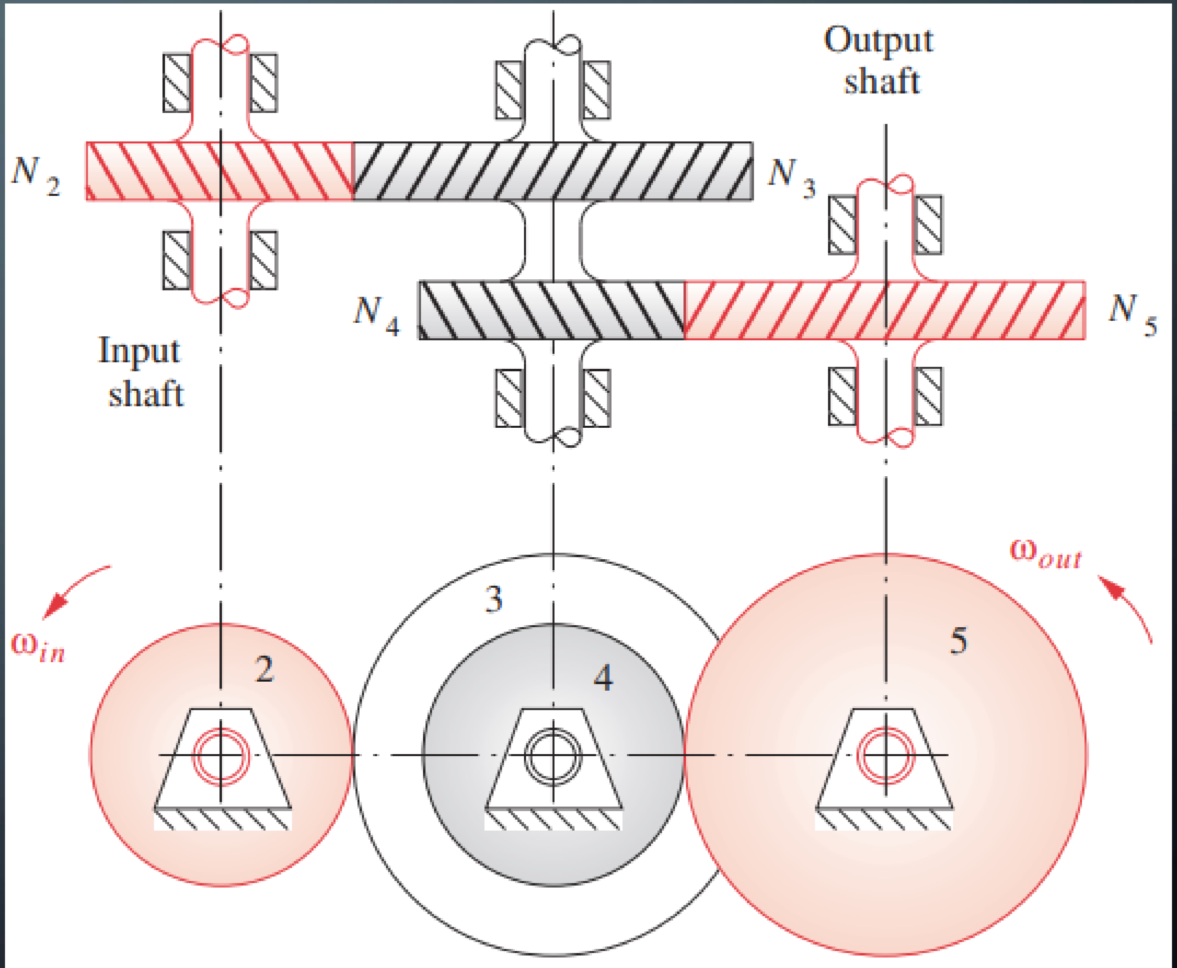
$$= \frac{\text{Velocity of Gear A}}{\text{Velocity of Gear B}} = \frac{\text{Teeth on Gear B}}{\text{Teeth on Gear A}} = \frac{\text{Diameter of Gear B}}{\text{Diameter of Gear A}}$$

COMPOUND GEARS

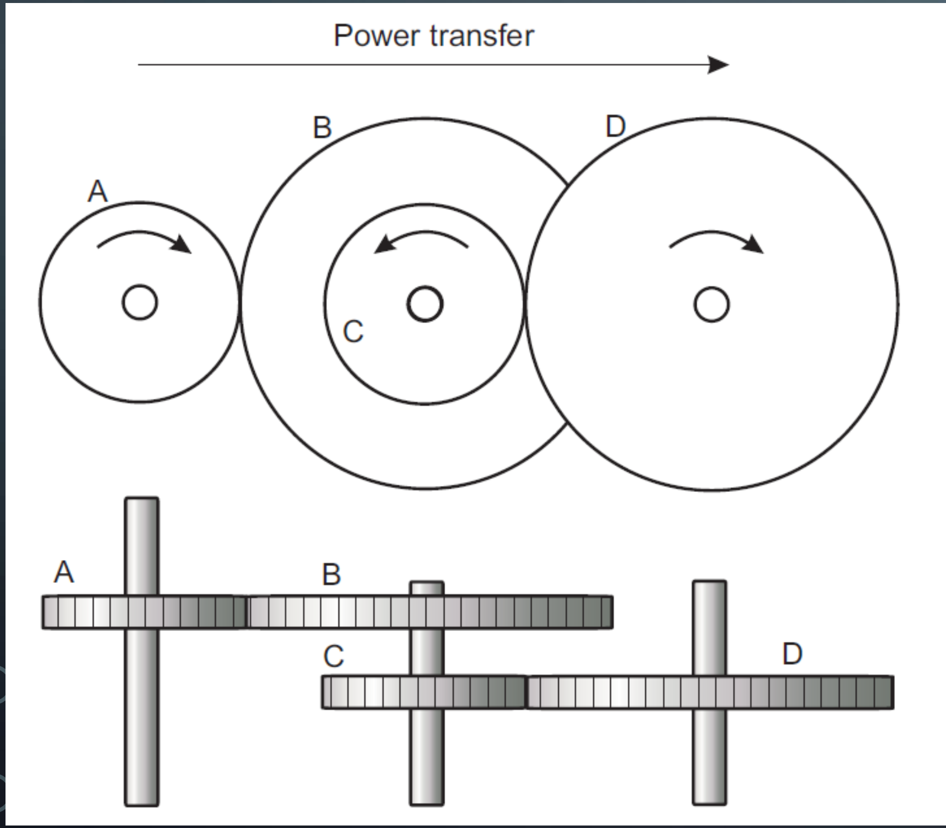
- A compound train is one in which at least one shaft carries more than one gear.
- Gear ratio

$$m_V = \left(-\frac{N_2}{N_3}\right)\left(-\frac{N_4}{N_5}\right)$$

$$m_V = \pm\frac{\text{product of number of teeth on driver gears}}{\text{product of number of teeth on driven gears}}$$



COMPOUND GEARS



$$\frac{\text{Velocity of Gear A}}{\text{Velocity of Gear B}} = \frac{\text{Teeth on Gear B}}{\text{Teeth on Gear A}}$$

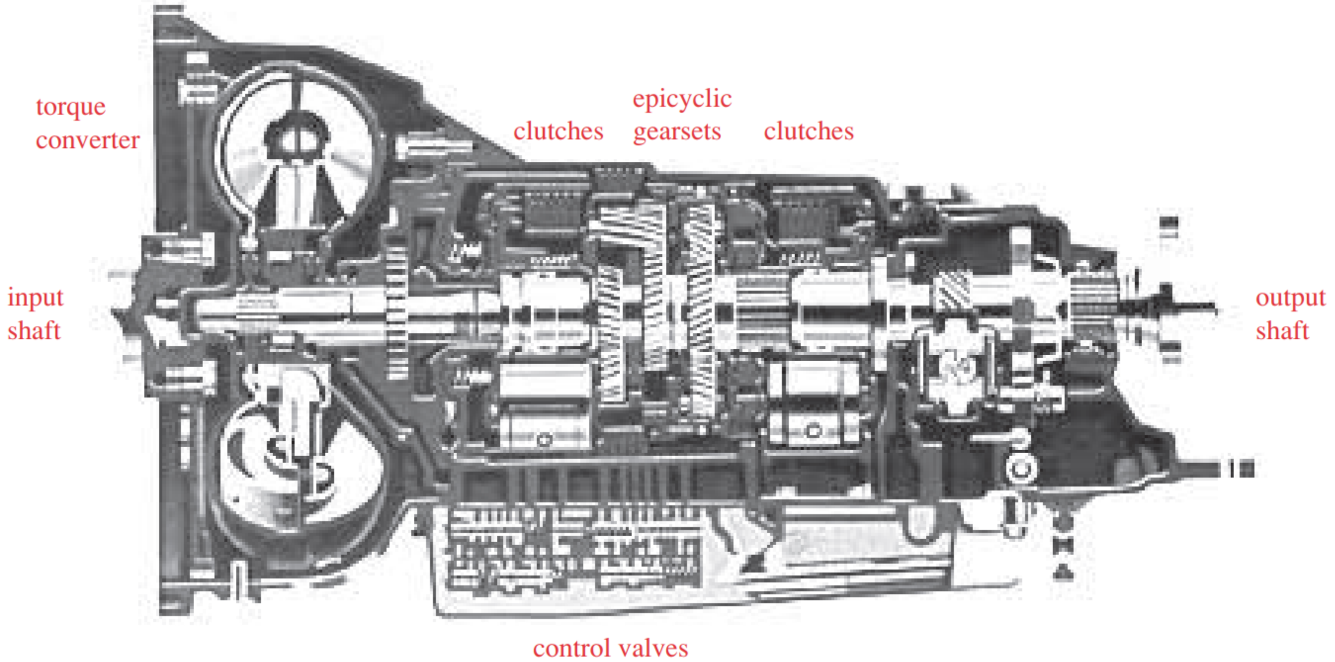
$$\frac{\text{Velocity of Gear C}}{\text{Velocity of Gear D}} = \frac{\text{Teeth on Gear D}}{\text{Teeth on Gear C}}$$

$$\text{Velocity of Gear D} = \text{Velocity of Gear A} \times \frac{\text{Teeth on Gear A}}{\text{Teeth on Gear B}} \times \frac{\text{Teeth on Gear C}}{\text{Teeth on Gear D}}$$

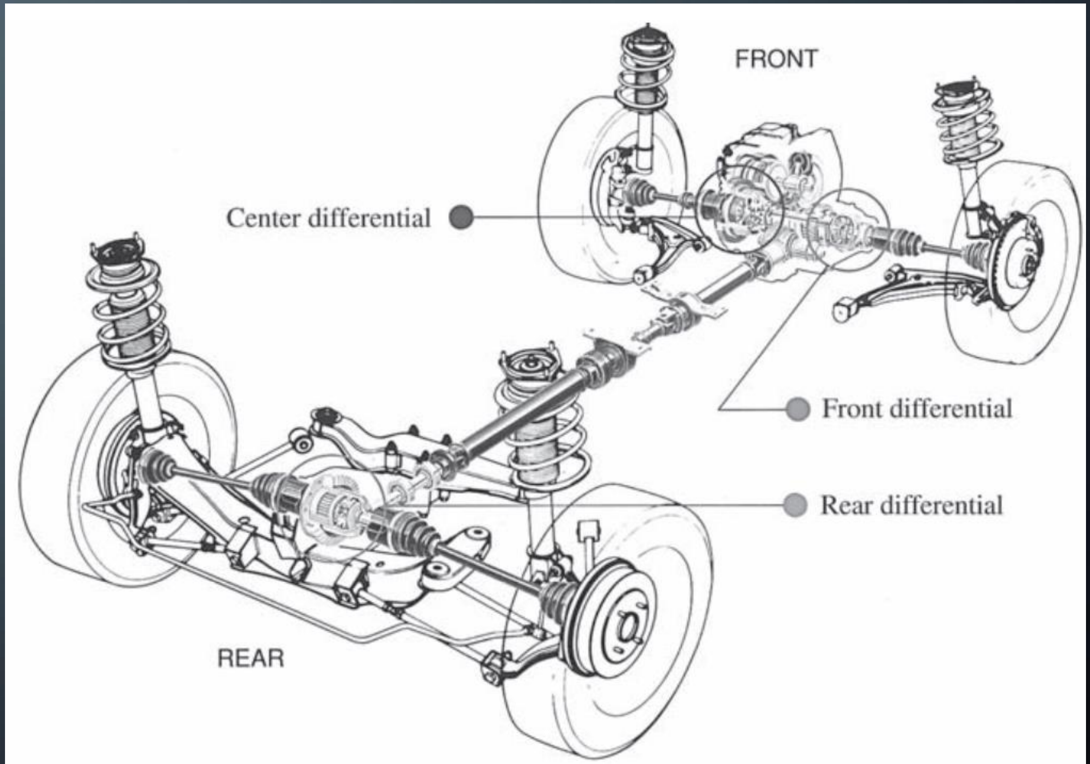
$$\text{Overall Velocity (Gear) Ratio} = \frac{\text{Product of Teeth or Diameters on Follower Gears}}{\text{Product of Teeth or Diameters on Driver Gears}}$$

OR

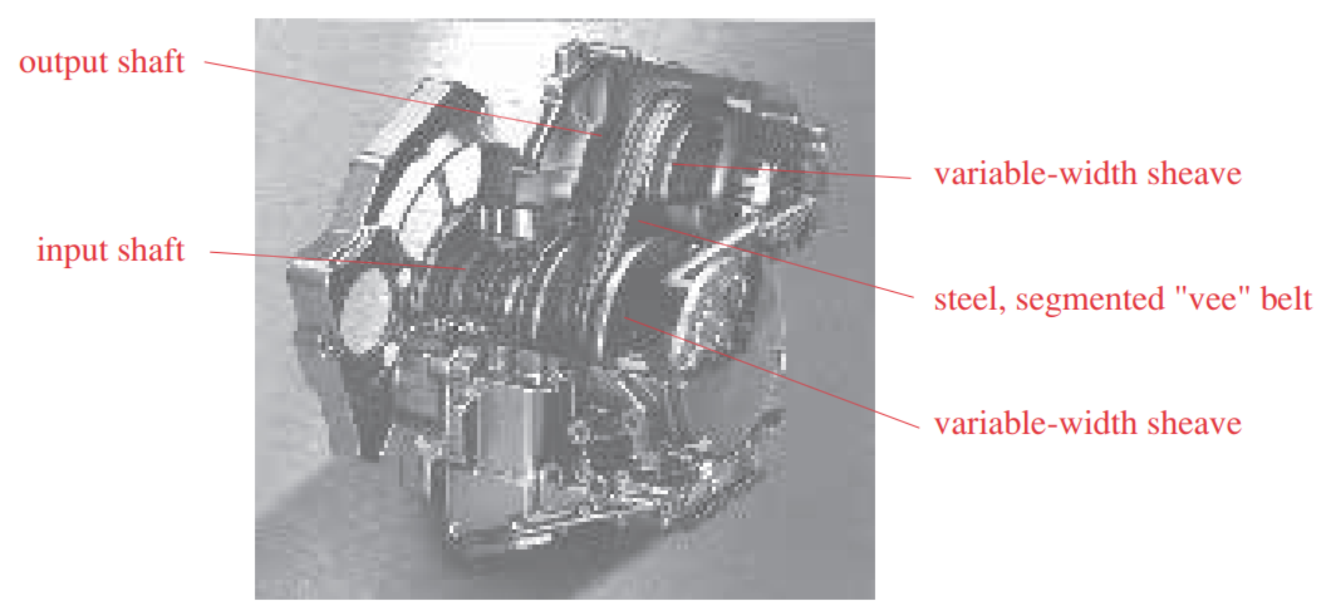
$$\text{Overall Velocity (Gear) Ratio} = \frac{\text{Product of Radii on Follower Gears}}{\text{Product of Radii on Driver Gears}}$$



torque
converter
clutches
epicyclic
gearsets
clutches
input
shaft
output
shaft
control valves

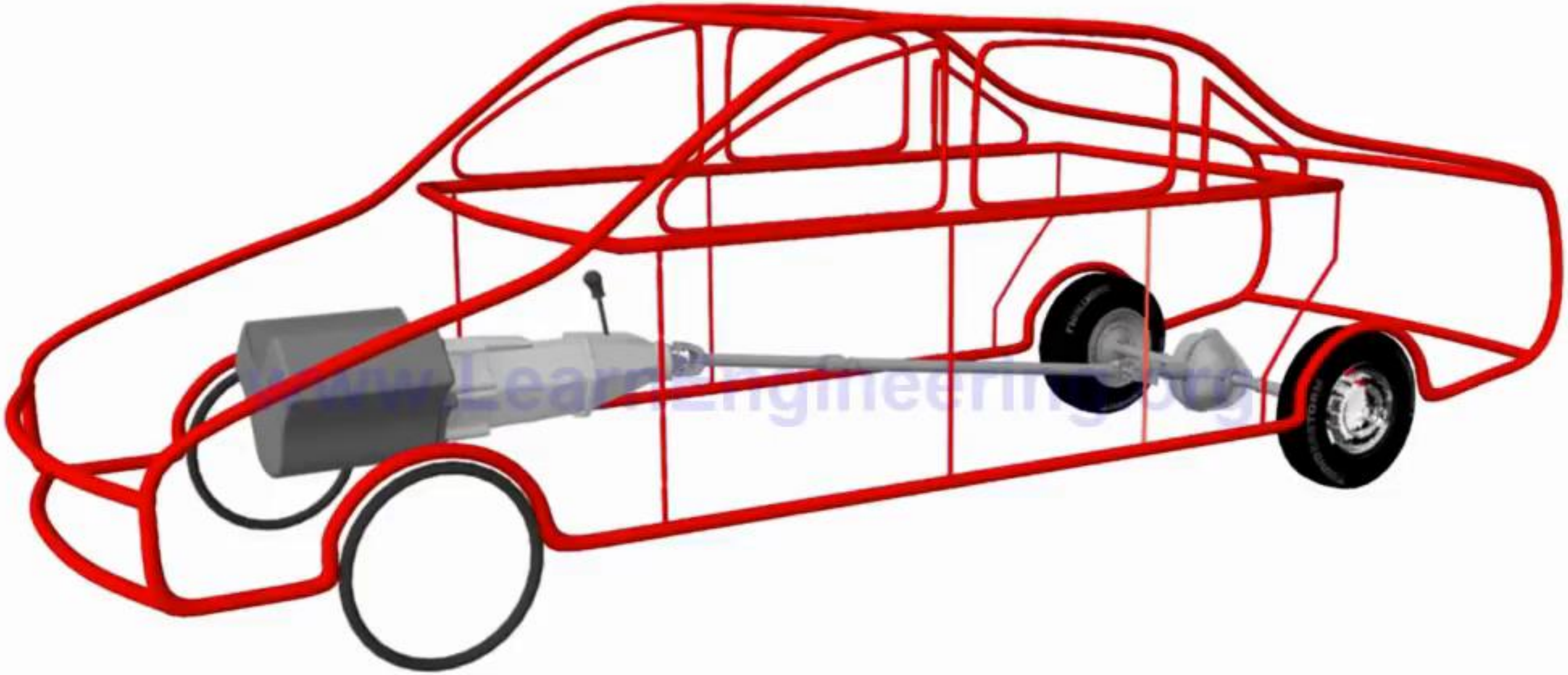


FRONT
Center differential
Front differential
Rear differential
REAR

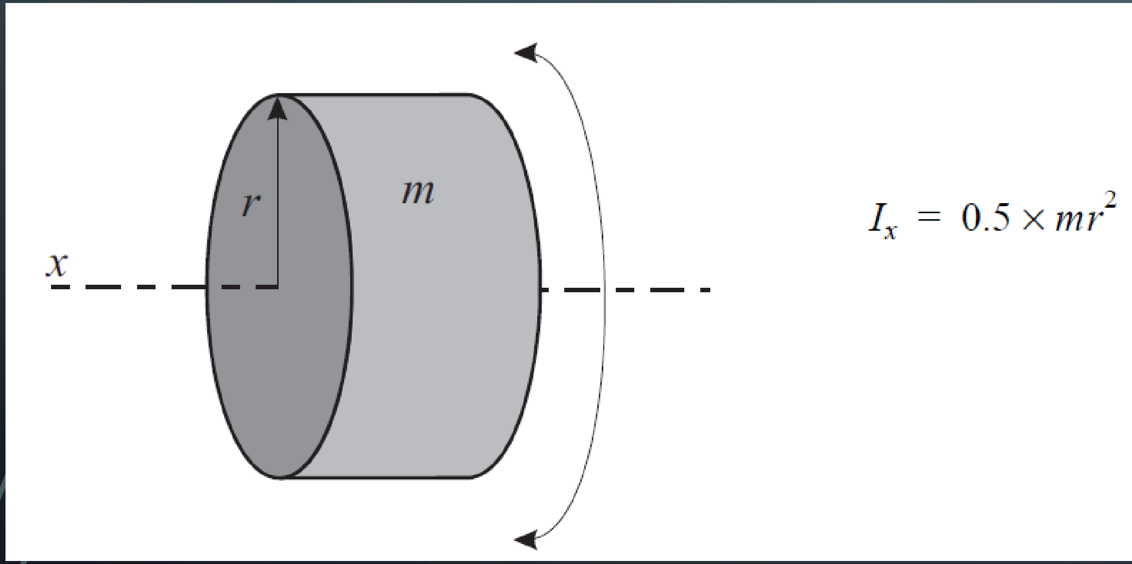


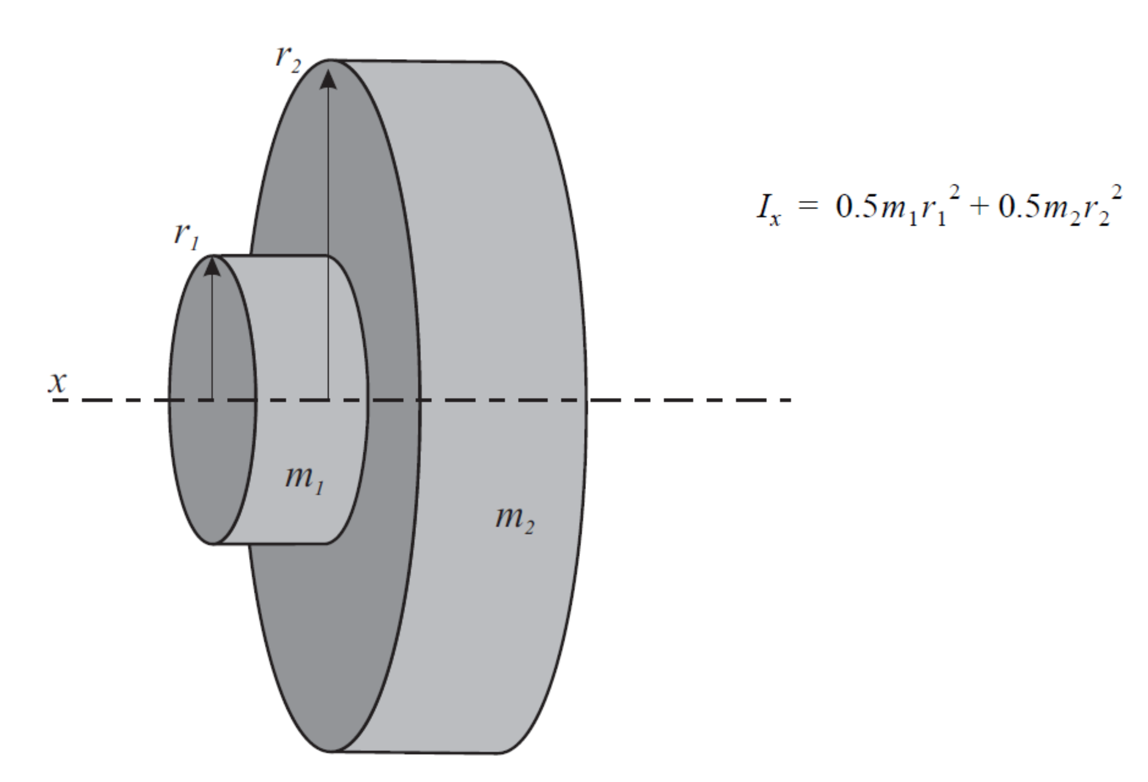
output shaft
input shaft
variable-width sheave
steel, segmented "vee" belt
variable-width sheave

MANUAL TRANSMISSION



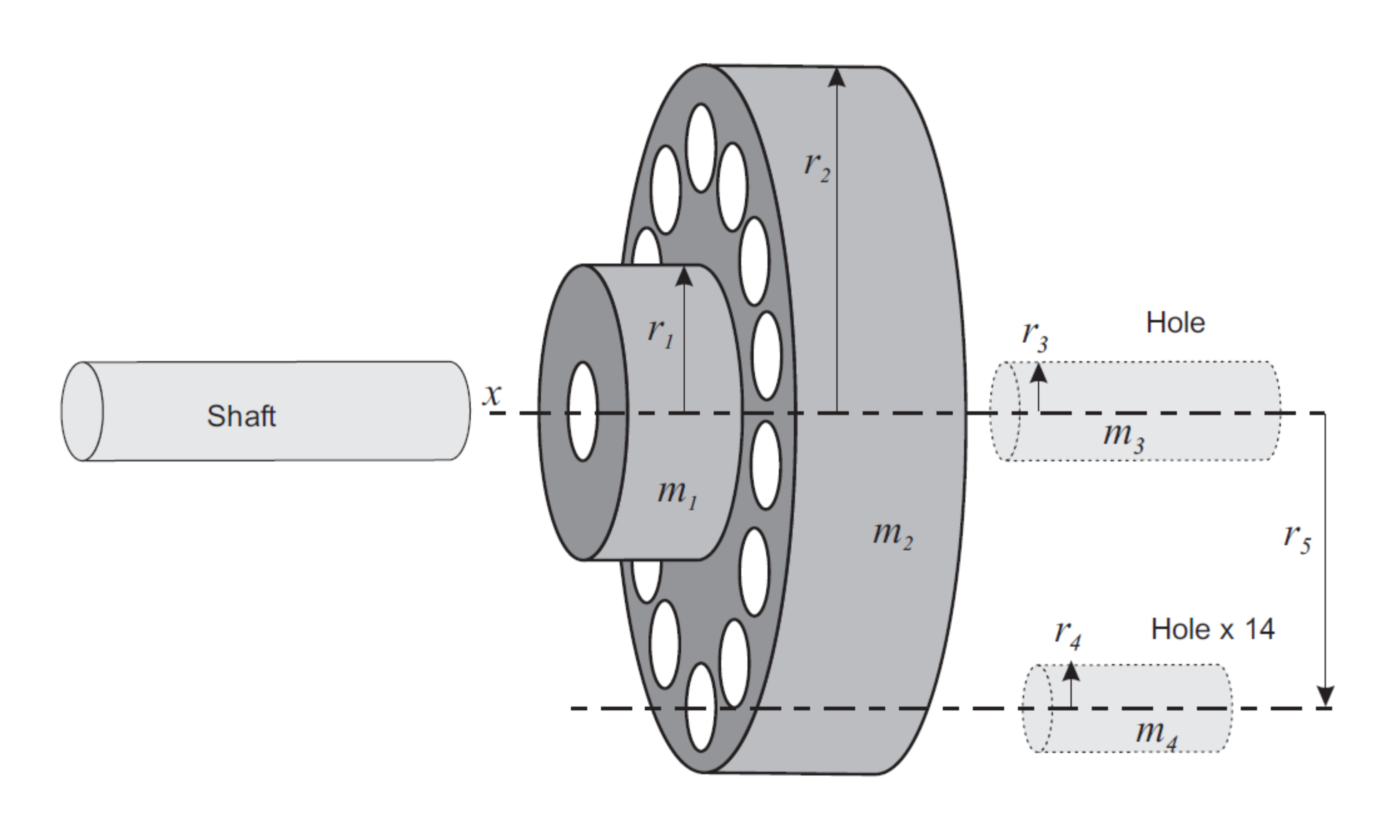
MOMENTS OF INERTIA



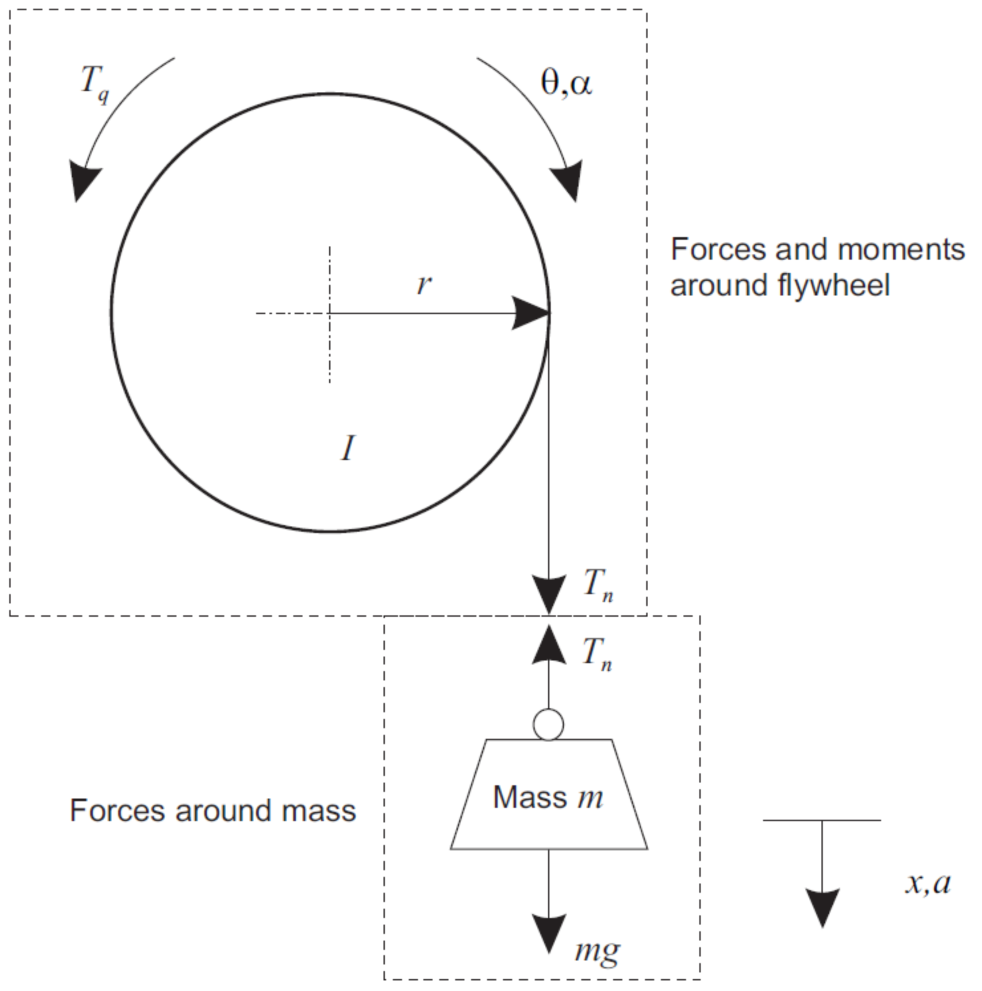


MOMENTS OF INERTIA

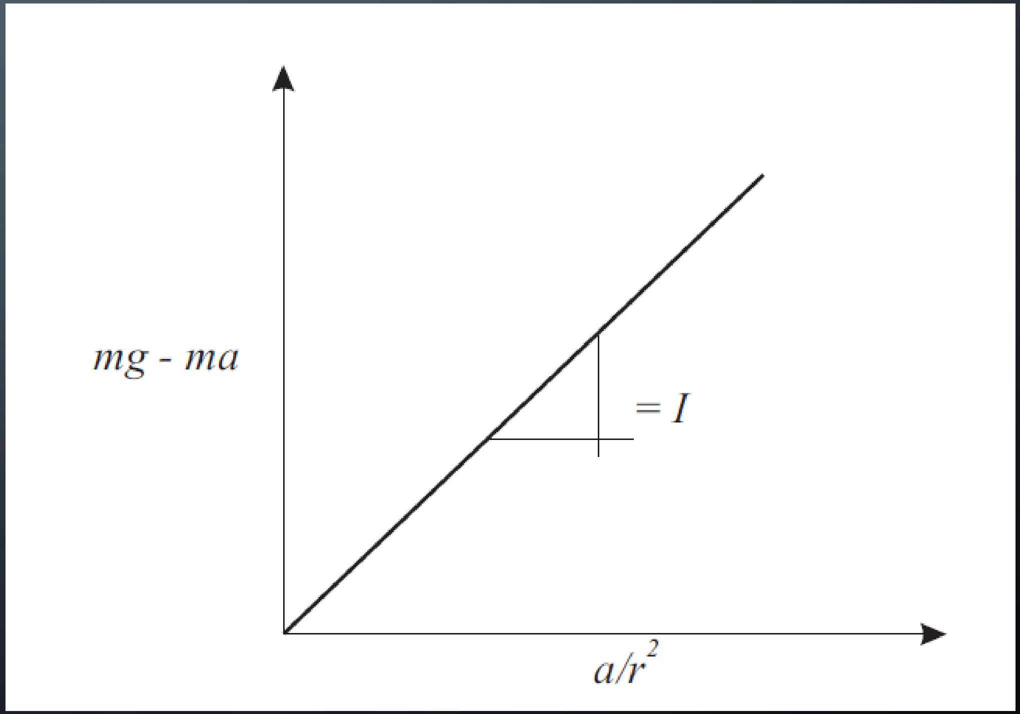
$$I_x = I_{shaft} + 0.5m_1r_1^2 + 0.5m_2r_2^2 - 0.5m_3r_3^2 - 14\{0.5(m_4r_4^2 + m_4r_5^2)\}$$



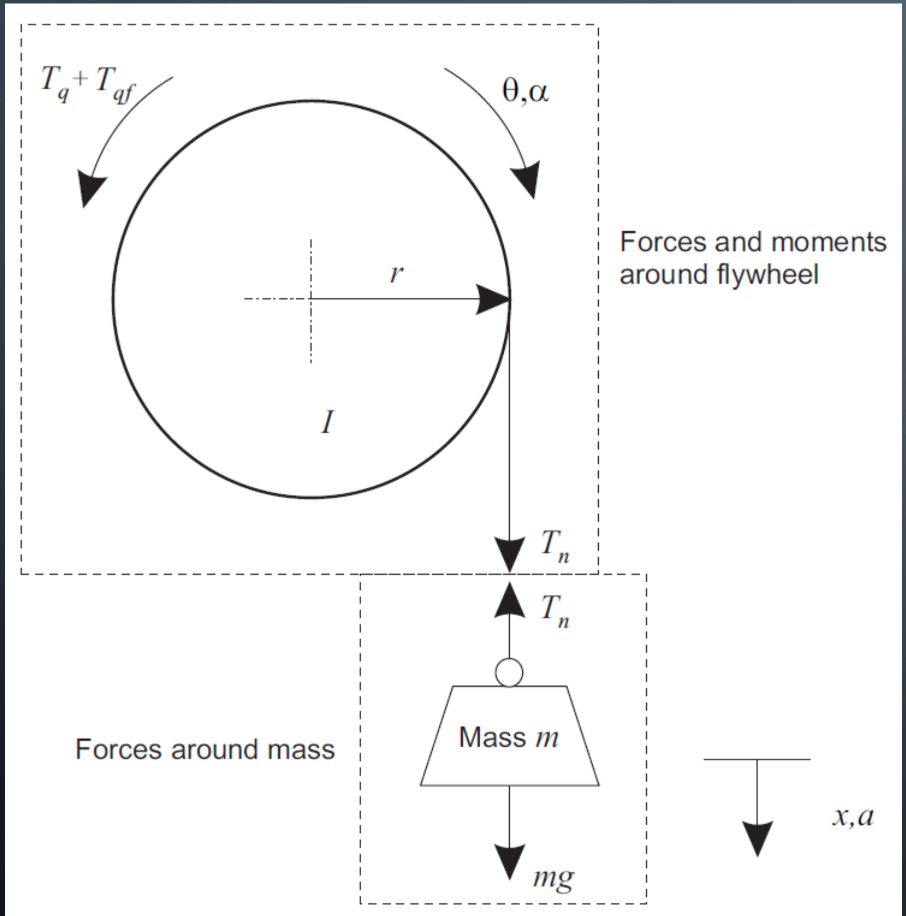
FINDING MOMENTS OF INERTIA



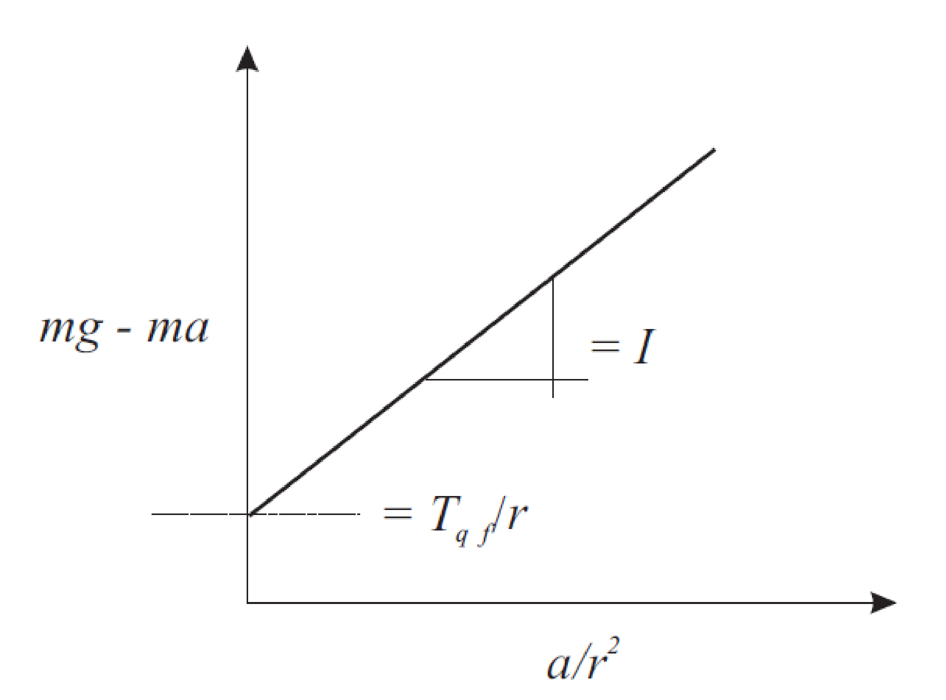
$$mg - ma = I\frac{a}{r^2}$$



FINDING MOMENTS OF INERTIA

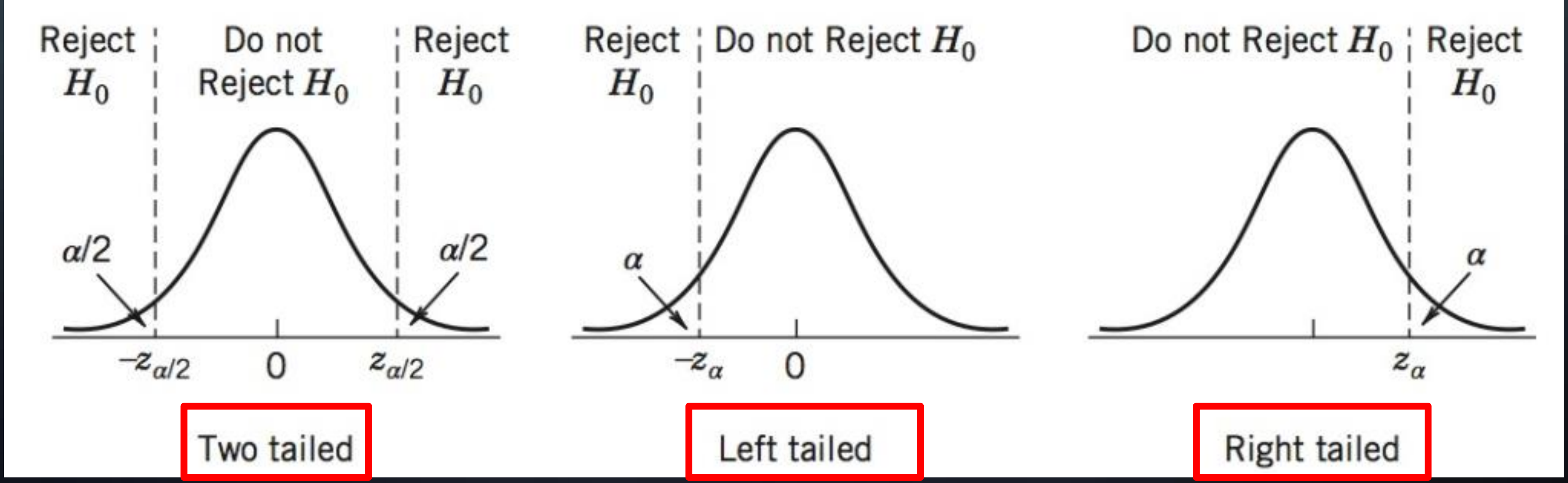


$$mg - ma = I\frac{a}{r^2} + \frac{T_{qf}}{r}$$



HYPOTHESIS TESTING

- Two-tailed test $H_0: x' = x_0,\ \ H_a: x' \neq x_0$
- Left-tailed test $H_0: x' \geq x_0,\ \ H_a: x' < x_0$
- Right-tailed test $H_0: x' \leq x_0,\ H_a: x' > x_0$



HYPOTHESIS TESTING

- z-test

$$z_0 = \frac{\bar{x} - x_0}{\sigma/\sqrt{N}}$$

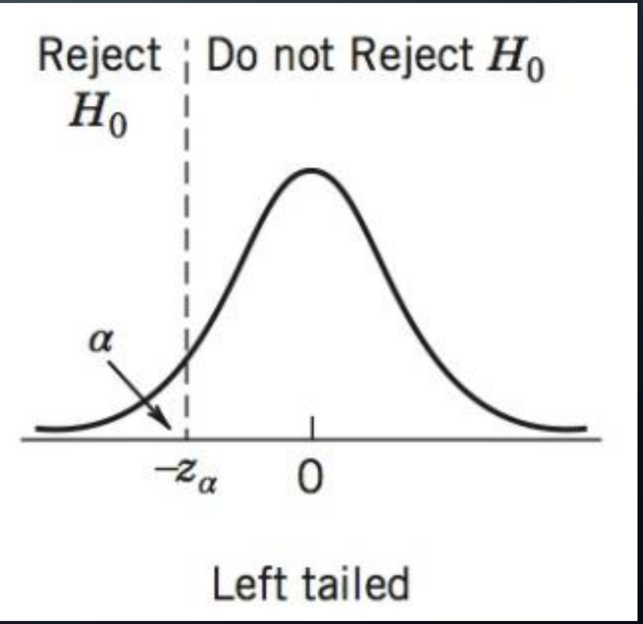
- t-test

$$t_0 = \frac{\bar{x} - x_0}{s_x/\sqrt{N}}$$

- level of significance α

$$P(z) \equiv 1 - \alpha$$

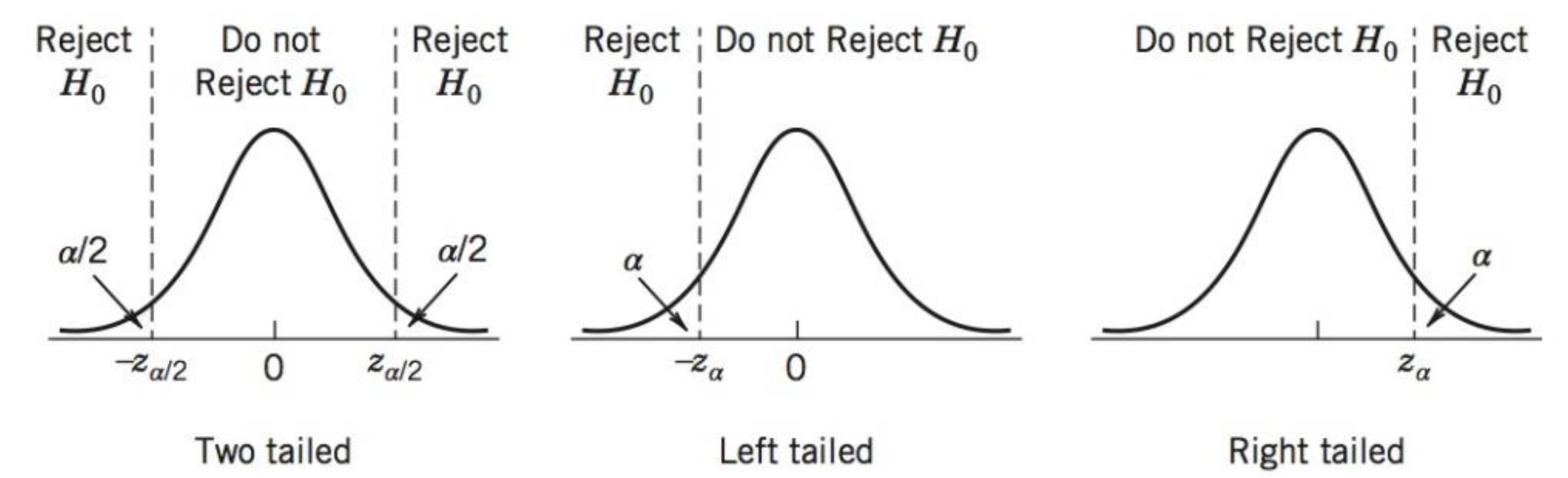
- p-value
  - Observed level of significance



HYPOTHESIS TESTING

Table 4.5 Critical Values Used in the z-test

Level of significance, α	0.10	0.05	0.01
Critical values of z One-tailed tests	−1.28 or 1.28	−1.645 or 1.645	−2.33 or 2.33
Critical values of z Two-tailed tests	−1.645 and 1.645	−1.96 and 1.96	−2.58 and 2.58



HYPOTHESIS TESTING

- Step 1
  - Establish the null hypothesis and the appropriate alternative hypothesis, such as $H_0: x' = x_0$, $H_a: x' \neq x_0$
- Step 2
  - Assign a level of significance α to determine critical values.
- Step 3
  - Calculate the observed value of the test statistic
- Step 4
  - Compare the observed test statistic to the critical values.

EXAMPLE 4.5

- From experience, a bearing manufacturer knows that it can produce roller bearings to the stated dimensions of 2.00 mm within a standard deviation of 0.061 mm. As part of its quality assurance program, engineering takes samples from boxes of bearings before shipment and measures them. On one occasion, a sample size of 25 bearings is obtained with the result $\bar{x} = 2.03 \text{ mm}$. Do the measurement support the hypothesis that $x' = 2.00 \text{ mm}$ for the whole box at a 5% level of significance?
- KNOWN: $\bar{x} = 2.03 \text{ mm}; x_0 = 2.00 \text{ mm}; \sigma = 0.061 \text{ mm}; N = 25$
- FIND: Apply hypothesis test at $\alpha = 0.05$

EXAMPLE 4.5

- KNOWN: $\sigma^2 = 3.15 \text{ mm}; N = 25$
- FIND: Apply hypothesis test at $\alpha = 0.05$

- Step 1:

$$H_0: x' = x_0 = 2.00 \text{ mm} \quad H_a: x' \neq 2.00 \text{ mm}$$

- Step 2:

$$P(-z_{0.025} \leq z_0 \leq z_{0.025}) = 1 - \alpha = 0.95$$

Critical value: -1.96 and 1.96

- Step 3:

Test statistic: $z_0 = \frac{\bar{x} - x_0}{\sigma/\sqrt{N}} = 2.459$

- Step 4:

$z_0 > z_c$ we reject H_0