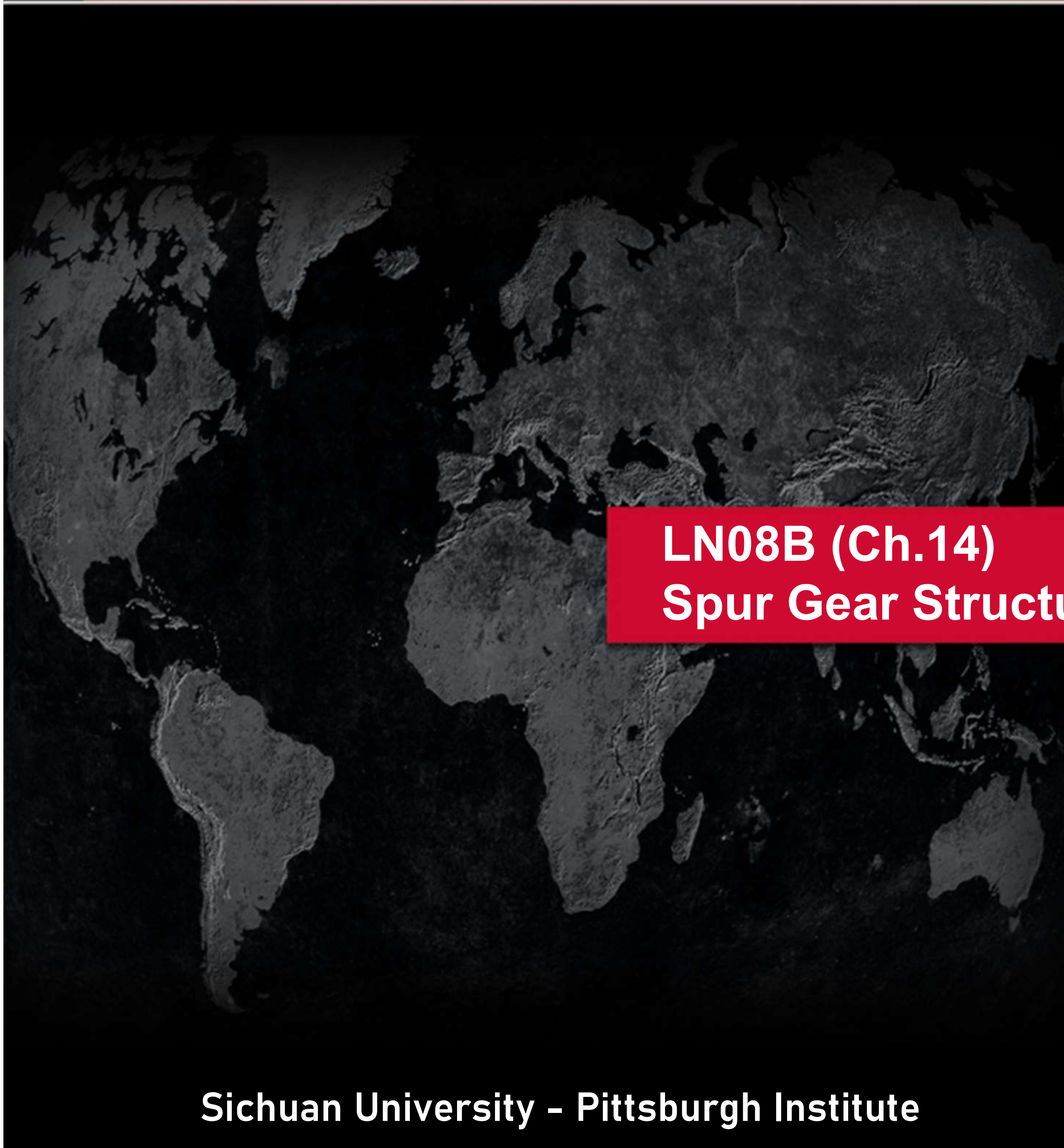
LN08B (Ch.14)
Spur Gear Structural Design Analysis



Instructor: Ping C. Sui, Ph.D.

ME 1029 Machine Design 2

Fall 2021

Learning Objectives

- Devoted primarily to analysis and design of spur gears to resist **bending failure** of the teeth as well as **pitting failure** of tooth surfaces.
  - Bending failure will occur when the significant tooth stress equals or exceeds either the yield strength or the bending endurance strength.
  - A surface pitting failure occurs when the significant contact stress equals or exceeds the surface endurance strength.

- Limit technical contents to
  - spur gears;
  - one single pressure angle (ϕ=20^o); and
  - by using only full-depth teeth.
- This simplification reduces the analysis complexity from design variations while constitutes an ideal introduction to the use of the general AGMA method.

14-3 AGMA Stress Equations

AGMA uses two fundamental stress equations (Stress Numbers)

- 14-1 The Lewis Bending Equation (Bending Stress, σ)
- 14-2 Surface Durability (Surface Contact Stress, σ_c)

14-4 AGMA Strength Equations

- Instead of strength, AGMA uses data termed allowable stress numbers (S_t and S_c)
- This textbook uses gear strength
- AGMA allowable strengths for bending and contact stress are for
  - Unidirectional loading
  - 10 million (10^7) stress cycles
  - 99 percent reliability

Summary (AGMA Guideline)

- Bending Stress Limit

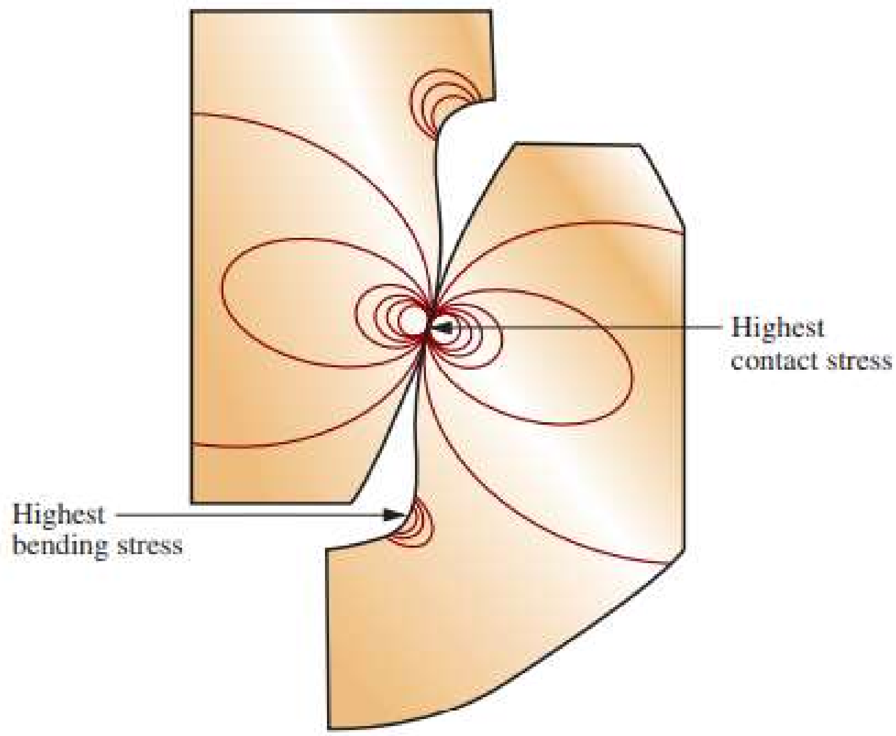
$$\sigma \leq \sigma_{all}$$

$$W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \leq \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

- Contact Stress Limit

$$\sigma_c \leq \sigma_{c,all}$$

$$C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I}} \leq \frac{S_c}{S_H} \frac{Z_N C_H}{K_T K_R}$$



In either form, the implication is

$$Working\ Stress \leq Limiting\ Strength$$

Spur Gear Structural Design Analysis

- 14–1 The Lewis Bending Equation
- 14–2 Surface Durability
- 14–3 AGMA Stress Equations
- 14–4 AGMA Strength Equations

- 14–5 Geometry Factors I and J (ZI and YJ)
- 14–6 The Elastic Coefficient Cp (ZE)
- 14–7 Dynamic Factor Kv
- 14–8 Overload Factor Ko
- 14–9 Surface Condition Factor Cf (ZR)
- 14–10 Size Factor Ks
- 14–11 Load-Distribution Factor Km (KH)
- 14–12 Hardness-Ratio Factor CH (ZW)
- 14–13 Stress-Cycle Factors YN and ZN
- 14–14 Reliability Factor KR (YZ)
- 14–15 Temperature Factor KT (Yu)
- 14–16 Rim-Thickness Factor KB
- 14–17 Safety Factors SF and SH

14-1 The Lewis Bending Equation

- Introduced by Wilfred Lewis in 1892, still remains the basis for most gear design today.

Beam Strength of Gear Teeth – Lewis Equation

Bending Moment $M = W^t l$

Section Modulus $I = \frac{F\,t^3}{12}$

Tooth Bending Stress $\sigma = \frac{Mc}{I} = \frac{6W^t l}{F\,t^2}$

(Note that symbol F is the gear width.)

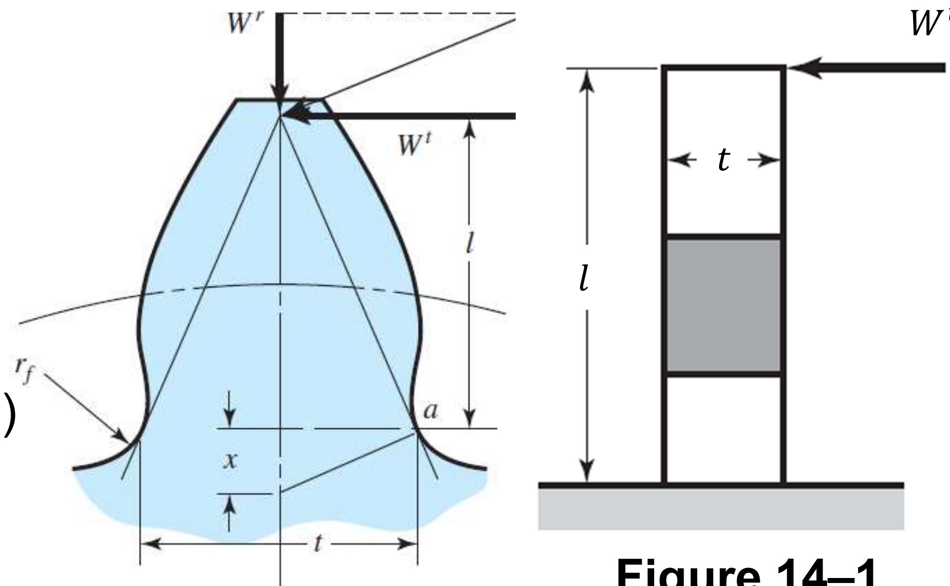
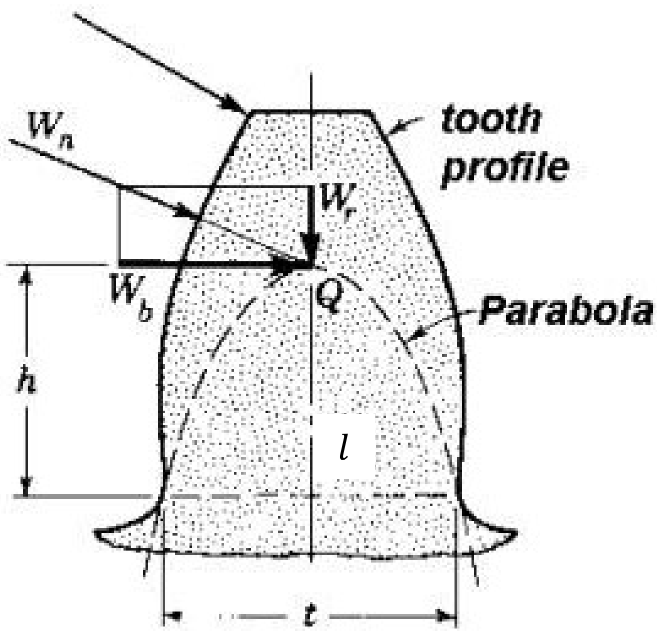
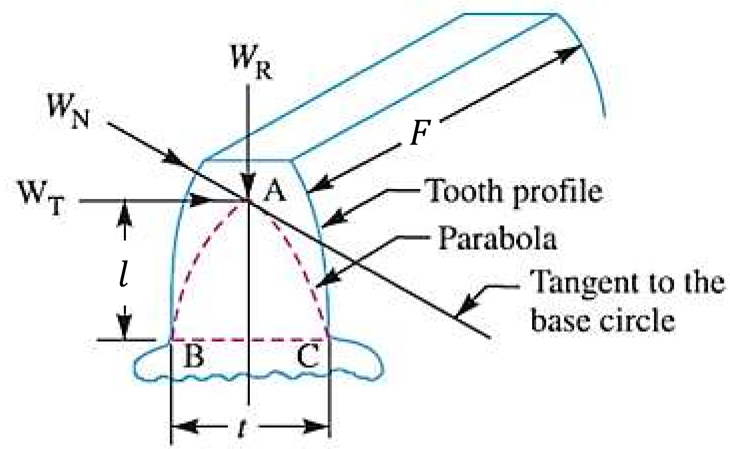


Figure 14–1





Assumptions of Lewis Equation

- The full load is applied to the tip of a single tooth.
  - This is obviously the most severe condition and is appropriate for gears of "ordinary" accuracy.
  - For high-precision gears, however, where contact ratio is usually greater than 1, the full load is never applied to a single tooth tip.
- The radial component, W_r, is negligible.
  - This is a conservative assumption, as W_r produces a compressive stress that subtracts from the bending tension
- The load is distributed uniformly across the full face width, which is a nonconservative assumption.
- Forces which are due to tooth sliding friction are negligible.
- Stress concentration in the tooth fillet is negligible.

Beam Strength of Gear Teeth – Lewis Equation

- Bending Moment $M = W^t l$
- Section Modulus $I = \frac{F\,t^3}{12}$
- Tooth Bending Stress $\sigma = \frac{Mc}{I} = \frac{6W^t l}{F\,t^2}$

Lewis developed an equivalent parabolic form from tooth profile which was assumed to be the only effective tooth volume resisting against tooth bending.

- Define **Lewis Form Factor (y)**

$$y = \frac{2x}{3p}$$

- Lewis Equation was expressed as

$$\sigma = \frac{W^t}{Fpy}$$

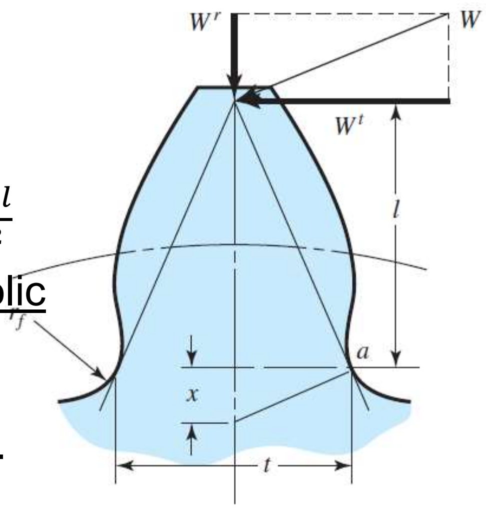
W^t: Tangential transmitted load

F: Tooth face width

t: tooth circumferential width at critical section

l : Height of Lewis parabola

p: circular pitch



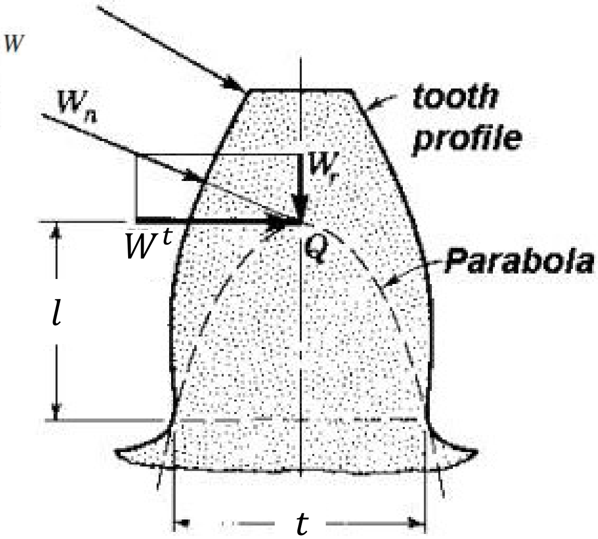
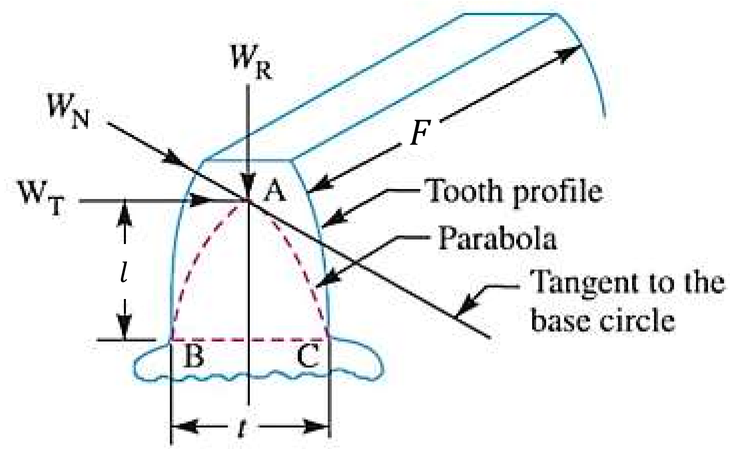
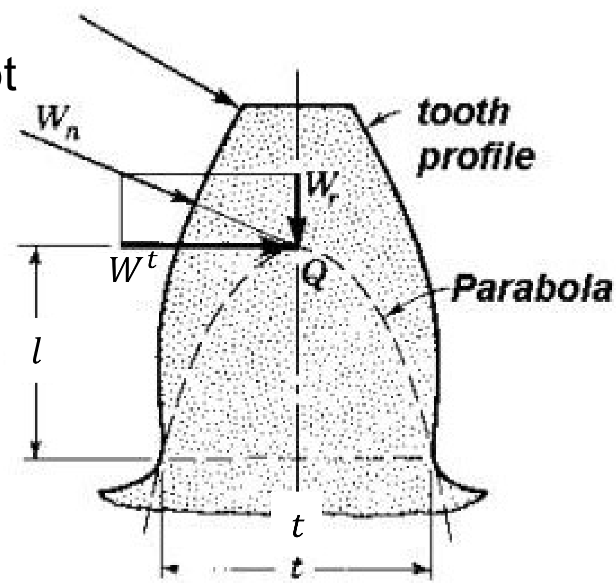


Figure 14–1



Beam Strength of Gear Teeth – Lewis Equation

Derivations of the Lewis parabola for gear tooth were omitted from the textbook. However, this same concept was largely adopted by AGMA in calculation of the tooth geometric form.



$$\sigma = \frac{W^t P}{FY} \qquad Y = \frac{2xP}{3}$$

P: Diametral pitch

Y: AGMA Lewis Form Factor

Table 14–2

Number of Teeth	Y	Number of Teeth	Y
12	0.245	28	0.353
13	0.261	30	0.359
14	0.277	34	0.371
15	0.290	38	0.384
16	0.296	43	0.397
17	0.303	50	0.409
18	0.309	60	0.422
19	0.314	75	0.435
20	0.322	100	0.447
21	0.328	150	0.460
22	0.331	300	0.472
24	0.337	400	0.480
26	0.346	Rack	0.485

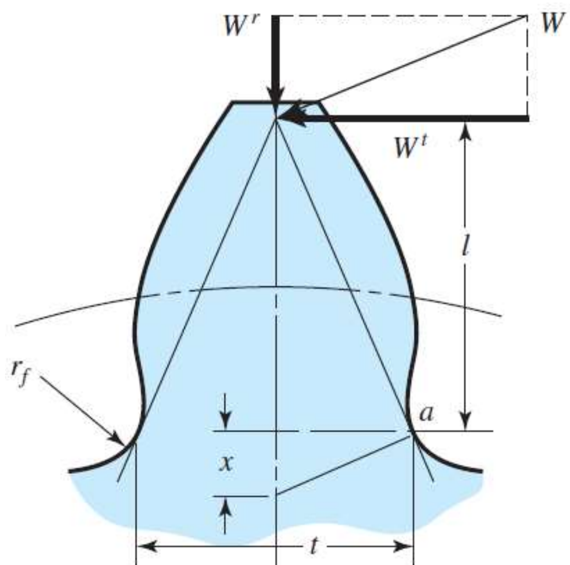
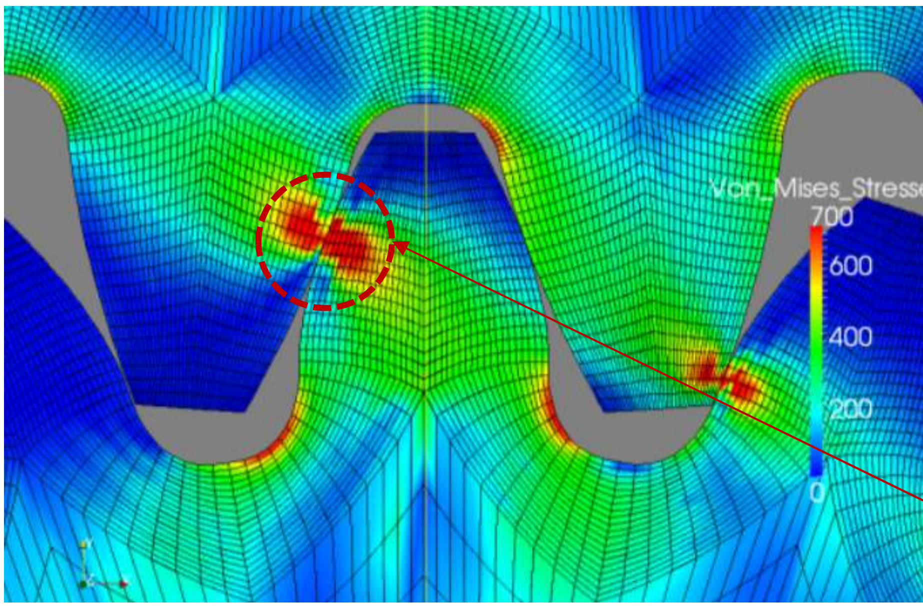
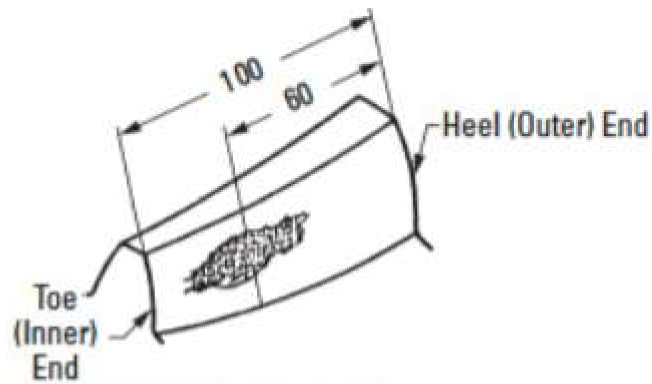


Figure 14–1

14-2 **Surface Durability**

- Failure of the surfaces of gear teeth.
- Pitting is a surface fatigue failure due to many repetitions of high contact stresses.

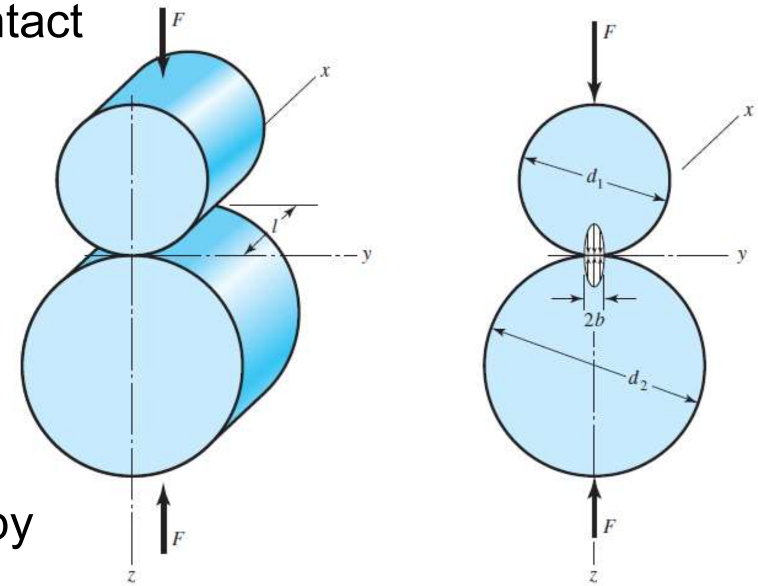




FEA stresses in a spur gear tooth.

Surface Contact Pressure (Hertz Theory)

Figure 3–38



Hertz Theory in Sec. 3-19 shows the surface contact pressure between two cylindrical bodies:

$$p_{max} = \frac{2F}{\pi b l}$$

p_{max}: Peak surface pressure

F: force pressing the two cylinders together

l: length of cylinders

b is the half Hertzian contact width and is given by

$$b = \left[\frac{2F}{\pi l}\frac{(1-\nu_1^2)/E_1+(1-\nu_2^2)/E_2}{1/d_1+1/d_2}\right]^{1/2}$$

Adapt to the gear nomenclatures, the peak surface contact pressure (Hertzian stress) is found to be

$$\sigma_c^2 = \frac{W^t}{\pi F\cos\phi}\frac{1/r_1+1/r_2}{(1-\nu_1^2)/E_1+(1-\nu_2^2)/E_2}$$

Surface Contact Pressure (Hertz Theory)

Surface contact pressure (Hertzian stress)

$$\sigma_c^2 = \frac{W^t}{\pi F \cos\phi} \frac{1/r_1 + 1/r_2}{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}$$

where r_1 and r_2 are the instantaneous values of the radii of curvature on the pinion and gear-tooth profiles, respectively, at the point of contact.

$$r_1 = \frac{d_P \sin\phi}{2} \qquad r_2 = \frac{d_G \sin\phi}{2}$$

where ϕ is the pressure angle and d_P and d_G are the pitch diameters of the pinion and gear, respectively. AGMA also defines an **elastic coefficient** C_p by

$$C_p = \left[\frac{1}{\pi\left[(1-\nu_p^2)/E_p + (1-\nu_G^2)/E_G\right]}\right]^{1/2}$$

Henceforth, surface contact pressure including the velocity factor (K_v) can be written as

$$\sigma_c = C_p\left[\frac{K_v W^t}{F\cos\phi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2}$$

Note that Hertzian stress is always a <u>compressive</u> stress.

EXAMPLE 14–3

The pinion of Examples 14–1 and 14–2 is to be mated with a 50-tooth gear of ASTM No. 50 cast iron. Tangential load is 382 lbf. Estimate the factor of safety of the drive based on the possibility of a surface fatigue failure.

Surface endurance strength of cast iron estimated from **S_c=0.32*HB** ksi for 10^8 cycles.

Table A–5 (ASTM No. 50 CI) ►E_P=30 Mpsi, ν_P=0.292, E_G =14.5 Mpsi, ν_G=0.211

$$C_p = \left[\frac{1}{\pi[(1-0.292^2)/(30\cdot10^6)+(1-0.21\ \ ^2)/(14.5\cdot10^6)]}\right]^{1/2} = 1817\ psi^{1/2}$$

Pinion pitch diameter d_P=2 in and Gear pitch diameter d_G=50/8=6.25 in

$$r_1 = \frac{d_P \sin\phi}{2} = \frac{2\ \mathrm{s}\qquad ^{\circ}}{2} = 0.342\ in \qquad r_2 = \frac{d_G \sin\phi}{2} = \frac{6.25 \sin 20^{\circ}}{2} = 1.069\ in$$

Face width F=1.5 in and $K_v = 1.52$

$$\sigma_c = C_p\left[\frac{K_v W^t}{F\cos\phi}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right]^{1/2} = 1817\left[\frac{1.52\cdot38}{1.5\cos 20^{\circ}}\left(\frac{1}{0.342}+\frac{1}{1.069}\right)\right]^{1/2} = 72400\ psi$$

Table A–24 H_B=262 for ASTM No. 50 CI ► S_C=0.32(262)= 83.8 kpsi

50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35

Safety factor $n = \frac{83.8}{72.4} = 1.16$

Please note the discussions of SF per loss-of-function load on P-737.

Table A-24

Mechanical Properties of Three Non-Steel Metals

(a) Typical Properties of Gray Cast Iron

[The American Society for Testing and Materials (ASTM) numbering system for gray cast iron is such that the numbers correspond to the *minimum tensile strength* in kpsi. Thus an ASTM No. 20 cast iron has a minimum tensile strength of 20 kpsi. Note particularly that the tabulations are *typical* of several heats.]

ASTM Number	Tensile Strength S_{ut}, kpsi	Compressive Strength S_{uc}, kpsi	Shear Modulus of Rupture S_{su}, kpsi	Modulus of Elasticity, Mpsi: Tension†	Modulus of Elasticity, Mpsi: Torsion	Endurance Limit* S_e, kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

*Polished or machined specimens.

†The modulus of elasticity of cast iron in compression corresponds closely to the upper value in the range given for tension and is a more constant value than that for tension.

14-3 AGMA Stress Equations

AGMA uses two fundamental stress equations (Stress Numbers)

- Bending stress (σ)
- Pitting resistance (contact stress, σ_c)

AGMA Fundamental Stress Equations

Bending Stress

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

W^t: tangential transmitted load, lbf (N)

K_o: overload factor

K_v: dynamic factor

K_s: size factor

$$\sigma = \frac{W^t P}{FY}$$

P_d: transverse diametral pitch

F (b): face width of the narrower member, in (mm)

K_m (K_H): load-distribution factor

KB: rim-thickness factor

J (Y_J): geometry factor for bending strength (which includes root fillet stress-concentration factor K_f)

(m_t): transverse metric module

Pitting Resistance (Contact Stress)

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

C_p (Z_E): elastic coefficient, $\sqrt{\frac{lbf}{in^2}}$ $\left(\sqrt{\frac{N}{mm^2}}\right)$

C_f (Z_R): surface condition factor

d_P (d_{w1}): pitch diameter of the pinion, in (mm)

I (Z_I): geometry factor for pitting resistance

$$\sigma_c = C_p \left[\frac{K_v W^t}{F \cos \phi}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\right]^{1/2}$$

14-4 AGMA Strength Equations

- Instead of strength, AGMA uses data termed allowable stress numbers (S_{at} and S_{ac})
- This textbook uses gear strength
- AGMA allowable strengths for bending and contact stress are for
  - Unidirectional loading
  - 10 million (10^7) stress cycles
  - 99 percent reliability

Summary (AGMA Guideline)

- Bending Stress Limit

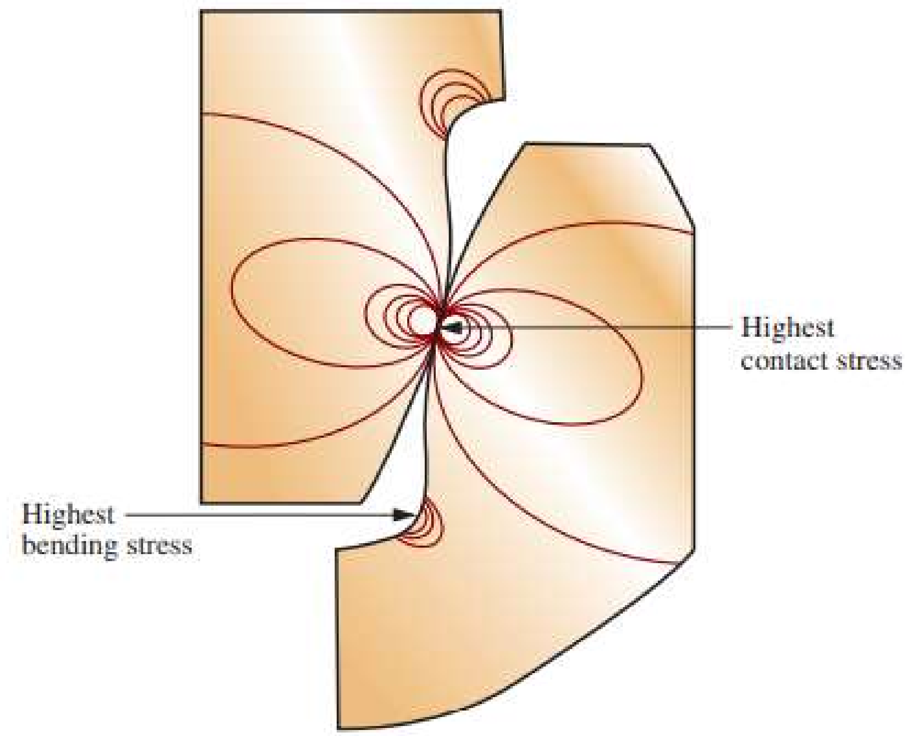
$$\sigma \le \sigma_{all}$$

$$W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \le \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

- Contact Stress Limit

$$\sigma_c \le \sigma_{c,all}$$

$$C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I}} \le \frac{S_c}{S_H} \frac{Z_N C_H}{K_T K_R}$$



In either form, the implication is

$$Working\ Stress\ \le Limiting\ Strength$$

Gear Bending Strength (S_t)

Gear strengths are not identified with other strengths such as S_{ut}, S_e, or S_y. Their use should be restricted to gear problems.

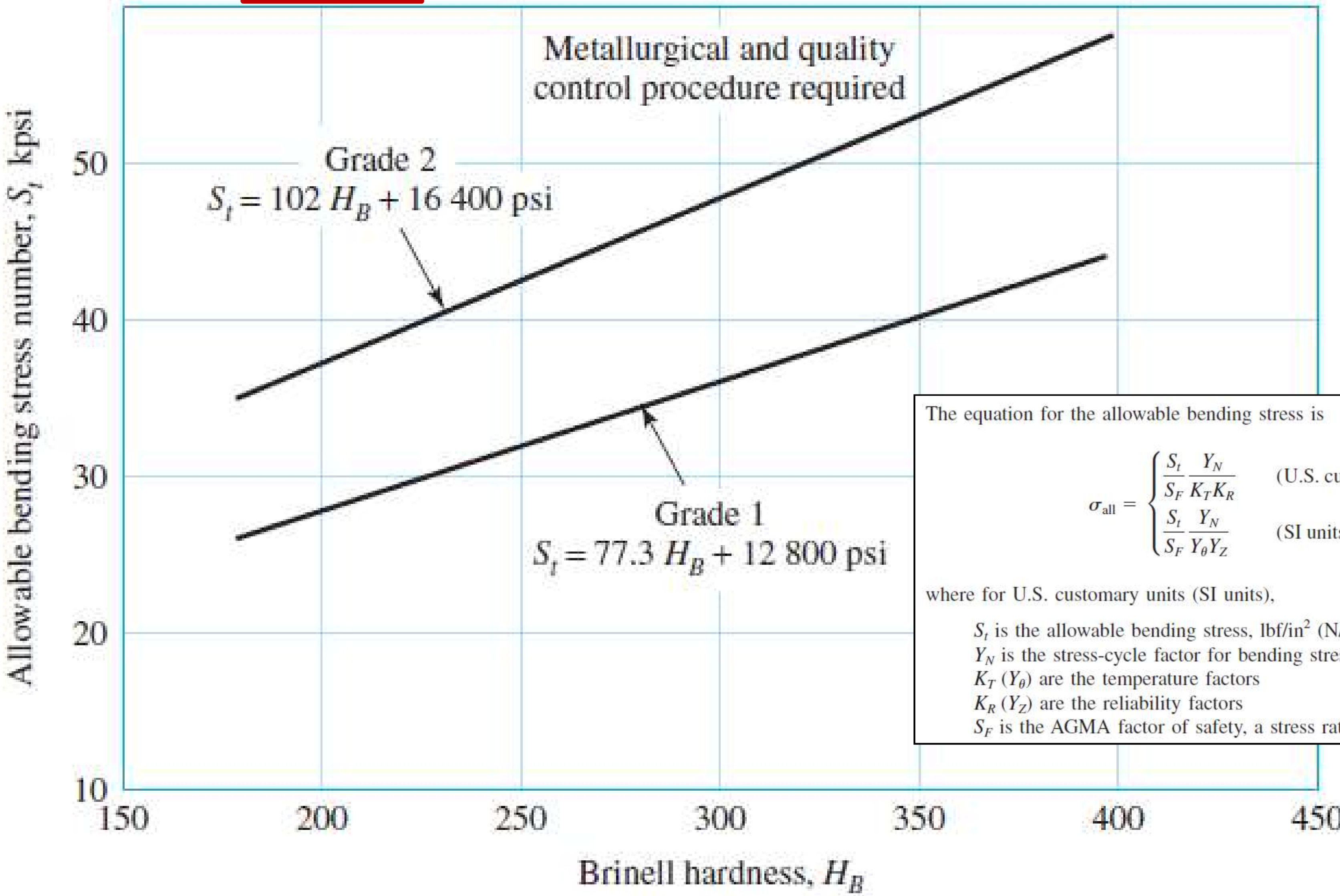
Table 14-3 Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Steel Gears

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Bending Stress Number S_t,[2] psi Grade 1	Grade 2	Grade 3
Steel[3]	Through-hardened	See Fig. 14–2	See Fig. 14–2	See Fig. 14–2	—
	Flame[4] or induction hardened[4] with type A pattern[5]	See Table 8*	45 000	55 000	—
	Flame[4] or induction hardened[4] with type B pattern[5]	See Table 8*	22 000	22 000	—
	Carburized and hardened	See Table 9*	55 000	65 000 or 70 000[6]	75 000
	Nitrided[4,7] (through-hardened steels)	83.5 HR15N	See Fig. 14–3	See Fig. 14–3	—
Nitralloy 135M, Nitralloy N, and 2.5% chrome (no aluminum)	Nitrided[4,7]	87.5 HR15N	See Fig. 14–4	See Fig. 14–4	See Fig. 14–4

Table 14-4 Bending Strength S_t at 10^7 Cycles and 0.99 Reliability for Iron/Bronze Gears

Gear Bending Strength (S_t)

Figure 14-2 Allowable bending stress number for through-hardened steels.



The equation for the allowable bending stress is

$$\sigma_{all} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

where for U.S. customary units (SI units),

- S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
- Y_N is the stress-cycle factor for bending stress
- K_T (Y_θ) are the temperature factors
- K_R (Y_Z) are the reliability factors
- S_F is the AGMA factor of safety, a stress ratio

Gear Bending Strength (S_t)

Figure 14-2 Through-Hardened Steels

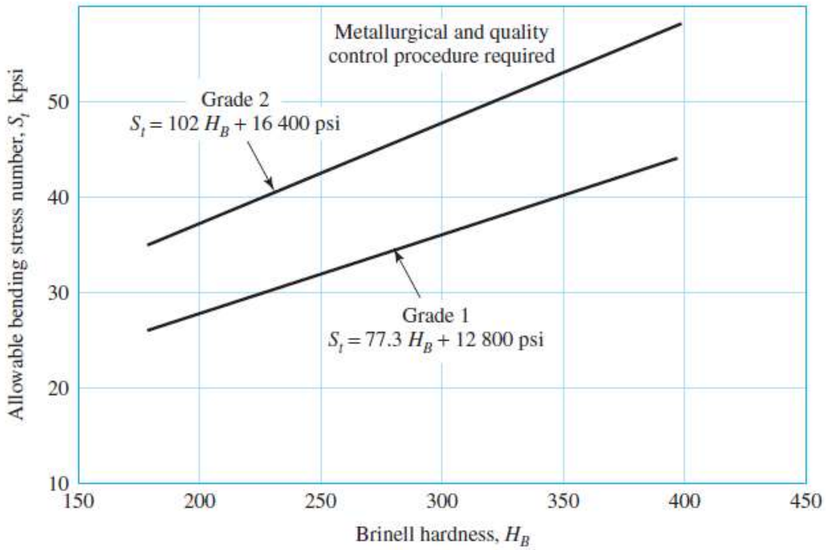


Figure 14-3 Nitrided Through-Hardened Steels

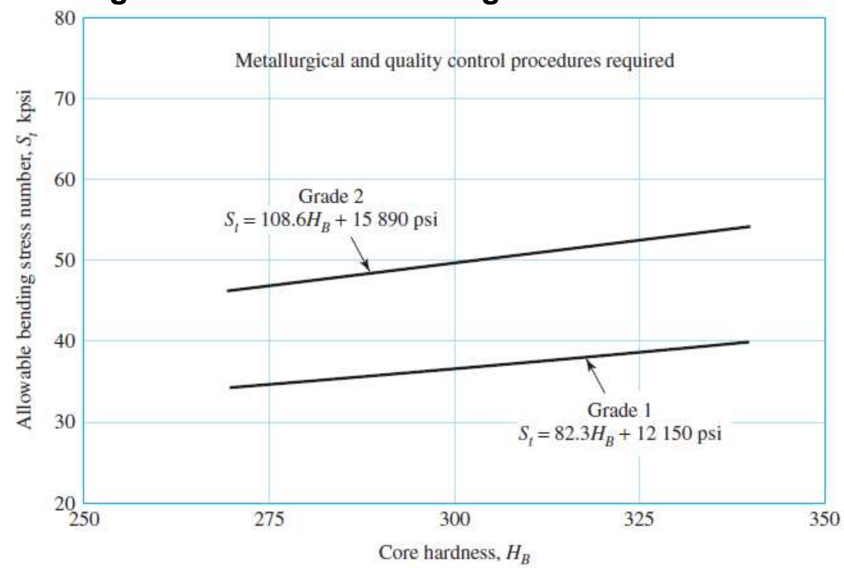
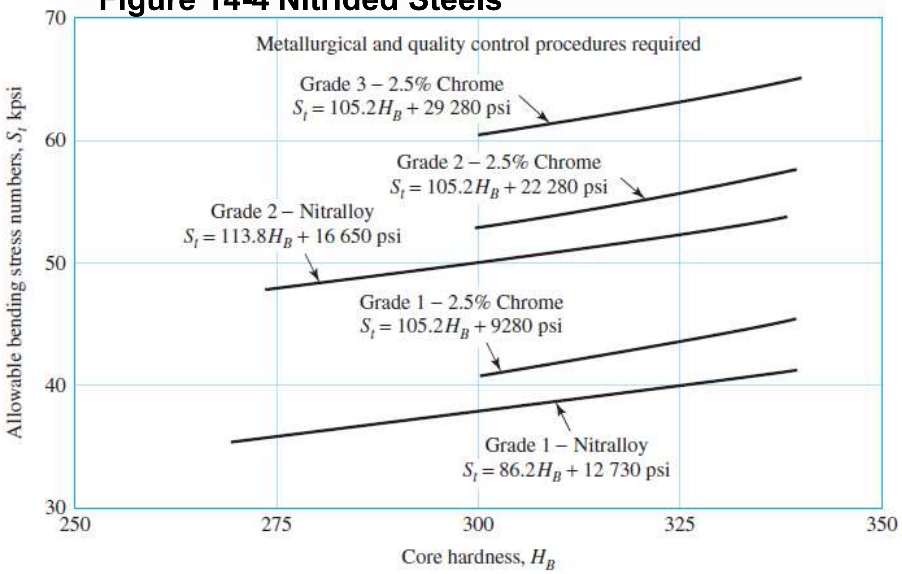


Figure 14-4 Nitrided Steels



The equation for the allowable bending stress is

(14-17)

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

where for U.S. customary units (SI units),

- S_t is the allowable bending stress, lbf/in^2 (N/mm^2)
- Y_N is the stress-cycle factor for bending stress
- K_T (Y_θ) are the temperature factors
- K_R (Y_Z) are the reliability factors
- S_F is the AGMA factor of safety, a stress ratio

Gear Contact Strength (S_C)

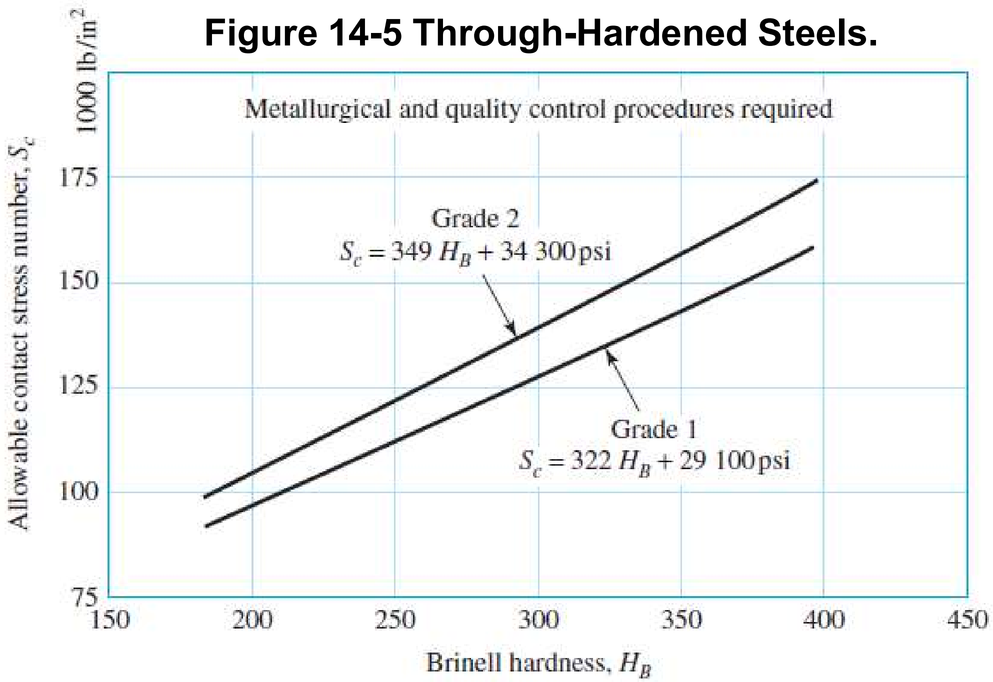
Table 14-6 Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Steel Gears

Material Designation	Heat Treatment	Minimum Surface Hardness[1]	Allowable Contact Stress Number,[2] S_c, psi Grade 1	Grade 2	Grade 3
Steel[3]	Through hardened[4]	See Fig. 14–5	See Fig. 14–5	See Fig. 14–5	—
	Flame[5] or induction hardened[5]	50 HRC	170 000	190 000	—
		54 HRC	175 000	195 000	—
	Carburized and hardened[5]	See Table 9*	180 000	225 000	275 000
	Nitrided[5] (through hardened steels)	83.5 HR15N	150 000	163 000	175 000
		84.5 HR15N	155 000	168 000	180 000
2.5% chrome (no aluminum)	Nitrided[5]	87.5 HR15N	155 000	172 000	189 000
Nitralloy 135M	Nitrided[5]	90.0 HR15N	170 000	183 000	195 000
Nitralloy N	Nitrided[5]	90.0 HR15N	172 000	188 000	205 000
2.5% chrome (no aluminum)	Nitrided[5]	90.0 HR15N	176 000	196 000	216 000

Table 14-7 Contact Strength S_c at 10^7 Cycles and 0.99 Reliability for Iron/Bronze Gears

Gear Contact Strength (S_C)

Figure 14-5 Through-Hardened Steels.



The equation for the allowable contact stress $\sigma_{c,\text{all}}$ is

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \qquad \textbf{(14-18)}$$

S_c: allowable contact stress, lbf/in^2 (N/mm^2)
Z_N: stress-cycle factor
C_H (Z_W): hardness ratio factors for pitting resistance
K_T (Y_u): temperature factors
K_R (Y_Z): reliability factors
S_H: AGMA factor of safety, a stress ratio

Summary (AGMA Guideline)

- Bending Stress Limit

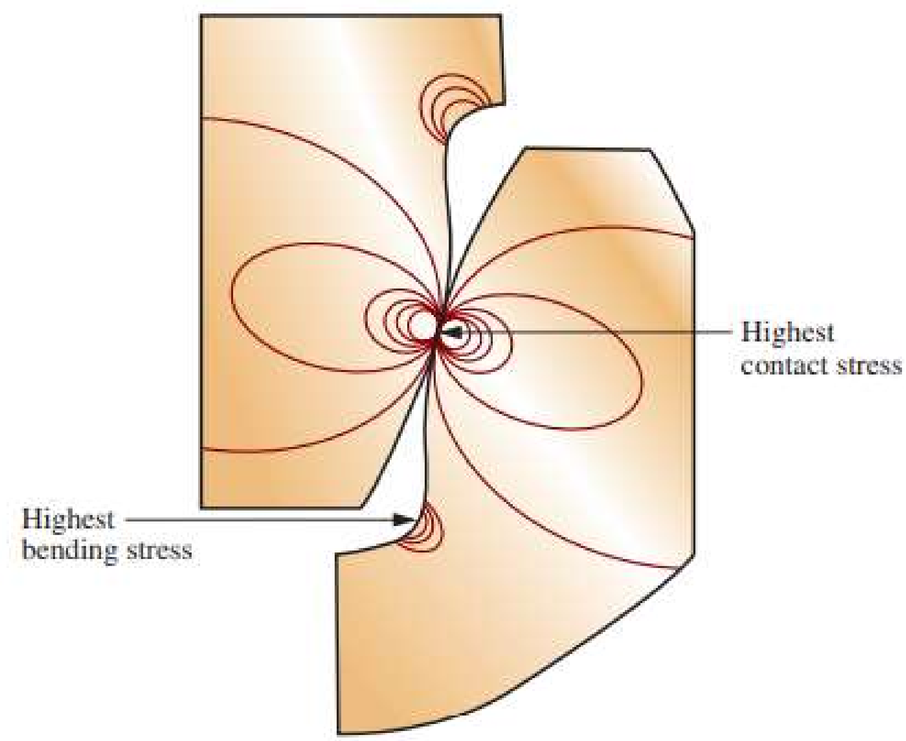
$$\sigma \leq \sigma_{all}$$

$$W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J} \leq \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

- Contact Stress Limit

$$\sigma_c \leq \sigma_{c,all}$$

$$C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I}} \leq \frac{S_c}{S_H} \frac{Z_N C_H}{K_T K_R}$$



In either form, the implication is

$$Working\ Stress\ \leq Limiting\ Strength$$

Modifying Factors (For Spur Gears)

- 14–5 Geometry Factors I and J (Z_I and Y_J)
- 14–6 The Elastic Coefficient C_p (Z_E)
- 14–7 Dynamic Factor K_v
- 14–8 Overload Factor K_o
- 14–9 Surface Condition Factor C_f (Z_R)
- 14–10 Size Factor K_s
- 14–11 Load-Distribution Factor K_m (K_H)
- 14–12 Hardness-Ratio Factor C_H (Z_W)
- 14–13 Stress-Cycle Factors Y_N and Z_N
- 14–14 Reliability Factor K_R (Y_Z)
- 14–15 Temperature Factor K_T (Y_u)
- 14–16 Rim-Thickness Factor K_B
- 14–17 Safety Factors S_F and S_H

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

14-5 Geometry Factors I and J (Z_I and Y_J)

- Bending-Strength Geometry Factor J (Y_J)

$$J = \frac{Y}{K_f m_N}$$

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

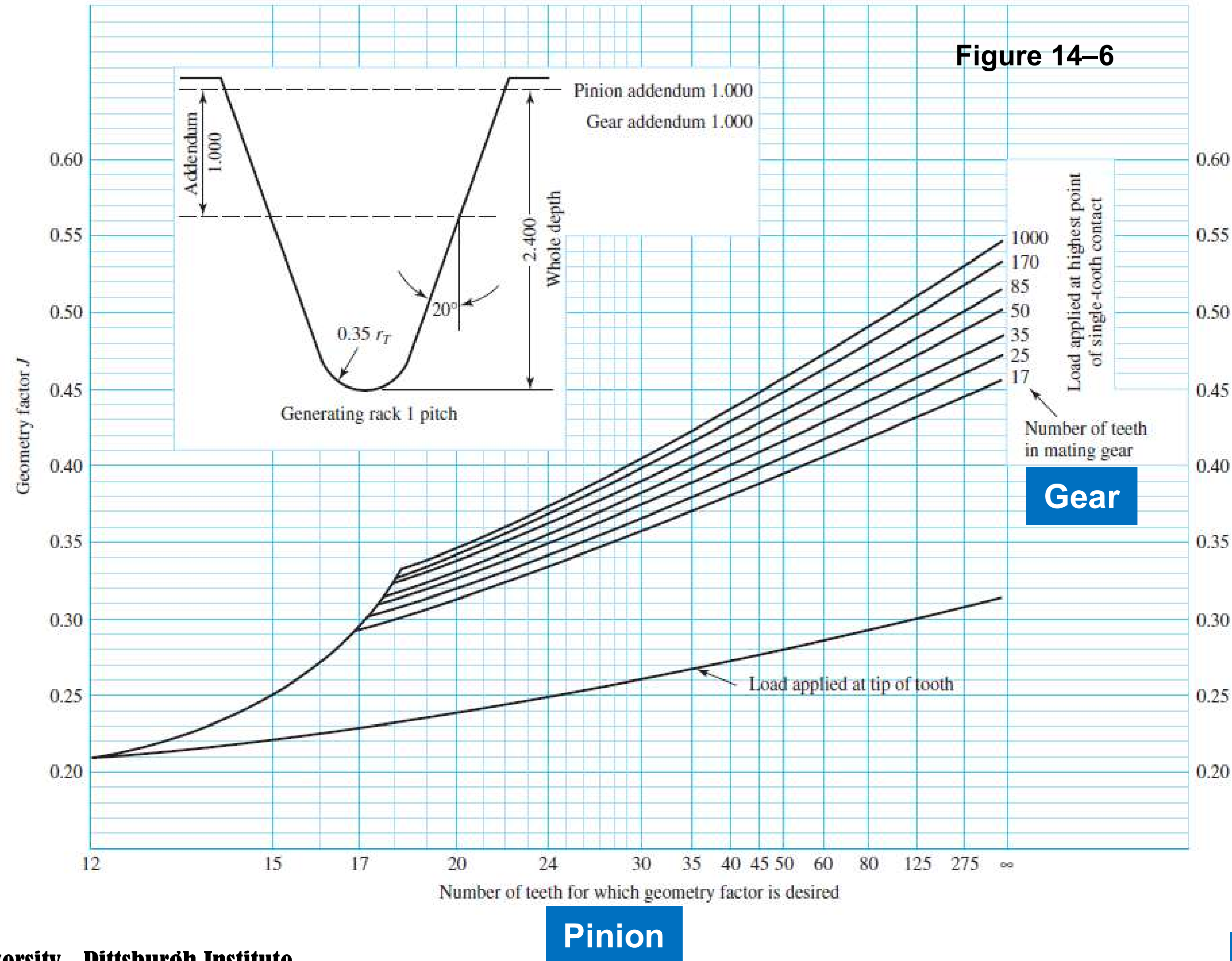
K_f is called a stress-correction factor by AGMA

Load-sharing ratio $m_N = \dfrac{tooth\ face\ width}{minimum\ length\ of\ line\ of\ contact}$

For spur gears, m_N=1

14-5 Geometry Factors I and J (Z_I and Y_J)

Bending-Strength Geometry Factor J (Y_J)



14-5 Geometry Factors I and J (Z_I and Y_J)

$$\sigma_c = \begin{cases} C_p\sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F}\dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E\sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b}\dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

<u>Surface-Strength Geometry Factor I (Z_I)</u>

Surface Stress $\sigma_c = C_p\left[\frac{K_v W^t}{F\cos\phi}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right]^{1/2}$

Define speed ratio $m_G = \frac{N_G}{N_P} = \frac{d_G}{d_P}$

$$\frac{1}{r_1}+\frac{1}{r_2} = \frac{2}{\sin\phi}\left(\frac{1}{d_P}+\frac{1}{d_G}\right) = \frac{2}{d_P\sin\phi}\left(\frac{m_G+1}{m_G}\right)$$

Surface stress $\sigma_c = C_p\left[\frac{K_v W^t}{d_P F}\frac{1}{\frac{\cos\phi\sin\phi}{2}\frac{m_G}{m_G+1}}\right]^{1/2} = C_p\left[\frac{K_v W^t}{d_P F}\frac{1}{I}\right]^{1/2}$

Geometry Factor I

External Gears $\mathrm{I} = \frac{\cos\phi\sin\phi}{2m_N}\frac{m_G}{m_G+1}$

Internal Gears $\mathrm{I} = \frac{\cos\phi\sin\phi}{2m_N}\frac{m_G}{m_G-1}$

For spur gears, $m_N=1$

14–6 The Elastic Coefficient C_p (Z_E)

$$C_p = \left[\frac{1}{\pi\left[\left(1-\nu_p^2\right)/E_p + \left(1-\nu_G^2\right)/E_G\right]}\right]^{1/2}$$

Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source: AGMA 218.01*

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in² (MPa)*: Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

14-7 Dynamic Factor K_v

- Dynamic factors are used to account for inaccuracies in the manufacture and meshing of gear teeth in action.

- Transmission error is defined as the departure from uniform angular velocity of the gear pair, such as:
  - Inaccuracies of the tooth profile
  - Vibration of the tooth
  - Magnitude of the pitch-line velocity
  - Dynamic unbalance of the rotating members
  - Wear and permanent deformation of the teeth
  - Gearshaft misalignment and linear/angular deflection of the shaft
  - Tooth friction

14-7 Dynamic Factor K_v (Cont'd)

- AGMA has defined a set of quality numbers, Q_v
  - Quality numbers 3 to 7 will include most commercial quality gears.
  - Quality numbers 8 to 12 are of precision quality.

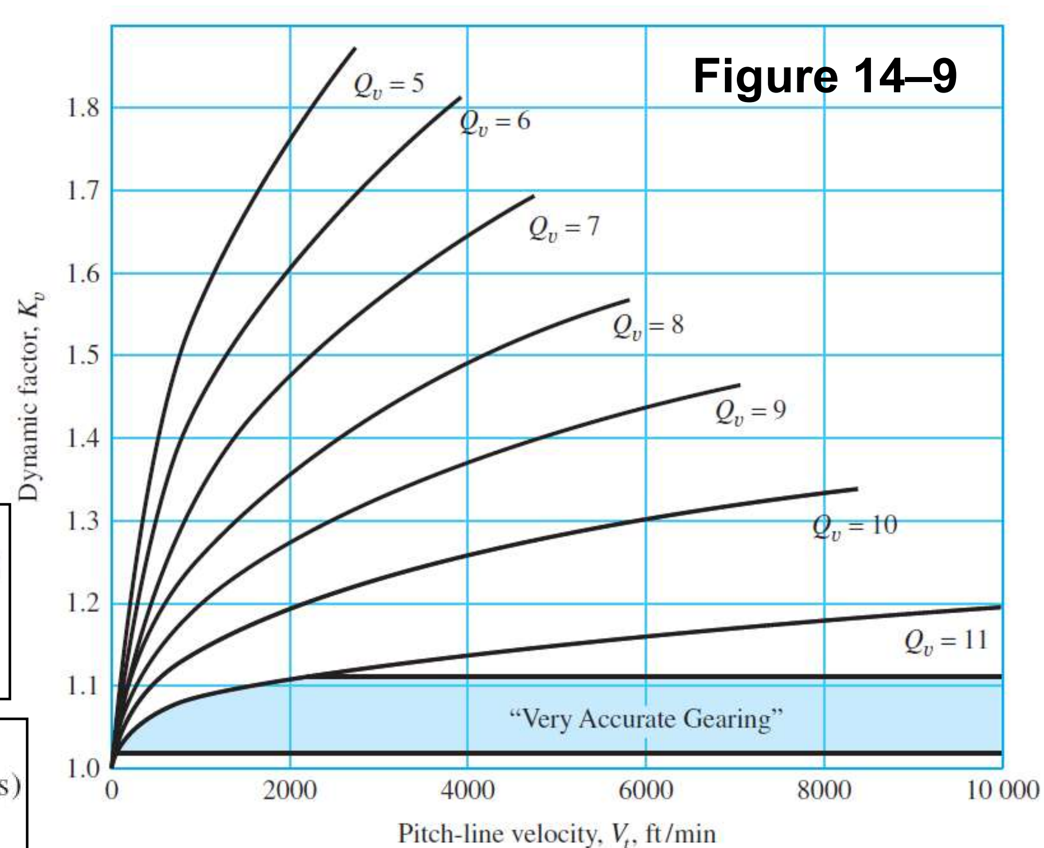
$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases}$$

$$A = 50 + 56(1 - B)$$

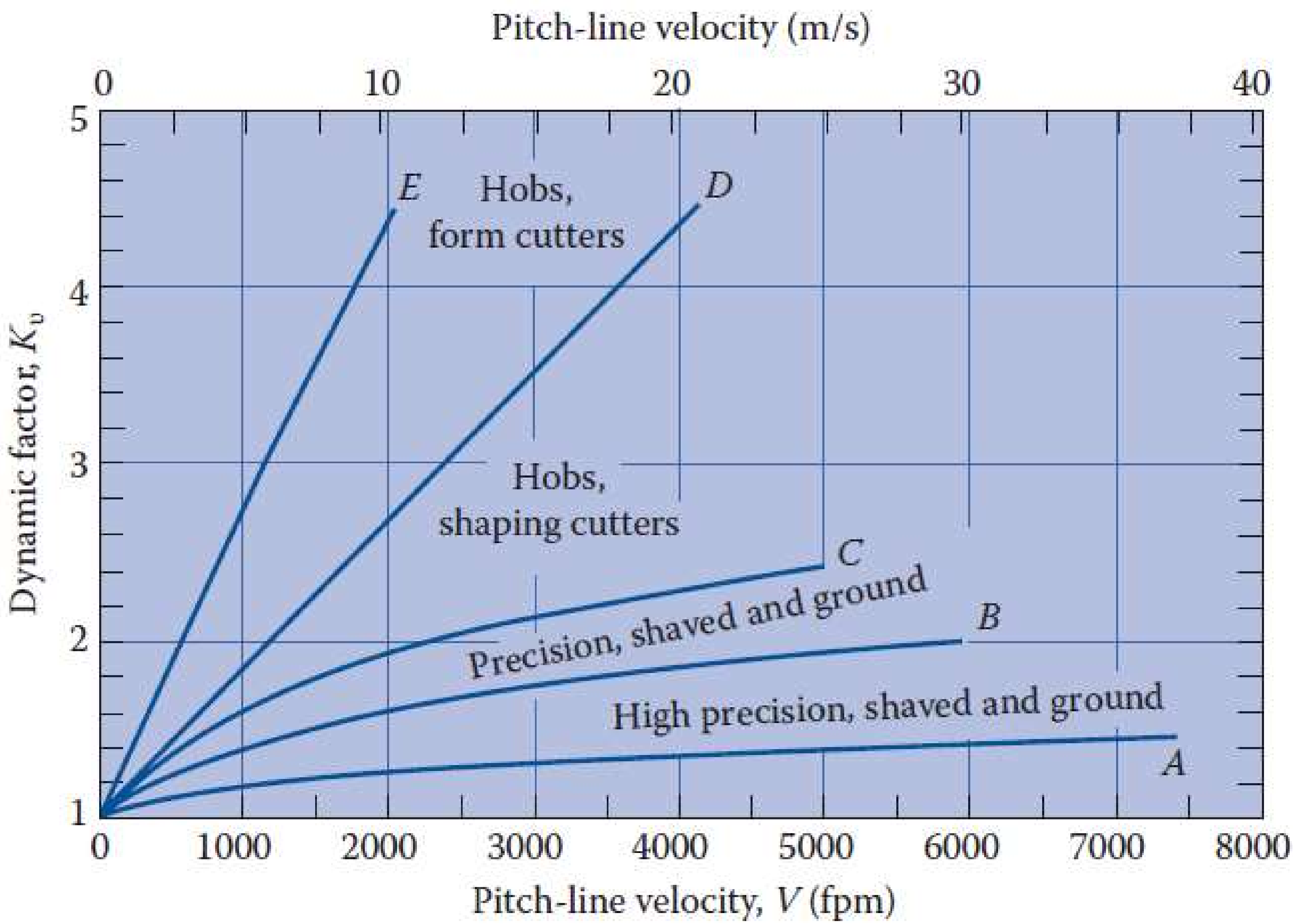
$$B = 0.25(12 - Q_v)^{2/3}$$

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$



14-7 Dynamic Factor K_v (Cont'd)



14-8 Overload Factor K_o

Overload factor K_o is intended to make allowance for all externally applied loads in excess of the nominal tangential load W_t, such as variations in torque from the mean value due to firing of cylinders in an internal combustion engine or reaction to torque variations in a piston pump drive.

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

14-9 Surface Condition Factor C_f (Z_R)

Surface condition factor is only for pitting resistance, which depends on

- Surface finish
- Residual stress
- Plastic effects (work hardening)

Standard surface conditions for gear teeth have not yet been established

$$\sigma_c = \begin{cases} C_p\sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F}\dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E\sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b}\dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

14-10 Size Factor K_s

Size factor reflects nonuniformity of material properties due to size. It depends upon

- Tooth size
- Diameter of part
- Ratio of tooth size to diameter of part
- Face width
- Area of stress pattern
- Ratio of case depth to tooth size
- Hardenability and heat treatment

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

AGMA hasn't established a standard size factors for gear teeth yet.

Textbook borrowed the size factor (k_b) for shaft HCF and recommended:

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$$

If K_s is less than 1, use K_s = 1.

Module, m, mm	Size factor K_s
≤ 5	1.00
6	1.05
8	1.15
12	1.25
20	1.40

14-11 Load Distribution Factor K_m (K_H)

Load-distribution factor is to reflect non-uniform distribution of load across the line of contact.

AGMA face load distribution factor $K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$

C_{mc}: Lead correction factor

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases}$$

C_{pm}: Pinion proportion modifier; consider **alignment** change due to pinion offset under a deflection of the bearings.

C_{pf}: Pinion proportion factor; based on **location** of the pinion relative to its bearing center line

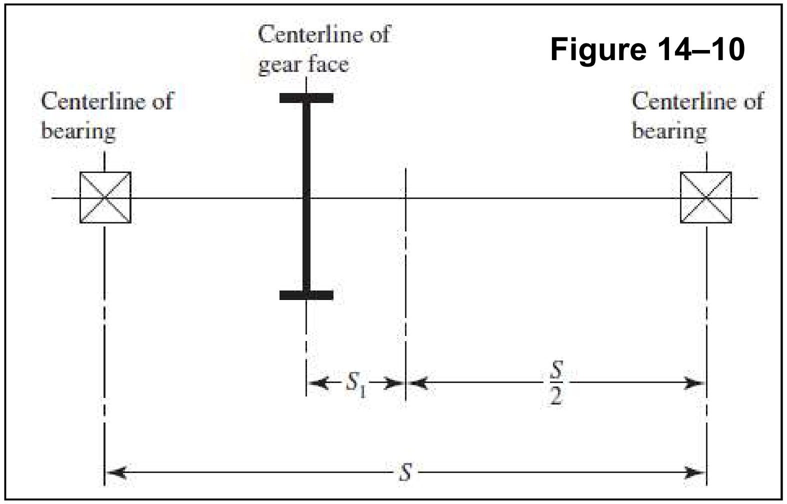


Figure 14–10

$$C_{pf} = \begin{cases} \dfrac{F}{10d_P} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d_P} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d_P} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases}$$

For values of $\frac{F}{10d_P} < 0.05$, use 0.05 instead.

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases}$$

14-11 Load Distribution Factor K_m (K_H) (Cont'd)

AGMA face load distribution factor $K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e)$

C_{ma}: Mesh alignment factor; accounts for the misalignment of the axes of rotation from all causes other than elastic deformation

$$C_{ma} = A + BF + CF^2$$

Table 14–9

Condition	A	B	C
Open gearing	0.247	0.0167	$-0.765(10^{-4})$
Commercial, enclosed units	0.127	0.0158	$-0.930(10^{-4})$
Precision, enclosed units	0.0675	0.0128	$-0.926(10^{-4})$
Extraprecision enclosed gear units	0.00360	0.0102	$-0.822(10^{-4})$

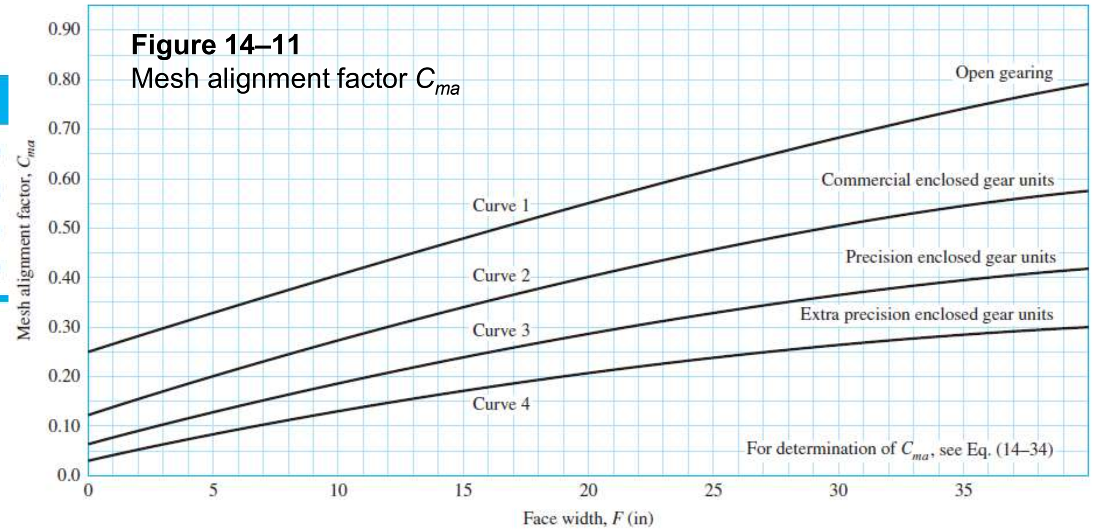


Figure 14–11
Mesh alignment factor C_{ma}

C_e: Mesh alignment correction factor

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases}$$

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases}$$

14-12 Hardness-Ratio Factor C_H (Z_W)

Pinion generally has a smaller number of teeth than gear and consequently is subjected to more cycles of contact stress. Therefore, a uniform surface strength can be obtained by making the pinion harder than the gear.

Hardness-ratio factor C_H is used only for the gear. For pinion, C_H=1.

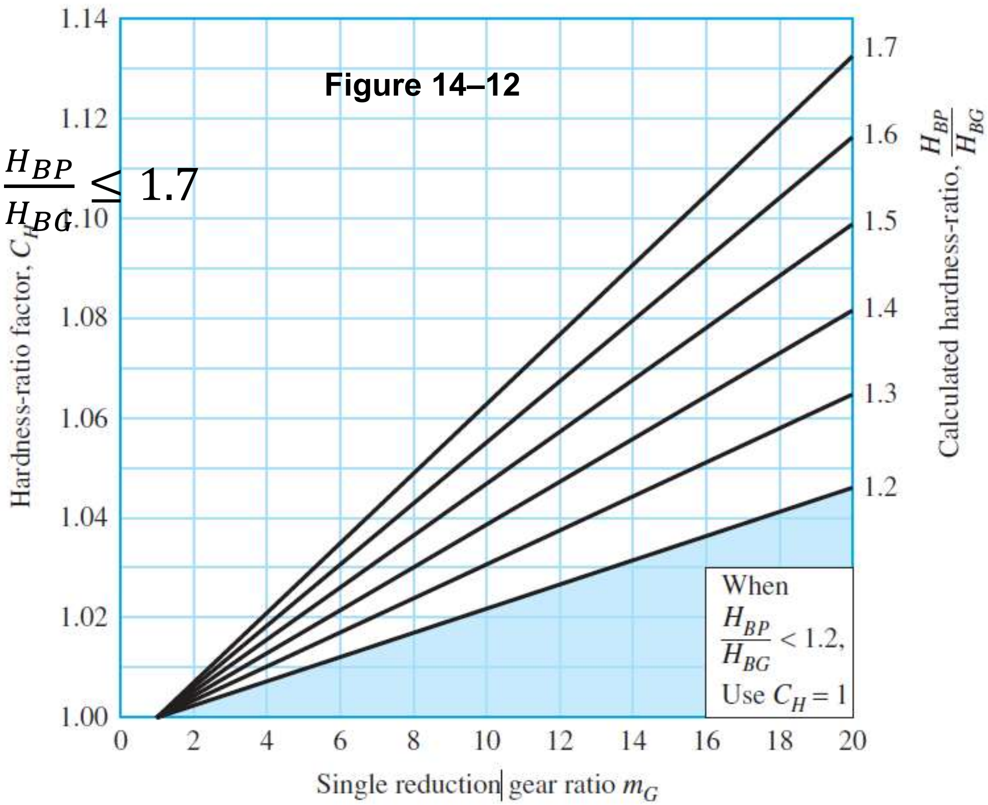
$$C_H = 1 + A'(m_G - 1)$$

$$A' = 0.00898\left(\frac{H_{BP}}{H_{BG}}\right) - 0.00829, \quad 1.2 \le \frac{H_{BP}}{H_{BG}} \le 1.7$$

$$A' = 0 \qquad \frac{H_{BP}}{H_{BG}} < 1.2,$$

$$A' = 0.00698 \qquad \frac{H_{BP}}{H_{BG}} > 1.7$$

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$



14-12 Hardness-Ratio Factor C_H (Z_W) (Cont'd)

When surface-hardened pinions with hardness of 48 Rockwell C scale (Rockwell C48) or harder are run with through-hardened gears (180–400 Brinell), a work hardening occurs. The C_H factor is a function of pinion surface finish f_P and the mating gear hardness.

$$C_H = 1 + B'(450 - H_{BG})$$

$$B' = 0.00075\, e^{(-0.0012 f_P)}$$

f_P: Pinion surface finish expressed as root-mean-square roughness R_a (μin)

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

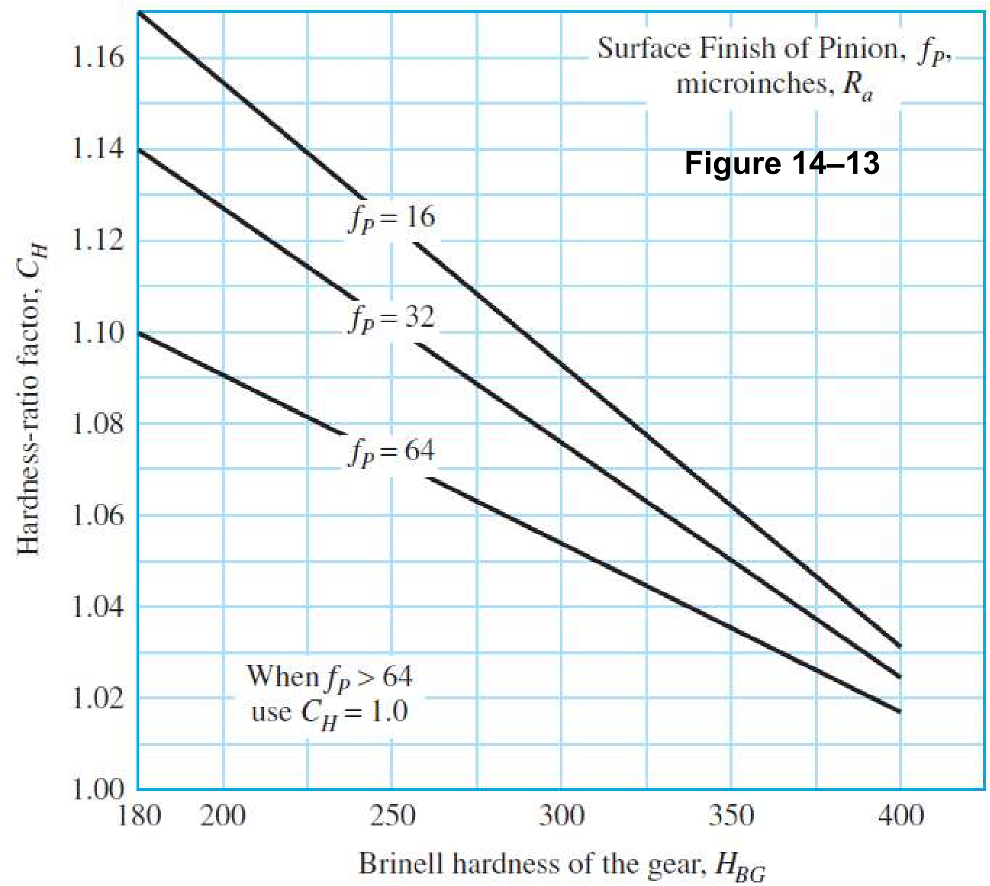


Figure 14–13

14-13 Stress-Cycle Factors Y_N and Z_N

Purpose of the stress-cycle factors Y_N and Z_N is to modify the gear strength for lives other than 10^7 cycles.

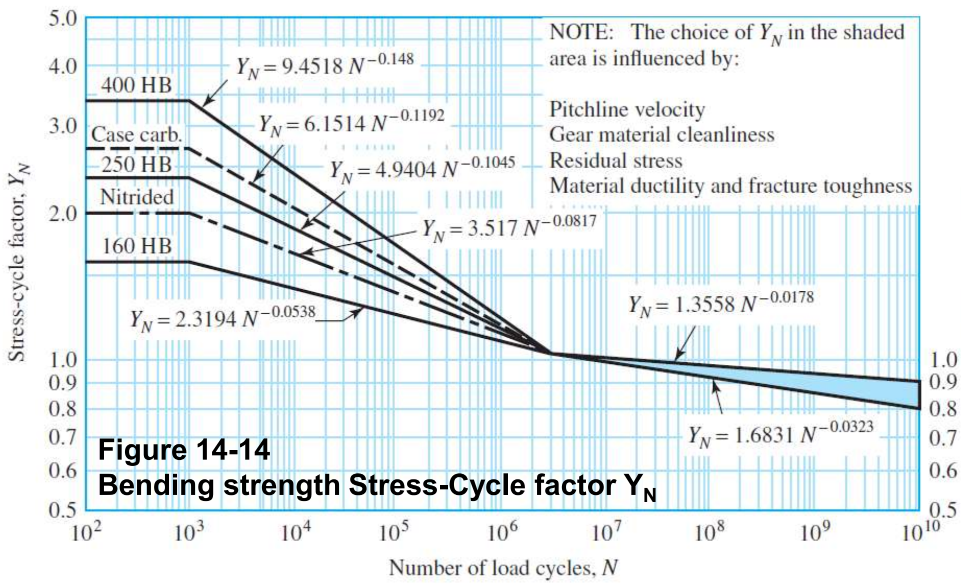


Figure 14-14
Bending strength Stress-Cycle factor Y_N

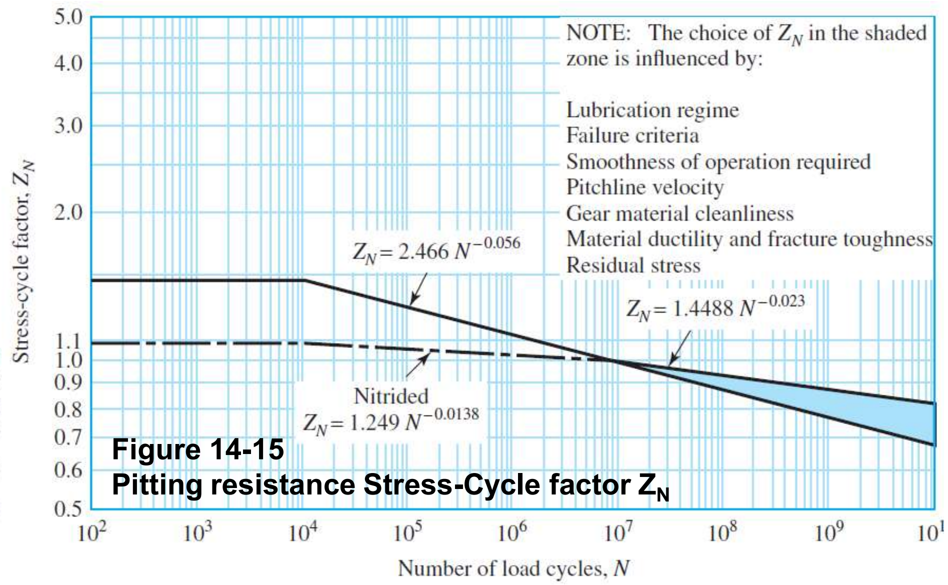


Figure 14-15
Pitting resistance Stress-Cycle factor Z_N

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\[2ex] \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

14-14 Reliability Factor K_R (Y_Z)

Gear strengths S_t and S_c are based on a reliability of 99 percent.

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \le R \le 0.9999 \end{cases}$$

Table 14-10

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F} \dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F} \dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H} \dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

14-15 Temperature Factor K_T (Y_θ)

For oil or gear-blank temperatures up to 250°F (120°C), use $K_T = Y_\theta = 1.0$.

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

14-16 Rim- Thickness Factor K_B

When the rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue failure may be through the gear rim rather than at the tooth fillet.

$$m_B = \frac{t_R}{h_t} = \frac{\textit{rim thickness below the tooth}}{\textit{tooth height}}$$

$$K_B = 1.6\ ln\frac{2.242}{m_B} \qquad m_B < 1.2$$

$$K_B = 1 \qquad m_B \geq 1.2$$

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{b m_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases}$$

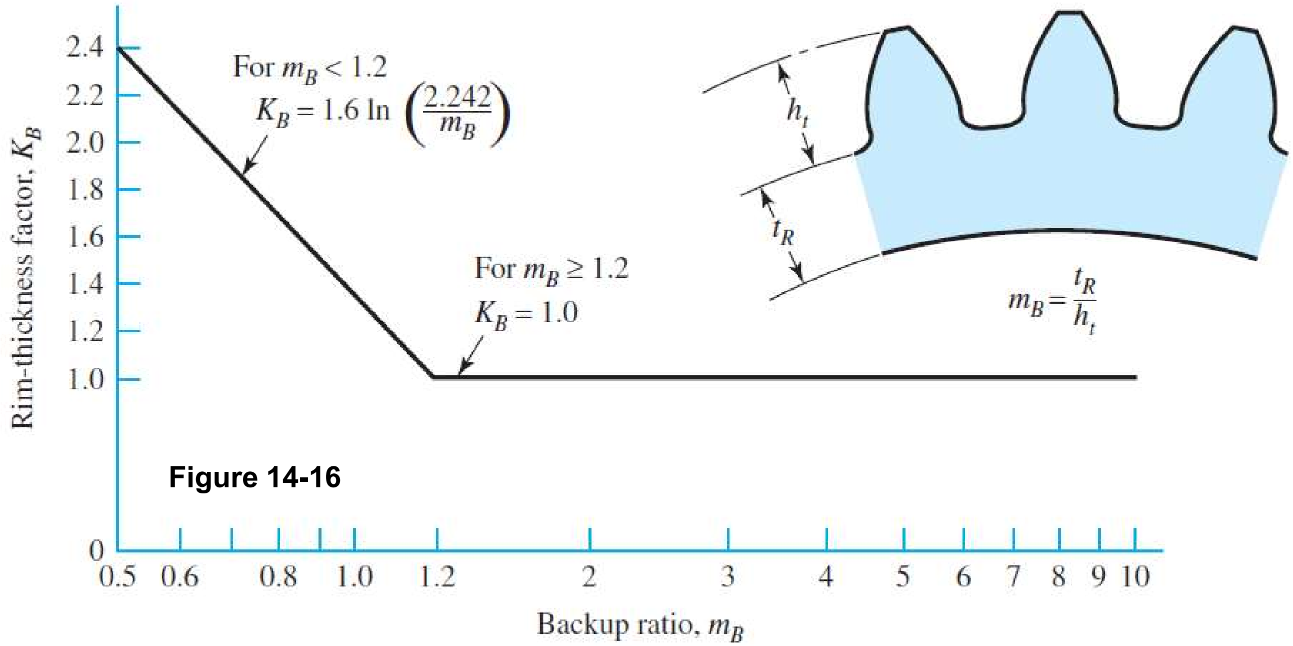


Figure 14-16

14-17 Safety Factors S_F and S_H

S_F : Safety factor guarding against bending fatigue failure

S_H: Safety factor guarding against pitting failure

$$S_F = \frac{S_t Y_N/(K_T K_R)}{\sigma} = \frac{fully\ corrected\ bending\ strength}{bending\ stress}$$

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c} = \frac{fully\ corrected\ contact\ strength}{contact\ stress}$$

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases}$$

Example 14-4

A 17-tooth 20° pressure angle spur pinion rotates at 1800 rev/min and transmits 4-hp to a 52-tooth disk gear. The diametral pitch is 10 teeth/in, the face width 1.5 in, and the quality standard is No. 6. The gears are straddle-mounted with bearings immediately adjacent. The pinion is a grade 1 steel with a hardness of 240 Brinell tooth surface and through-hardened core. The gear is steel, through-hardened also, grade 1 material, with a Brinell hardness of 200, tooth surface and core. Poisson's ratio is 0.30, J_P=0.30, J_G=0.40, and Young's modulus is $30(10^6)$ psi. The loading is smooth because of motor and load. Assume a pinion life of 10^8 cycles and a reliability of 0.90, and use $Y_N = 1.3558\, N^{-0.0178}$, $Z_N = 1.4488\, N^{-0.023}$. The tooth profile is uncrowned. This is a commercial enclosed gear unit.

- Find the factor of safety of the gears in bending.
- Find the factor of safety of the gears in wear.
- By examining the factors of safety, identify the threat to each gear and to the mesh.

	Design Notes	Tech Refs
14–5 Geometry Factors I and J (Z_I and Y_J)		
14–6 The Elastic Coefficient C_p (Z_E)		Eq 14-23
14–7 Dynamic Factor K_v	Q_v=6; quality standard No. 6	Eq 14-28 Eq 14-27, Kv=1.377
14–8 Overload Factor K_o	The loading is smooth because of motor and load	K_o=1
14–9 Surface Condition Factor C_f (Z_R)	TBD	C_f=1
14–10 Size Factor K_s	NG=52 teeth Np=17-tooth,	Table 14-2, Yp=0.303, F=1.5" $K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535} = 1.043$ YG=0.412, Ks=1.052
14–11 Load-Distribution Factor K_m (K_H)	The tooth profile is uncrowned. The gears are straddle-mounted with bearings immediately adjacent. This is a commercial enclosed gear unit.	Eq. (14–30): C_{mc} =1 Cpm=1 (Fig. 14–11): Cma = 0.15 Eq. (14–35): Ce = 1
14–12 Hardness-Ratio Factor C_H (Z_W)	grade 1 steel, HBP=240 & HBG=200	Eq 14-36
14–13 Stress-Cycle Factors Y_N and Z_N	$Y_N = 1.3558\, N^{-0.0178}$ $Z_N = 1.4488\, N^{-0.023}$	
14–14 Reliability Factor K_R (Y_Z)	reliability of 0.9	Table 14.10, K_R=0.85
14–15 Temperature Factor K_T (Y_u)	TBD	K_T=1
14–16 Rim-Thickness Factor K_B	Assuming constant thickness gears	K_B=1
S_t	grade 1 steel, HBP=240 & HBG=200	Table 14–3, Fig. 14–2
S_c	grade 1 steel, HBP=240 & HBG=200	Table 14–6, Fig. 14–5,

Matching Pinion and Gear Bending Safety Factor

It is desired to have equivalent SF between pinion and gear.

For bending stress

$$\sigma_P = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}\right)_P \qquad \sigma_G = \left(W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}\right)_G$$

$$(S_F)_P = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_P \qquad (S_F)_G = \left(\frac{S_t Y_N/(K_T K_R)}{\sigma}\right)_G$$

Substituting for stress/strength and cancelling identical terms:

$$(S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G} \frac{J_P}{J_G}$$

Speed ratio: $m_G = \frac{N_G}{N_P}$

Stress-cycle factor: $Y_N = \alpha\, N^{-\beta}$ (Fig. 14-14)

Stress-cycle factor, for pinion: $(Y_N)_P = \alpha\, N_P^{\beta}$, for gear: $(Y_N)_G = \alpha \left(\frac{N_P}{m_G}\right)^{\beta}$

Simplifying gives: $(S_t)_G = (S_t)_P \left(\frac{N_G}{N_P}\right)^{\beta} \frac{J_P}{J_G} = (S_t)_P\, m_G^{\beta} \frac{J_P}{J_G}$

Normally, $m_G>1$ and $J_G>J_P$, so the relationship shows that the gear can be less strong (lower Brinell hardness) than the pinion for the same safety factor.

Matching Pinion and Gear Pitting Resistance Safety Factor

For surface contact stress:

$$(\sigma_c)_P = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_P^{1/2} \qquad (\sigma_c)_G = C_p\left(W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}\right)_G^{1/2}$$

$$(S_H)_P = \left(\frac{S_c Z_N/(K_T K_R)}{\sigma_c}\right)_P \qquad (S_H)_G = \left(\frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}\right)_G$$

Substituting for stress/strength and cancelling identical terms:

$$(S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G}\left(\frac{1}{C_H}\right)_G = (S_t)_P\, m_G^{\beta}\left(\frac{1}{C_H}\right)_G$$

, where $\frac{(Z_N)_P}{(Z_N)_G} = m_G^{\beta}$

Value of β comes from Fig. 14-15.

Since C_H is so close to unity, it is usually neglected; therefore,

$$(S_c)_G = (S_c)_P\, m_G^{\beta}$$

Design Procedure and Guidelines

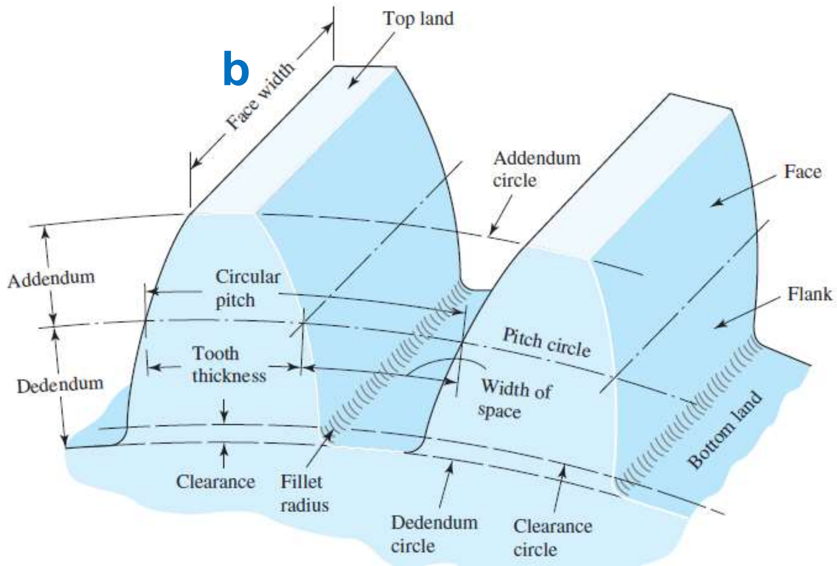
- Select Gear Type, Materials, Accuracy Grades, Heat Treatments, and Manufacturing Methods

- Initial Selection of Design Variables
  - Pinion number of teeth, N_P
  - Generally speaking, gears with more teeth tend to run more smoothly and quietly than gears with fewer teeth.
  - N_P should be as small as possible to keep a transmission system compact. But the possibility of interference or undercut is great with fewer teeth.
  - For enclosed gearings, initially select N_P =20–40
  - For open gearings, initially select N_P =17–20.

Selection Criteria of Face Width (b)

- Increasing face width reduces both bending stress and contact stresses and improves surface durability.
- Normally not greater than the pitch diameter of pinion, as a wide face width increases possibility of misalignment and uneven load distribution along the gear teeth.
- The wider the face width, the more difficult it is to manufacture and mount the gears so that contact is uniform across the full face width.
- Face width, b, is not standardized, but generally,

$$\frac{9}{P} < b < \frac{14}{P} \quad or \quad 9m < b < 14m$$



- Or depending from type of bearing support:

$$b_{gear} = \varphi_d \cdot d_p$$

$$b_{pinion} = b_{gear} + (5{\sim}10)mm$$

, where

$\varphi_d = 0.9{\sim}1.4$ Symmetric Support

$\varphi_d = 0.7{\sim}1.15$ Asymmetric Support

$\varphi_d = 0.4{\sim}0.6$ Cantilever Support