

Instructor: Ping C. Sui, Ph.D. ME 1029 Mechanical Design 2

Fall 2021

Topics Covered

- 8–1 Thread Standards and Definitions
- 8–2 The Mechanics of Power Screws
- 8–3 Threaded Fasteners
- 8–4 Joints-Fastener Stiffness
- 8–5 Joints Member Stiffness
- 8–6 Bolt Strength
- 8–7 Tension Joints-The External Load
- 8–8 Relating Bolt Torque to Bolt Tension
- 8–9 Statically Loaded Tension Joint with Preload
- 8–11 Fatigue Loading of Tension Joints
- 8–10 Gasketed Joints
- 8–12 Bolted and Riveted Joints Loaded in Shear



Introduction

- Nonpermanent Joints/ Threaded Fasteners
 - Screws
 - Bolts/Nuts
 - Rivets
- Different names but similar working principle: change angular motion to linear motion
 - to transmit power, or
 - to develop large forces for position locking, liquid/gas sealing, etc.





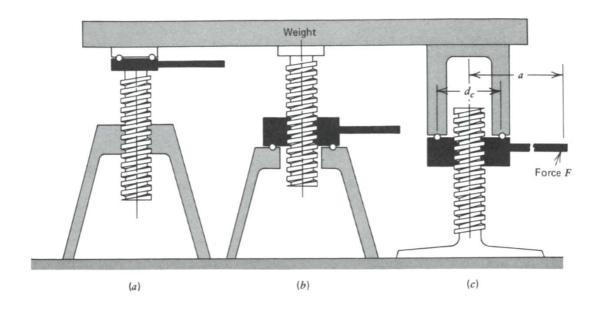


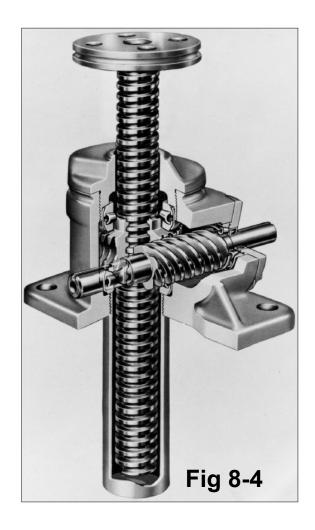




Power Screws

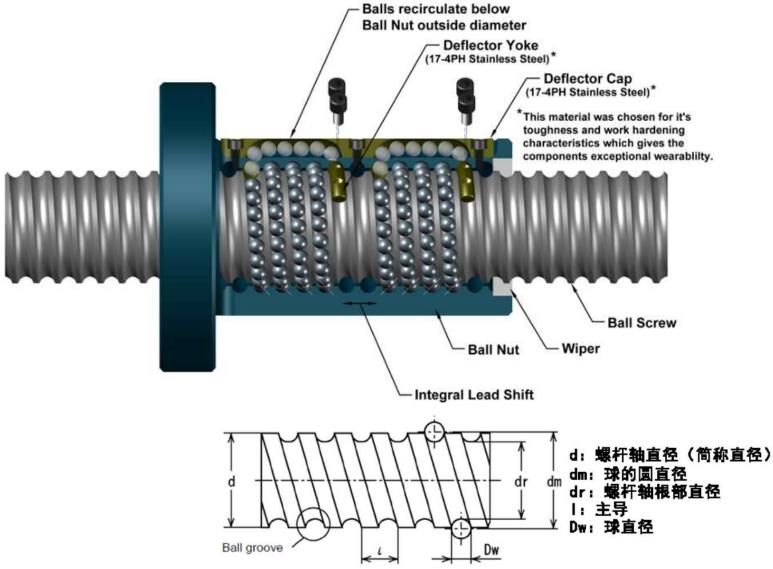
 A power screw is a device used in machinery to change angular motion into linear motion, and, usually, to <u>transmit power</u>.







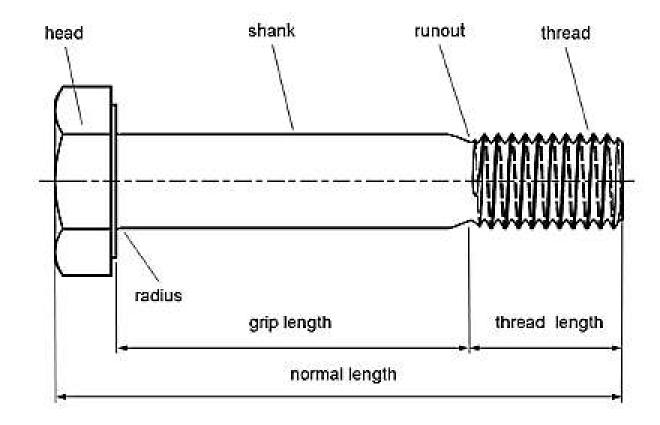
High Precision Application: Ball Screws



8-1 Thread Standards and Definitions: Geometry and Terminology of External Screw Thread

Thread Details

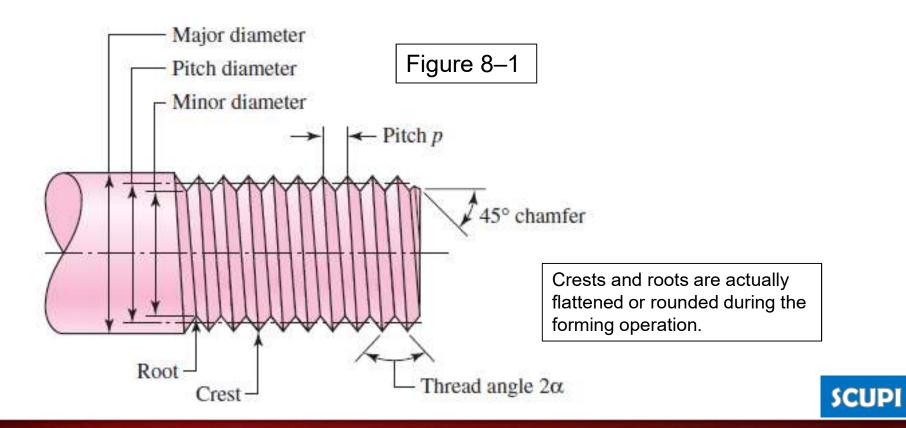
 All threads are made according to the right-hand rule unless otherwise noted. That is, if the bolt is turned clockwise, the bolt advances toward the nut.





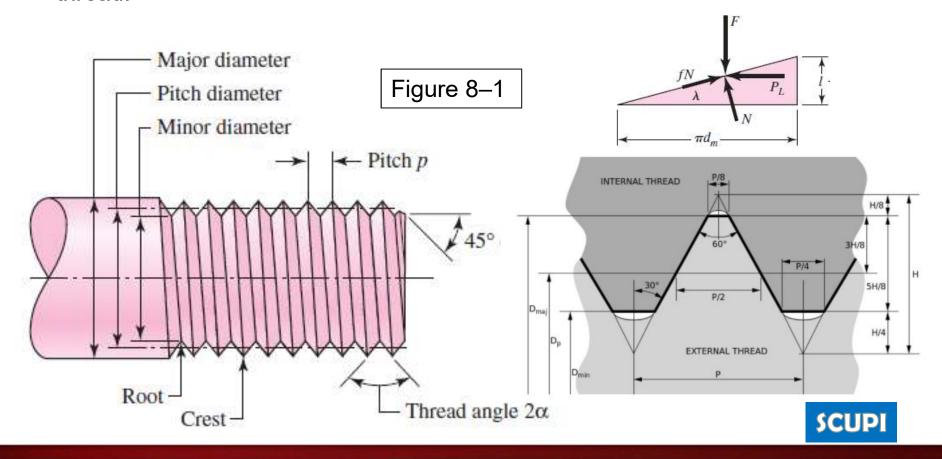
Terminology – Thread Profile

- Pitch (p): distance between adjacent thread forms measured parallel to the thread axis
- Major Diameter (d): the largest diameter of a screw thread
- Minor (or root) diameter (d_r): the smallest diameter of a screw thread
- Pitch diameter (d_p): A theoretical diameter between the major and minor diameters (the diameter where the width of the thread and groove are equal)



Terminology – Thread Profile

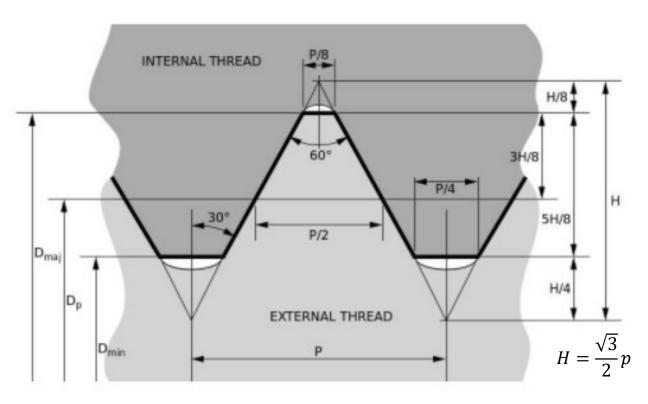
- Lead (I): axial distance the mating thread (or nut) advancing in one revolution.
- Thread height (h): radial distance between major diameter and minor diameter.
- Thread angle (α) : angle between flanks of adjacent threads measured in an axial plane.
- Lead angle (λ) : angle between perpendicular of the screw axis and rise of the thread.

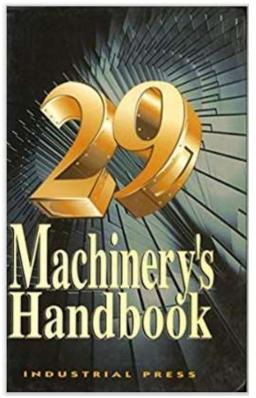


Unified Thread Profiles

American National (Unified) Thread Standard

- United States and Great Britain (Unit: inch)
- M and MJ profiles, etc. (Unit: mm)
- Thread angle is 60° and the crests of the thread may be either flat or rounded.







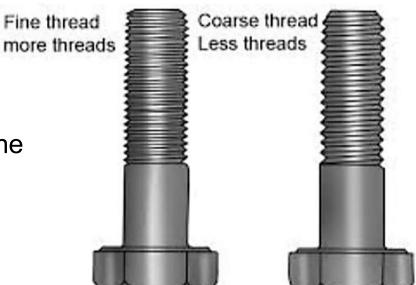
Unified Thread Series

UNC: Coarse-Pitch Series

UNF: Fine-Pitch Series

UNEF: Extra-Fine-Pitch Series

UNRF: Unified National Round Fine



- Unified threads are specified by stating the <u>nominal major diameter</u>, the <u>number of threads per inch</u> (TPI), and the <u>thread series</u>.
 - 5/8 in-18 UNRF (0.625 in-18 UNRF): nominal major diameter of 0.625 in and 18 threads per inch
- Metric threads are specified by writing the <u>nominal major diameter</u> and <u>pitch</u> in millimeters, in that order.
 - M12X1.75: nominal major diameter of 12 mm and pitch of 1.75 mm.



Unified Screw Threads UNC and UNF (Metric Threads)

Nominal	C	oarse-Pitch	Series		Fine-Pitch S	eries
Major Diameter d mm	Pitch P mm	Tensile- Stress Area A, mm²	Minor- Diameter Area A, mm²	Pitch P mm	Tensile- Stress Area A, mm²	Minor- Diamete Area A, mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			0.4
3.5	0.6	6.78	6.00		Tabl	e 8-1
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

ANSI B1. 1-1974 and B18. 3. 1-1978

Minor Diameter: d_r =d-1.226869p Pitch Diameter: d_p =d-0.649519p

Tensile Stress Area =
$$\frac{\pi}{4} \left(\frac{d_r + d_p}{2} \right)^2$$

Unified Screw Threads UNC and UNF

Table 8-2
Diameters and Area of Unified Screw Threads UNC and UNF*

Size Designation		Coc	ırse Series-	-UNC	Fi	Fine Series—UNF			
	Nominal Major Diameter in	Threads per Inch N	Tensile- Stress Area A _f in ²	Minor- Diameter Area A _r in ²	Threads per Inch N	Tensile- Stress Area A _f in ²	Minor- Diameter Area A _r in ²		
0	0.0600				80	0.001 80	0.001 51		
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37		
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39		
3	0.0990	48	0.004 87	0.004.06	56	0.005 23	0.004 51		
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66		
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16		
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74		
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85		
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5		
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.022 6		
1/4	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.032 6		
5	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.0524		
1 4 5 16 3 8 7 16 1 2 9 16	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.0809		
7	0.4375	14	0.1063	0.093 3	20	0.1187	0.1090		
1 2	0.5000	13	0.141 9	0.125 7	20	0.1599	0.148 6		
16	0.5625	12	0.182	0.162	18	0.203	0.189		
<u>5</u>	0.6250	11	0.226	0.202	18	0.256	0.240		
3	0.7500	10	0.334	0.302	16	0.373	0.351		
5 8 3 4 7	0.8750	9	0.462	0.419	14	0.509	0.480		
1	1.0000	8	0.606	0.551	12	0.663	0.625		
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024		
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521		

ANSI B1. 1-1974 and B18. 3. 1-1978

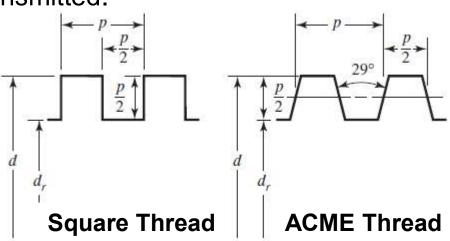
Minor Diameter: d_r =d-1.299038p Pitch Diameter: d_p =d-0.649519p

Tensile Stress Area =
$$\frac{\pi}{4} \left(\frac{d_r + d_p}{2} \right)^2$$

Power Screws - Square Thread and ACME Thread

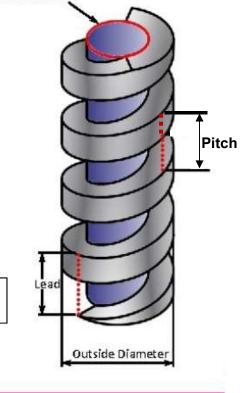
Square and Acme threads are used on screws when power is to be

transmitted.



Major Diameter = d Minor Diameter d_r = d-p Mean Diameter $d_m = d - \frac{p}{2}$

Tensile Stress Area = $\frac{\pi d_r^2}{4}$

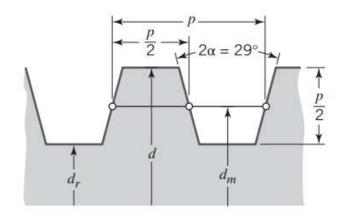


Internal Diameter

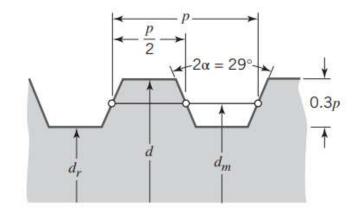
Table 8-3 Preferred Pitches for ACME Thread

d, in	<u>1</u>	<u>5</u>	3/8	$\frac{1}{2}$	<u>5</u> 8	<u>3</u>	<u>7</u> 8	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3
p, in	1 16	1 14	1 12	1 10	1/8	1 6	<u>1</u>	<u>1</u> 5	1/5	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

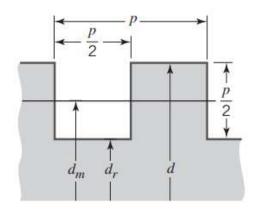
Alternative Power Screw Profiles



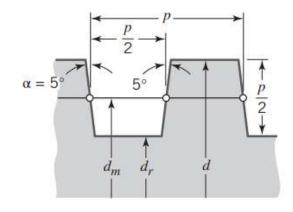




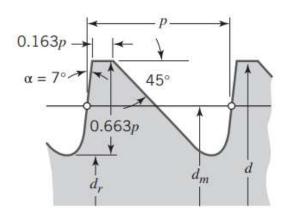
(b) Acme stub



(c) Square



(d) Modified square



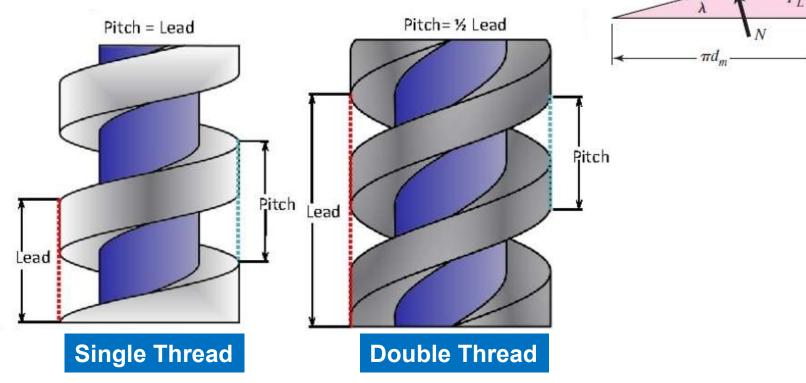
(e) Buttress

Relationship between Lead vs. Pitch

• For l as the lead, and λ as the lead angle, and p as the pitch

$$\tan \lambda = \frac{l}{\pi d_m}$$

- Single Thread: l = p
- Double Thread: l = 2p





8-2 The Mechanics of Power Screws

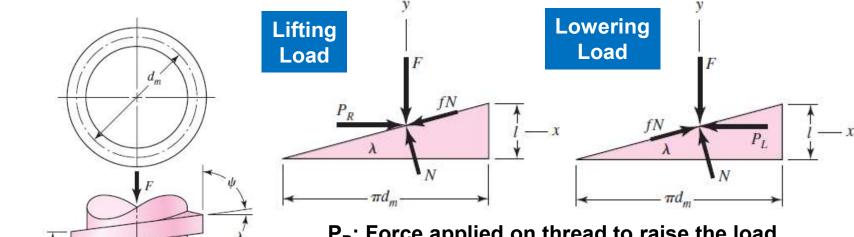
Objectives:

- Estimate needed torque to raise the load; and
- Evaluate structural integrity of the threads.



Mechanics of A Power Screw (Square Thread Form)

- Torque required to raise this load (tightening the bolt/screw), and
- Torque required to <u>lower</u> the load (loosening the bolt/screw)



P_R: Force applied on thread to raise the load.

P_L: Force applied on thread to lower the load.

Table 8-4 Coefficient of Friction, f

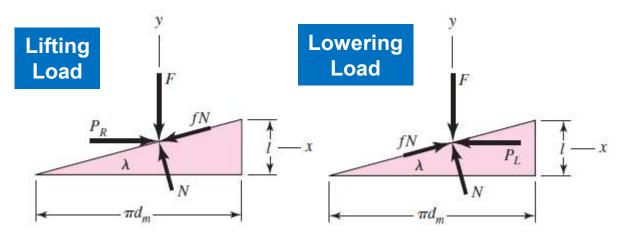
Screw				
Material	Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15-0.25	0.15-0.23	0.15-0.19	0.15-0.25
Steel, machine oil	0.11-0.17	0.10-0.16	0.10-0.15	0.11-0.17
Bronze	0.08-0.12	0.04-0.06		0.06-0.09

 ψ : helix angle λ : lead angle

Nut

F/2





$$P_R - N \sin \lambda - fN \cos \lambda = 0$$

$$-F - fN \sin \lambda + N \cos \lambda = 0$$

$$-P_L - N\sin\lambda + fN\cos\lambda = 0$$

$$-F + fN \sin \lambda + N \cos \lambda = 0$$

By cancelling normal force N,

$$P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda}$$

$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda}$$

Divide both numerator and denominator by $\cos \lambda$ and use $\tan \lambda = \frac{\iota}{\pi d_m}$

$$P_R = \frac{F(l/\pi d_m + f)}{1 - fl/\pi d_m}$$

$$P_L = \frac{F(f - l/\pi d_m)}{1 + fl/\pi d_m}$$

Multiply by the mean radius $d_m/2$ to get torque

$$T_R = \frac{Fd_m}{2} \frac{\pi f d_m + l}{\pi d_m - fl}$$

$$T_L = \frac{Fd_m}{2} \frac{\pi f d_m - l}{\pi d_m + fl}$$

Power Screw Torque Requirement

Torque required to overcome thread friction and to <u>raise</u> the load

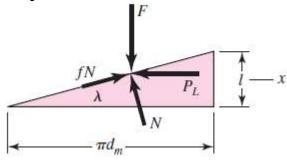
$$T_R = \frac{Fd_m}{2} \frac{\pi f d_m + l}{\pi d_m - fl}$$

Torque required to overcome thread friction and to <u>lower</u> the load

$$T_L = \frac{Fd_m}{2} \frac{\pi f d_m - l}{\pi d_m + fl}$$

- Self-Locking: Prevent screw from slipping when lowering load
 - For self-locking condition when lowering the load, $\pi f d_m > l$, the following condition must satisfy

$$f > \frac{l}{\pi d_m} = \tan \lambda$$



Power Screw Torque Efficiency

• Let COF f = 0, torque on frictionless surface:

$$T_0 = \frac{Fl}{2\pi}$$

- since thread friction has been eliminated, this is the torque required only to raise the load.
- Define torque efficiency as

$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi T_R}$$

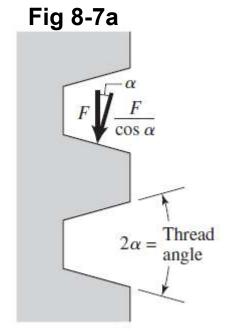
Is T_0 smaller or greater than T_R ?

2nd Torque Contribution Term (Wedge Friction)

- For non-square threads, like ACME, additional friction is introduced by the thread angle. This is known as the "Wedge" effect.
- Effect of thread angle is simplified by neglecting the compound effect of lead angle on thread angle. In this way, normal force component on thread should be represented by $\frac{N}{\cos \alpha}$
- The frictional torque for raising the load is:

$$T_R = \frac{Fd_m}{2} \left(\frac{l + \pi f}{\pi d_m - f l \sec \alpha} \right)$$

• Note that for ACME Thread: $2\alpha = 29^{\circ}$



3rd Torque Contribution Term (Collar/Shoulder Friction)

In case a collar or shoulder is used to support the axial load or exert preload on joints, this friction also needs to be overcome in order to tighten or loosen the joint.

$$T_c = \frac{Ff_c d_c}{2}$$

Henceforth, the total torque for tightening a joint is

$$T_R = \frac{Fd_m}{2} \left(\frac{l + \pi f d_m \sec \alpha}{\pi d_m - f l \sec \alpha} \right) + \frac{Ff_c d_c}{2}$$

Table 8-6 Coefficient of Friction, fc

Combination	Running	Starting	
Soft steel on cast iron	0.12	0.17	
Hard steel on cast iron	0.09	0.15	
Soft steel on bronze	0.08	0.10	
Hard steel on bronze	0.06	0.08	

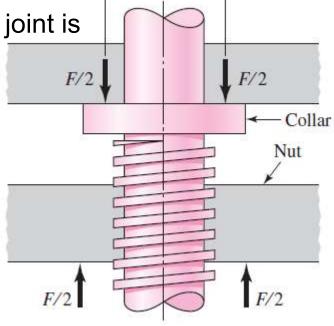


Fig 8-7b

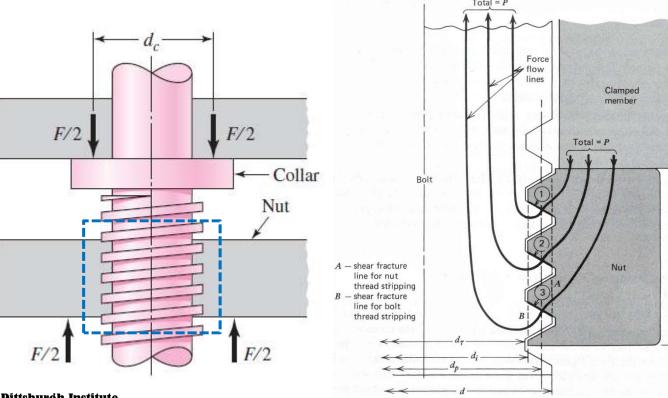
Load Sharing Among Power Screw Threads

The engaged threads don't share the load equally.

Experiments showed that the first engaged thread (FET) carries 0.38 of the load, the second 0.25, the third 0.18, and the seventh is free of load.

Use 0.38F and n₁ = 1 for the maximum stress on power screw thread

form



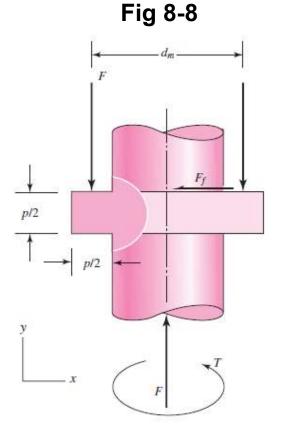
Static Thread Stresses

On Thread Body

- Torsional stress during tightening $au = \frac{16 \, T}{\pi d_r^3}$
- Axial stress

$$\sigma = \frac{F}{A_t}$$

- Threaded Fastener: See Table 8-1, 8-2 for A_t
- Power Screw: $A_t = \frac{\pi d_r^2}{4}$



Static Thread Stresses

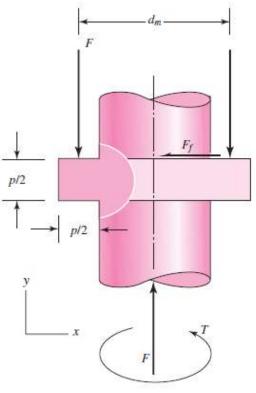
On Thread Flank Face

- Bearing stress: $\sigma_B = -\frac{F}{\pi d_m n_t(p/2)} = -\frac{2F}{\pi d_m n_t p}$
 - n_t: number of engaged threads

Table 8-4 Screw Bearing Pressure

Screw Material	Nut Material	Safe p _b , psi	Notes
Steel	Bronze	2500-3500	Low speed
Steel	Bronze	1600-2500	≤10 fpm
	Cast iron	1800-2500	$\leq 8 \text{ fpm}$
Steel	Bronze	800-1400	20-40 fpm
	Cast iron	600-1000	20-40 fpm
Steel	Bronze	150-240	≥50 fpm

Fig 8-8



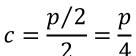
Static Thread Stresses

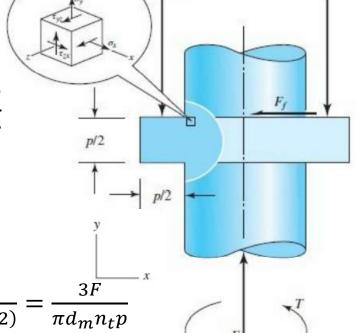
At Thread Root

Bending Stress

$$I = \frac{\pi d_r n_t p^3}{96} \qquad M = \frac{Fp}{4} \qquad c = \frac{p/2}{2} = \frac{p}{4}$$

$$\sigma_b = \frac{Mc}{I} = \frac{6F}{\pi d_r n_t p}$$





Transverse (or Punching) Shear

- @Center of Root
$$\tau = \frac{3V}{2A} = \frac{3}{2} \frac{F}{\pi d_m n_t(p/2)} = \frac{3F}{\pi d_m n_t p}$$

Shear force acting at the root radius balances the torsion, T.

$$\tau = \frac{T}{\left(\pi d_r n_t \frac{p}{2}\right) \frac{d_r}{2}} = \frac{4T}{\pi d_r^2 n_t p}$$

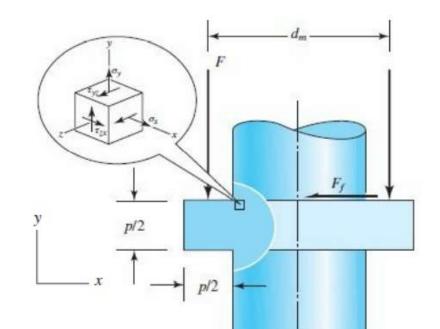
Failure Assessment for Static Power Screw Threads

Component Stresses at Thread Root:

$$\sigma_{x} = \frac{6F}{\pi d_{r} n_{t} p} \qquad \tau_{xy} = 0$$

$$\sigma_{y} = -\frac{4F}{\pi d_{r}^{2}} \qquad \tau_{yz} = \frac{16 T}{\pi d_{r}^{3}}$$

$$\sigma_{z} = 0 \qquad \tau_{zx} = -\frac{4T}{\pi d_{r}^{2} n_{t} p}$$



 Typical static failure assessment methods (MSST or DET) can be applied for power screw threads analysis.

$$\sigma_e = \frac{1}{\sqrt{2}} \left[\left(\sigma_x - \sigma_y \right)^2 + \left(\sigma_y - \sigma_z \right)^2 + (\sigma_z - \sigma_x)^2 + 6 \left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 \right) \right]^{1/2}$$

Fig 8-8



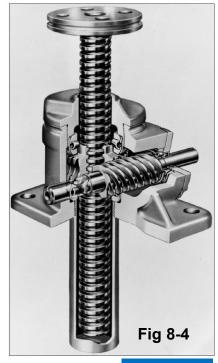
Example 8-1

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with <u>double threads</u>, and it is to be used in an application similar to that in Fig. 8–4.

Given $f = f_c = 0.08$, $d_c = 40$ mm, and F = 6.4 kN per screw.

Find:

- (a) thread depth, thread width, pitch diameter, minor diameter, and lead.
- (b) torque required to raise and lower the load
- (c) efficiency during lifting the load
- (d) body stresses, torsional and compressive
- (e) bearing stress
- (f) thread bending stress at the root of the thread
- (g) von Mises stress at the root of the thread
- (h) maximum shear stress at the root of the thread.





Example 8-1 (Cont'd)

Solution:

(a) Find thread depth, thread width, pitch diameter, minor diameter, and lead.

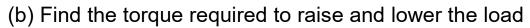
Major diameter d=32mm; Pitch p=4mm;

Minor diameter d_r=d-p=32-4=28mm

Mean diameter $d_m=d-p/2=32-4/2=30$ mm

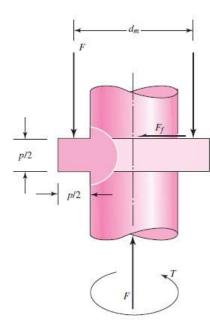
Lead I=np=2(4)=8mm

Thread width= $\frac{p}{2} = 2mm$; Thread depth= $\frac{p}{2} = 2mm$



Torque required to raise the load

$$\begin{split} T_R &= \frac{Fd_m}{2} \left(\frac{l + \pi f d_m}{\pi d_m - f l} \right) + \frac{Ff_c d_c}{2} \\ &= \frac{6400 \cdot 30}{2} \left(\frac{8 + \pi \cdot 0.08 \cdot 30}{\pi 30 - 0.08 \cdot 8} \right) + \frac{6400 \cdot 0.08 \cdot 40}{2} = 26180 \, N \cdot mm = 26.18 \, N \cdot n \iota \end{split}$$



Torque required to lower the load

$$\begin{split} T_L &= \frac{Fd_m}{2} \left(\frac{\pi f d_m - l}{\pi d_m + f l} \right) + \frac{Ff_c d_c}{2} \\ &= \frac{6400 \cdot 30}{2} \left(\frac{\pi \cdot 0.08 \cdot 30 - 8}{\pi 30 + 0.08 \cdot 8} \right) + \frac{6400 \cdot 0.08 \cdot 40}{2} = 9770 \ N \cdot mm = 9.77 \ N \cdot m \end{split}$$

Example 8-1 (Cont'd)

Solution:

(c) Find the efficiency during lifting the load

Overall efficiency
$$e = \frac{Fl}{2\pi T_R} = \frac{6400 \cdot 8}{2\pi \cdot 26180} = 0.31$$

(d) Find the body stresses, torsional and compressive.

Thread body shear stress
$$\tau = \frac{16 T_R}{\pi d_r^3} = \frac{16 \cdot 26180}{\pi \cdot 28^3} = 6.1 MPa$$

Power Screw
$$A_t = \frac{\pi d_r^2}{4} = \frac{\pi^2}{4} = 616 \text{ } mm^2$$

Axial nominal stress
$$\sigma = -\frac{6400}{616} = -10.39 MPa$$

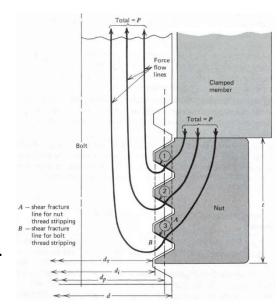
(e) Find the bearing stress

Assume one thread (n_t=1) carrying 0.38F

$$\sigma_B = -\frac{F}{\pi d_m n_t(p/2)} = \frac{0.38 \cdot 6400}{\pi \cdot 30 \cdot 1 \cdot 2} = -12.9 MPa$$

(f) Find the thread bending stress at the root of the thread.

$$\sigma_b = \frac{Mc}{I} = \frac{6F}{\pi d_r n_t p} = \frac{6 \cdot 0.38 \cdot 6400}{\pi \cdot 28 \cdot 1.4} = 41.5 MPa$$



Example 8-1 (Cont'd)

Solution:

Tangential shear stress:

$$\tau_{zx} = -\frac{4T}{\pi d_r^2 n_t p} = -4.04 MPa$$

(g) Determine the von Mises stress at the root of the thread.

$$\sigma_x$$
 = 41.5 MPa au_{xy} = 0 au_{yz} = -10.39 MPa au_{yz} = 6.07 MPa au_{zx} = -4.04 MPa

Von Mises Stress is:

$$\sigma_e = \frac{1}{\sqrt{2}} \left[\left(\sigma_x - \sigma_y \right)^2 + \left(\sigma_y - \sigma_z \right)^2 + \left(\sigma_z - \sigma_x \right)^2 + 6 \left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2 \right) \right]^{1/2}$$

$$\sigma_e = \frac{1}{\sqrt{2}} \left[\left(41.5 - (-10.39) \right)^2 + (-10.39 - 0)^2 + (0 - 41.5)^2 + 6(0 + 6.07^2 + 4.04^2) \right]^{1/2}$$

$$= 49.2 \, MPa$$

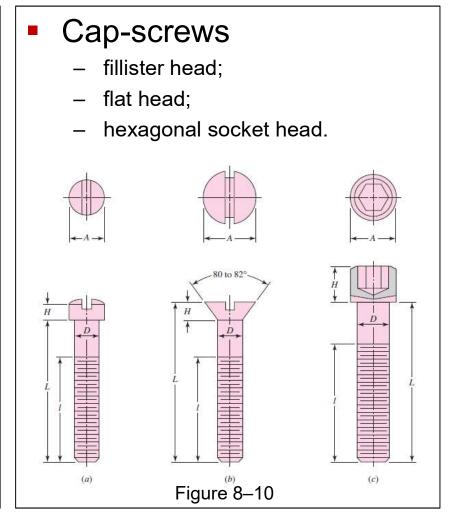
8-3 Threaded Fasteners



Examples of Threaded Fasteners

Hexagon-head bolt and nuts Figure 8–9 ← Approx.

1/64 in Figure 8–12 Approx. 1/4 in → H→



Unified Screw Threads UNC and UNF (Metric Threads)

Nominal	C	oarse-Pitch	Series		Fine-Pitch Series			
Major Diameter d mm	Pitch P mm	Tensile- Stress Area A, mm²	Minor- Diameter Area A, mm²	Pitch P mm	Tensile- Stress Area A, mm²	Minor- Diameter Area A, mm²		
1.6	0.35	1.27	1.07					
2	0.40	2.07	1.79					
2.5	0.45	3.39	2.98					
3	0.5	5.03	4.47			0.4		
3.5	0.6	6.78	6.00		Table	e 8-1		
4	0.7	8.78	7.75					
5	0.8	14.2	12.7					
6	1	20.1	17.9					
8	1.25	36.6	32.8	1	39.2	36.0		
10	1.5	58.0	52.3	1.25	61.2	56.3		
12	1.75	84.3	76.3	1.25	92.1	86.0		
14	2	115	104	1.5	125	116		
16	2	157	144	1.5	167	157		
20	2.5	245	225	1.5	272	259		
24	3	353	324	2	384	365		
30	3.5	561	519	2	621	596		
36	4	817	759	2	915	884		
42	4.5	1120	1050	2	1260	1230		
48	5	1470	1380	2	1670	1630		
56	5.5	2030	1910	2	2300	2250		
64	6	2680	2520	2	3030	2980		
72	6	3460	3280	2	3860	3800		
80	6	4340	4140	1.5	4850	4800		
90	6	5590	5360	2	6100	6020		
100	6	6990	6740	2	7560	7470		
110				2	9180	9080		

ANSI B1. 1-1974 and B18. 3. 1-1978

Minor Diameter: d_r =d-1.226869p Pitch Diameter: d_p =d-0.649519p

Tensile Stress Area =
$$\frac{\pi}{4} \left(\frac{d_r + d_p}{2} \right)^2$$

Unified Screw Threads UNC and UNF

Table 8-2
Diameters and Area of Unified Screw Threads UNC and UNF*

Size Designation		Coarse Series—UNC			Fi	Fine Series—UNF		
	Nominal Major Diameter in	Threads per Inch N	Tensile- Stress Area A _f in ²	Minor- Diameter Area A _r in ²	Threads per Inch N	Tensile- Stress Area A _f in ²	Minor- Diameter Area A _r in ²	
0	0.0600				80	0.001 80	0.001 51	
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37	
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39	
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51	
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66	
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16	
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74	
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85	
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5	
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.0226	
1/4	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.0326	
14516 38716 12916	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.0524	
3	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.0809	
7	0.4375	14	0.1063	0.093 3	20	0.1187	0.1090	
1/2	0.5000	13	0.141 9	0.125 7	20	0.1599	0.148 6	
78	0.5625	12	0.182	0.162	18	0.203	0.189	
5	0.6250	11	0.226	0.202	18	0.256	0.240	
3	0.7500	10	0.334	0.302	16	0.373	0.351	
58 34 7 8	0.8750	9	0.462	0.419	14	0.509	0.480	
1	1.0000	8	0.606	0.551	12	0.663	0.625	
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024	
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521	

ANSI B1. 1-1974 and B18. 3. 1-1978

Minor Diameter: d_r =d-1.299038p Pitch Diameter: d_p =d-0.649519p

Tensile Stress Area $=\frac{\pi}{4}\left(\frac{d_r+d_p}{2}\right)^2$

Bolt/Screw Geometric and Structural Information

Metric	USCS	Provided Information	
Table 8-1	Table 8-2	Diameters, Areas, TPI of Coarse-Pitch and Fine- Pitch Threads	
	Table 8–9	SAE Specifications for Steel Bolts (Min Proof/Tensile/Yield Strength)	
	Table 8-10	ASTM Specifications for Steel Bolts (Min Proof/Tensile/Yield Strength)	
Table 8-11		Steel Bolts, Screws, and Studs (Min Proof/Tensile/Yield Strength)	
Table A–29	Table A–29	Bolt head dimensions of Square and Hexagonal Bolts	
Table A–30	Table A–30	Screw head dimensions of Hexagonal Cap Screws and Heavy Hexagonal Screws	
Table A–31	Table A–31	Dimensions of Hexagonal Nuts	
Table A–33	Table A–32	Dimensions of Plain Washers	
Table 8–4	Table 8–4	Screw Bearing Pressure P _b	
Table 8–5	Table 8–5	Coefficients of Friction f for Threaded Pairs	
Table 8–6	Table 8–6	Thrust-Collar Friction Coefficients	
Table 8–7	Table 8–7	Procedure for Finding Fastener Stiffness	
Table 8–8	Table 8–8	Simplified Member Stiffness Calculation: Parameters of Various Member Materials	
	Table 8-12	Load Sharing based on Bolt and Member Stiffnesses.	
Table 8-15	Table 8-15	Torque Factors K for Use with Eq 8-27	
Table 8-16	Table 8-16	Fatigue Stress-Concentration Factors K _f for Threaded Elements	
Table 8-17	Table 8-17 (P.445)	Fully Corrected Endurance Strengths for Bolts and Screws with Rolled Threads	

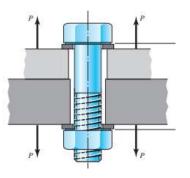


Structural Risks of Nonpermanent Joints

- In most cases the threat is from overproof loading of fasteners.
- The threat from fatigue is low and deterministic method is adequate.
- Washers must always be used under the bolt head
- During tightening, the first thread of the nut tends to take the entire load; but yielding occurs, which eventually further distributed the loading over about three nut threads. For this reason, <u>nuts should</u> never be re-used.









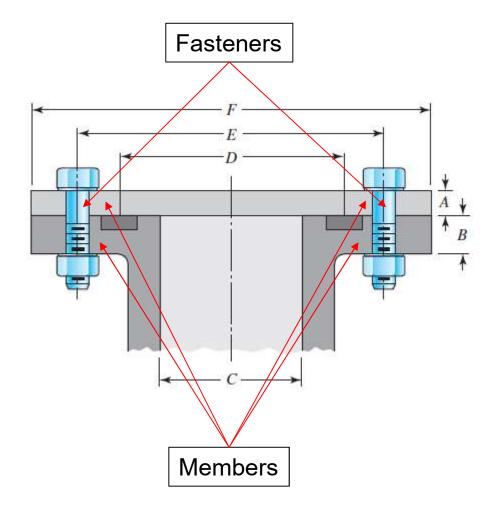
8-4 Joints: Fastener Stiffness

8-5 Joints: Member Stiffness

Final joint stiffness is the combination of fastener and member.







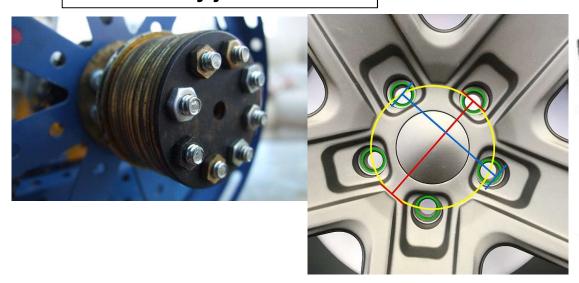
Joint = Fastener + Members

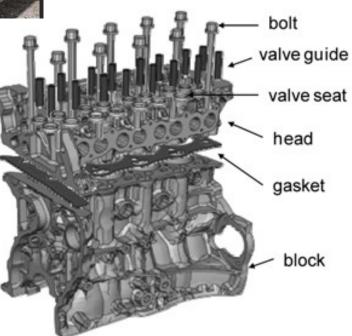






Q: How many joints needed?

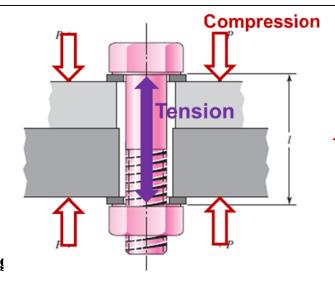


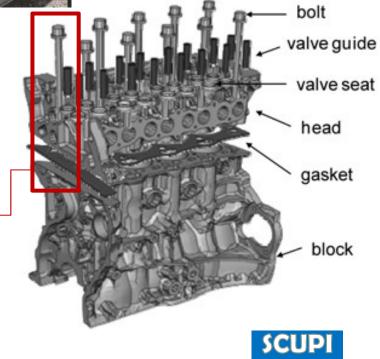


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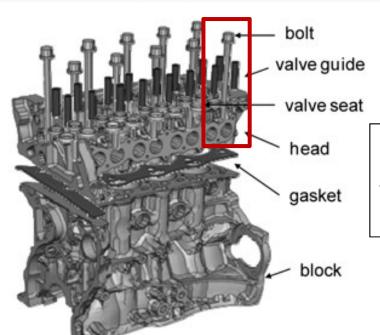


Q: What are the tension/compression acting on each joint?



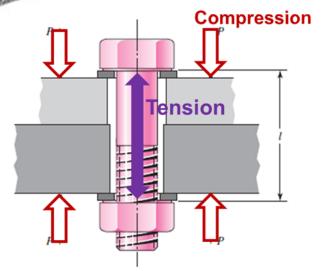


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Questions:

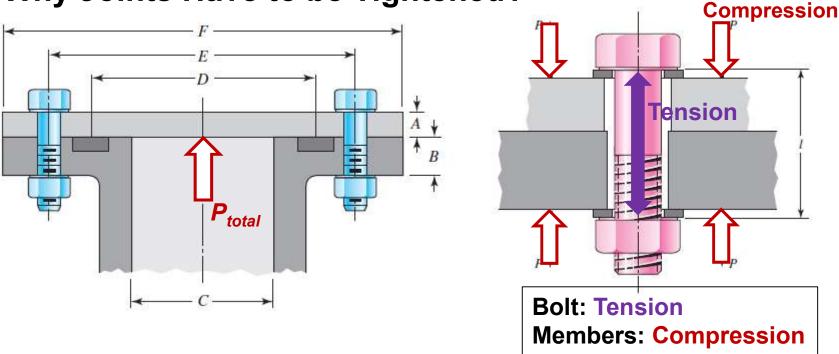
How many joints are needed? What are the tension/compression acting on each joint?



$$\frac{1}{K_{joint}} = \frac{1}{K_{fastener}} + \frac{1}{K_{member}}$$



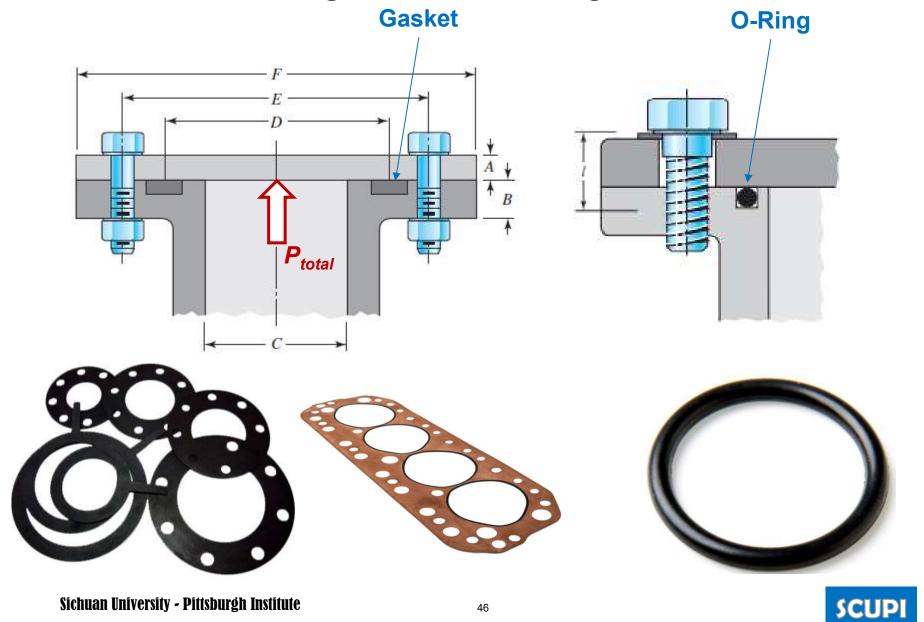
Why Joints Have to be Tightened?



- Purpose of a joint is to clamp two, or more, parts together.
- Clamping force is produced by twisting the nut stretches the bolt to produce the clamping force.
- This clamping force is called the pretension or bolt preload.



Gaskets or O-Ring Seals for Leakage Prevention



Fastener Stiffness Calculation

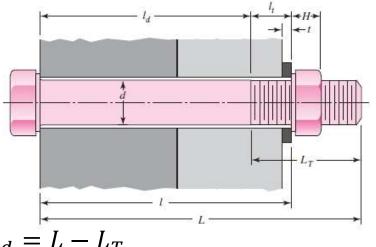
- Fastener Length: L
- Thread Length: L_{τ}
- Grip Length: *l*



- Length of Threaded Portion in Grip: $l_t = l l_d$
- Stress Area of Unthreaded Portion: $A_d = \pi d^2/4$
- Stress Area of Threaded Portion: A_t (Table 8-1 & 8-2)
- Stiffness of Unthreaded Portion: $k_d = \frac{A_d E}{L_d}$
- Stiffness of Threaded Portion: $k_t = \frac{A_t E}{I}$
- Fastener Effective Stiffness k_h

Fastener Effective Stiffness
$$k_b$$

$$k_b = \frac{1}{\frac{1}{k_d} + \frac{1}{k_t}} = \frac{k_d k_t}{k_d + k_t} = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$



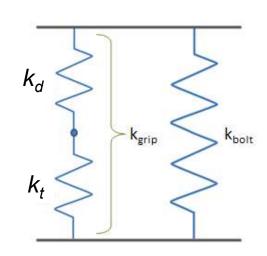


Table 8-7 Procedure for Finding Fastener Stiffness

Given fastener diameter d and pitch p in mm or number of threads per inch

Given fastener diameter d and pitch p in mm or number of threads per inch

Washer thickness: t from Table A-32 or A-33

Nut thickness [Fig. (a) only]: H from Table A-31

Grip length:

For Fig. (a): l = thickness of all material squeezedbetween face of bolt and face of nut

For Fig. (b): $l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \ge d \end{cases}$

Fastener length (round up using Table A-17*):

For Fig. (a): L > l + H

For Fig. (*b*): L > h + 1.5d

Threaded length L_T : Inch series:

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in,} & L \le 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in,} & L > 6 \text{ in} \end{cases}$$

Metric series:

$$L_T = \begin{cases} 2d + 6 \text{ mm}, & L \le 125 \text{ mm}, d \le 48 \text{ mm} \\ 2d + 12 \text{ mm}, & 125 < L \le 200 \text{ mm} \\ 2d + 25 \text{ mm}, & L > 200 \text{ mm} \end{cases}$$

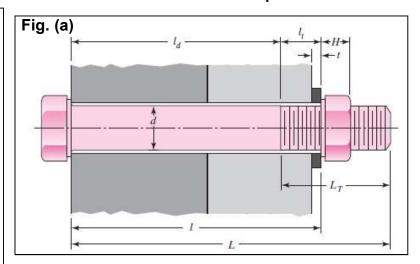
Length of unthreaded portion in grip: $l_d = L - L_T$

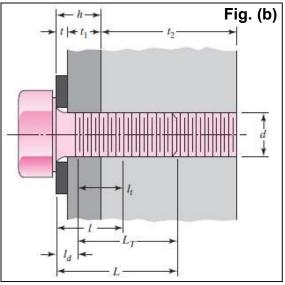
Length of threaded portion in grip: $l_t = l - l_d$

Area of unthreaded portion: $A_d = \pi d^2/4$

Area of threaded portion: A_t from Table 8–1 or 8–2

Fastener stiffness: $k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$







8-5 Joints: Member Stiffness

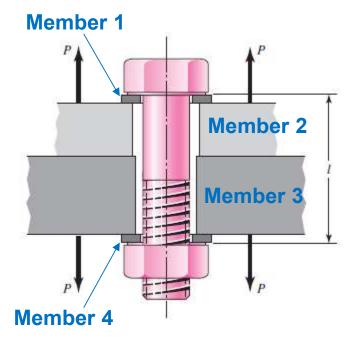


Member Stiffness Calculation

- Multiple members can be included in the grip of the fastener. All together these act like compressive springs in series.
- Total Member Stiffness

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_i}$$

 Experimental technique was used to determine the pressure distribution at member interface. Its effective zone is approximated by the pressure cone method.

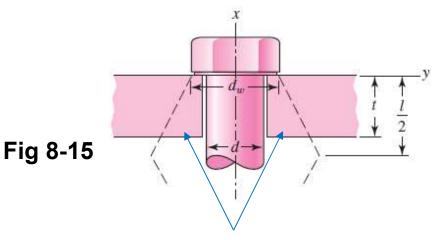


Compression of an conical element with thickness dx is $d\delta = \frac{P dx}{EA}$

Area of the element
$$A = \pi \left(r_o^2 - r_i^2\right) = \pi \left[\left(x \tan \alpha + \frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right]$$

Frustum compression
$$\delta = \int_0^t d\delta = \frac{P}{\pi E d \tan \alpha} \ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}$$

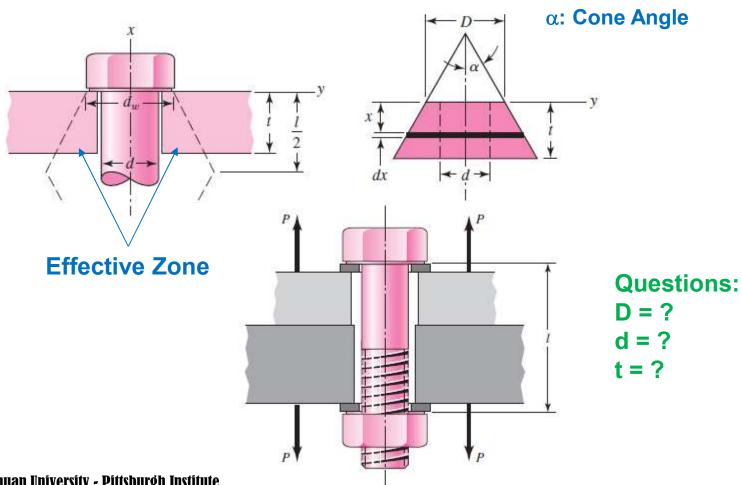
Frustum Stiffness
$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D +)(D - d)}}$$



 α : Cone Angle

Effective Zone

Frustum Stiffness
$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}}$$



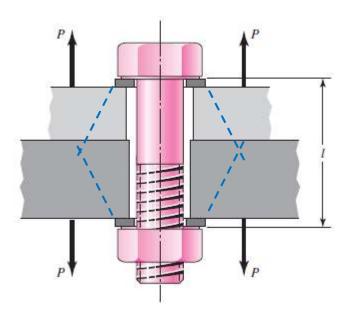
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Frustum Stiffness
$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D +)(D - d)}}$$

Must be solved separately for each frustum in the joint and assembled to obtain member stiffness k_m .

Total Member Stiffness: $\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_i}$



Questi

Regardless of member thickness, depth of the frustum is always half of the grip length.

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Question: How many frustums in this case?

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$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}}$$
For most combinations, 25°≤α≤33°

- In this book $\alpha=30\deg$ was recommended for hardened steel, cast iron, or aluminum members
- Simplification #1: Joint members have the same Young's modulus E with symmetrical frusta back to back $k_m = k/2$

$$k_m = \frac{\pi E d \tan \alpha}{\ln \frac{(l \tan \alpha - w - d)(d_w + d)}{(l \tan \alpha - w + d)(d_w - d)}}$$

$$= \frac{1 - 2t}{\ln (l \tan \alpha - w - d)(d_w + d)}$$

$$= \frac{1 - 2t}{\ln (l \tan \alpha - w - d)(d_w + d)}$$

$$= \frac{1 - 2t}{\ln (l \tan \alpha - w - d)(d_w + d)}$$

<u>Simplification #2</u>: For α =30deg and washer diameter d_w=1.5d, member stiffness k_m is reduced to

$$k_m = \frac{0.5774 \, \pi Ed}{2 \ln \frac{5(0.5774 \, l + 0.5d)}{(0.5774 \, l + 2.5d)}}$$

Refer to P.427- 428 for more discussions of cone angle α

Member Stiffness Calculation (Simplification #3)

FEA was used to seek simplification in calculation procedures. Assumptions:

- α=30deg
- washer diameter D=1.5d
- Ignore washer stiffness
- Two members with same thickness and use same material

Member Equivalent Stiffness

$$\frac{k_m}{Ed} = A \exp\left(\frac{Bd}{l}\right)$$

Table 8-8

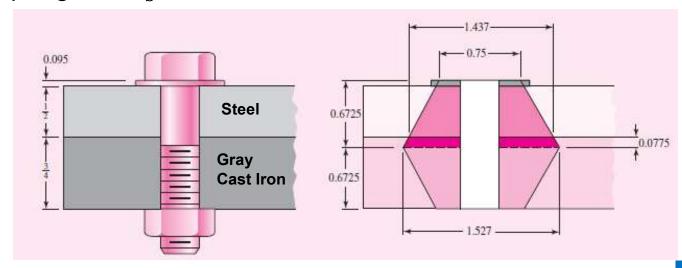
Material Used	Poisson Ratio	Elastic GPa	Modulus Mpsi	A	В
Steel	0.291	207	30.0	0.787 15	0.628 73
Aluminum	0.334	71	10.3	0.796 70	0.638 16
Copper	0.326	119	17.3	0.795 68	0.635 53
Gray cast iron	0.211	100	14.5	0.778 71	0.616 16
General expression				0.789 52	0.629 14

Example 8-2 (Bolt with Nut)

Given: Two plates are clamped by washer-faced $\frac{1}{2}$ in-20 UNF × 1 $\frac{1}{2}$ in SAE grade 5 bolts each with a standard steel plain washer.

Calculate:

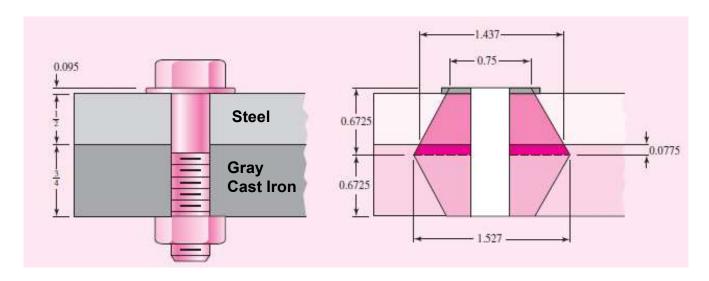
- member spring rate k_m if the top plate is steel and the bottom plate is gray cast iron
- member spring rate k_m if both plates are steel and have equal thickness by method of conical frusta
- member spring rate k_m if both plates are steel using Eq. (8–23)
- bolt spring rate k_b



Example 8-2 (Cont'd)

Table A–32 ► thickness of a standard 1/2 N plain washer is 0.095 in. As shown in figure, the frusta extend halfway into the joint.

frustum thickness =
$$\frac{1}{2}(0.5 + 0.75 + 0.095) = 0.6725$$
 in



Example 8-2 (Cont'd)

Member Spring Rate

Upper frustum thickness: Washer + Steel Plate=0.6725-0.0775=0.595"

$$k = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}} = \frac{\pi (30 \cdot 10^6) \cdot 0.5 \cdot 0.5774}{\ln \frac{(2 \cdot 0.595 \cdot 0.5774 + 0.75 - 0.5)(0.75 + 0.5)}{(2 \cdot 0.595 \cdot 0.5774 + 0.75 + 0.5)(0.75 - 0.5)}} = 30.8 \cdot 10^6 \ lbf/in$$

Upper cast iron frustum thickness=0.0775"

$$k = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}} = \frac{\pi (14.5 \cdot 10^6) \cdot 0.5 \cdot 0.5774}{\ln \frac{(2 \cdot 0.0775 \cdot 0.5774 + 1.437 - 0.5)(1.437 + 0.5)}{(2 \cdot 0.0775 \cdot 0.5774 + 1.437 + 0.5)(1.437 - 0.5)}} = 285.6 \cdot 10^6 \ lbf/in$$

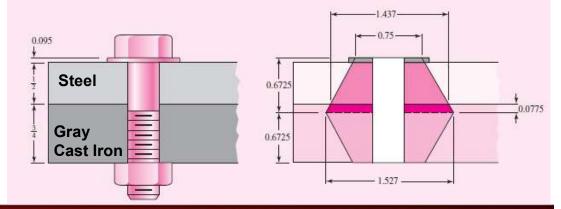
Lower cast iron frustum thickness=0.6725"

$$k = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}} = \frac{\pi (14.5 \cdot 10^6) \cdot 0.5 \cdot 0.5774}{\ln \frac{(2 \cdot 0.6725 \cdot 0.5774 + 0.75 - 0.5)(0.75 + 0.5)}{(2 \cdot 0.6725 \cdot 0.5774 + 0.75 + 0.5)(0.75 - 0.5)}} = 14.15 \cdot 10^6 \ lbf/in$$

Total frustum spring rate:

$$\frac{1}{k_m} = \frac{1}{30.8 \cdot 10^6} + \frac{1}{285.6 \cdot 10^6} + \frac{1}{14.15 \cdot 10^6}$$

 $k_m = 9.378 \cdot 10^6 \ lbf/in$



Example 8-2 (Cont'd)

Simplification #2: Member Spring Rate (Both are steel and equal thickness)

Frustum thickness l = 2 * 0.6725 = 1.345 in

$$k_m = \frac{0.5774 \,\pi Ed}{2 \ln \frac{5(0.5774 \,l + 0.5d)}{(0.5774 \,l + 2.5d)}} = \frac{0.5774 \,\pi (30 \cdot 10^6) \,0.5}{2 \ln \frac{5(0.5774 \cdot .345 + 0.5 \cdot 0.5)}{(0.5774 \cdot .345 + 2.5 \cdot 0.5)}} = 14.64 \cdot 10^6 \,lbf/in$$

Simplification #3: Member Spring Rate (Both are steel and equal thickness)

$$\frac{k_m}{Ed} = A \exp\left(\frac{Bd}{l}\right) = 0.78715 \exp\left(\frac{0.62873 \cdot .5}{1.345}\right)$$
$$k_m = 14.92 \cdot 10^6 \ lbf/in$$

Bolt Spring Rate (Table 8-7)

$$L_T = 2d + \frac{1}{4} = 2 \cdot 0.5 + 0.25 = 1.25 in$$

$$l_d = L - L_T = 1.5 - 1.25 = 0.25 in$$

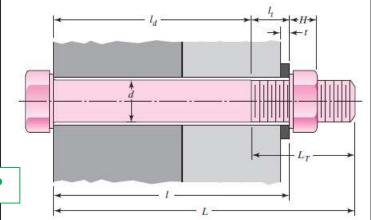
$$l_t = l - l_d = 1.345 - 0.25 = 1.095 in$$

$$A_d = \frac{\pi \cdot 0.5^2}{4} = 0.1963 in^2$$

$$A_t = 0.1599 in^2$$
 (Table 8-2)

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.1963 \cdot 0.1599}{0.1963 \cdot 1.095 + 0.1599 \cdot 0.25}$$

$$k_b = 3.69 \cdot 10^6 \, lbf/in$$
 (Generally $k_b < k_m$)



Sichuan University - Pittsburg Question: Which should fail first?

Example 8-5 (Tapped Joint)

Figure 8–21 shows a connection using cap screws. The joint is subjected to a fluctuating force whose maximum value is 5 kip per screw.

- Cap screw, 5/8 in-11 UNC, SAE 5;
- Hardened-steel washer, t_w = 1/16 in thick;
- Steel cover plate, t₁ =5/8in, E_s = 30 Mpsi;
- Cast-iron base, t₂ = 5/8in, E_{ci} = 16 Mpsi.

Find k_b and k_m using the assumptions given in the caption of Fig. 8–21.

Solution:

Number of frustums in the joint = ??

- the upper two frusta are steel and
- the lower one is cast iron.

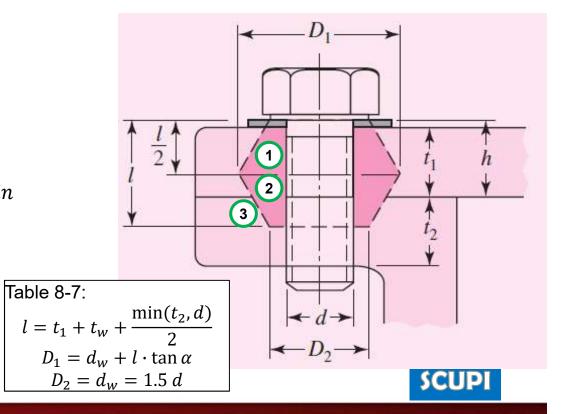
$$h = t_1 + t_w = \left(\frac{5}{8} + \frac{1}{16}\right) = \frac{11}{16} = 0.6875in$$

$$D_2 = 1.5d = 1.5 \cdot \frac{5}{8} = 0.9375in$$

Since t₂=d, effective grip

$$l = h + \frac{d}{2} = \left(\frac{11}{16} + \frac{15}{28}\right) = 1 \text{ in}$$

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Example 8-5 (Cont'd)

<u>Upper Frustum #1</u>: $t = \frac{l}{2} = 0.5 in$, D = 0.9375 in, $d = \frac{5}{8} in$, $\alpha = 30 deg$, $E = E_s$

$$k_1 \coloneqq \frac{\pi \cdot E \cdot d \cdot \tan(\alpha)}{\ln\left(\frac{(2 \cdot t \cdot \tan(\alpha) + D - d) \cdot (D + d)}{(2 \cdot t \cdot \tan(\alpha) + D + d) \cdot (D - d)}\right)} = \left(46.46 \cdot 10^6\right) \frac{lbf}{in}$$

Frustum dia @Interface: $d_w + 2(l - h) \cdot \tan \alpha = 1.2983 in$

<u>Middle Frustum #2</u>: $t = t_2 = h - \frac{l}{2} = 0.1875$ in, D = 1.2983 in, $d = \frac{5}{8}$ in, $\alpha = 30$ deg, $E = E_s$

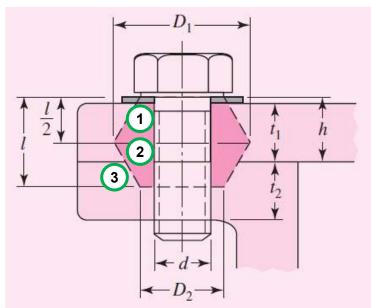
$$k_2 \coloneqq \frac{\boldsymbol{\pi} \cdot E \cdot d \cdot \tan(\alpha)}{\ln\left(\frac{(2 \cdot t \cdot \tan(\alpha) + D - d) \cdot (D + d)}{(2 \cdot t \cdot \tan(\alpha) + D + d) \cdot (D - d)}\right)} = \left(197.58 \cdot 10^6\right) \frac{\boldsymbol{lbf}}{\boldsymbol{in}}$$

Bottom Frustum #3: t = l - h = 0.3125 in, D = 0.9375 in, $d = \frac{5}{8} in$, $\alpha = 30 deg$, $E = E_{ci}$

$$k_3 \coloneqq \frac{\pi \cdot E \cdot d \cdot \tan(\alpha)}{\ln\left(\frac{(2 \cdot t \cdot \tan(\alpha) + D - d) \cdot (D + d)}{(2 \cdot t \cdot \tan(\alpha) + D + d) \cdot (D - d)}\right)} = (32.397 \cdot 10^6) \frac{lbf}{in}$$

Total member stiffness:

$$k_m := \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)^{-1} = \left(17.406 \cdot 10^6\right) \frac{lbf}{in}$$



Example 8-5 (Cont'd)

Bolt Stiffness:

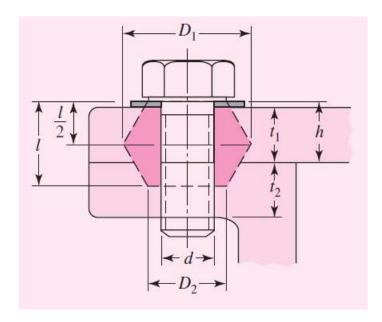
Cap screw is threaded all the way

From Table 8-2, stress area in the thread: $A_t = 0.226 in^2$

Effective grip length:
$$l = h + \frac{d}{2} = \left(\frac{11}{16} + \frac{15}{28}\right) = 1$$
 in

Hence, bolt stiffness is

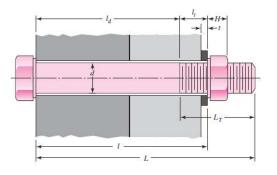
$$k_b = \frac{A_t \cdot E}{l} = \left(6.78 \cdot 10^6\right) \frac{lbf}{in}$$



Summary of Pressure Cone Method

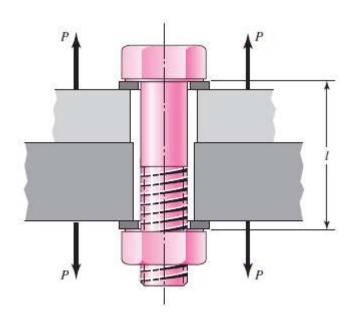


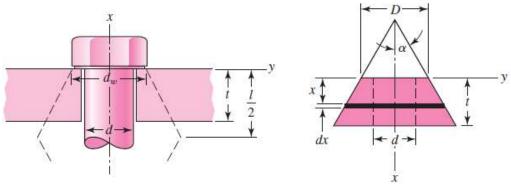
Joint Stiffness Calculation



1. Bolt Stiffness k_b

$$k_b = \frac{k_d k_t}{k_d + k_t} = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$





2. Member Stiffness (Pressure Cone Method)

$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}}$$
 (Method #1)

Let joint members have the same material, k_m =k/2; I=2t; α =30deg and washer diameter d_w =1.5d

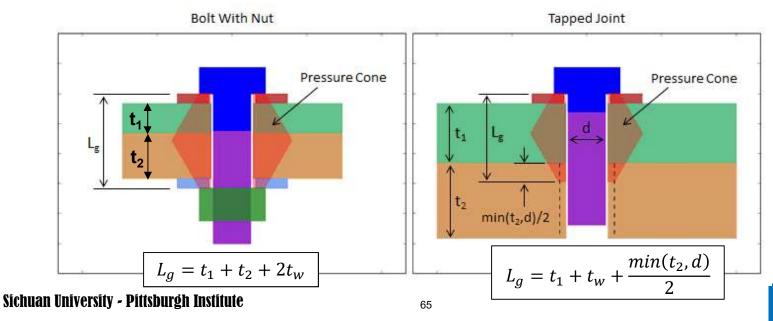
$$k_m = \frac{0.5774 \, \pi E d}{2 \ln \frac{5(0.5774 \, l + 0.5 d)}{(0.5774 \, l + 2.5 d)}}$$
 (Method #2)

or
$$\frac{k_m}{Ed} = A \exp\left(\frac{Bd}{l}\right)$$
 (Method #3)



Effective Grip Length

- Height of the pressure cone depends on the grip length, L_g, which is the combined thickness of the parts being clamped in the joint.
- Bolt with Nut
 - pressure cone starts under the head of the bolt and ends under the nut
 - frustum diameters in this case is determined using the diameters of the bearing faces
- Tapped Joint (Cap Screw)
 - pressure cone starts under the head of the bolt and ends in the threaded portion of the final plate.



SCUPI

8-6 Bolt Strength

- Typically the bolt fails first.
 - It is the least expensive
 - It is the most easily replaced



Bolt Strength

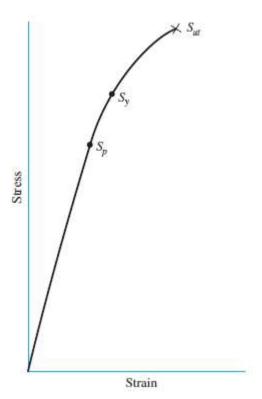
- Proof Load: Maximum load that a bolt can withstand without acquiring a permanent set (force)
- Proof Strength: proportional limit and corresponds to 0.0001-in permanent set in the fastener (stress)
- Minimum Strength Specification of Steel Bolts
 - Table 8-9 (SAE), p.425
 - Table 8-10 (ASTM), p.426
 - Table 8-11 (Metric Bolts), p.427
- The grade of the nut should be the same grade as the bolt.

Min Proof Strength/Min Tensile Strength/Min Yield Strength

Table 8-9

SAE Specifications for Steel Bolts

		,				
SAE Grade No.	Size Range Inclusive, in	Minimum Proof Strength,* kpsi	Tensile	Minimum Yield Strength,* kpsi	Material	Head Marking
1	$\frac{1}{4} - 1\frac{1}{2}$	33	60	36	Low or medium carbon	
2	$\frac{1}{4} - \frac{3}{4}$	55	74	57	Low or medium carbon	Ž.
	$\frac{7}{8} - 1\frac{1}{2}$	33	60	36		
4	$\tfrac{1}{4} - I \tfrac{1}{2}$	65	115	100	Medium carbon, cold-drawn	
5	$\frac{1}{4}$ -1	85	120	92	Medium carbon, Q&T	
	$1\frac{1}{8}-1\frac{1}{2}$	74	105	81		
5.2	$\frac{1}{4}$ -1	85	120	92	Low-carbon martensite, Q&T	
7	$\frac{1}{4} - 1\frac{1}{2}$	105	133	115	Medium-carbon alloy, Q&T	Ö
8	$\tfrac{1}{4} \! - \! 1 \tfrac{1}{2}$	120	150	130	Medium-carbon alloy, Q&T	
8.2	$\frac{1}{4}$ -1	120	150	130	Low-carbon martensite, Q&T	



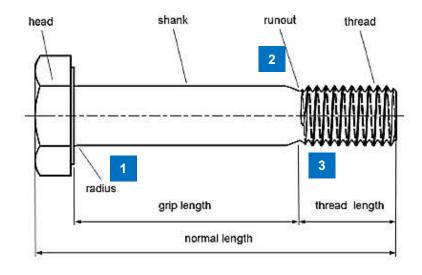
Proof strength is less than yield strength

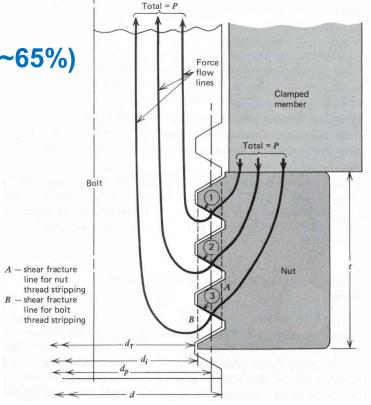


Bolt Failures

- With washer protection of the shoulder fillet and thread runout ≤15°. Bolts in fatigue axial loading failed mostly at:
 - 1. Fillet under head (~15%)
 - 2. Thread runout (~20%)

3. First engaged thread in the nut (~65%)

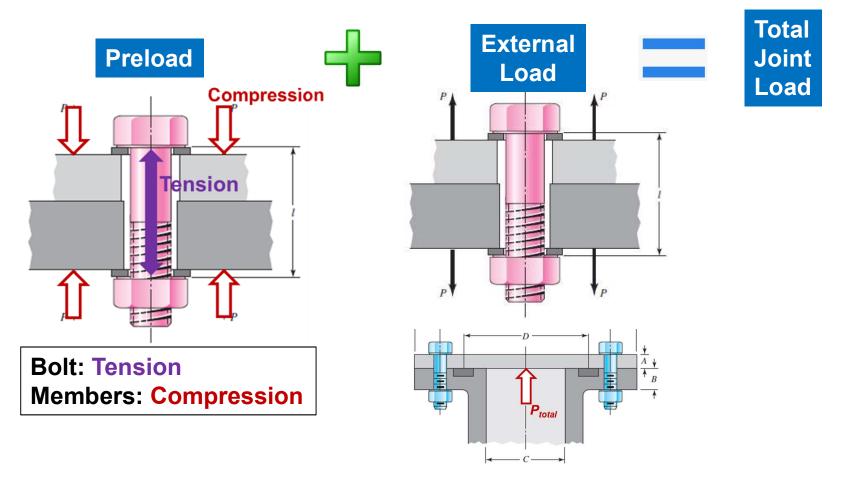






8-7 Tension Joints: The External Load

Total load acting on each joint is the summation of preload and external load



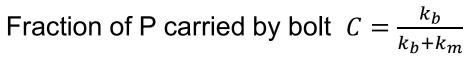
Load-Sharing Between Bolt and Members

 P_{total} = Total external tensile load applied to the joint P = external tensile load per bolt

$$P = P_b + P_m = \frac{P_{total}}{N} = (k_b + k_m) \cdot \delta$$

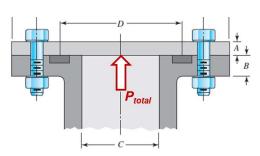
 P_b = portion of P taken by bolt

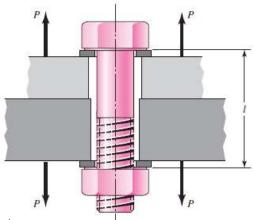
 P_m = portion of P taken by members



$$P_b = CP$$
 $P_m = (1 - C)P$

C is also known as the Stiffness Constant of the Joint





 F_i = bolt preload

Question: Why is the sign different?

Resultant Bolt Load $F_b = P_b + F_i = CP + F_i$

Resultant Member Load $F_m = P_m - F_i = (1 - C)P - F_i$

Load-Sharing Between Bolt and Members

■ Table 8–12: Steel members clamped using a $\frac{1}{2}$ in – 13 NC bolt

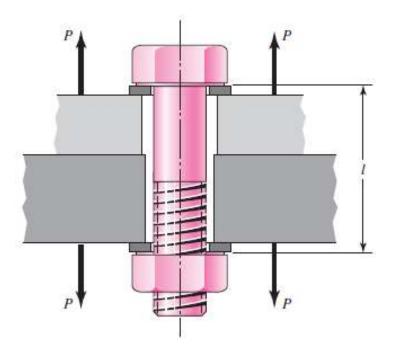
Stiffnesses, M lbf/in							
Bolt Grip, in	kь	k _m	c	1 – C			
2	2.57	12.69	0.168	0.832			
3	1.79	11.33	0.136	0.864			
4	1.37	10.63	0.114	0.886			

- Noted that:
 - members take 80+ percent of the external load
 - making the <u>bolt grip</u> longer causes the members to take an even greater percentage of the external load. Why?

8-8 Relating Bolt Torque to Bolt Tension



Question: Preload is applied to any fastener. How to ensure the preload is developed and at its desired magnitude?





Bolt Preload Control

- In industrial applications, bolt preload is generated and controlled by
 - Torque wrench (most controllable and accurate)
 - Pneumatic-impact wrench
 - Turn-of-the-nut method (very time-consuming with wishy-washy outcome)







Relationship Between Bolt Preload and Torque

$$T_R = \frac{F_i d_m}{2} \left(\frac{l + \pi f d_m \sec \alpha}{\pi d_m - f l \sec \alpha} \right) + \frac{F_i f_c d_c}{2}$$

Given

$$T_R = \frac{F_i d_m}{2} \left(\frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + \frac{F_i f_c d_c}{2}$$

• Washer dia= $1.5d_m \sim 1.5d$ $d_c = \frac{d+1.5d}{2} = 1.25d$

$$T_R = \left[\frac{d_m}{2d} \left(\frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha} \right) + 0.625 f_c \right] F_i d = K F_i d$$

$$K = \frac{d_m}{2d} \left(\frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec} \right) + 0.625 f_c$$

- K is the torque factor
- K=0.2 is the most commonly used torque factor

Bolt Condition	K	
Nonplated, black finish	0.30	
Zinc-plated	0.20	
Lubricated	0.18	
Cadmium-plated	0.16	
With Bowman Anti-Seize	0.12	
With Bowman-Grip nuts	0.09	



Table 8-15

Example 8-3

 $A_{\frac{3}{4}}$ in-16 UNF × $2\frac{1}{2}$ in SAE grade 5 bolt is subjected to a load P of 6 kip in a tension joint. The initial bolt tension is $F_i = 25$ kip.

Bolt stiffness $k_b = 6.50$ Mlbf/in and Joint Stiffness $k_m = 13.8$ Mlbf/in

- Determine the preload and service load stresses in the bolt. Compare these to the SAE minimum proof strength of the bolt.
- Specify the torque necessary to develop the preload, using Eq. (8–27).
- Specify the torque necessary to develop the preload, using Eq. (8–26) with f=fc = 0.15.

Solution:

For $\frac{3}{4}$ in-16 UNF × $2\frac{1}{2}$ in bolt, from Table 8-2, $A_t=0.373~in^2$

Preload stress in threaded area: $\frac{F_i}{A_t} = \frac{25}{0.373} = 67.02 \text{ ksi}$

Stiffness Constant
$$C = \frac{k_b}{k_b + k_m} = \frac{6.5}{6.5 + 13.8} = 0.32$$

Stress under service load $\frac{F_b}{A_t} = \frac{CP + F_i}{A_t} = \frac{0.32 \cdot 6 + 25}{0.373} = 72.17 \text{ ksi}$

Table 8-2
Diameters and Area of Unified Screw Threads UNC and UNF*

		Coarse Series—UNC		Fi	ne Series—	UNF		
Size Designation	Nominal Major Diameter in	Threads per Inch N	Tensile- Stress Area A _f in ²	Minor- Diameter Area A _r in ²	Threads per Inch N	Tensile- Stress Area A _t in ²	Minor- Diameter Area A _r in ²	
0	0.0600				80	II ANTON	D1 1 10 1	1 7 1 0 0 1
1	0.0730	64	0.002 63	0.002 18	72	ANSI	B1. 1-1974 and	I B18. 3. 1-
2	0.0860	56	0.003 70	0.003 10	64	(
3	0.0990	48	0.004 87	0.004 06	56	Minor	Diameter: d_r	=d-1. 299
4	0.1120	40	0.006 04	0.004 96	48		n Diameter: $d_p^{^{'}}$	
5	0.1250	40	0.007 96	0.006 72	44	(i Diame eet i ap	a
6	0.1380	32	0.009 09	0.007 45	40	(/ 1 .
8	0.1640	32	0.014 0	0.011 96	36	Tonsi	le Stress Area	$-\frac{\pi}{a_r} \left(\frac{a_r + a_r}{a_r} \right)$
10	0.1900	24	0.017 5	0.014 50	32	101131	ic stress med	-4 \ 2
12	0.2160	24	0.024 2	0.020 6	28	(`
1	0.2500	20	0.031 8	0.026 9	28	0.030 4	0.032 0	
5	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.0524	
3	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.0809	
7	0.4375	14	0.1063	0.093 3	20	0.1187	0.1090	
1 2	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6	
14 5 16 3 8 7 16 1 2 9 16	0.5625	12	0.182	0.162	18	0.203	0.189	
	0.6250	11	0.226	0.202	18	0.256	0.240	
5 3 4 7 8	0.7500	10	0.334	0.302	16	0.373	0.351	
7 8	0.8750	9	0.462	0.419	14	0.509	0.480	
1	1.0000	8	0.606	0.551	12	0.663	0.625	
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024	_
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521	scul

Example 8-3 (Cont'd)

For $\frac{3}{4}$ in-16 UNF × $2\frac{1}{2}$ in SAE grade 5 bolt, min proof strength S_p=85 ksi (Table 8-9)

Stress under service load $\frac{F_b}{A_t} = \frac{CP + F_i}{A_t} = \frac{0.32 \cdot 6 + 25}{0.373} = 72.17 \ ksi < S_p$

Torque for preload $T = KF_i d = 0.2 \cdot 25000 \cdot 0.75 = 3750 in \cdot lbf$

Use Eq 8-26 for preload calculation

Table 8-2, Minor Dia Area $A_r = 0.351 in^2$

Minor Dia:
$$d_r = \sqrt{\frac{4A_r}{\pi}} = 0.6685 in$$

Mean Dia:
$$d_m = \frac{0.75 + 0.6685}{2} = 0.7093 in$$

For UNC/UNF thread, $\alpha = 30^{\circ}$

$$\tan \lambda = \frac{l}{\pi d_m} = \frac{1/1}{\pi \cdot 0.7093} = 0.0280 \quad \lambda = 1.6066^{\circ}$$

$$T_R = \left[\frac{d_m}{2d} \left(\frac{\tan \lambda + f \sec \alpha}{1 - f \tan \lambda \sec \alpha}\right) + 0.625 f_c\right] F_i d$$

$$T_R = \left[\frac{0.7093}{2 \cdot 0.75} \left(\frac{\tan 1.6066^\circ + 0.15 \cdot \sec 30^\circ}{1 - 0.15 \cdot \tan 1.6066^\circ \sec 30^\circ}\right) + 0.625 \cdot 0.15\right] 25000 \cdot 0.75 = 3551 \ in \cdot lbf$$

$$K = 0.189 < 0.2$$

SAE Grade No.	Size Range Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Markin
ľ	$\frac{1}{4} - 1\frac{1}{2}$	33	60	36	Low or medium carbon	
2	$\frac{1}{4} - \frac{3}{4}$	55	74	57	Low or medium carbon	
	$\frac{7}{8} - 1\frac{1}{2}$	33	60	36		
4	$\frac{1}{4}$ -1 $\frac{1}{2}$	65	115	100	Medium carbon, cold-drawn	
5	$\frac{1}{4}$ -1	85	120	92	Medium carbon, Q&T	
	$1\frac{1}{8} - 1\frac{1}{2}$	74	105	81		
5.2	$\frac{1}{4}$ -1	85	120	92	Low-carbon martensite, Q&T	
7	$\frac{1}{4}$ - $1\frac{1}{2}$	105	133	115	Medium-carbon alloy, Q&T	Ŏ
8	$\tfrac{1}{4} 1 \tfrac{1}{2}$	120	150	130	Medium-carbon alloy, Q&T	
8.2	$\frac{1}{4}$ -1	120	150	130	Low-carbon martensite, Q&T	



Risk Assessment 8-9 Statically Loaded Tension Joint with Preload

- Static applications
- Joint with preload
- External load exerts additional tensile force on bolt



1. Safety Factor Against Proofing Strength

Tensile stress in bolt
$$\sigma_b=rac{F_b}{A_t}=rac{CP+F_i}{A_t}$$
 Tensile load in bolt $n_p(CP+F_i)=n_p\sigma_bA_t=S_pA_t$

Yielding factor of safety against proof strength: $n_p = \frac{S_p}{\sigma_b}$ S_p is proof strength of the bolt material

$$n_p = \frac{S_p A_t}{CP + F_i}$$

Design Target: n_p = ~1 (it is typical to load the joint up to proof strength)

2. Safety Factor Against Overloading by External Load

Apply safety factor to external loading only: $C(n_L P) + F_i = S_p A_t = F_b$

Tensile stress in bolt up to proof strength $S_p = \frac{F_b}{A_t} = \frac{C(n_L P) + F_i}{A_t}$

$$n_L = \frac{S_p A_t - F_i}{C \cdot P}$$

3. Safety Factor Against Separation

Find limit of external load to prevent joint from separation.

P₀: Minimum external load that causes joint separation

When separation occurs:
$$F_m = 0$$
 $(1 - C)P_0 - F_i = 0$

$$(1-C)P_0 - F_i = 0$$

Re-arrange the equation and express it as $P_0 = \frac{F_i}{1-C}$

Define factor of safety against joint separation $n_0 = \frac{P_0}{P}$

Henceforth,
$$n_0 = \frac{F_i}{P(1-C)}$$



To further simplify the design calculations, several proof strength had been proposed for joint preload design.

A common practice for both static and fatigue loading is to use

$$F_i = \begin{cases} 0.75F_p & \text{for nonpermanent connections, reused fasteners} \\ 0.90F_p & \text{for permanent connections} \end{cases}$$
(8–31)

, where F_p is the proof load and

$$F_p = S_p A_t$$

- Get S_p from Tables 8-9 to 8-11
- For other materials, use $S_p = 0.85S_y$



Example 8-4

Figure 8-19 is a cross section of a grade 25 cast-iron pressure vessel. A total of N bolts are to be used to resist a separating force of 36 kip.

- Determine k_b, k_m, and C.
- Find the number of bolts required for a load factor of 2 where the bolts may be reused when the joint is taken apart.
- With the number of bolts obtained above, determine the realized load factor for overload, the yielding factor of safety, and the load factor for joint separation.

Solution:

Grip
$$l = 1.5 in$$

From Table A–31, nut thickness = $\frac{35}{64}in$ (p.1063)

Bolt length
$$L = 1.5 + \frac{35}{64} + \frac{2}{11} = 2.229 in$$

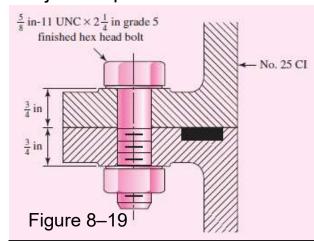
Table A-17 (p.1043), the closest bolt length L = $2\frac{1}{4}$ in

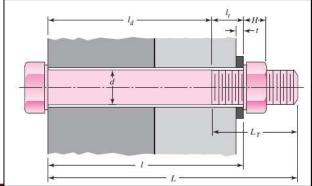
Since
$$L < 6$$
 in, $L_t = 2d + \frac{1}{4} = 2 \cdot 0.625 + 0.25 = 1.5$ in $l_d = 2.25 - 1.5 = 0.75$ in $l_t = l - l_d = 1.5 - 0.75 = 0.75$ in

Table 8-2,
$$A_t = 0.226 in^2$$

Major Dia Area
$$A_d = \frac{\pi \cdot 0.625^2}{4} = 0.3068 \ in^2$$

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Example 8-4 (Cont'd)

Bolt Stiffness
$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{0.3068 \cdot 0.226 \cdot 30}{0.3068 \cdot 0.75 + 0.226 \cdot 0.75} = 5.21 \ Mlbf/in$$

Table A-24, No. 25 CI: E=14 Msi

Member Stiffness
$$k_m = \frac{0.5774 \, \pi Ed}{2 \ln \frac{5(0.5774 \, l + 0.5d)}{(0.5774 \, l + 2.5d)}} = \frac{0.5774 \, \pi (14 \cdot 10^6) \, 0.62}{2 \ln \frac{5(0.5774 \cdot 1.5 + 0.5 \cdot 0.625)}{(0.5774 \cdot 1.5 + 2.5 \cdot 0.625)}} = 8.95 \, Mlbf/in$$

Stiffness constant
$$C = \frac{k_b}{k_b + k_m} = \frac{5.21}{5.21 + 8.95} = 0.368$$

Table 8-9, $S_p = 85 \, ksi$

Recommended preload

$$F_i = 0.75 S_p A_t = 0.75 \cdot 85 \cdot 0.226 = 14.4 \ kip$$

To resist overloading
$$n_L = \frac{S_p A_t - F_i}{C \cdot (P_{total}/N)} = 2$$

Calculate N=5.52; so six bolts should be sufficient.

Using N=6;

Actual load factor
$$n_L = \frac{S_p A_t - F_i}{C \cdot P} = \frac{85 \cdot 0.226 - .4}{0.368(36/)} = 2.18$$

SF against proof strength
$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{85 \cdot 0.226}{0.368 \cdot 6 + 14.4} = 1.16$$

SF against joint separation
$$n_0 = \frac{F_i}{P(1-C)} = \frac{14.4}{6(1-0.368)} = 3.80$$

