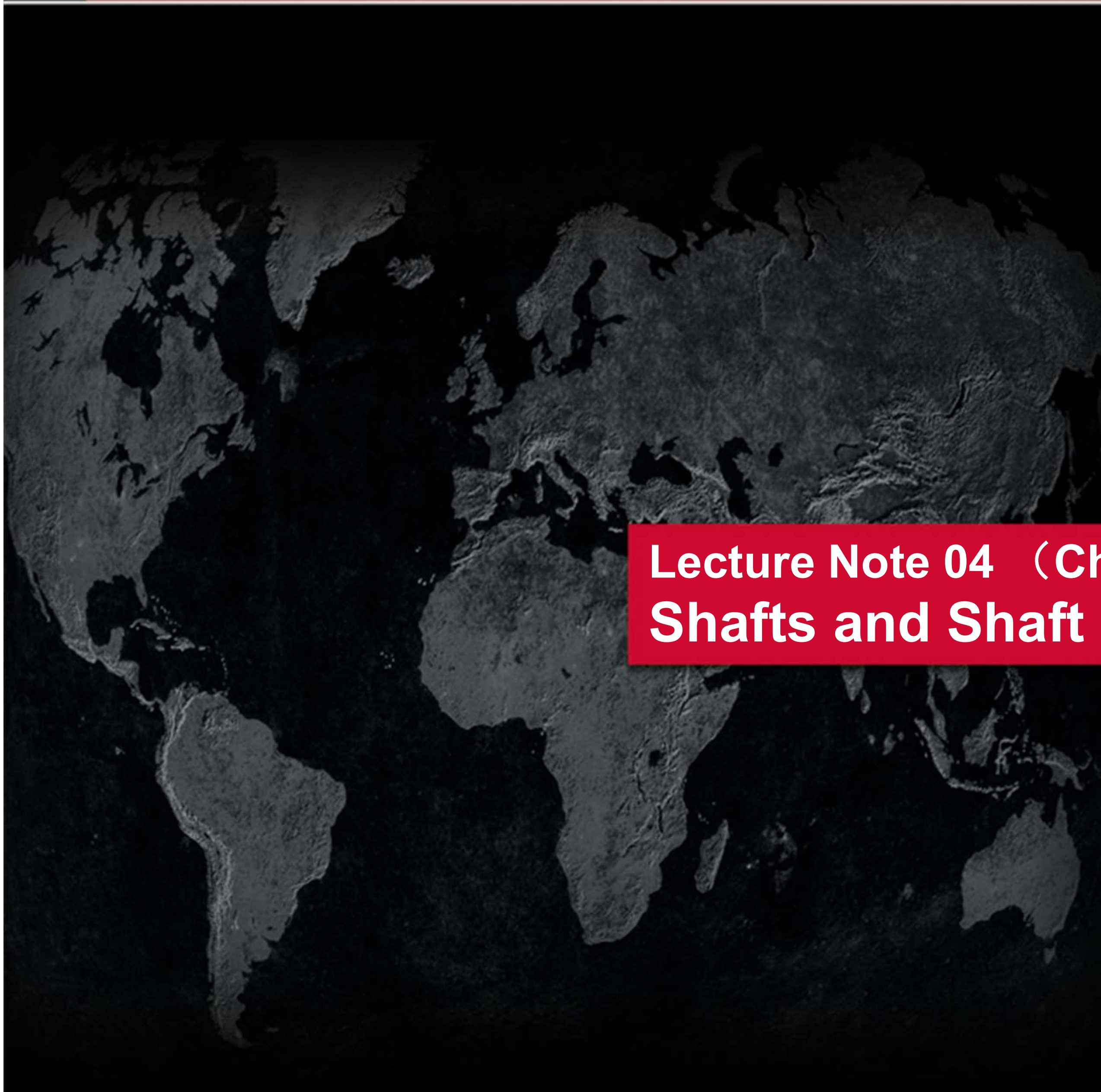

Lecture Note 04 （Ch.07)
Shafts and Shaft Components

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ME 1029 Mechanical Design 2

Fall 2021

Sichuan University - Pittsburgh Institute

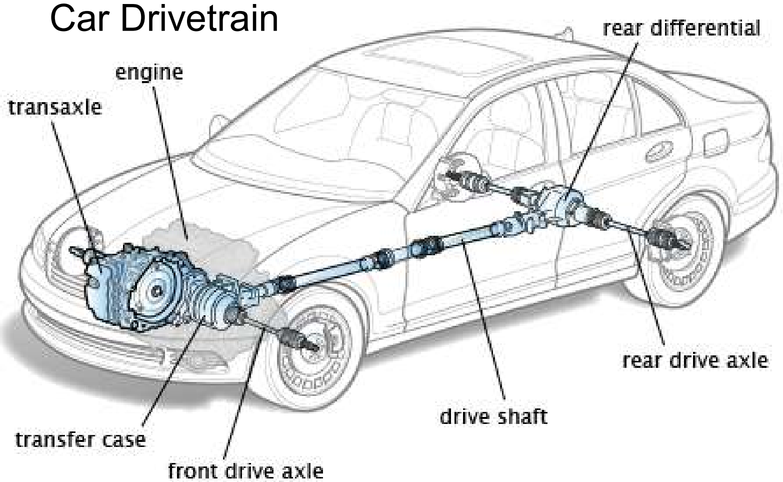
Topics Covered

- 7-3 Shaft Layout
- 7-2 Shaft Materials
- 7-4 Shaft Design for Stress
- 7-5 Deflection Considerations
- 7-6 Critical Speeds for Shafts
- 7-7 Miscellaneous Shaft Components
- 7-8 Shaft Limits and Fits (LN01)

7-3 Shaft Layout

Examples of Shaft Application

Car Drivetrain



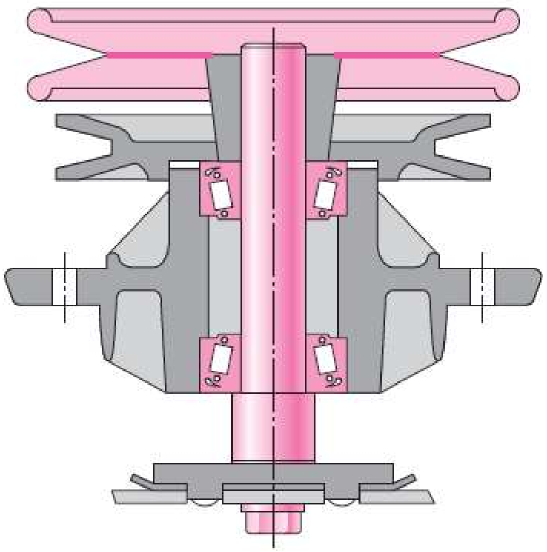
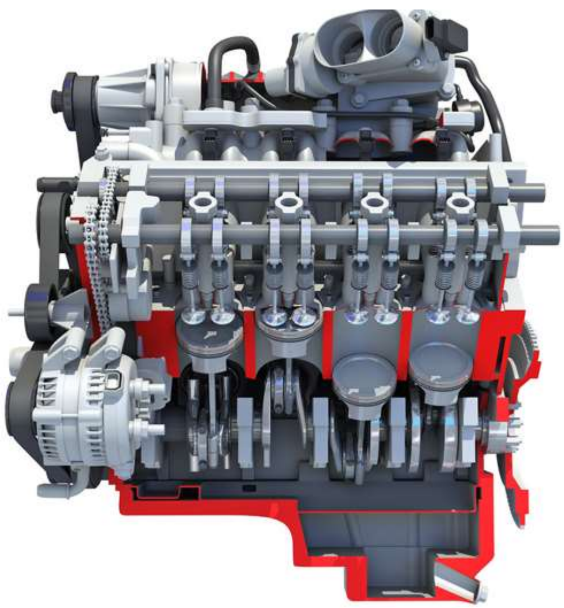


Fig 7-3 Mowing Machine Spindle

Airplane Turbine



Engine Crankshaft

Shaft Types

- **Shaft**: A shaft is a rotating member, usually of circular cross section, used to transmit power or motion.
- **Axle**: An axle is a nonrotating member that carries no torque and is used to support rotating wheels, pulleys, and the like.

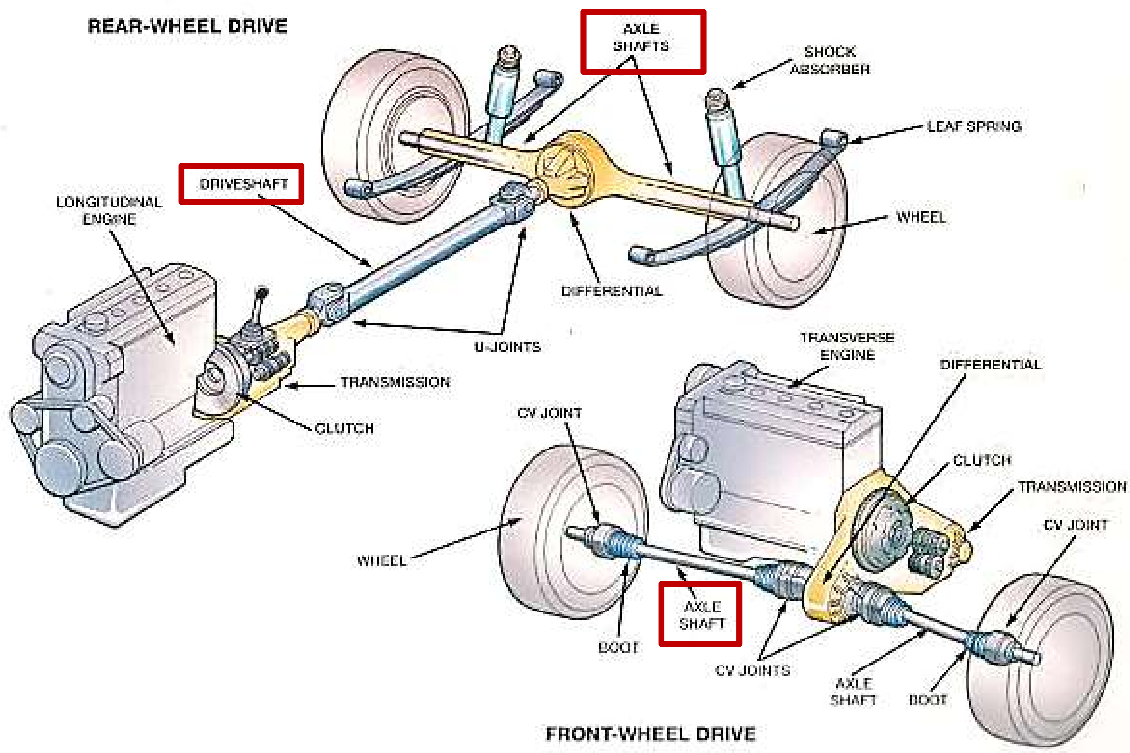
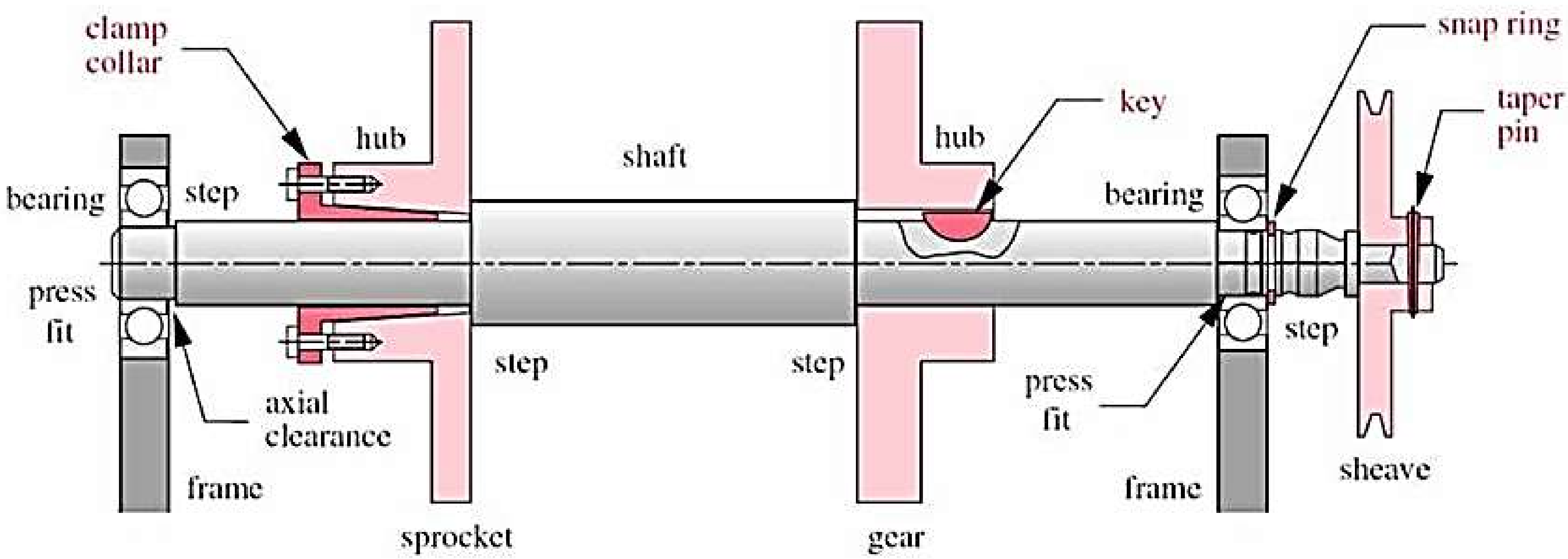


Illustration of a Shaft with Various Attachments/Details



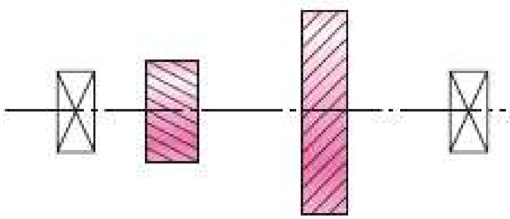
Shaft Elements:

- Shaft
- Gears
- Pulleys
- Bearings
- Hubs
- Keys, etc.

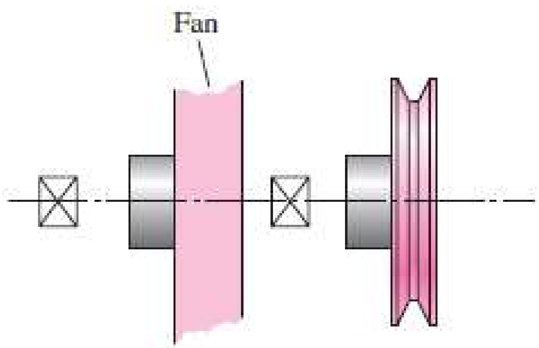
Detailing Shaft Design

Schematics →

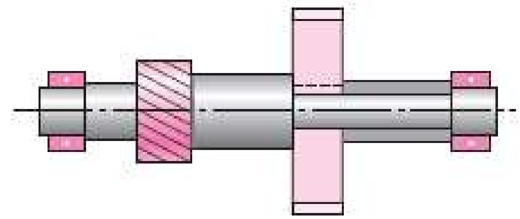
Detailing Shaft Design
- Axial Layout of Components
- Supporting Axial Loads
- Providing for Torque Transmission
- Assembly/Disassembly



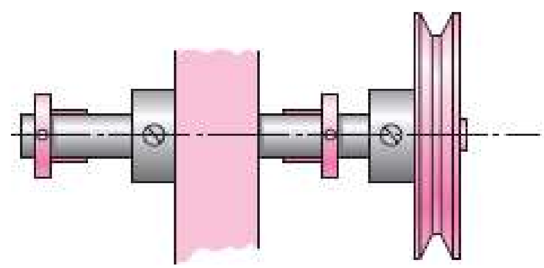
Gears/Support Bearings



Fan & Shaft



(b)

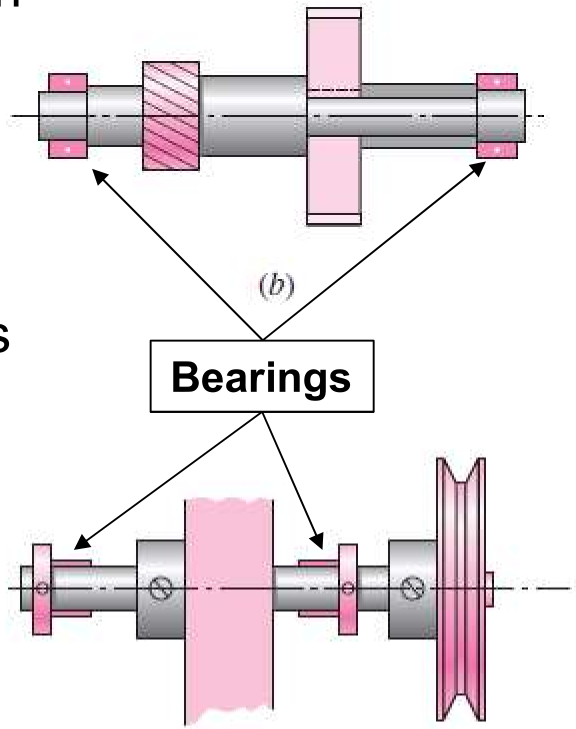


(d)

Figure 7–2

Detailing Shaft Design - Axial Layout of Components

- Best to support load-carrying components between bearings rather than cantilevered outboard of the bearings.
- Shaft loads are support with two bearings in most designs.
- Place load-carrying components near the bearings
- Keep shaft as short as possible.
- Use shoulder to position components.
- Press fits, pins, or collars with setscrews can be used to maintain axial locations where axial loads are small.



Detailing Shaft Design - Supporting Axial Loads

- Axial loads are not always trivial.
- Countermeasures are necessary to provide a means to transfer the axial loads into the shaft, then through a bearing to the ground.
- Often, the same means of providing axial location, e.g., shoulders, retaining rings, and pins, will be used to also transmit the axial load into the shaft.
- Generally best to have only <u>one bearing</u> carry the axial load.

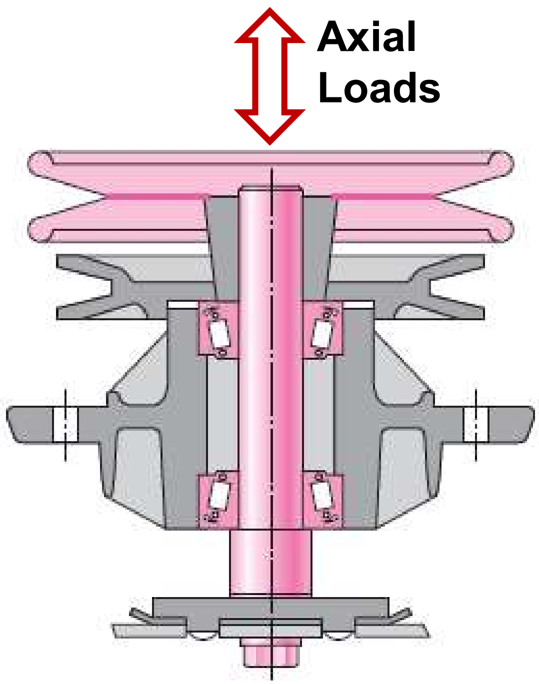
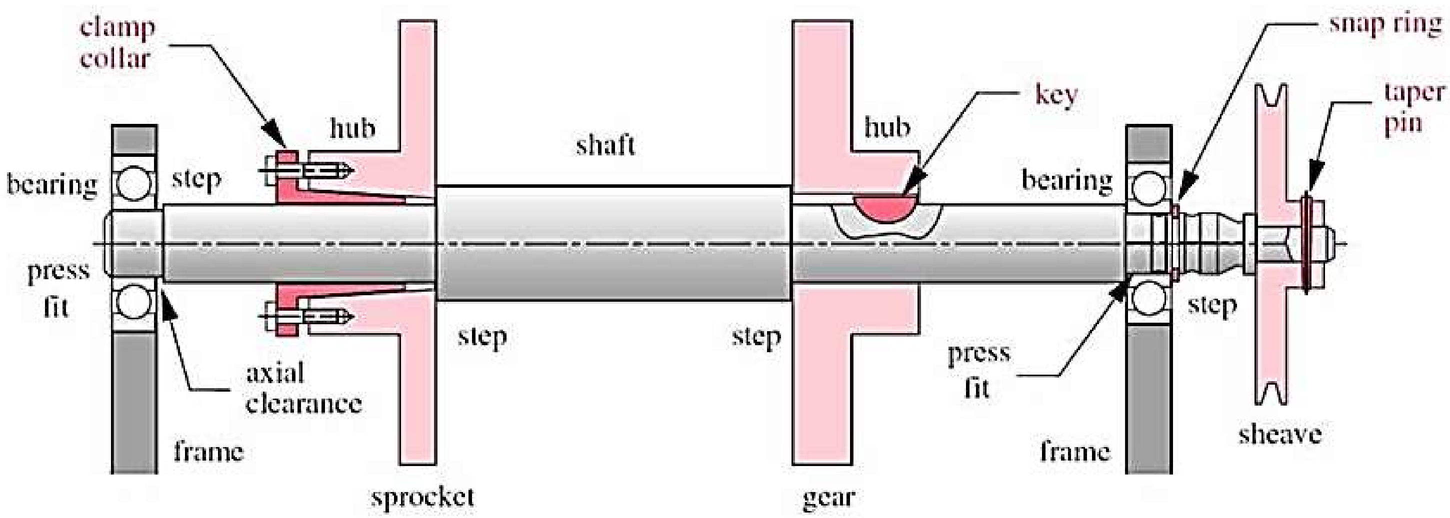


Figure 7–3

Detailing Shaft Design - Providing for Torque Transmission

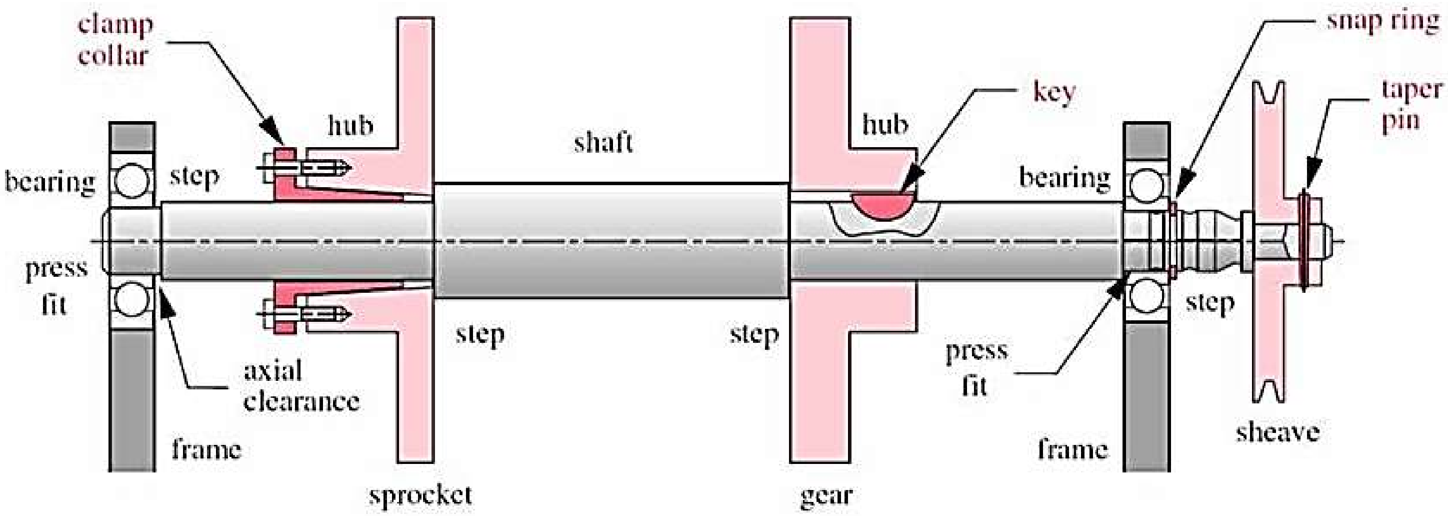
- Size shaft to support the torsional stress and torsional deflection.
- Provide means of transmitting torque between shaft and gears.
- Size torque-transfer elements to fail first if torque exceeds acceptable operating limits, protecting more expensive components.
- Torque-transfer elements:
  - Keyed components are the most effective and economical means of transmitting moderate to high levels of torque.
  - Press and shrink fits for securing hubs to shafts are used both for torque transfer and for preserving axial location.
  - Tapered fits are often used on the overhanging end of a shaft.



Detailing Shaft Design - Assembly and Disassembly

Detailing Shaft Design - Assembly and Disassembly

- A make or break step that is very cumbersome to foresee.
- Concerns:
  - Shaft components are usually inexpensive and not reused.
  - Shaft is very expensive and typically capitalized for depreciation. They will be reused for multiple operations before written off.
  - Therefore, preserving shaft dimensions through repetitive assembly and disassembly is one primary consideration during shaft system design.



7-2 Shaft Material Selection

- Rotating fatigue is the primary factor in driving shaft material selection.
- Low carbon steels, cold-drawn (CD) or hot-rolled (HR) steel, such as ANSI 1020-1050 steels are first to be considered for cost reason.
- When warranted for fatigue concern, alloy steels like ANSI 1340-50, 3140-50, 4140, 4340, 5140, and 8650 are selected for convenience in heat treatment.
- Surface hardening is needed when used as a bearing surface. Typical material choices for surface hardening include carburizing grades of ANSI 1020, 4320, 4820, and 8620.

7-4 Shaft Design for Stress

Shaft Design for Stress

- Typical loadings on a transmission shaft
  - Bending moments
  - Torsions
  - Axial loads

Uniaxial or multi-axial loading?

- Sophisticated analysis tools such as FEA are frequently used to get a stress distribution of a complete shaft.
- During conceptual design stage for sizing a shaft, it is more efficient to use hand calculations as presented here.
- It is not necessary to evaluate the stresses in a shaft at every point; a few potentially critical locations will suffice.
  - Critical locations are usually on the outer surface,
  - At axial locations where the bending moment is large, where the torque is present, and where stress concentrations exist.
- Bending moment is usually the culprit in causing shafts to fail

Example 6-9

A rotating shaft simply supported in ball bearings at A and D and loaded by a nonrotating force F of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

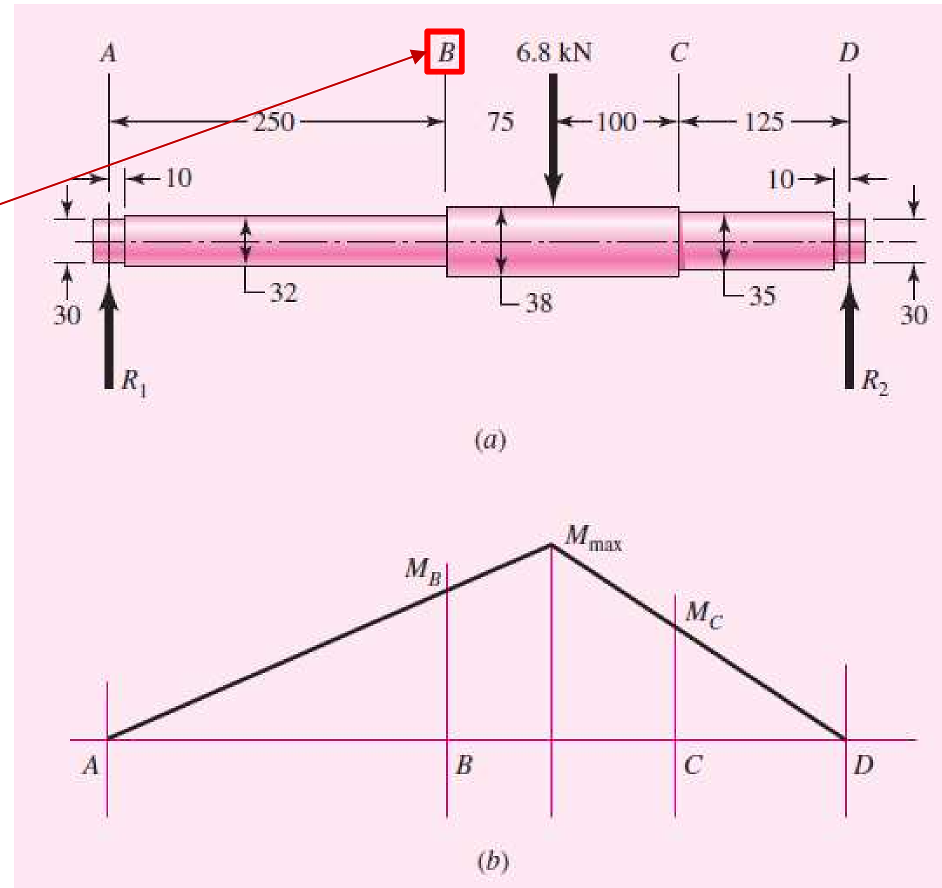
All fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel.

Material: SAE 1050 CD

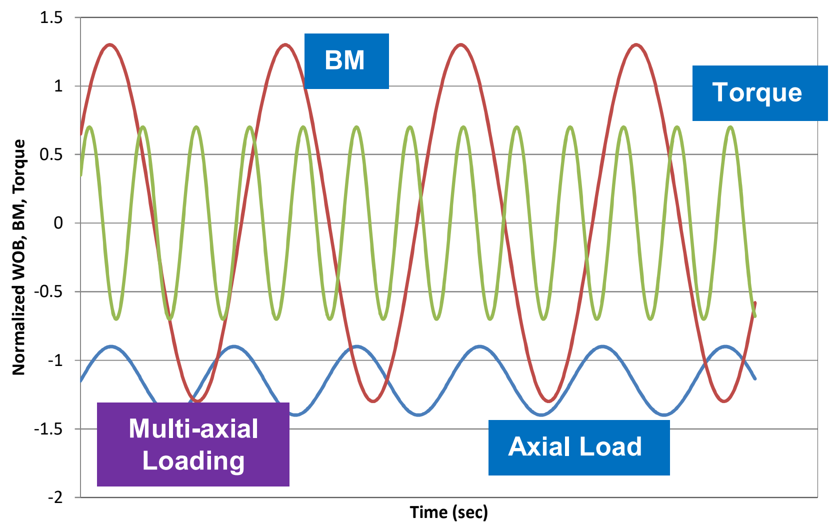
Table A-20: S_{ut}=690 MPa; S_y=580 MPa

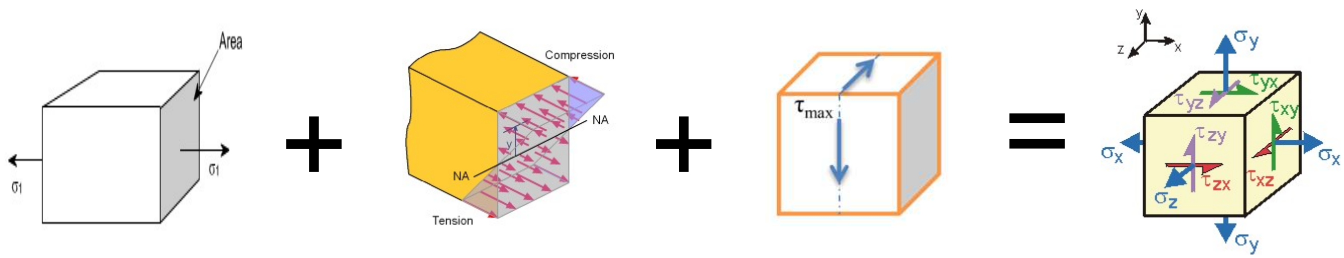
Identify critical stress location

In this case: Point B



Shaft Stresses Under Multi-Axial Loading



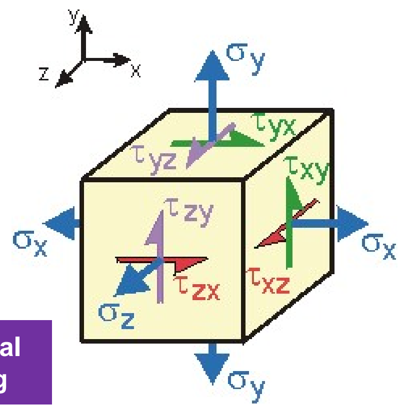


$$\sigma_{axial} + \sigma_{BM} + \sigma_{torsion} = \frac{F}{A} + \frac{M \cdot y}{I} + \frac{T \cdot r}{J} = \sigma$$

Combined Stress

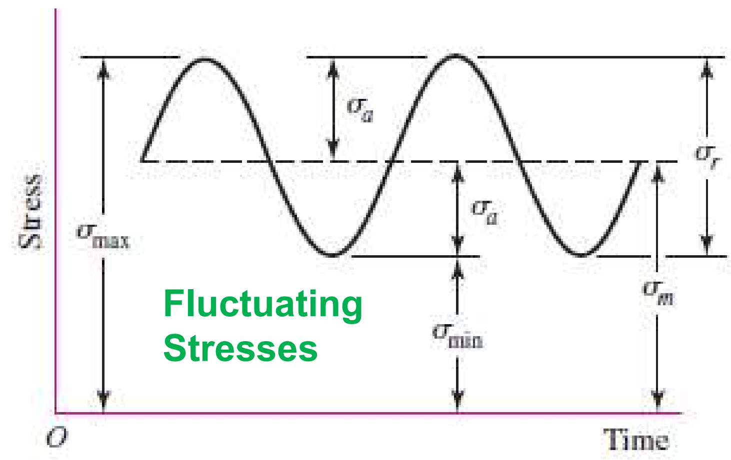
Contribution from axial loads is usually very small at critical locations. So they are commonly ignored during feasibility study.

Shaft Stresses Under Multi-Axial Loading

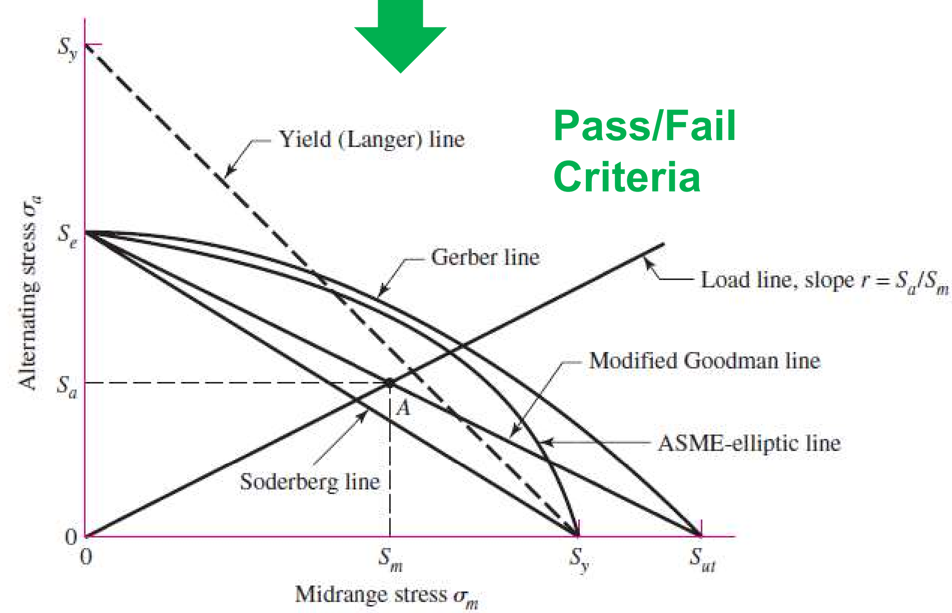


Multi-axial Loading

?



Fluctuating Stresses



Pass/Fail Criteria

Question:
How to convert the 6 stress components to equivalent σ_m & σ_a?

Calculation of Shaft Stresses

- Applied loads on an element rotating with the shaft can be further separated into portions contributing from mean loads and alternating loads:

$$M = M_m + M_a \qquad T = T_m + T_a$$

- Assume contribution of fatigue damaging from axial load is negligible.
- Fluctuating stresses due to bending and torsion are given by

$$\sigma_a = K_f \frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3}$$

$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3}$$

for rotating round, solid shafts

M_m and M_a: mean and alternating bending moments
T_m and T_a: mean and alternating torques
K_f and K_{fs}: fatigue stress-concentration factors for bending and torsion

Steps for Fatigue Stress Concentration Calculation

- Calculate stress concentration (K_t or K_{ts}) first
- Find q per Fig. 6-20 for bending/axial loading, or q_{shear} shear per Fig. 6-21 for torsional loading. (For steel material)
- Calculate (K_f or K_{fs})

$$K_f = 1 + q(K_t - 1) \qquad \text{or} \qquad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

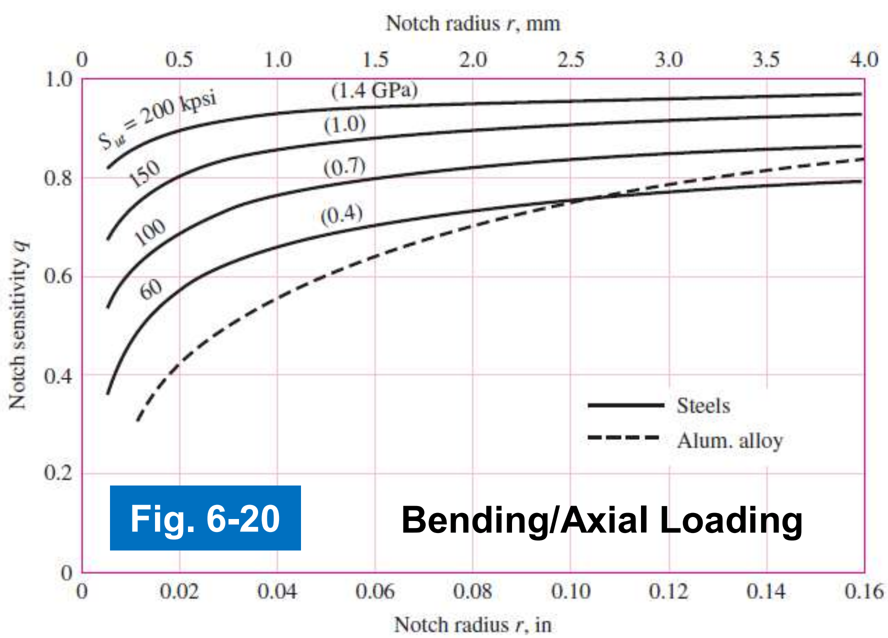


Fig. 6-20 Bending/Axial Loading

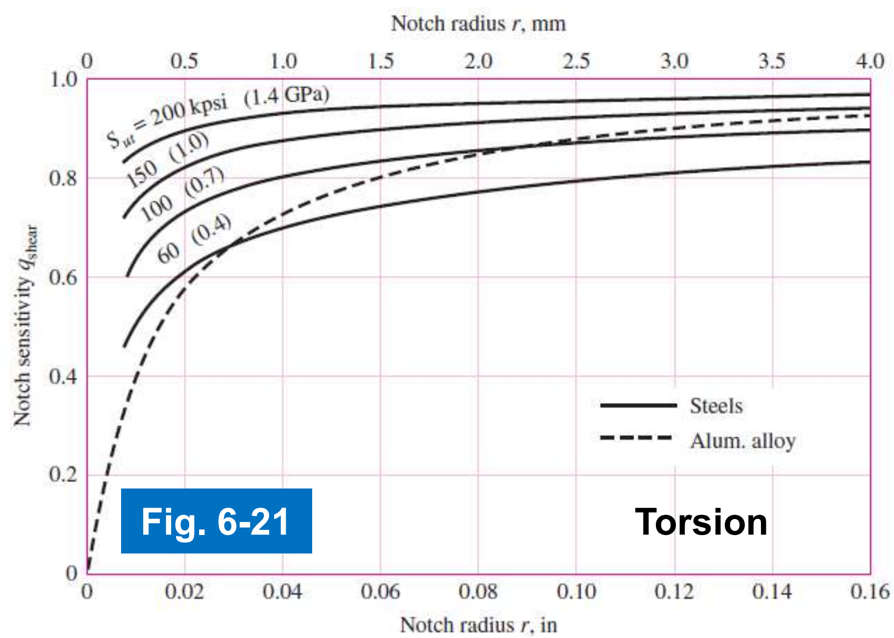
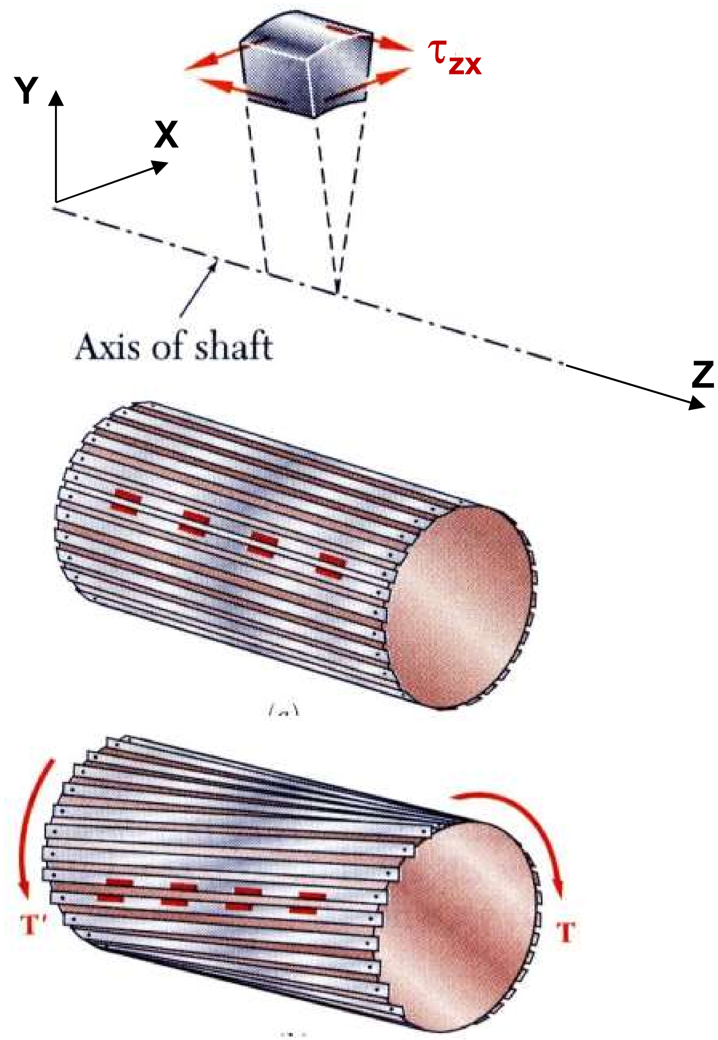


Fig. 6-21 Torsion

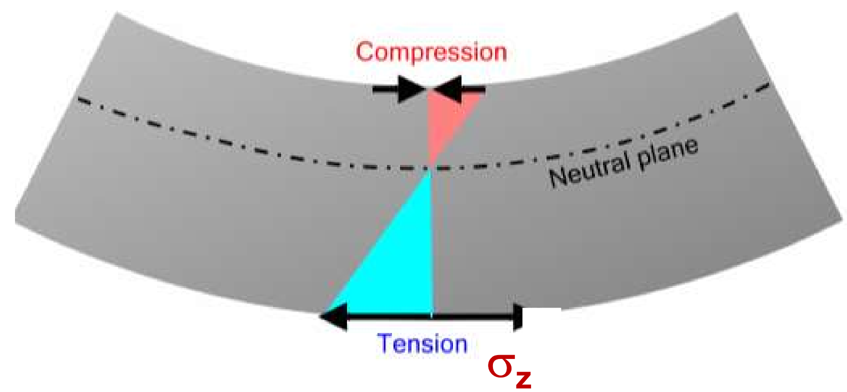
- For conservative approach: use K_f=K_t

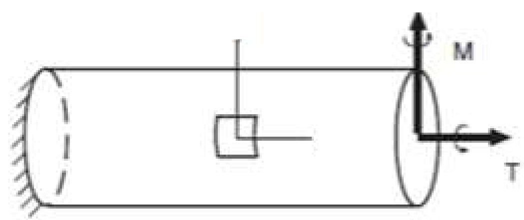
Stresses Resulting from Moment and Torsion

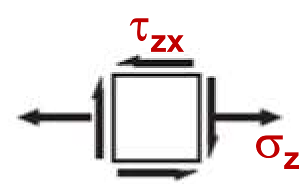
Torsion



Bending Moment







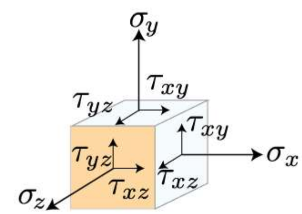
Calculation of Mean and Alternating Stresses

- von Mises Stresses per distortion energy failure theory (DET)

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2}$$

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$



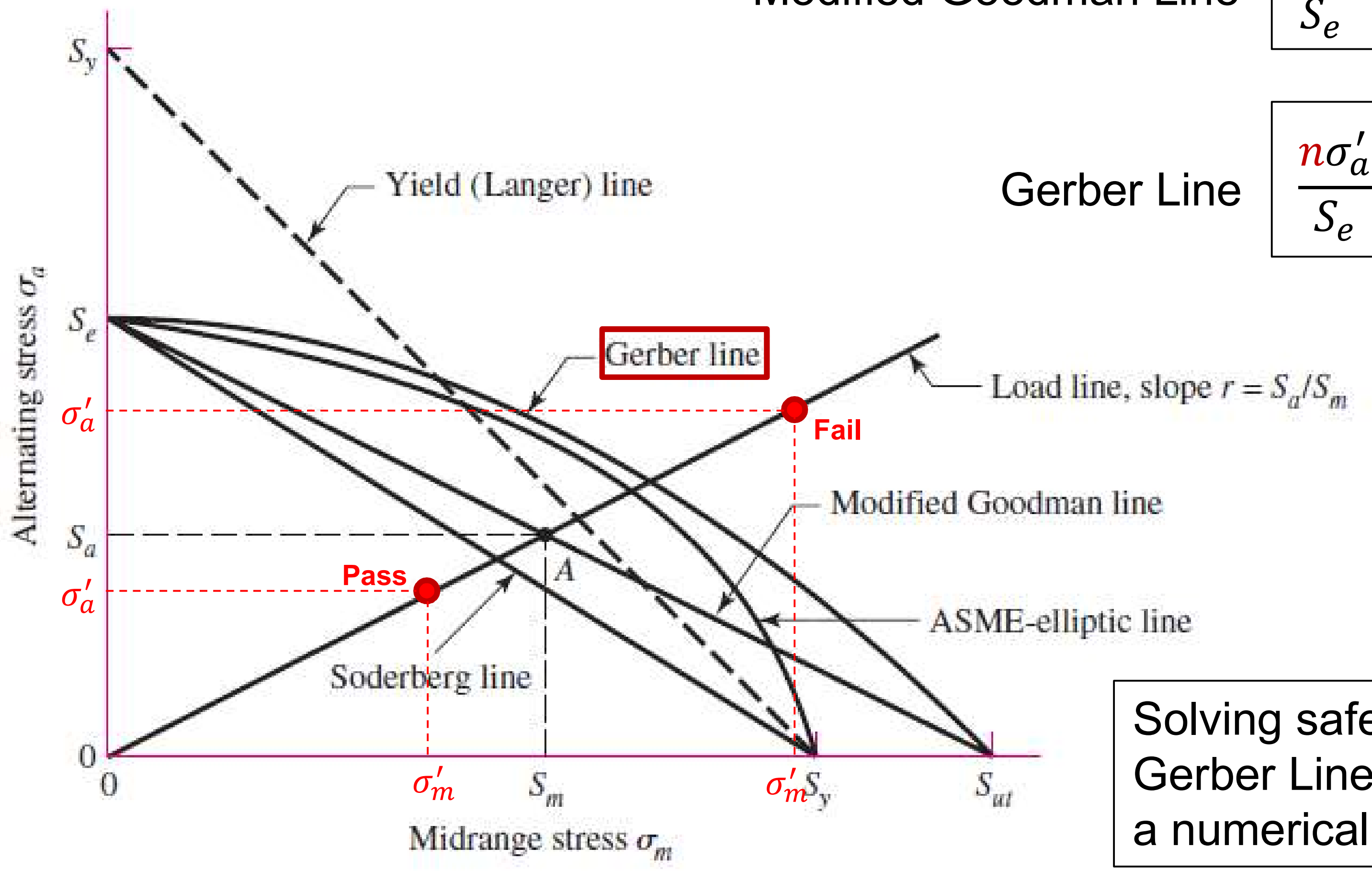
DET is used to integrate multiaxial stress components into equivalent stresses σ'_a and σ'_m.

6-12 Fatigue Failure Criteria for Fluctuating Stress

- Soderberg Line
- Modified Goodman Line
- Gerber Line
- ASME-Elliptic Line

Fatigue Failure Criteria and Safety Factor (n)

Fig. 6-27



Modified Goodman Line

$$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n}$$

Gerber Line

$$\frac{n\sigma'_a}{S_e} + \left(\frac{n\sigma'_m}{S_{ut}}\right)^2 = 1$$

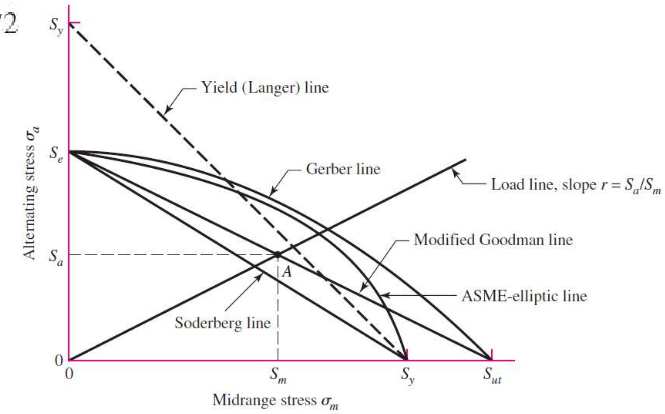
Solving safety factor per Gerber Line is easier with a numerical method.

Calculation of Safety Factor Using DE-Goodman

- von Mises Stresses per distortion energy failure theory (DET)

$$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3} \right)^2 + 3 \left(\frac{16K_{fs} T_a}{\pi d^3} \right)^2 \right]^{1/2}$$

$$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3} \right)^2 + 3 \left(\frac{16K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$



- Safety factor using modified Goodman Line

$$\frac{1}{n} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

- Substitution of σ_a' and σ_m' results in

$$\frac{1}{n} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} \left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2 \right]^{1/2} \right\}$$

Sizing Shaft Diameter Using DE-Goodman

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}$$

- Above equation requires shaft diameter as given. However, this is usually not the case at early stage of design.
- Rearrange the previous equation to get shaft diameter

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}\left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2\right]^{1/2}\right\}\right)^{1/3}$$

DE-Goodman

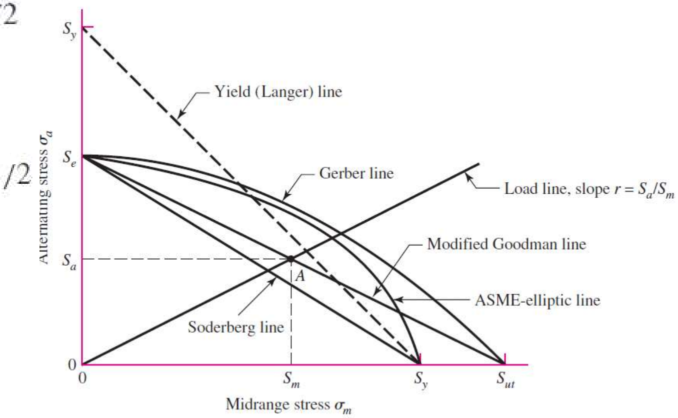
- This shaft diameter is calculated for
  - a solid, round shaft,
  - a prescribed safety factor n,
  - von Mises stress from distortion energy theory as critical stress
  - modified Goodman as the failure criteria

Safety Factor and Shaft Diameter Using DE-Gerber

- von Mises Stresses per distortion energy failure theory (DET)

$$\sigma_a' = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma_m' = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{1/2}$$



- Safety factor using Gerber Line

$$\frac{n\sigma_a'}{S_e} + \left(\frac{n\sigma_m'}{S_{ut}}\right)^2 = 1$$

- Safety Factor (n) and Shaft Diameter (d)

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\} \qquad d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3}$$

where

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs}T_a)^2} \qquad B = \sqrt{4(K_f M_m)^2 + 3(K_{fs}T_m)^2}$$

Special Case: Constant Bending Moment and Torsion

- A common operating case for a rotating shaft is to have constant bending and torsion. If so, the bending stress is completely reversed and the torsion is steady.

$$M = M_m + M_a \qquad T = T_m + T_a$$

$$M = M_a \qquad T = T_m \qquad (M_m = T_a = 0)$$

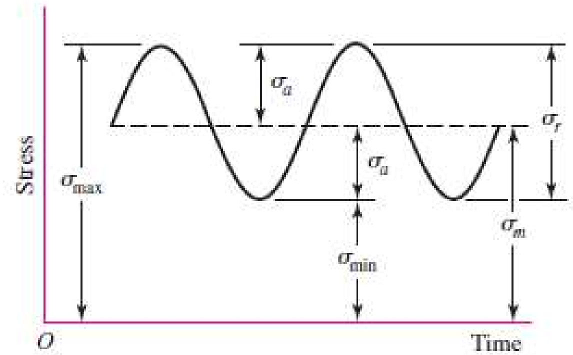
$$d = \left(\frac{16n}{\pi} \left\{ \frac{1}{S_e} \left[4(K_f M_a)^2 + 3(K_{fs} T_a)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[4(K_f M_m)^2 + 3(K_{fs} T_m)^2 \right]^{1/2} \right\} \right)^{1/3}$$

- Calculation of shaft diameter under this circumstance per DE-Goodman can be simplified to

$$d = \left[\frac{16n}{\pi} \left(\frac{2K_f M_a}{S_e} + \frac{\sqrt{3} K_{fs} T_m}{S_{ut}} \right) \right]^{1/3}$$

First Cycle Failure Per Static Failure Theory

- It is always necessary to consider the possibility of static failure in the first load cycle.
- The Gerber and modified Goodman criteria do not guard against yielding, requiring a separate check for yielding. A von Mises maximum stress is calculated for this purpose.



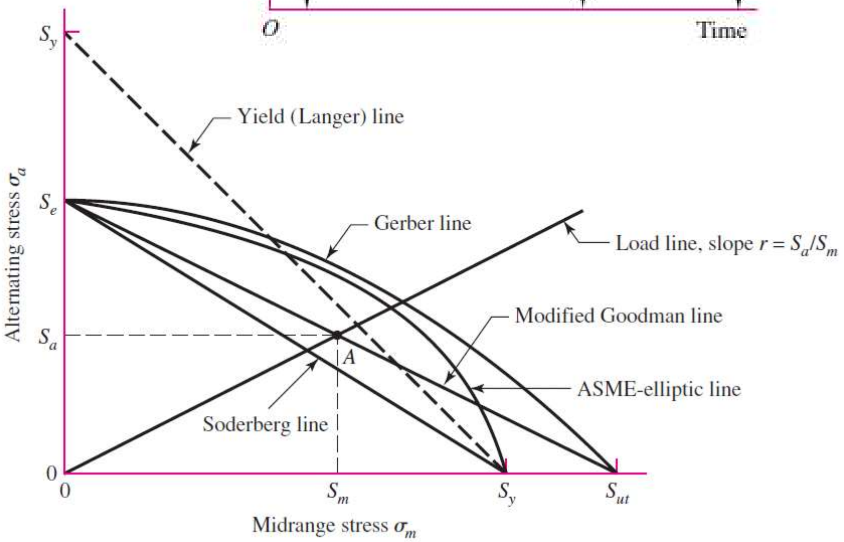
$$\sigma'_{max} = \left[(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2\right]^{1/2}$$

$$= \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2}$$

- Safety Factor: $n = \frac{S_y}{\sigma'_{max}}$

- A conservative estimation for SF:

$$n = \frac{S_y}{\sigma'_m + \sigma'_a}$$



Example 7-1

At a machined shaft shoulder the small diameter *d* is 1.100 in, the large diameter *D* is1.65 in, and the fillet radius is 0.11 in. The bending moment is 1260 lbf-in and the steady torsion moment is 1100 lbf-in.

The heat-treated steel shaft has an ultimate strength of S_{ut} = 105 ksi and a yield strength of S_y = 82 ksi. The reliability goal is 0.99.

- Determine the fatigue factor of safety of the design using DE-Goodman fatigue failure criteria.
- Determine the yielding factor of safety.

M_m = ?

M_a = ?

T_m = ?

T_a = ?

Example 7-1 (Cont'd)

Fatigue stress concentration factors:

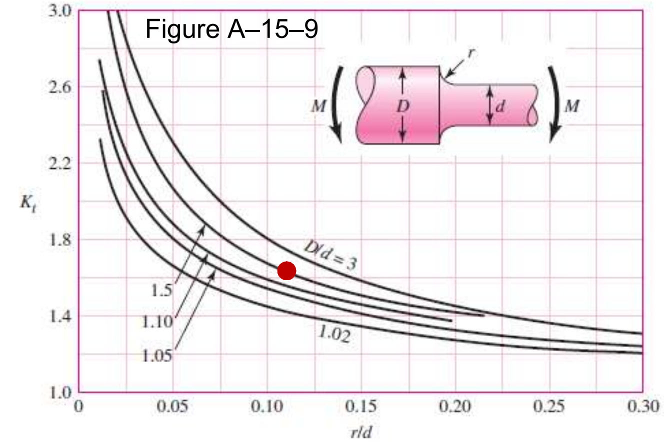
$$\frac{D}{d} = \frac{1.65}{1.10} = 1.5 \quad \frac{r}{d} = \frac{0.11}{1.1} = 0.1$$

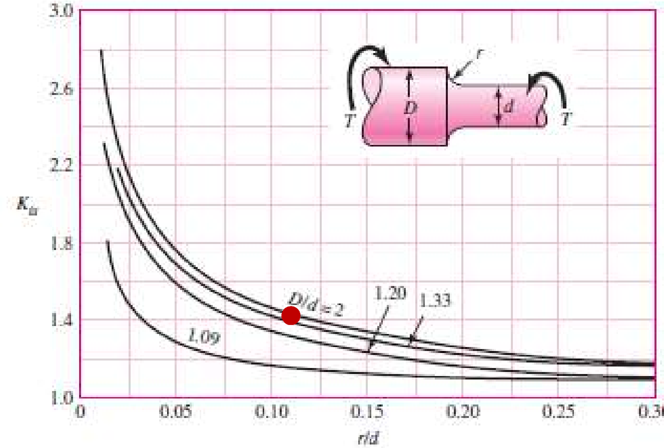
$K_t = 1.68 \quad K_{ts} = 1.42$

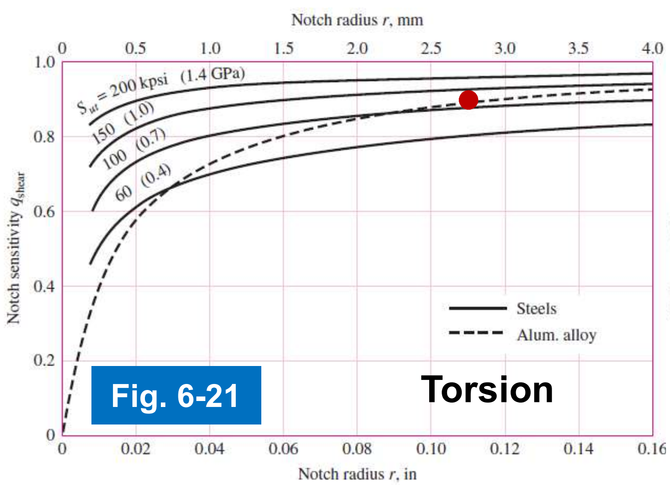
$q = 0.85 \ \text{(Fig 6} - 20) \quad q_{ts} = 0.88 \ \text{(Fig 6} - 21)$

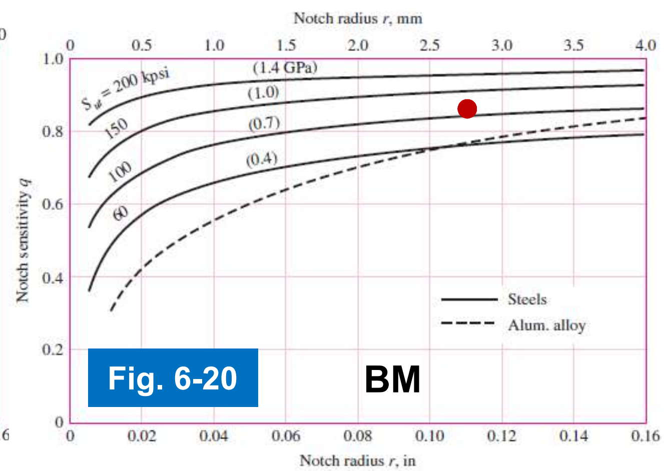
$K_f = 1 + 0.85(1.68 - 1) = 1.58$

$K_{fs} = 1 + 0.88(1.42 - 1) = 1.37$









Example 7-1 (Cont'd)

Fully corrected endurance limit:

S_{ut} = 105 ksi

S_e=29.3 ksi

	(Ksi)	Correction Factor	Parameter
S_{ut} @RT	105		
S_e' @RT	52.5	0.5	Eq. 6-8
	41.3	$k_a = aS_{ut}^b = 2.7 \cdot 105^{-0.265} = 0.787$	Machined Surface
	35.9	$k_b = \left(\frac{1.1}{0.3}\right)^{-0.107} = 0.87$	Size
	35.9	$k_c = 1$	Loading: Bending
	35.9	$k_d = 1$	Temperature
	29.3	$k_e = 0.814$	Reliability: 99%
S_e @RT	**29.3**		

Example 7-1 (Cont'd)

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

M_a=1260 in-lbf M_m=0 $\sigma'_a = \sqrt{\left(\frac{32\cdot 1.58\cdot 1260}{\pi\cdot 1.1^3}\right)^2} = 15.24Ksi$

T_a=0 T_m=1100 in-lbf $\sigma'_m = \sqrt{3\left(\frac{16\cdot 1.37\cdot 1100}{\pi\cdot 1.1^3}\right)^2} = 9.99Ksi$

Safety factor per DE-Goodman

$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_{ut}} = \frac{1}{n} = \frac{15.24}{29.3} + \frac{9.99}{105} = 0.615$ n=1.63

For yielding factor of safety

$$\sigma_{max} = \sqrt{\left(\frac{32\cdot 1.58\cdot 1260}{\pi\cdot 1.1^3}\right)^2 + 3\left(\frac{16\cdot 1.37\cdot 1100}{\pi\cdot 1.1^3}\right)^2} = 18.22Ksi$$

$n_y = \frac{S_y}{\sigma_{max}} = \frac{82}{18.22} = 4.5$ or $n_y = \frac{S_y}{\sigma'_m + \sigma'_a} = \frac{82}{9.99+15.24} = 3.25$

Estimating Stress Concentrations (K_t and K_{ts})

- Estimating stress concentrations require many fine details, such as fillet radii, etc., which usually are not available during early stage of design.
- Fortunately, since these elements are usually of standard proportions, it is possible to estimate the stress-concentration factors for initial design.
- For example: Shoulders for bearing and gear support should match the catalog recommendation. Bearing catalogs shows that a typical ratio of D/d ≈ 1.2-1.5 with r/d ≈ 0.02-0.06.
- For a first approximation, the worst case of D/d =1.5 and r/d=0.02 can be assumed.

	Bending	Torsional	Axial
Shoulder fillet—sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet—well rounded (r/d = 0.1)	1.7	1.5	1.9
End-mill keyseat (r/d = 0.02)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Table 7–1 First Iteration for K_t and K_{ts}

Example 7-2

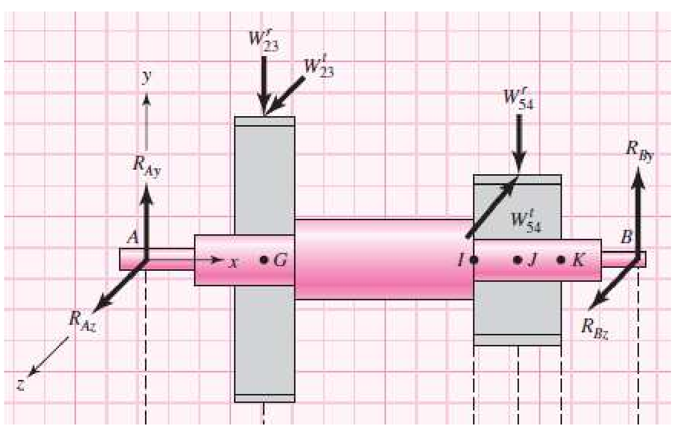
A double reduction gearbox design has developed to the point that the general layout and axial dimensions of the countershaft carrying two spur gears has been proposed.

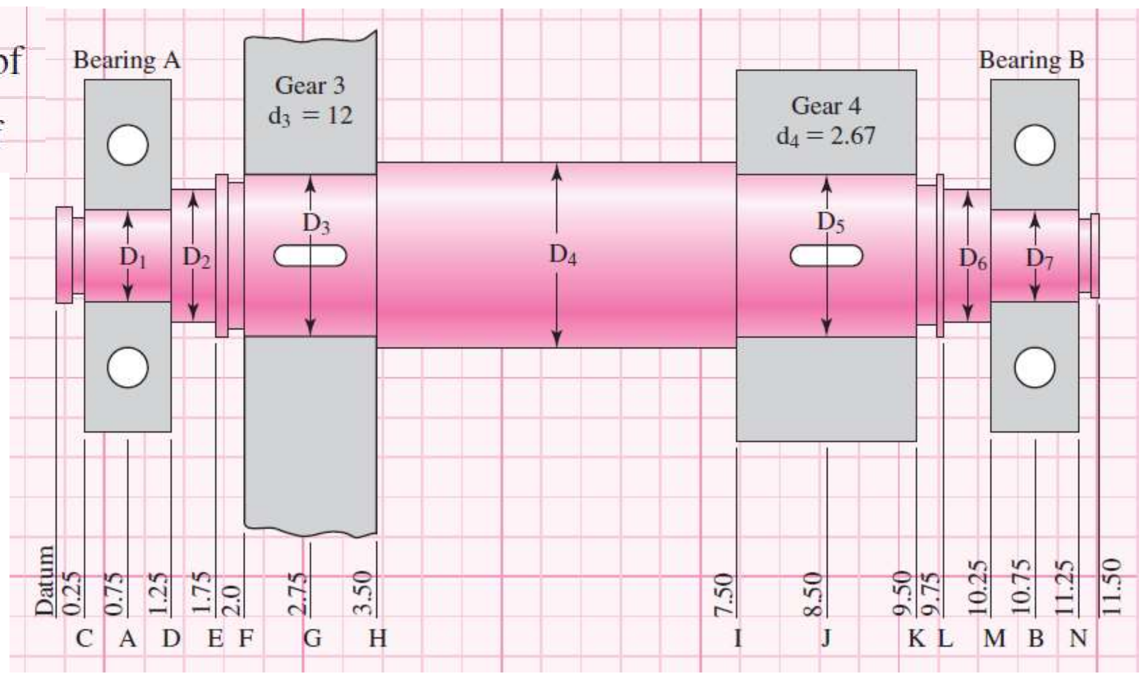
The gears and bearings are located and supported by shoulders, and held in place by retaining rings. The gears transmit torque through keys.

Design for infinite life of the shaft, with minimum safety factors of 1.5

$W_{23}^t = 540\ \text{lbf}$ $\quad$ $W_{54}^t = 2431\ \text{lbf}$

$W_{23}^r = 197\ \text{lbf}$ $\quad$ $W_{54}^r = 885\ \text{lbf}$





Example 7-2 (Cont'd)

Shaft torque: $T = W_{23}^{t}\frac{d_3}{2} = 540\frac{12}{2} = 3240\ in \cdot lbf$

@Point I, total moment

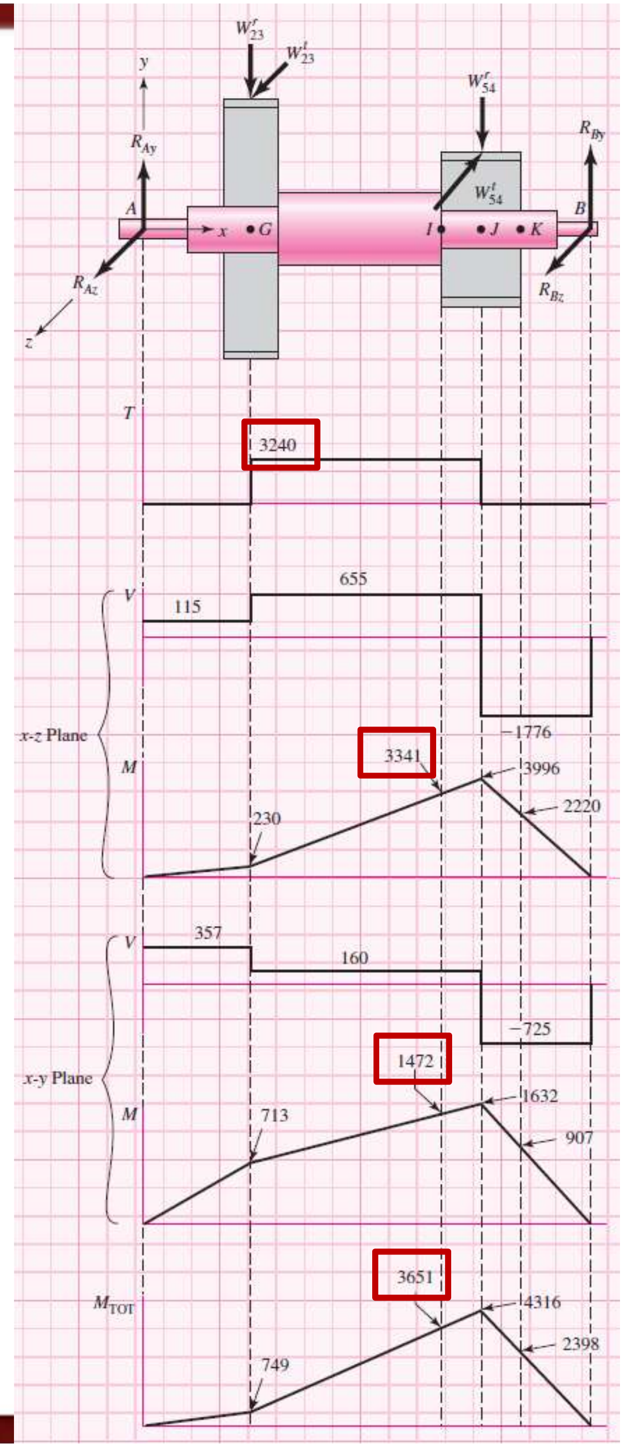
$M_a = \sqrt{3341^2 + 1472^2} = 3651 in \cdot lbf$
$M_m = T_a = 0$

► From Table 7-1, assume $K_t = 1.7$, $K_{ts} = 1.5$

Shaft material 1020 CD: S_{ut}=68ksi, S_y=57ksi (A-20)

►Assume k_b=0.9 since diameter @I is unknown.

	(Ksi)	Correction Factor	Parameter
S_{ut} @RT	68		
S_e' @RT	34	0.5	Eq. 6-8
	30	$k_a = aS_{ut}^{b} = 2.7 \cdot 68^{-0.265} = 0.883$	Machined Surface
	27	$k_b = 0.9$	Size
	27	$k_c = 1$	Loading: Bending
	27	$k_d = 1$	Temperature
	27	$k_e = 1$	Reliability: 50%
S_e @RT	**27**		



Example 7-2 (Cont'd)

Estimate the small diameter at the shoulder of point I

$$d = \left[\frac{16n}{\pi}\left(\frac{2K_f M_a}{S_e} + \frac{\sqrt{3}K_{fs}T_m}{S_{ut}}\right)\right]^{1/3}$$

$$= \left[\frac{16 \cdot 1.5}{\pi}\left(\frac{2 \cdot 1.7 \cdot 3651}{27000} + \frac{\sqrt{3} \cdot 1.5 \cdot 3240}{68000}\right)\right]^{1/3}$$

d=1.65 in

use d=1.625 in for below calculation

Revise S_e=25ksi

► Assume $\frac{D}{d} = 1.2$,

$D = 1.2 \cdot 1.625 = 1.95\ in$

Round D to 2 in

	(Ksi)	Correction Factor	Parameter
S_{ut} @RT	68		
S_e' @RT	34	0.5	Eq. 6-8
	30	$k_a = aS_{ut}^b = 2.7 \cdot 68^{-0.265} = 0.883$	Machined Surface
	25	$k_b = \left(\frac{1.625}{3}\right)^{-0.107} = 0.835$	Size
	25	$k_c = 1$	Loading: Bending
	25	$k_d = 1$	Temperature
	25	$k_e = 1$	Reliability: 50%
S_e @RT	**25**		

Example 7-2 (Cont'd)

Check if estimates are acceptable: $\frac{D}{d} = \frac{2}{1.625} = 1.23$

► Assume fillet radius $\frac{r}{d} = 0.1$, $r = 0.16\ in$

Revise $K_t = 1.6$, $q = 0.82$, $K_f = 1 + 0.82(1.6 - 1) = 1.49$

Revise $K_{ts} = 1.35$, $q_s = 0.85$, $K_{fs} = 1 + 0.85(1.35 - 1) = 1.30$

$$\sigma_a' = \frac{32K_f M_a}{\pi d^3} = \frac{32 \cdot 1.49 \cdot 3651}{\pi 1.625^3} = 12.9ksi$$

$$\sigma_m' = \frac{16\sqrt{3}K_{fs}T_m}{\pi d^3} = \frac{16\sqrt{3} \cdot 1.3 \cdot 3240}{\pi 1.625^3} = 8.66ksi$$

Per DE-Goodman

$$\frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{1}{n} = \frac{12.9}{25} + \frac{8.66}{68} = 0.643$$

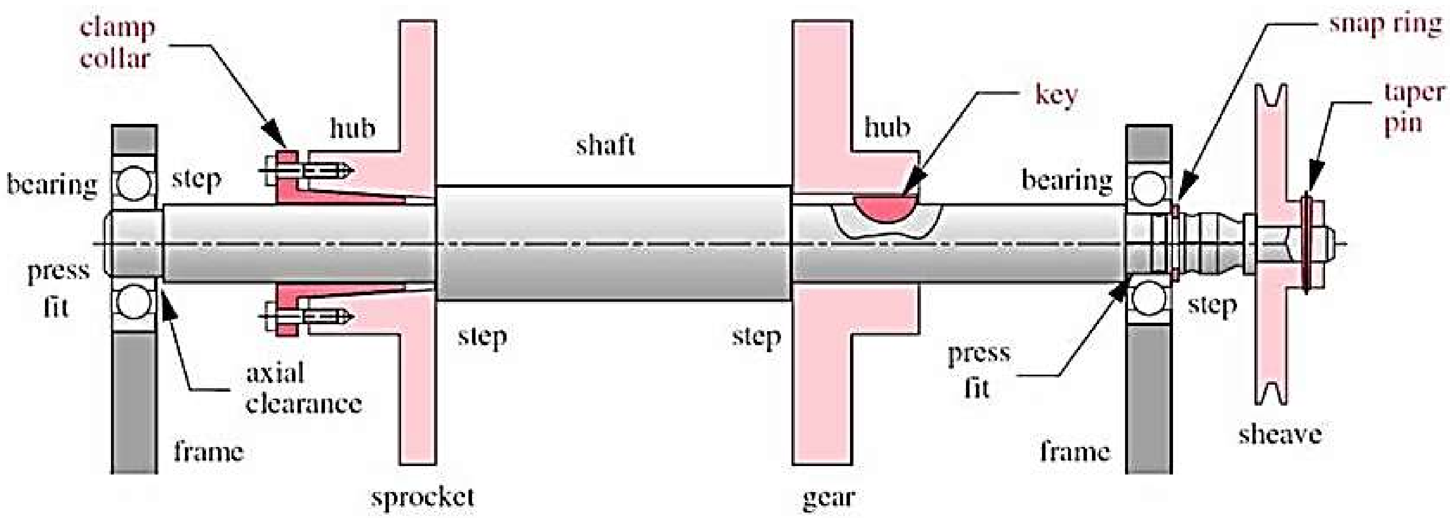
$$n = 1.55 > 1.5$$

Check yielding

$$n_y = \frac{S_y}{\sigma_m + \sigma_a} = \frac{57}{12.9 + 8.66} = 2.64$$

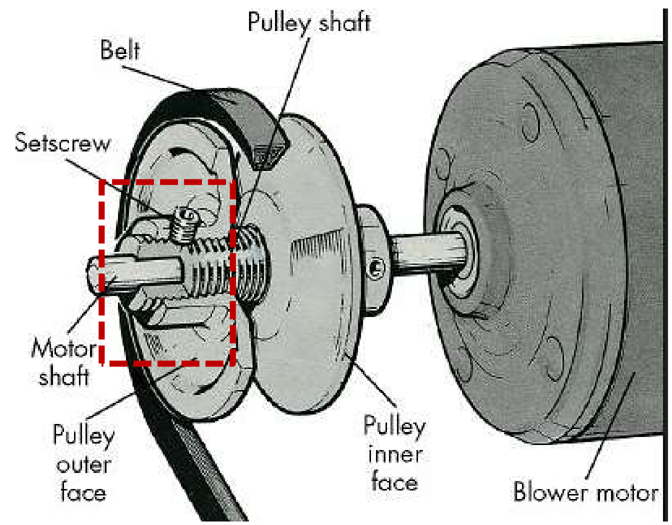
Part 2. Shafts and Shaft Components

- Axial Layout of Components
- Supporting Axial Loads
- Providing for Torque Transmission
- Assembly/Disassembly



7-7 Miscellaneous Shaft Components

Shaft Components - Setscrews



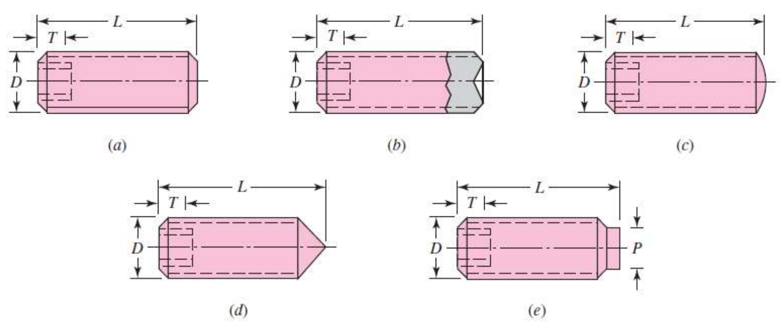
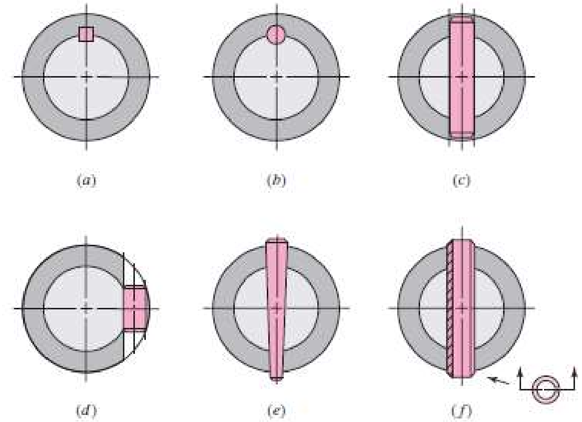


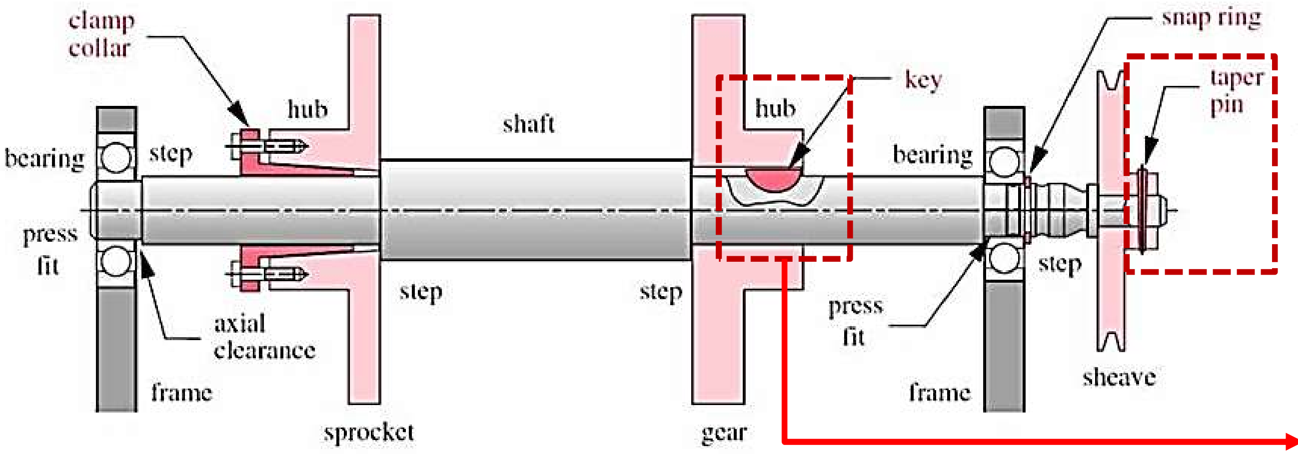
Table 7-4

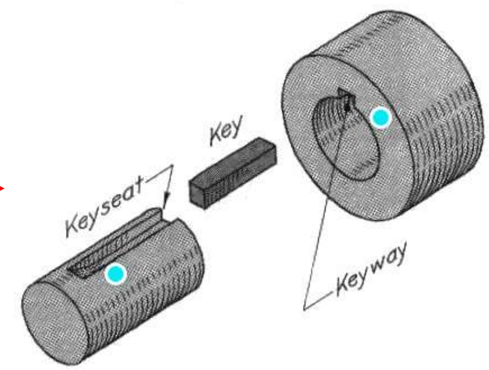
Size, in	Seating Torque, lbf · in	Holding Power, lbf
#0	1.0	50
#1	1.8	65
#2	1.8	85
#3	5	120
#4	5	160
#5	10	200
#6	10	250
#8	20	385
#10	36	540
$\frac{1}{4}$	87	1000
$\frac{5}{16}$	165	1500
$\frac{3}{8}$	290	2000
$\frac{7}{16}$	430	2500
$\frac{1}{2}$	620	3000
$\frac{9}{16}$	620	3500
$\frac{5}{8}$	1325	4000
$\frac{3}{4}$	2400	5000
$\frac{7}{8}$	5200	6000
1	7200	7000

- Setscrew depends on compression to develop the clamping force.
- **Holding Power**: Resistance to axial motion of the collar or hub relative to the shaft.
- Typical factors of safety: 1.5 to 2.0 for static loads; 4-8 for dynamic loads
- Setscrews should have a length of about half of the shaft diameter.

Shaft Components - Keys and Pins





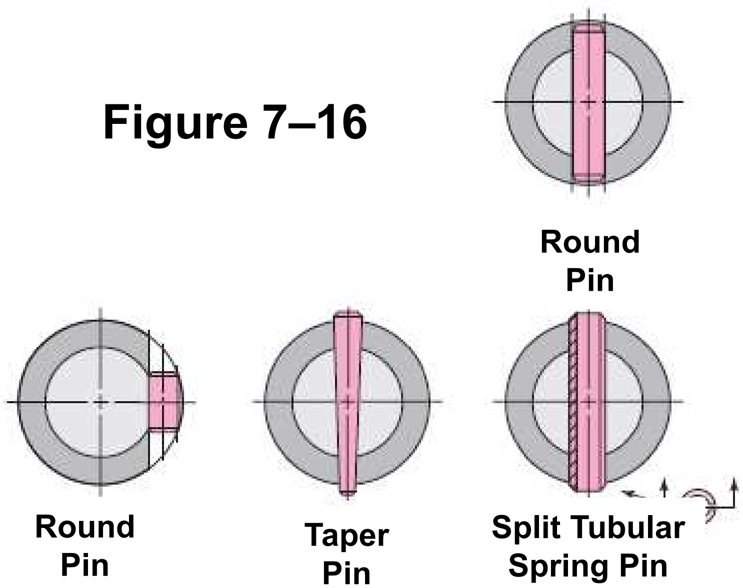


- Keys and pins are used on shafts to secure rotating elements, such as gears, pulleys, or other wheels.
  – Keys are used to enable the **transmission of torque** from the shaft to the shaft-supported element.
  – Pins are used for **axial positioning** and for the transfer of torque or thrust or both

Shaft Components - Pins

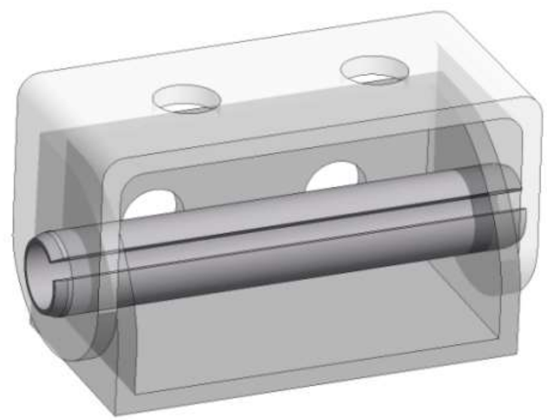
- Pins are used for **axial positioning** and for the transfer of **torque or thrust** or both

Figure 7–16



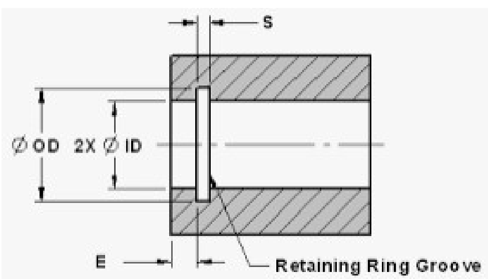
$$d = D - 0.0208L$$

d: diameter at small end, in
D: diameter at large end, in
L: length, in

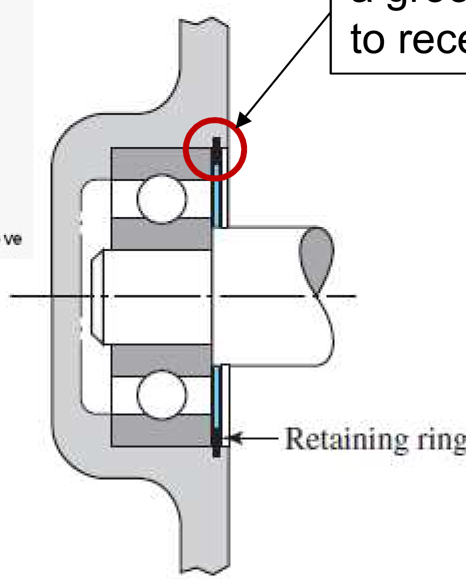


Shaft Components – Retaining Rings

- Frequently used to <u>axially position</u> a component on a shaft or in a housing bore. Capable of supporting light axial loads as well.
- Weakness:
  - Radius in the bottom of the groove typically about 1/10 of the groove width. This causes comparatively high stress concentration factors, around 5 for bending and axial, and 3 for torsion. (Appendix Tables A–15–16 and A–15–17)

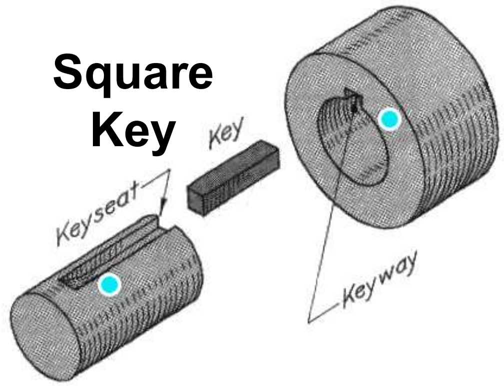


a groove is cut in the shaft or bore to receive the spring retainer



Shaft Components – Square & Rectangular Keys

Square Key



- Designer chooses an appropriate key length to carry the torsional load.
- Maximum length of a key is limited by the hub length of the attached element, and should generally <u>not exceed 1.5 times the shaft diameter</u>.
- Minimum key length should be ≥ 25 % larger than the shaft diameter.

$$1.5D \geq L \geq 1.25D$$

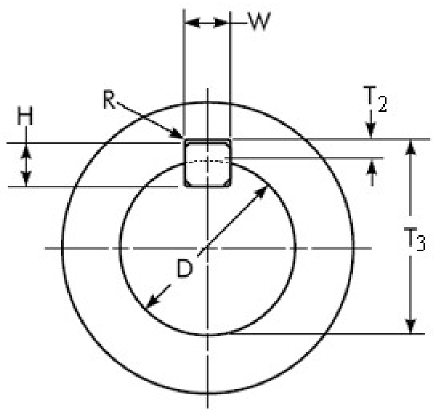
- **TechRef: ANSI B17.1 Keys and Keyseats**

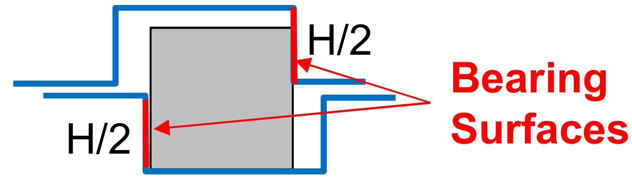
Table 7–6

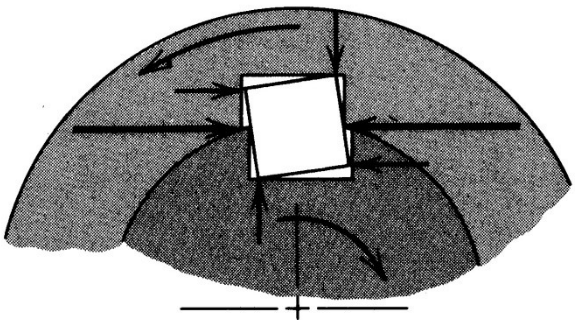
Shaft Diameter		Key Size		
Over	To (Incl.)	w	h	Keyway Depth
$\frac{5}{16}$	$\frac{7}{16}$	$\frac{3}{32}$	$\frac{3}{32}$	$\frac{3}{64}$
$\frac{7}{16}$	$\frac{9}{16}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{3}{64}$
		$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{16}$
$\frac{9}{16}$	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$
		$\frac{3}{16}$	$\frac{3}{16}$	$\frac{3}{32}$
$\frac{7}{8}$	$1\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{3}{32}$
		$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$
$1\frac{1}{4}$	$1\frac{3}{8}$	$\frac{5}{16}$	$\frac{1}{4}$	$\frac{1}{8}$
		$\frac{5}{16}$	$\frac{5}{16}$	$\frac{5}{32}$
$1\frac{3}{8}$	$1\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
		$\frac{3}{8}$	$\frac{3}{8}$	$\frac{3}{16}$
$1\frac{3}{4}$	$2\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{8}$	$\frac{3}{16}$
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
$2\frac{1}{4}$	$2\frac{3}{4}$	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{7}{32}$
		$\frac{5}{8}$	$\frac{5}{8}$	$\frac{5}{16}$
$2\frac{3}{4}$	$3\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
		$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{8}$

Two Primary Failure Modes

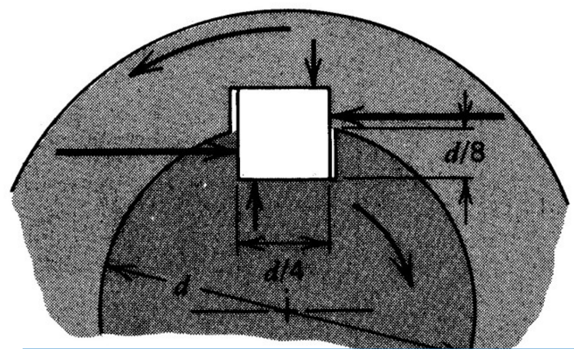
- Failure of the key can be by direct shear, or by bearing stress.





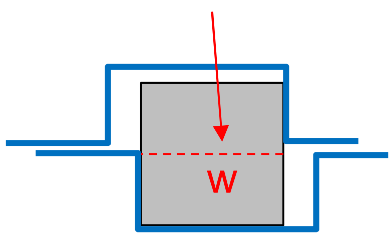


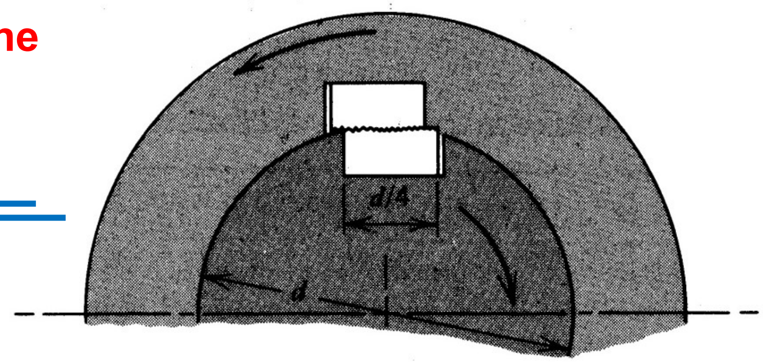
(a) Loosely fitted key



2. It can be crushed due to the compressive bearing forces

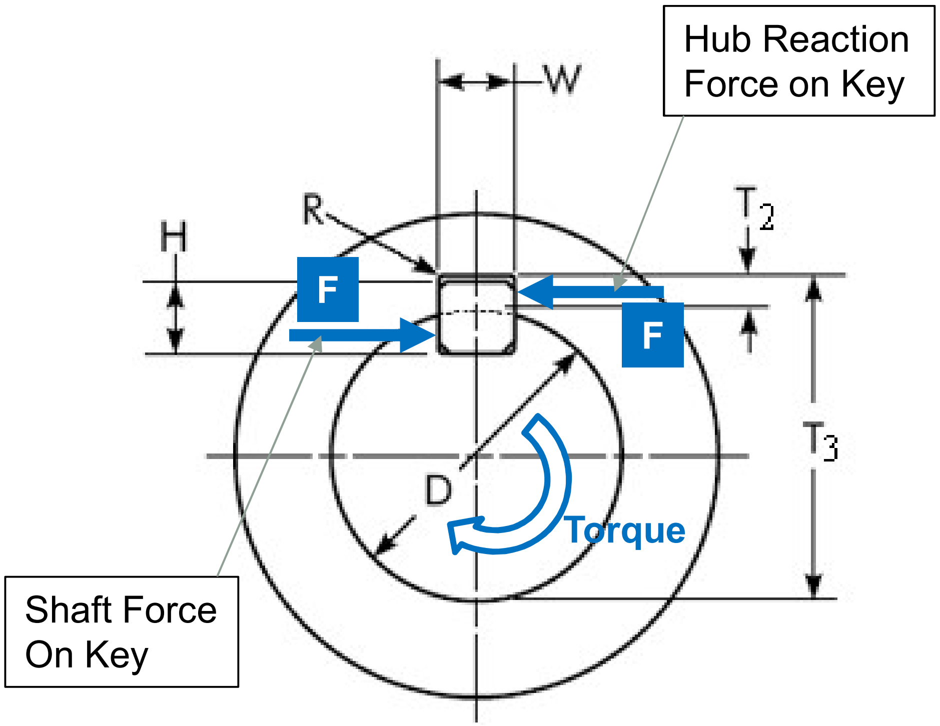
Shearing Plane

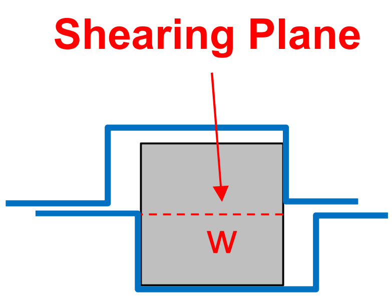




1. It can be sheared off

Forces Acting on Key/Keyway: Shearing Stress





Force $F = \frac{T}{D/2}$

Key Shear Area $A_s = w \cdot L$

Average Shear Stress:

$$\tau_{avg} = \frac{F}{A_s} = \frac{2T}{wDL}$$

<u>Required Key Length in Shear:</u>

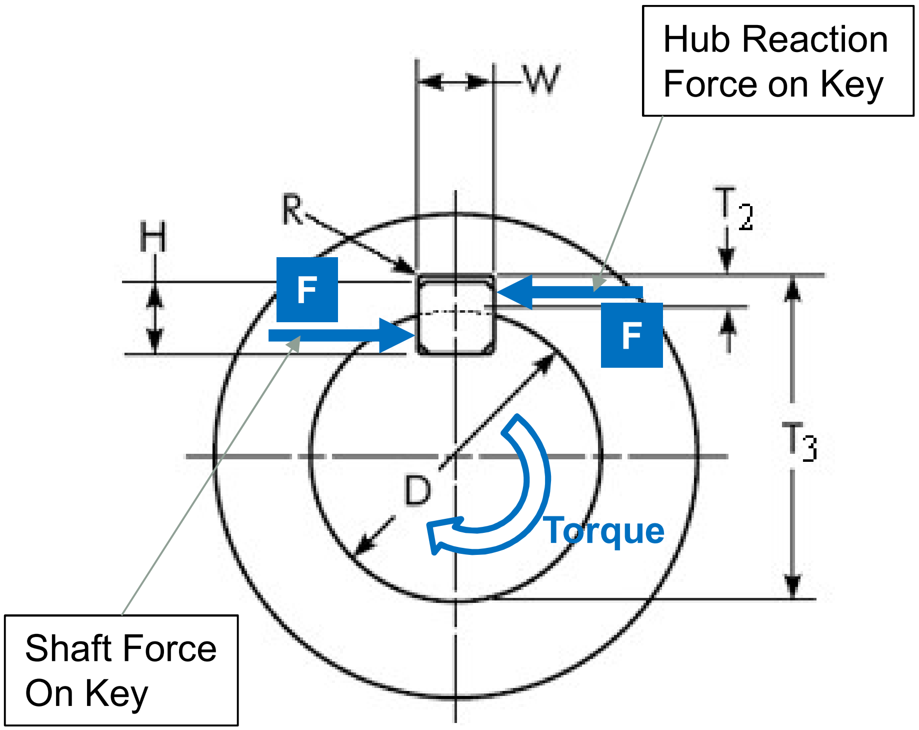
- $S_{sy} = 0.577 \cdot S_y$ per DET
- $S_{sy} = 0.5 \cdot S_y$ per MSS
- Safety factor n

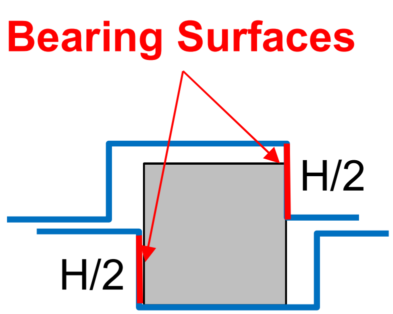
$$\tau_{avg} = \frac{F}{A_s} = \frac{2T}{wDL} = \frac{S_{sy}}{n}$$

$$L = \frac{2nT}{wDS_{sy}}$$

$$L = \frac{4nT}{wDS_y}$$

Forces Acting on Key/Keyway: Bearing Stress





Force $F = \frac{T}{D/2}$

Bearing Stress Area $A_b = \frac{H}{2} \cdot L$

Average Shear Stress:

$$\sigma_{b,avg} = \frac{F}{A_b} = \frac{4T}{HDL}$$

More realistically,

$$\sigma_b = K\sigma_{b,avg} = \frac{4KT}{HDL}$$

K: Triaxial Stress Factor $(1 \leq K \leq 1.5)$

Safety factor n

<u>Required Key Length in Bearing:</u>

$$\sigma_b = \frac{F}{A_b} = \frac{4KT}{HDL} = \frac{S_y}{n}$$

$$L = \frac{4nKT}{HDS_y}$$

Required Key Length

Shearing Failure

$$L = \frac{4nT}{wDS_y}$$ (Per MSS)

Bearing Stress Failure

$$L = \frac{4nKT}{HDS_y}$$

- If K=1, these equations give the same key length result for a square key. (H = W)
- In general K≥1.0; therefore, the calculated key length for bearing stress resistance will be greater.

Example 7-6

A UNS G10350 steel shaft, heat-treated to a minimum yield strength of 75 ksi, has a diameter of 1-7/16”. The shaft rotates at 600 rev/min and transmits 40 hp through a gear. Select an appropriate key for the gear with safety factor=2.8.

<u>Solution:</u>

From Table 7–6, a 3/8” square key is selected

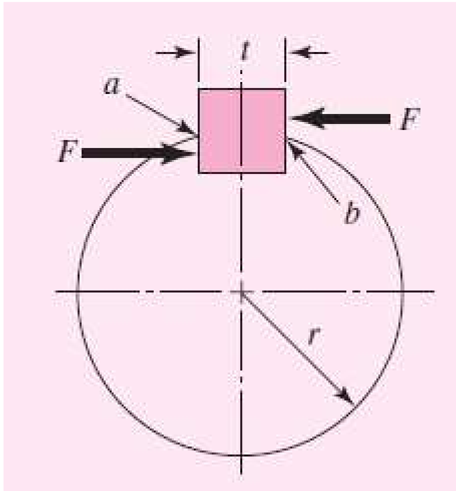
$t = H = \frac{3}{8} in,\ depth = \frac{3}{16} in$ (Square key)

Key material: UNS G10200 cold-drawn steel

Transmitted torque: $T = \frac{63025 \cdot Hp}{rpm} = \frac{63025 \cdot 40}{600} = 4200\ in \cdot lbf$

Shaft diameter $2r = 1\frac{7}{16} in = 1.4375\ in \quad r = 0.71875\ in$

Force $F = \frac{T}{r} = \frac{4200}{0.71875} = 5850\ lbf$



Allowable shear stress (DET): $S_{sy} = 0.577 \cdot S_y = 0.577 \cdot 65 = 37.5ksi$

Shear stress: $\tau = \frac{F}{t \cdot L}$ $\quad n = 2.8 = \frac{S_{sy}}{\tau} = \frac{37.5 \cdot 100}{5850/(0.375 \cdot L)}$ $\quad L = 1.165in$

Bearing stress for half of key $\sigma = \frac{F}{(H/2) \cdot L} = \frac{5850}{0.5 \cdot 0.375 \cdot L}$

$n = 2.8 = \frac{S_y}{\sigma} = \frac{65 \cdot 1000}{5850/(0.5 \cdot 0.375 \cdot L)}$ $\quad L = 1.344in$

Shaft Components – Square & Rectangular Keys

Safety Factor of Key Design*

- Because the stress distribution for keyed connections is not completely understood, a factor of safety of **1.5** should be used when the torque is static.
- For minor shock loads, a factor of safety of **2.5** should be used
- For high shock loads (especially if the loads are reversible), and a factor of safety of **4.5** should be used.

* Technical Manual Keyway Joints, ETP Transmission AB

Shaft Components – Square & Rectangular Keys

Stress Concentrations in an End-Milled Keyseat

- For fillets cut by standard milling-machine cutters,

$$\frac{r}{d} = \frac{\text{radius at bottom of the groove}}{\text{shaft diameter}}$$

- with a ratio of $\frac{r}{d} = 0.02$,
  - K_t = 2.14 for bending
  - K_{ts} = 2.62 for torsion without the key in place,
  - or K_{ts} = 3.0 for torsion with the key in place.

- Keeping the end of a keyseat at least a distance of $\frac{Shaft\ Diameter}{10}$ from the start of the shoulder fillet

Shaft Components – Woodruff Keys

Table 7-8

Key Size w	Key Size D	Height b	Offset e	Keyseat Depth Shaft	Keyseat Depth Hub
$\frac{1}{16}$	$\frac{1}{4}$	0.109	$\frac{1}{64}$	0.0728	0.0372
$\frac{1}{16}$	$\frac{3}{8}$	0.172	$\frac{1}{64}$	0.1358	0.0372
$\frac{3}{32}$	$\frac{3}{8}$	0.172	$\frac{1}{64}$	0.1202	0.0529
$\frac{3}{32}$	$\frac{1}{2}$	0.203	$\frac{3}{64}$	0.1511	0.0529
$\frac{3}{32}$	$\frac{5}{8}$	0.250	$\frac{1}{16}$	0.1981	0.0529
$\frac{1}{8}$	$\frac{1}{2}$	0.203	$\frac{3}{64}$	0.1355	0.0685
$\frac{1}{8}$	$\frac{5}{8}$	0.250	$\frac{1}{16}$	0.1825	0.0685
$\frac{1}{8}$	$\frac{3}{4}$	0.313	$\frac{1}{16}$	0.2455	0.0685
$\frac{5}{32}$	$\frac{5}{8}$	0.250	$\frac{1}{16}$	0.1669	0.0841
$\frac{5}{32}$	$\frac{3}{4}$	0.313	$\frac{1}{16}$	0.2299	0.0841
$\frac{5}{32}$	$\frac{7}{8}$	0.375	$\frac{1}{16}$	0.2919	0.0841
$\frac{3}{16}$	$\frac{3}{4}$	0.313	$\frac{1}{16}$	0.2143	0.0997
$\frac{3}{16}$	$\frac{7}{8}$	0.375	$\frac{1}{16}$	0.2763	0.0997
$\frac{3}{16}$	1	0.438	$\frac{1}{16}$	0.3393	0.0997
$\frac{1}{4}$	$\frac{7}{8}$	0.375	$\frac{1}{16}$	0.2450	0.1310
$\frac{1}{4}$	1	0.438	$\frac{1}{16}$	0.3080	0.1310
$\frac{1}{4}$	$1\frac{1}{4}$	0.547	$\frac{5}{64}$	0.4170	0.1310
$\frac{5}{16}$	1	0.438	$\frac{1}{16}$	0.2768	0.1622
$\frac{5}{16}$	$1\frac{1}{4}$	0.547	$\frac{5}{64}$	0.3858	0.1622
$\frac{5}{16}$	$1\frac{1}{2}$	0.641	$\frac{7}{64}$	0.4798	0.1622
$\frac{3}{8}$	$1\frac{1}{4}$	0.547	$\frac{5}{64}$	0.3545	0.1935
$\frac{3}{8}$	$1\frac{1}{2}$	0.641	$\frac{7}{64}$	0.4485	0.1935

Table 7-7

Keyseat Width, in	Shaft Diameter, in From	Shaft Diameter, in To (inclusive)
$\frac{1}{16}$	$\frac{5}{16}$	$\frac{1}{2}$
$\frac{3}{32}$	$\frac{3}{8}$	$\frac{7}{8}$
$\frac{1}{8}$	$\frac{3}{8}$	$1\frac{1}{2}$
$\frac{5}{32}$	$\frac{1}{2}$	$1\frac{5}{8}$
$\frac{3}{16}$	$\frac{9}{16}$	2
$\frac{1}{4}$	$\frac{11}{16}$	$2\frac{1}{4}$
$\frac{5}{16}$	$\frac{3}{4}$	$2\frac{3}{8}$
$\frac{3}{8}$	1	$2\frac{5}{8}$

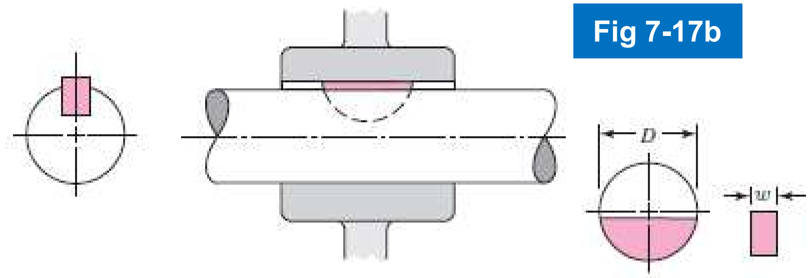
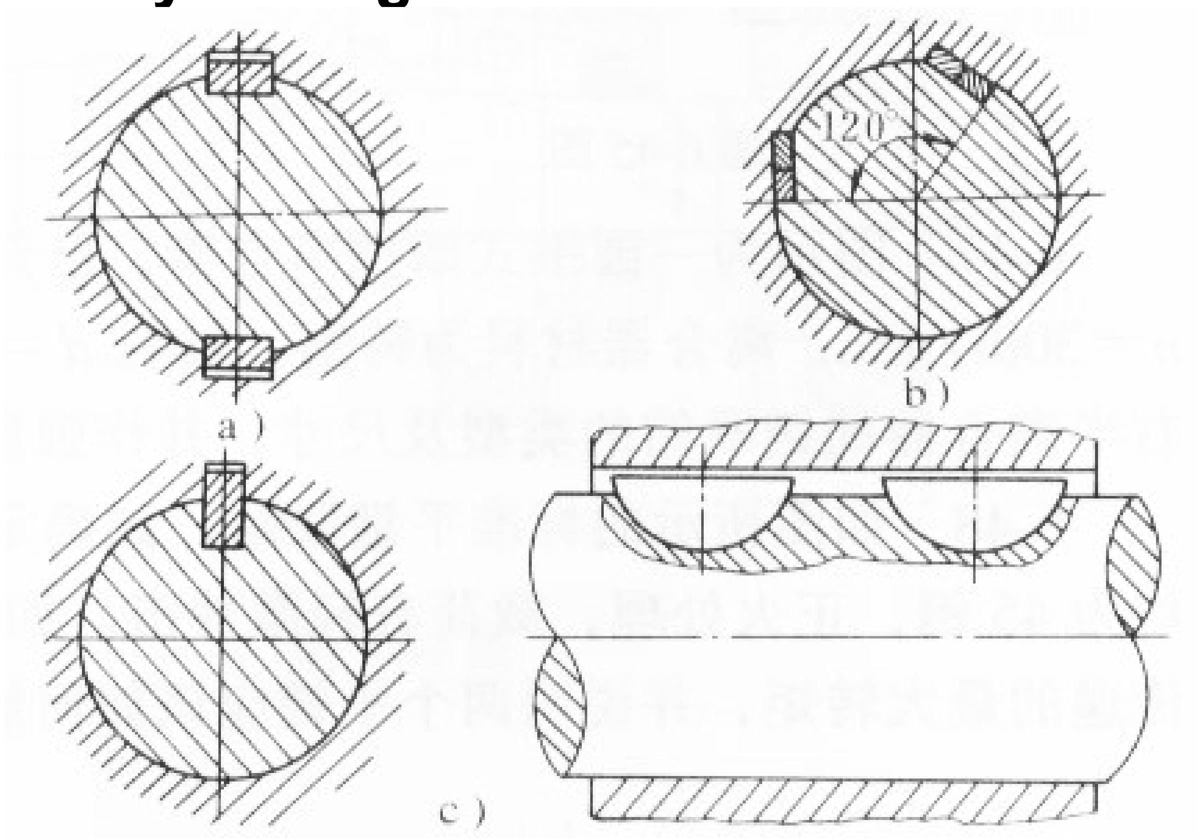


Fig 7-17b

The most commonly used part for shaft-to-hub torque-transmission.

Double-Key Arrangement



- Mostly serve for the redundancy purpose.

Other Shaft Design Concerns

- 7-5 Deflection Considerations
- 7-6 Critical Speeds for Shafts
- 7-8 Limits and Fits (LN01 Press Fits)

Shaft Deflection Considerations

- Deflection of the shaft, both linear and angular, should be checked at components demand higher accuracy like **gears** and **bearings**.
- Typical ranges for maximum slopes and transverse deflections of the shaft centerline are given in Table 7–2.
- Shaft deflection analysis is usually evaluated with the assistance of software
  - (Please review Sec 4-4 to 4-6 for beam deflection methods.)

Table 7–2

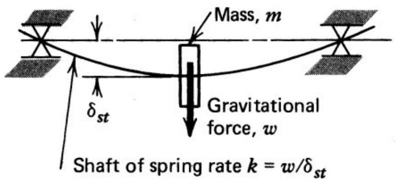
Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical ball	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	< 0.0005 rad
Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in

$$Diametral\ Pitch\ P = \frac{Number\ of\ Teeth}{Pitch\ Diameter}$$

7-6 Critical Speeds for Shafts

Configuration	Critical speed equation

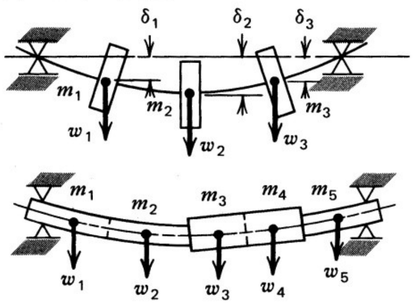
(a) Single mass



$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{w}} = \sqrt{\frac{g}{\delta_{st}}} \qquad (17.1)$$

$$n_c = \frac{30}{\pi}\sqrt{\frac{k}{m}} = \frac{30}{\pi}\sqrt{\frac{kg}{w}} = \frac{30}{\pi}\sqrt{\frac{g}{\delta_{st}}} \qquad (17.1a)$$

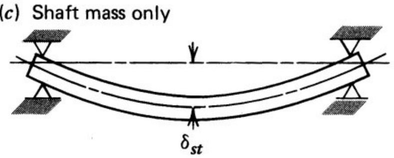
(b) Multiple masses



$$n_c \approx \frac{30}{\pi}\sqrt{\frac{g(w_1\delta_1 + w_2\delta_2 + \dots)}{w_1\delta_1^2 + w_2\delta_2^2 + \dots}}$$

$$n_c \approx \frac{30}{\pi}\sqrt{\frac{g\Sigma w\delta}{\Sigma w\delta^2}} \qquad (17.2)$$

(c) Shaft mass only



$$n_c \approx \sqrt{\frac{5g}{4\delta_{st}}} \qquad (17.3)$$

Quantity	Symbol	SI	English
Mass	m	kg	lb-sec²/in.
Gravitational force	w	N	lb
Static deflection	δ_{st}	m	in.
Shaft spring rate	k	N/m	lb/in.
Accel. of gravity	g	m/s²	in./sec²
Natural frequency	ω_n	rad/s	rad/sec
Critical speed	n_c	rpm	rpm

n_c: critical speed
δ: shaft deflection

Ref: R.C. Juvinall, Fundamentals of Machine Component Design, John, Wiley & Sons, 1983.

7-8 Limits and Fits (LN01 Press Fits)