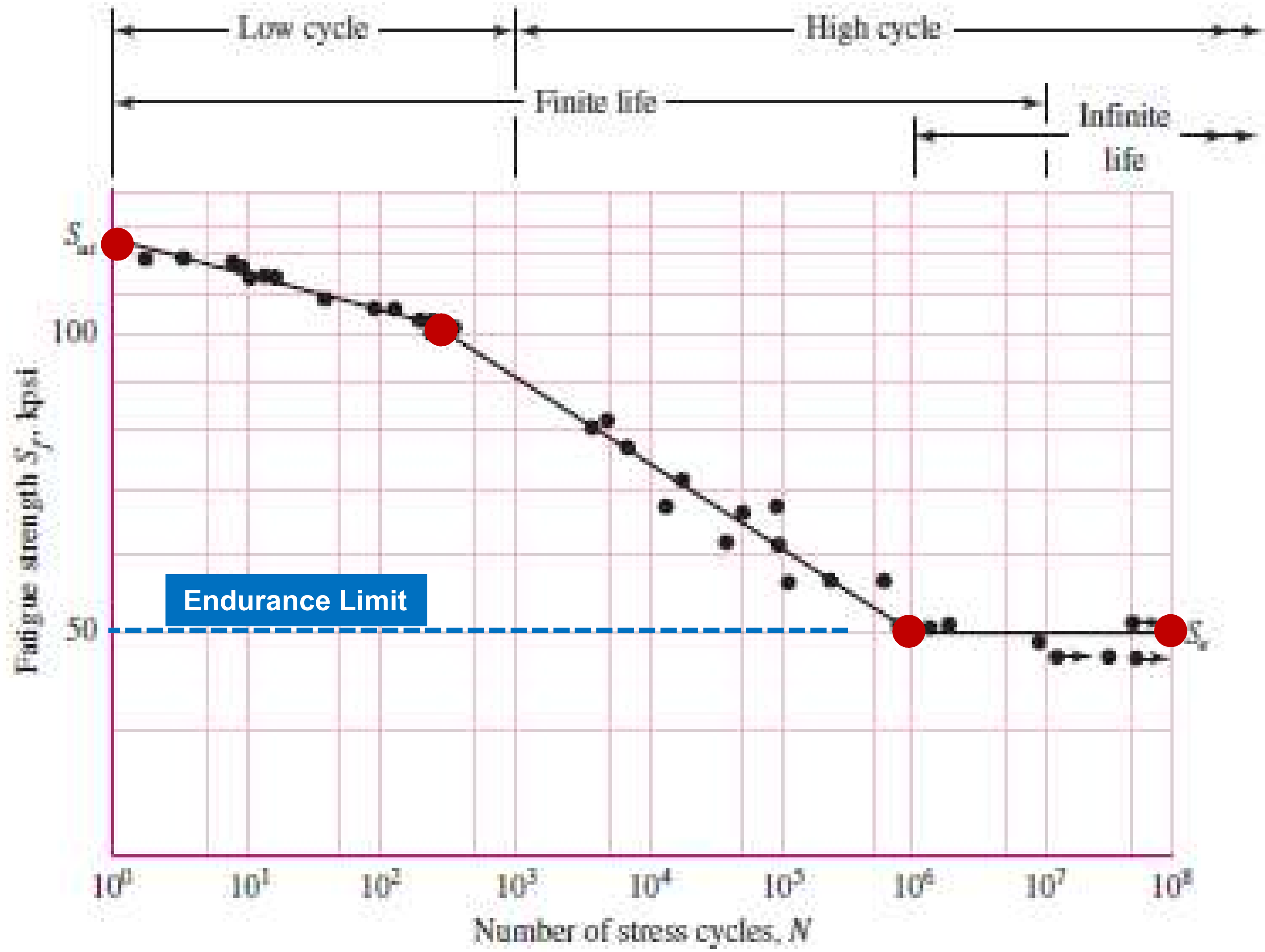
6-8 Fatigue Strength (Analytical Approach)

- ► Unmodified Endurance Limit (S_e')
- ► S-N Diagram

Example of S-N Diagram

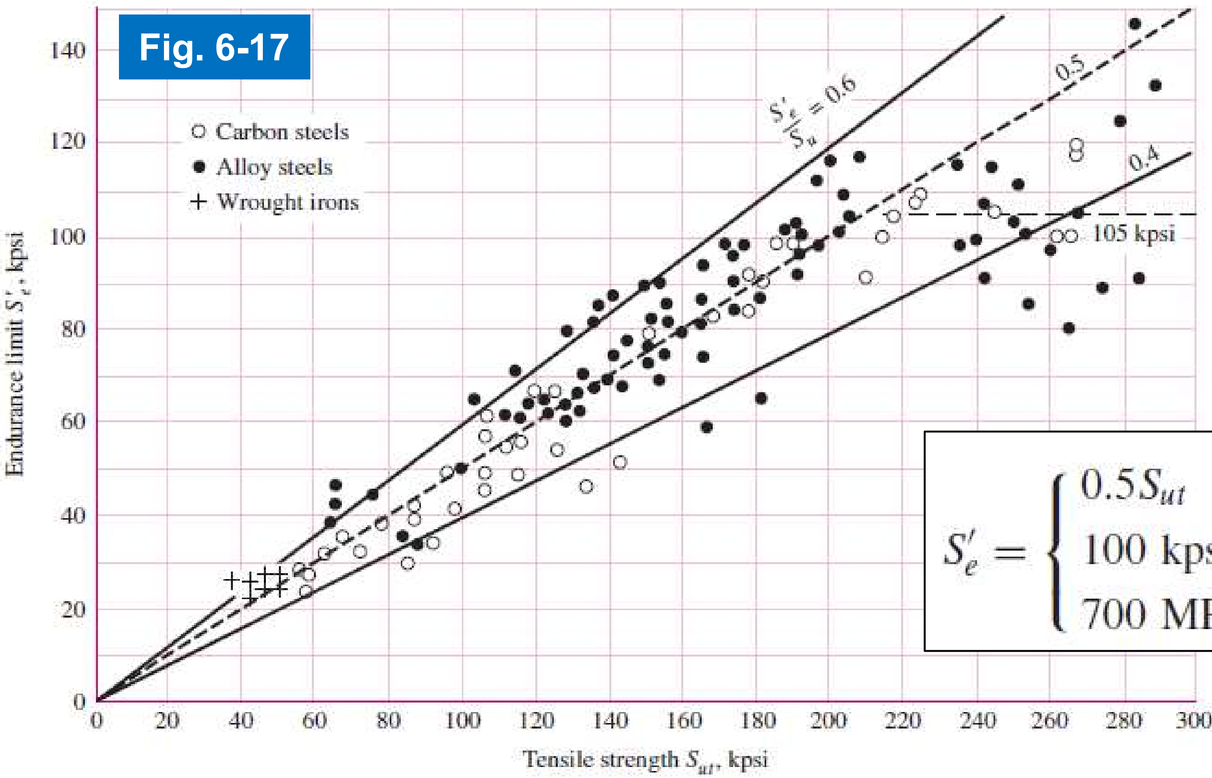


Estimate Endurance Limit (S'_e) – Analytical Approach

Analytical approach is also commonly used in engineering practice when lab-tested S'_e is not available

Correlation indicated endurance limit ranges from about 40 to 60 percent of the tensile strength for steels up to about 210 ksi

Fig. 6-17



Step 1

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Calculate Parameters for S-N Curve Construction

Step 2: Estimate <u>Fatigue Strength Fraction (f)</u>

- f=0.9 (S_{ut} < 70 ksi)
- Use Fig. 6-18 (S_{ut} ≥ 70 ksi)

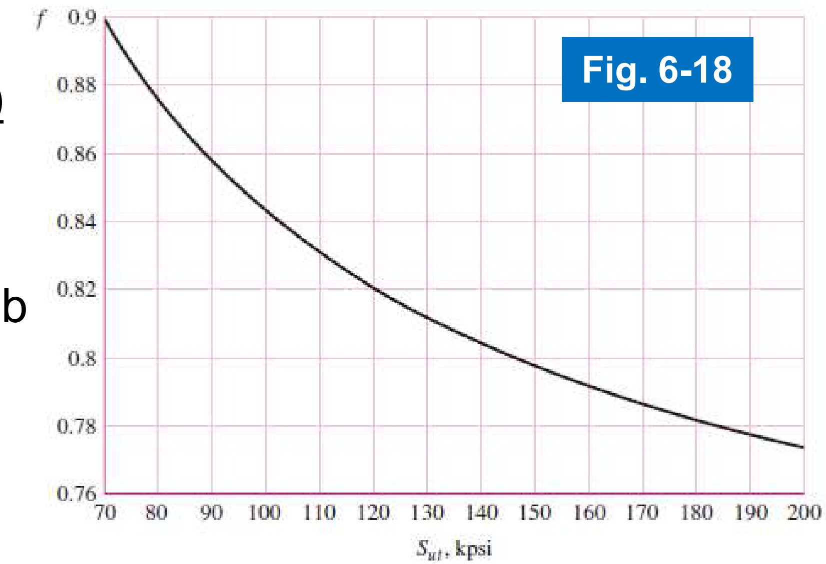
Step 3: Calculate fatigue life constants a and b

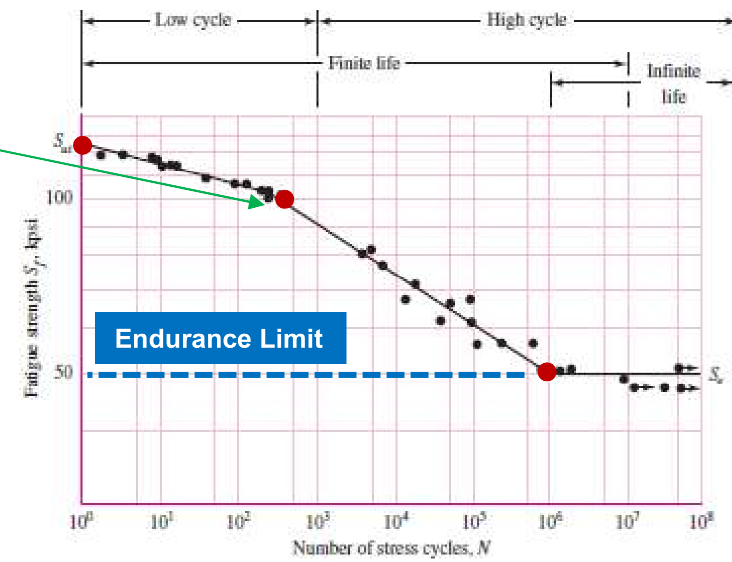
$$a = \frac{(f\,S_{ut})^2}{S_e} \qquad b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e}\right)$$

Step 4: Draw S-N

- @ Cycle=1, $(S_f)_1 = S_{ut}$
- @ Cycle= 10^3, $(S_f)_{1000} = f\,S_{ut}$
- @ Cycle= 10^6, $(S_f)_{1M} = S_e$

Cycle Count (N)	S-N Curve (S_f)
$1 \le N \le 10^3$	$S_f = S_{ut} N^{(\log f)/3}$
$10^3 \le N \le 10^6$	$S_f = aN^b$
$N > 10^6$	$S_f = S_e$





Note: this only applies to purely reversing stresses where σ_m = 0.

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Example 6-2

Given a 1050 HR steel.

- Estimate the endurance strength @10^4 cycles
- Estimate the S-N curve.
- Expected life of a polished rotating-beam speci under a completely reversed stress of 55 ksi.

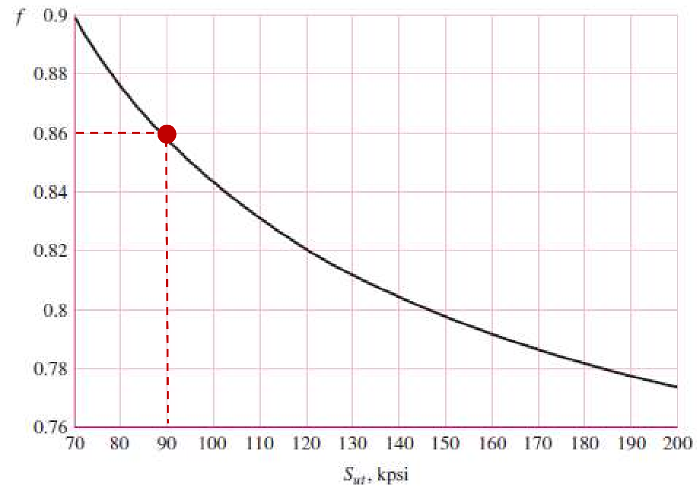
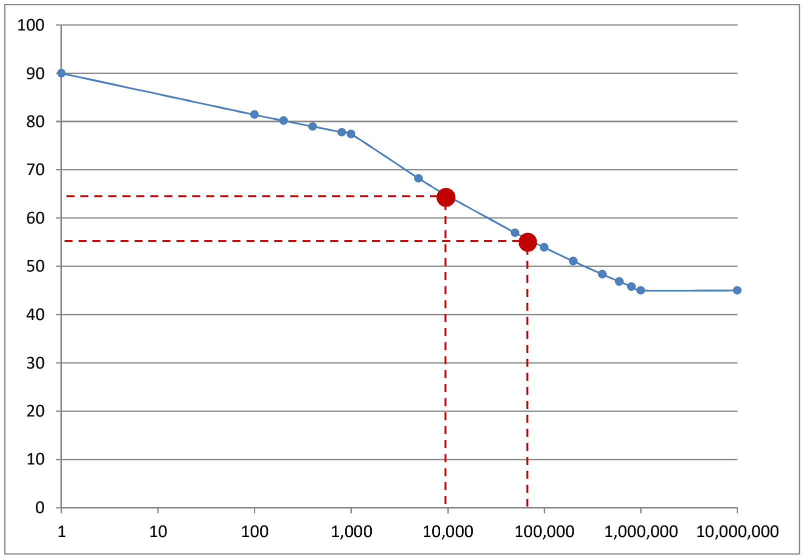


Table A-20: S_{ut}=90Ksi; $S'_e = 0.5 \cdot 90 = 45Ksi$

f=0.86

$$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{(0.86 \cdot 90)^2}{45} = 133.1Ksi$$

$$b = -\frac{1}{3}\log\left(\frac{f\ S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.86 \cdot 90}{45}\right) = -0.0785$$



$$S_f = aN^b = 133.1(10000)^{-0.0785} = 64.6Ksi$$

Since σ_a= S_f = 55 ksi > S'_e

$$N = \left(\frac{S_f}{a}\right)^{1/b} = \left(\frac{55}{133.1}\right)^{1/-0.0785} = 77500$$

Question: Should S'_e or S_e be used in calculations of a & b?

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Modification Factor: Temperature Effect (k_d)

- If ultimate strength is known for operating temperature (S_T), then just use that strength. Let k_d = 1 and proceed as usual.
- If ultimate strength is known only at room temperature (S_{RT}), then use Table 6–4 to estimate ultimate strength at operating temperature (S_T). With that strength, let k_d = 1 and proceed as usual.
- Alternatively, use ultimate strength at room temperature and apply temperature factor from Table 6–4 to the endurance limit.

$$S_T = k_d S_{RT}$$

- Use Table 6-4 or the curve-fitted polynomial to get k_d

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

Example 6-8

A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70,000 cycles to failure in an operating environment of 550°F. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70,000 cycles.

Material 1015 hot-rolled steel; S_{ut} = 50 ksi at 70°F (A-20)

	(Ksi)	Correction Factor	Parameter
S_{ut} @70°F	50		Table A-20
S_{ut} @550°F	49	0.983	Temperature 550°F
S_e' @550°F	24.5	0.5	Eq. 6-8
	23.6	$k_a = aS_{ut}^b = 2.7 \cdot 49^{-0.265} = 0.963$	Machined Surface
	23.6	$k_b = 1$	Size
	20.1	$k_c = 0.85$	Loading: Axial
	20.1	$\boldsymbol{k_d} = ??$	Temperature
	16.3	$k_e = 0.814$	Reliability: 99%
S_e @550°F	??		

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S_e @550°F	**16.3**		

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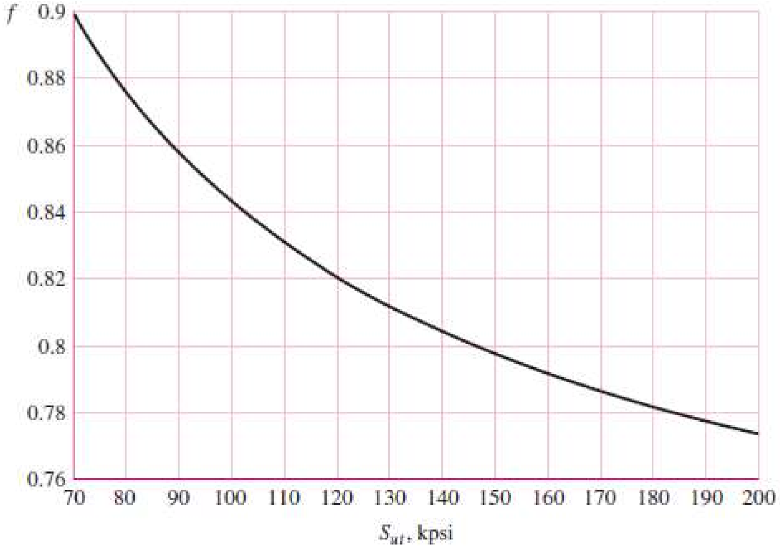
	(Ksi)	Correction Factor	Parameter
S_{ut} @70°F	50		Table A-20
S_e' @70°F	25.0	0.5	Eq. 6-8
	24.1	$k_a = aS_{ut}^b = 2.7 \cdot 49^{-0.265} = 0.963$	Machined Surface
	24.1	$k_b = 1$	Size
	20.5	$k_c = 0.85$	Loading: Axial
	20.1	$\boldsymbol{k_d = 0.983}$	Temperature 550°F
	16.3	$k_e = 0.814$	Reliability: 99%
S_e @550°F	**16.3**		

Example 6-8 (Cont'd)

Estimate fatigue strength at 70,000 cycles.



Since S_{ut} = 49 < 70 ksi, then f=0.9

$$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{(0.9 \cdot 45)^2}{16.3} = 119.3\ ksi$$

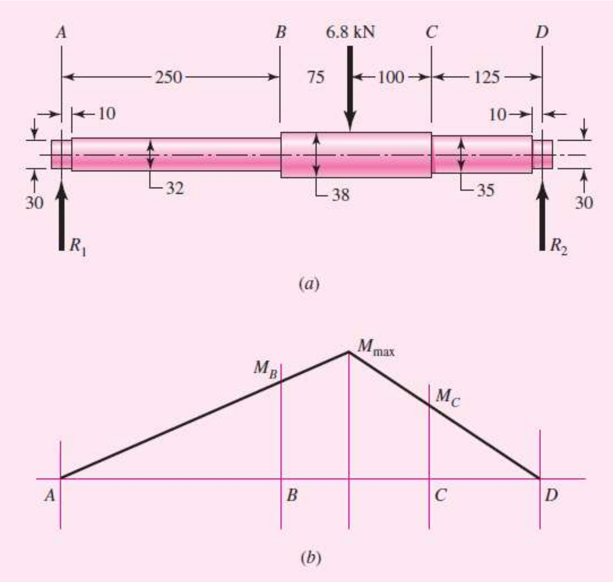
$$b = -\frac{1}{3}\log\left(\frac{f\ S_{ut}}{S_e}\right) = -\frac{1}{3}\log\left(\frac{0.9 \cdot 45}{16.3}\right) = -0.1441$$

$$S_f = aN^b = 119.3\ (70000)^{-0.1441} = 23.9\ ksi$$

Example 6-9

A rotating shaft simply supported in ball bearings at A and D and loaded by a nonrotating force F of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

All fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel.



Material: SAE 1050 CD

Table A-20: S_{ut}=690 MPa; S_y=580 MPa

Critical stress location?

Identify critical stress location: Point B

	(MPa)	Correction Factor	Parameter
S_{ut} @RT	690		Table A-20
S_e' @RT	345	0.5	Uncorrected Endurance Limit
	275.3	$k_a = aS_{ut}^b = 4.51 \cdot 690^{-0.265} = 0.798$	Machined Surface
	236.2	$k_b = (32/7.62)^{-0.107} = 0.858$	Size (d=32mm)
	236.2	$k_c = 1$	Loading: Bending
	236.2	$k_d = 1$	Temperature
	236.2	$k_e = 1$	Reliability: 50%
S_e @RT	**236.2**		Fully Corrected Endurance Limit

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Example 6-9 (Cont'd)

From Fig A-15-9, K_t=1.65

$$\left(\frac{D}{d} = \frac{38}{32} = 1.1875, \frac{r}{d} = \frac{3}{32} = 0.09375\right);$$

Notch sensitivity *q*=0.84 (r=3mm)

$$K_f = 1 + q(K_t - 1) = 1 + 0.84(1.65 - 1) = 1.55$$

Bending moment @Point B:

$$M_B = \frac{6800 \cdot 225 \cdot 250}{550} = 695.5\ N - m$$

Tensile bending stress @Point B:

$$\sigma_B = K_f \frac{M_B \cdot (d/2)}{(\pi d^4/64)} = 1.55 \frac{(695.5 \cdot 1000) \cdot 32}{\pi 32^3} = 335.1 MPa$$

Fig. 6-18: f=0.844

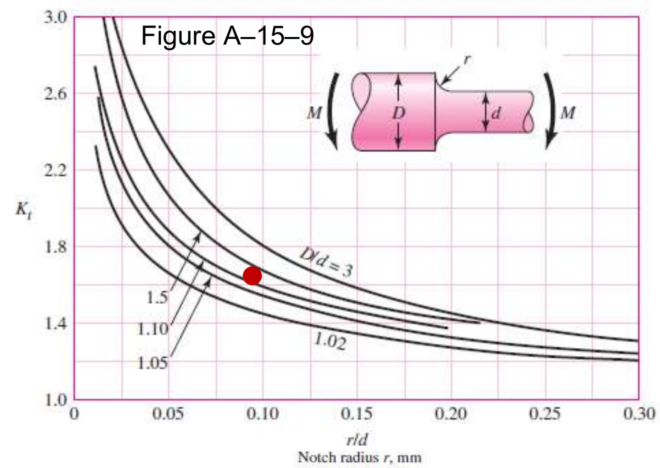
$$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{(0.844 \cdot 690)^2}{236.2} = 1437 MPa$$

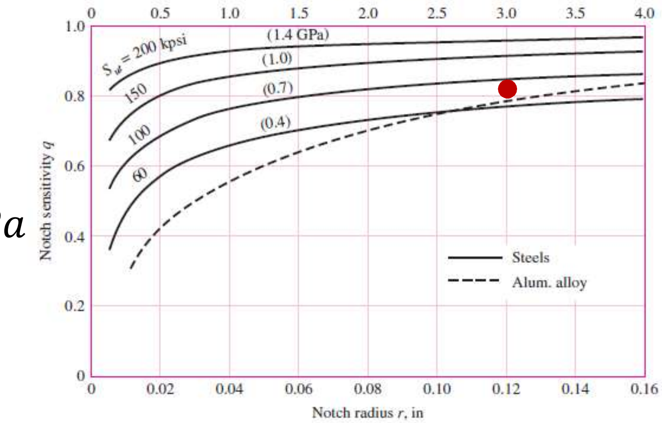
$$b = -\frac{1}{3}\log\left(\frac{0.844 \cdot 690}{236.2}\right) = -0.1308$$

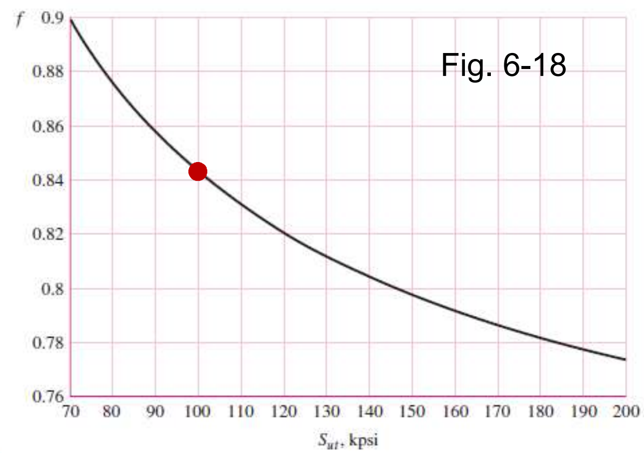
Calculate # of cycles under fully reversed stress (σ_B):

$$S_f = aN^b = \sigma_B \qquad 335.1 = 1437 \cdot N^{-0.1308}$$

N=68,000 cycles

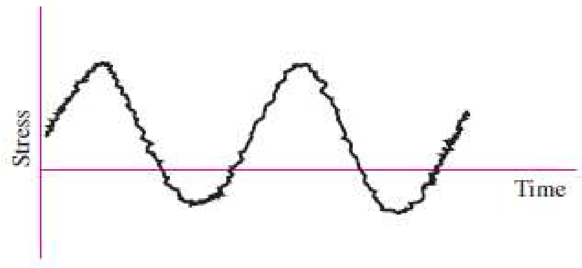




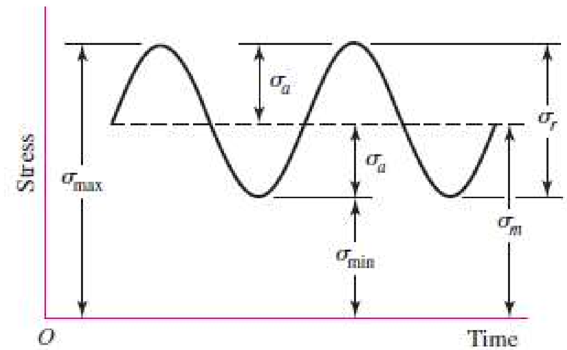


6-11 Characterizing Fluctuating Stresses

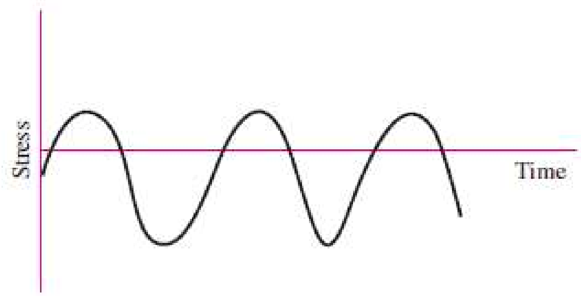
Characterizing Fluctuating Stresses



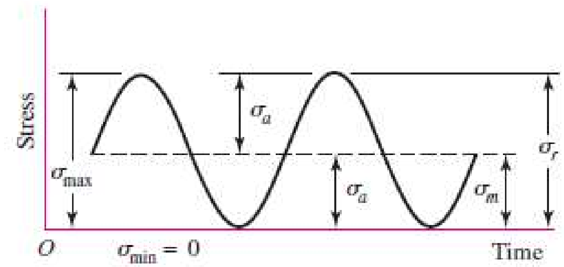
(a)

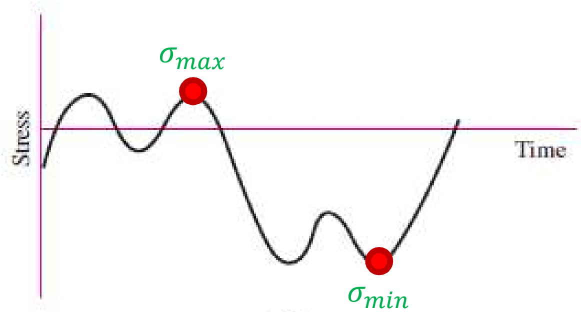


(d)

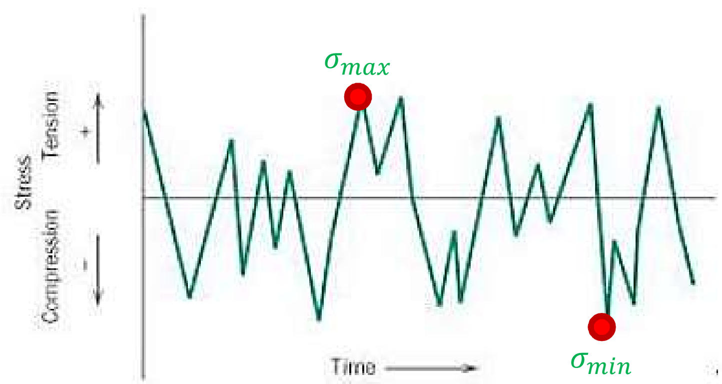


(b)





(c)



- Mean Stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

- Alternating Stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

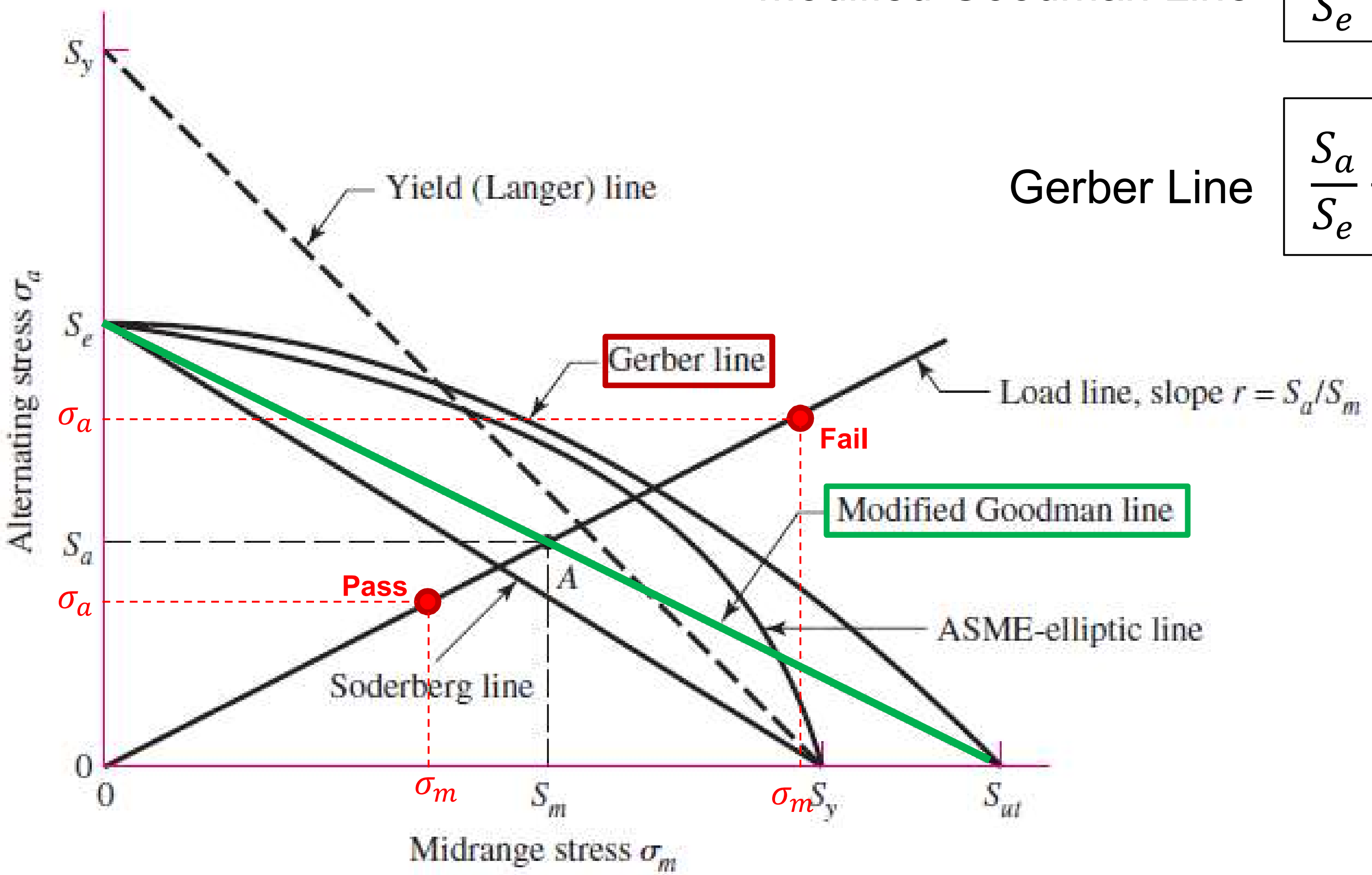
Shape of the wave is not important, but maximum and minimum are important

6-12 Fatigue Failure Criteria for Fluctuating Stress

- Soderberg Line
- Modified Goodman Line
- Gerber Line
- ASME-Elliptic Line

Fatigue Failure Criteria

Fig. 6-27



Modified Goodman Line

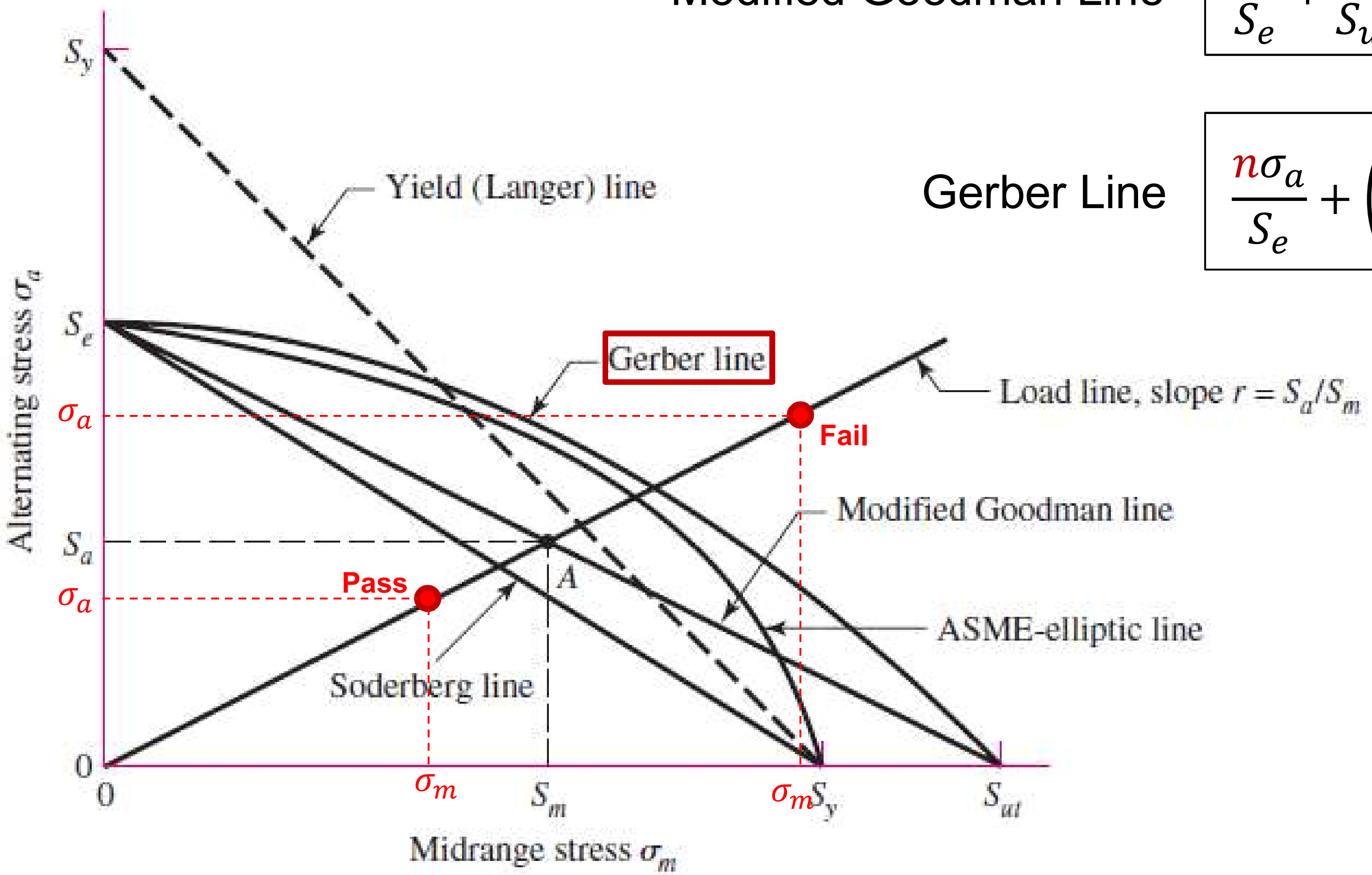
$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

Gerber Line

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Fatigue Factor of Safety (n)

Fig. 6-27



Modified Goodman Line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Gerber Line

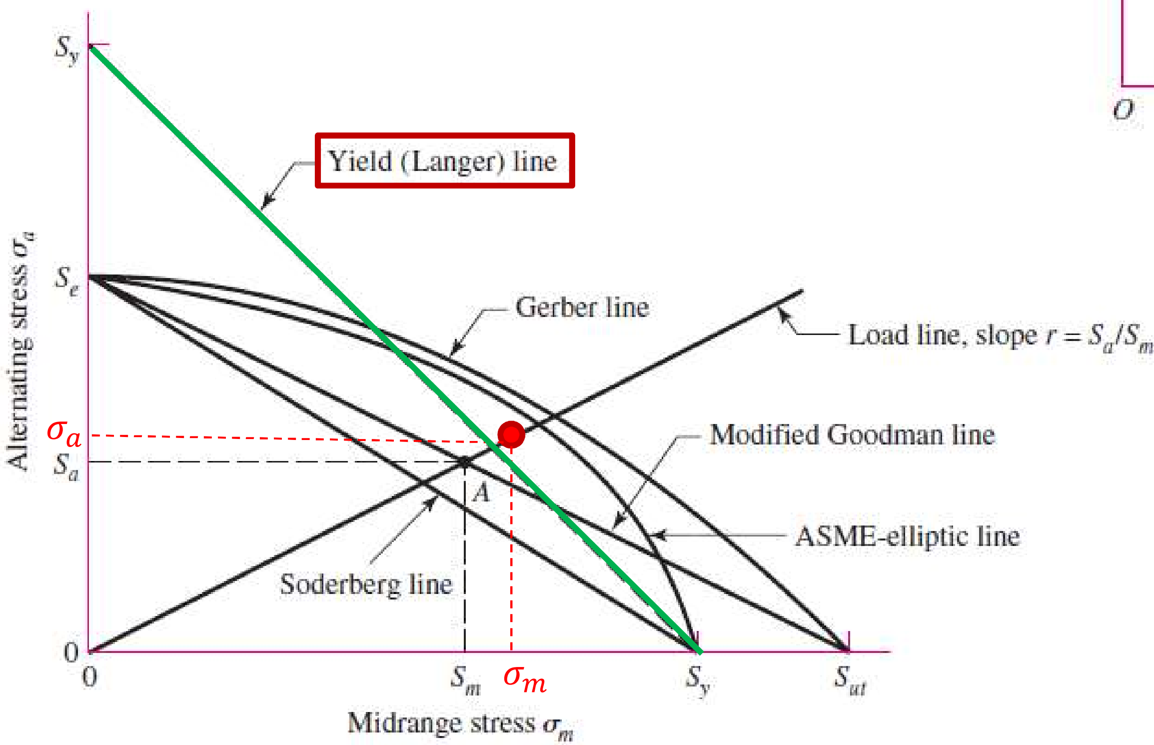
$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

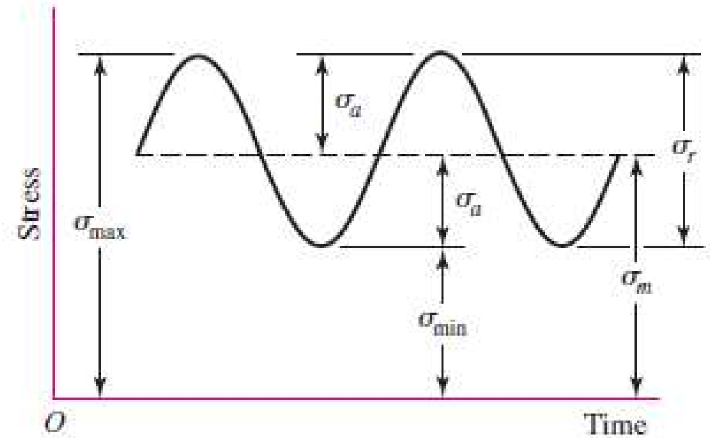
Local Yielding Factor of Safety (n)

Does σ_{max} exceed yield stress (S_y)?

Langer first-cycle-yielding:

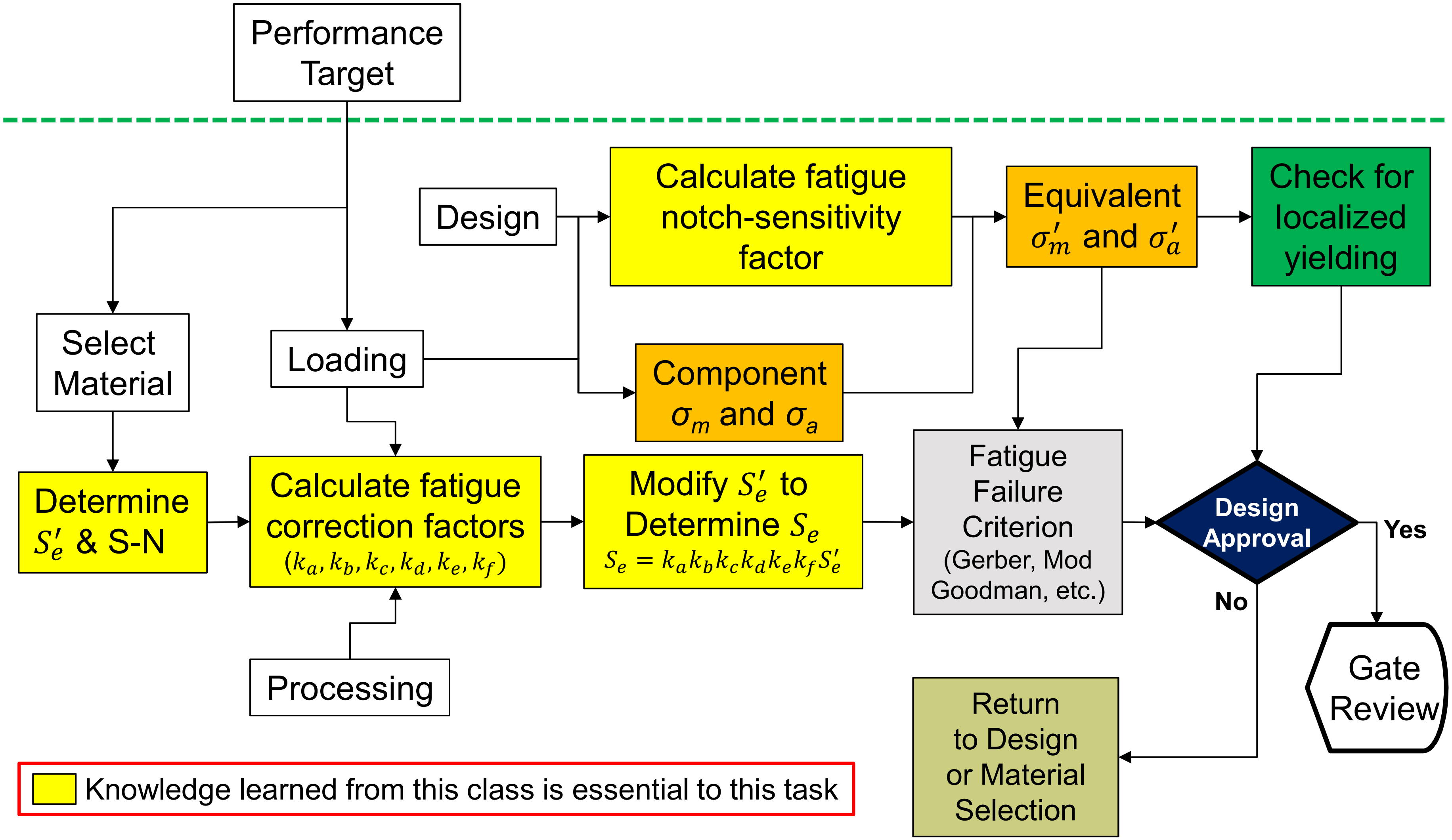
$$\sigma_m + \sigma_a = \sigma_{max} = \frac{S_y}{n}$$





Sec 6-18 Road Map for Stress-Life Method

Workflow for Dynamic Fatigue Analysis



Summary

- Differentiate between uniaxial and multiaxial
- How to get S_e'
  - Rotating beam test
  - Analytical approach
- Apply modification factors to get S_e
- Draw Modified Goodman and Gerber Line