Lecture Note 03
Fatigue Failure Analysis from Fluctuating Loading

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ME 1029 Mechanical Design 2

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Sichuan University - Pittsburgh Institute

Palace of Versailles, France

Lesson-Learned from Prominent Failures in History

- The Versailles Train Crash (May 1842)

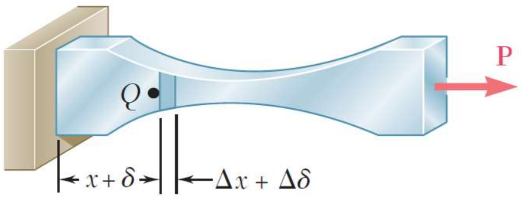
- Often, machine members are found to have failed under the action of repeated or fluctuating stresses.
- Analysis reveals that the actual maximum stresses were well below the ultimate strength of the material, and quite frequently even below the yield strength.
- Most distinguishing characteristic of these failures is that the stresses have been repeated a very large number of times.
- Hence the failure is called a fatigue failure.

Covered Topics

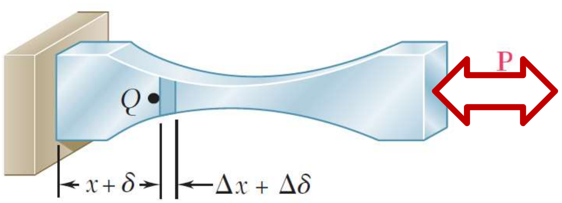
- **Fatigue-Life Methods**
  - Stress-Life Method (Sec 6-4)
- **Part I: Fatigue Analysis Under Simple Loading**
  - Fatigue Strength and the Endurance Limit (Secs 6-7 & 6-8)
  - Endurance Limit Modifying Factors (Sec 6-9)
  - Characterizing Fluctuating Stresses (Sec 6–11)
  - Fatigue Failure Criteria for Fluctuating Stress (6-12)
    - Modified Goodman Line
    - Gerber Line
- **Part II: Fatigue Analysis Under Multiaxial Loading**
  - Combinations of Loading Modes (Sec. 6–14, LN04 Shaft Design)
  - Road Maps and Important Design Equations for the Stress-Life Method (Sec 6-18)

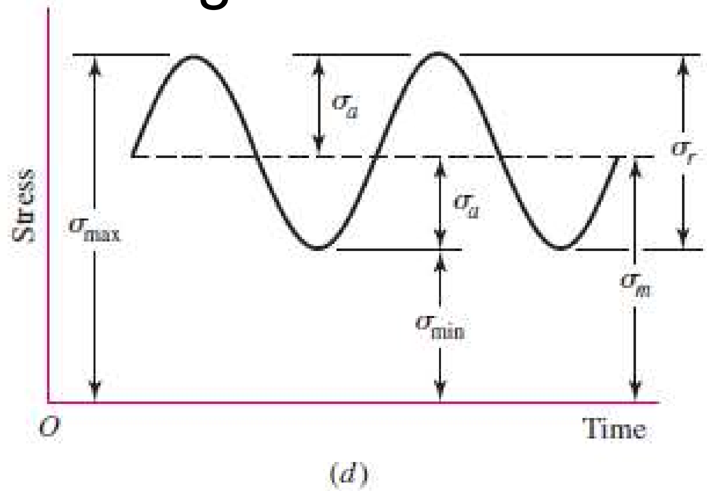
Fluctuating Loading vs. Static Loading

- Static Loading



- Fluctuating (or Dynamic) Loading





Design Analysis for Fluctuating Loading

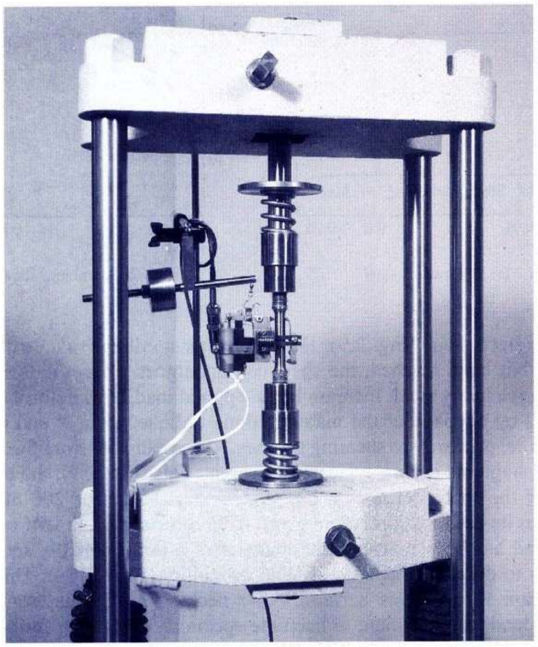
6-3 Fatigue-Life Analysis Methods

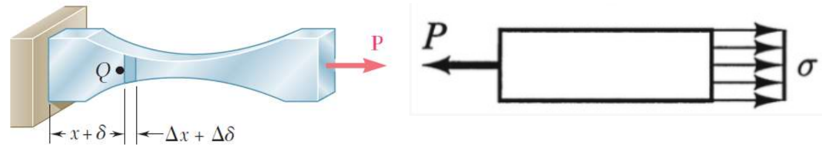
- The three major methods used in fatigue-life design and analysis are:
  - **stress-life method**,
  - strain-life method, and
  - linear-elastic fracture mechanics (LEFM) method

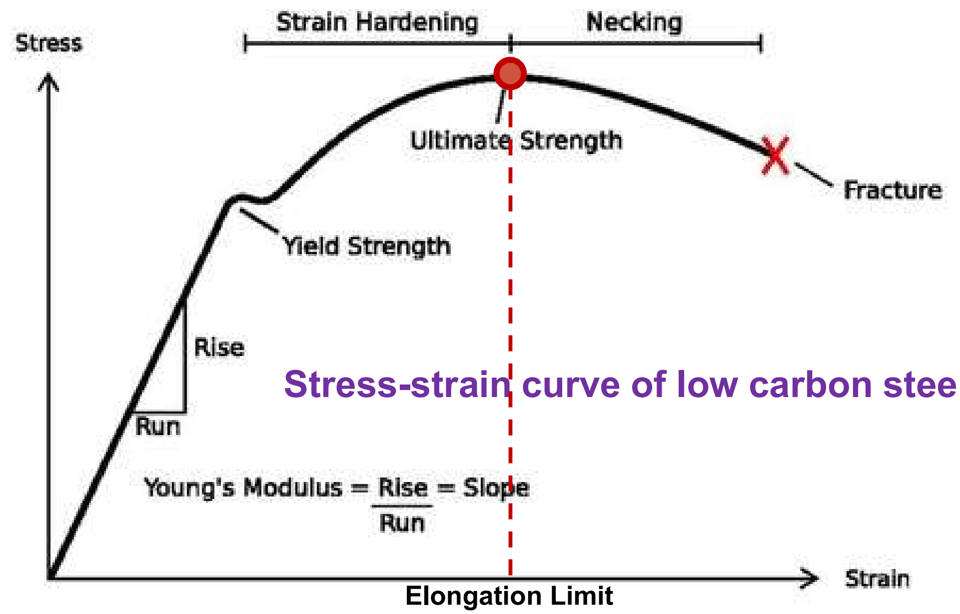
6-4 Stress-Life Method

6-7 Endurance Limit Per Lab Testing

Common Test Method for Static Failure Criteria (Uniaxial Tensile Test, ASTM E-8)







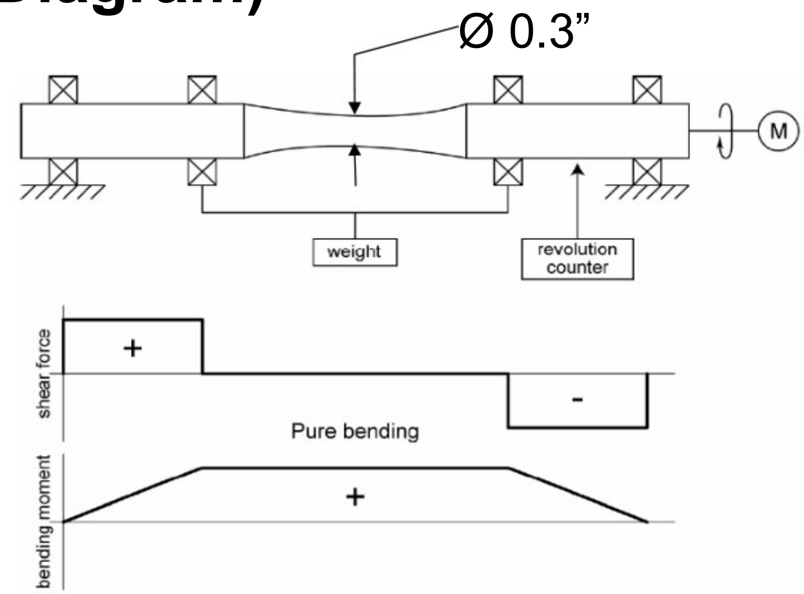
Maximum-Shear-Stress Theory: $(\tau_{max})_{design} \geq (\tau_{max})_{E8} = \frac{S_y}{2}$

Distortion-Energy Theory (DET): $(\tau_{oct})_{design} \geq (\tau_{oct})_{E8} = \frac{\sqrt{2}}{3} S_y$

Common Test Method for Dynamic Failure Criteria (Rotating Beam Testing for S-N Diagram)

- Test Setup
  - Standard R.R. Moore test specimen
  - 0.3 in. diameter at thinnest section
  - Specimen is polished
- Moment (**pure bending**) is uniform, the narrowing specimen has max bending stress at the middle.
- Each revolution gives one fully reversed cycle
- Outcome of the Rotating Beam Test is a complete **S-N Diagram**

- Advantage: fast and easy setup
- Example: At 3600RPM (60 Hz), get 216,000 cycles per hour, 5.2 million cycles per one 24-hour day



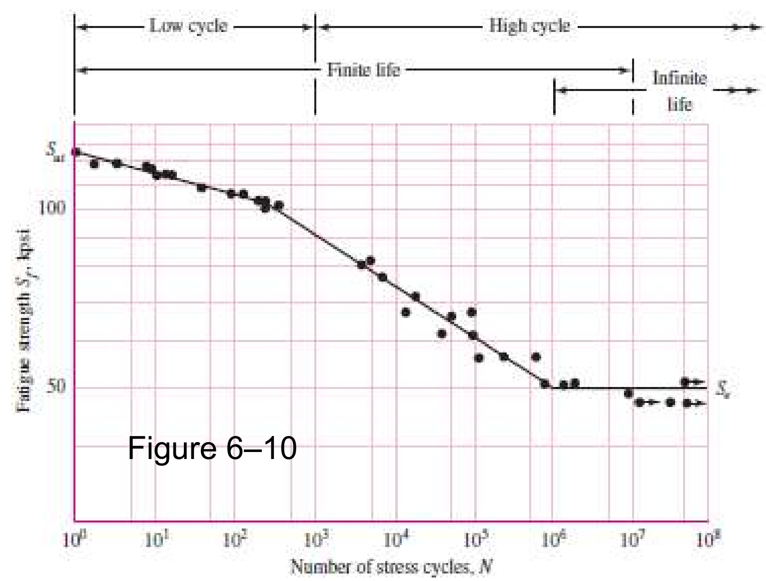


Figure 6–10

Failure Criteria for Static and Dynamic Loading

- Failure Criteria Under **Static** Loading

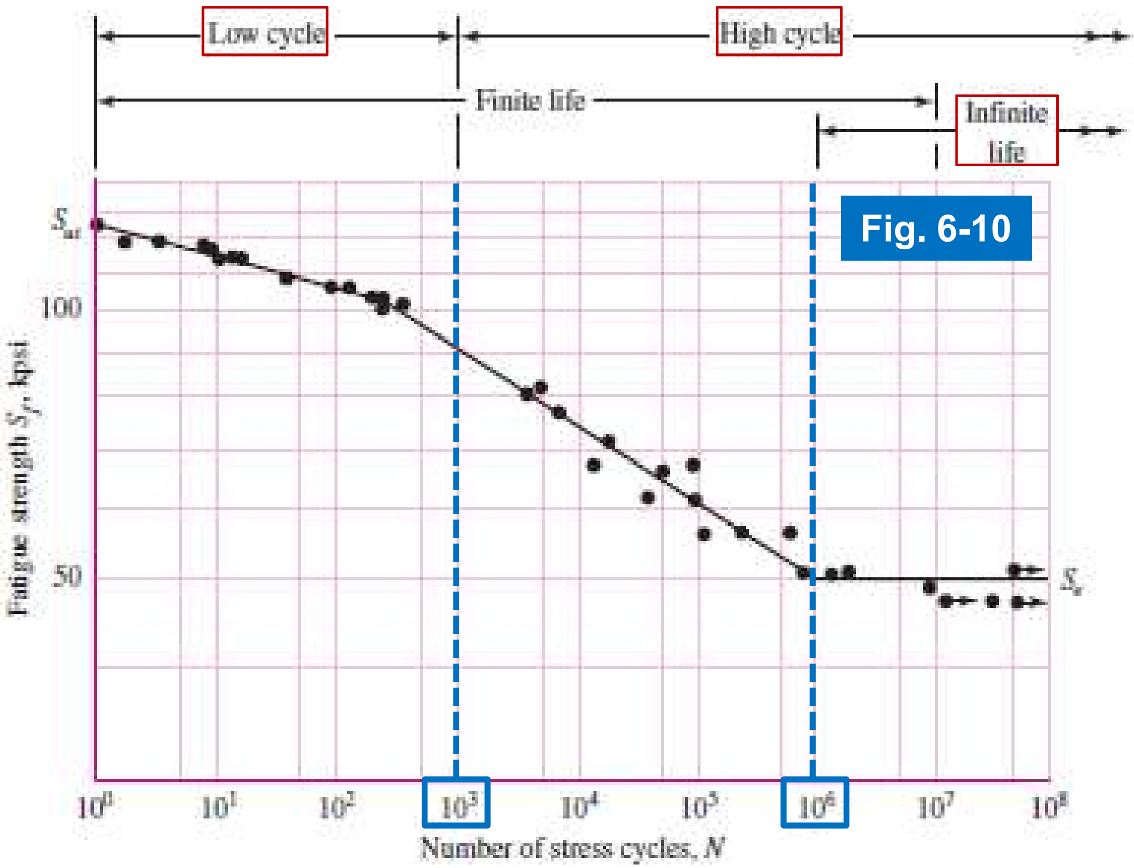
Tensile Testing (**Uniaxial**)	→	Failure Criteria (**Multiaxial**)
• Yield Strength • Ultimate Strength		• Max Shear Stress • Distortion Energy

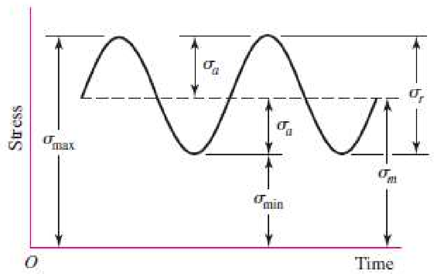
- Failure Criteria Under **Dynamic** Loading

R. R. Moore Rotating-Beam Testing (**Uniaxial**)	→	Failure Criteria (**Multiaxial**)
• Endurance Limit • Correction Factors		• Stress-Life Method • Gerber, mod Goodman, etc.

S-N Diagram – Three Cycle Regions

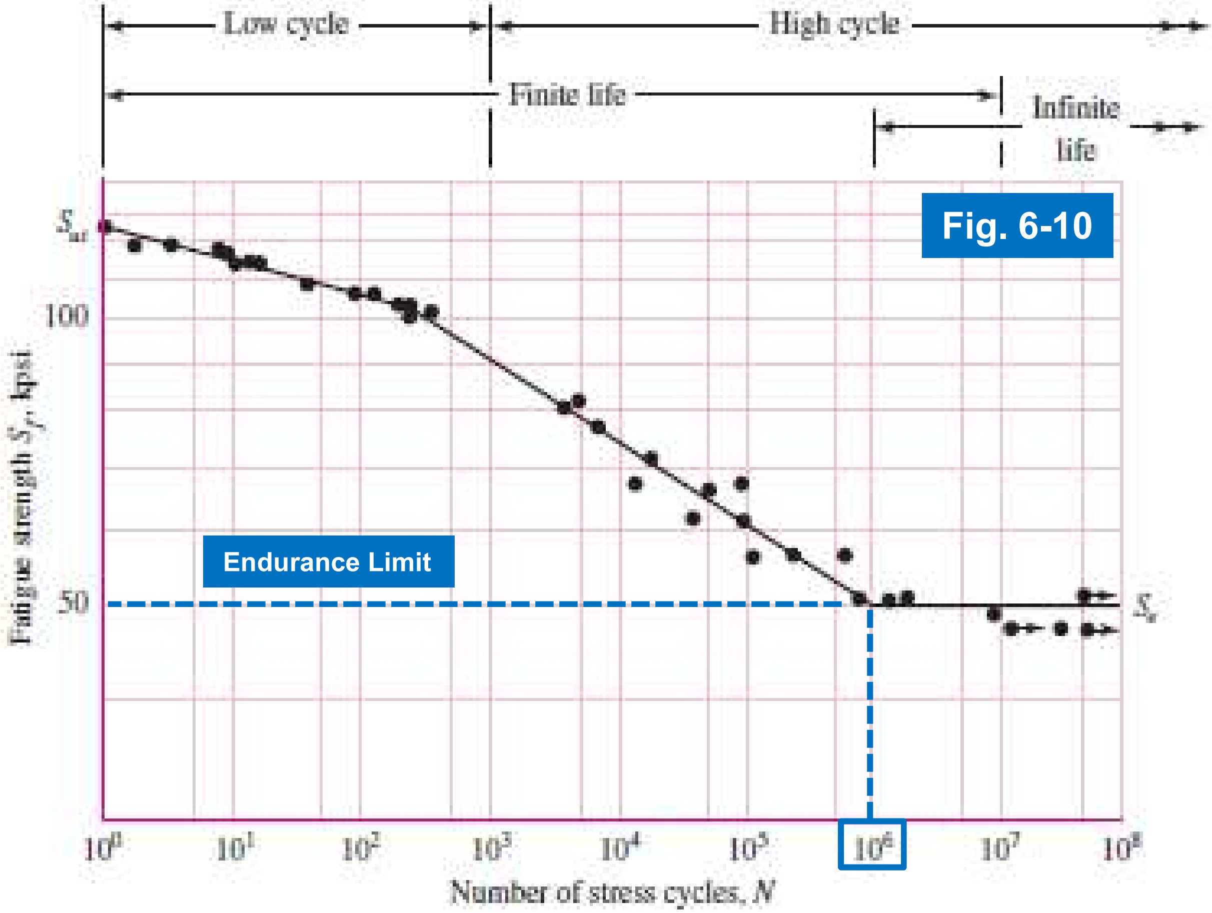
- The S-N diagram has three regions:
  - Low Cycle (<1K), High Cycle (1K-1MM), and Infinite Life (>1MM)
- In many engineering practices, Infinite Life demands ≥10 MM (10^7) cycles.

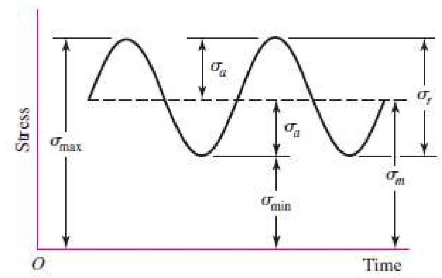




S-N Diagram

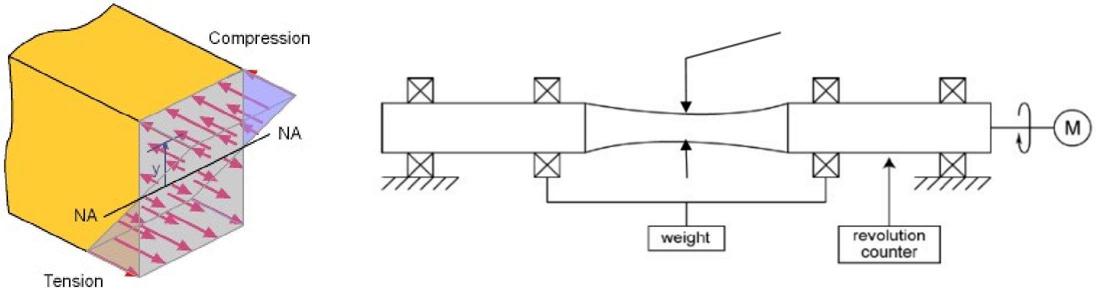
- S-N diagram shows the existence of an **Endurance Limit (S'_e)** – a stress level below which an infinite number of loading cycles can be applied to a material without causing fatigue failure.



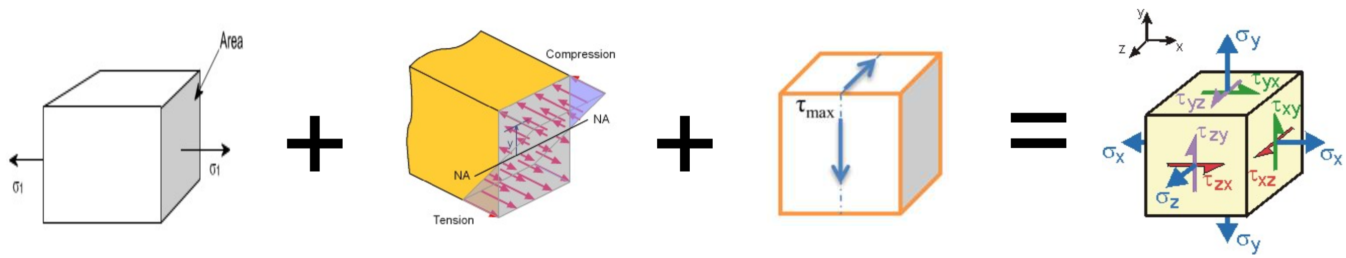


Generalization of Bending Endurance Limit

- The introduced endurance limit and S-N diagram are based on rotating bending testing.
- Henceforth, the data are theoretically the most accurate for bending fatigue.



- Extensive correlations had been done to generalize this bending endurance limit so that it can be applied for fatigue analysis under multiaxial loading applications.

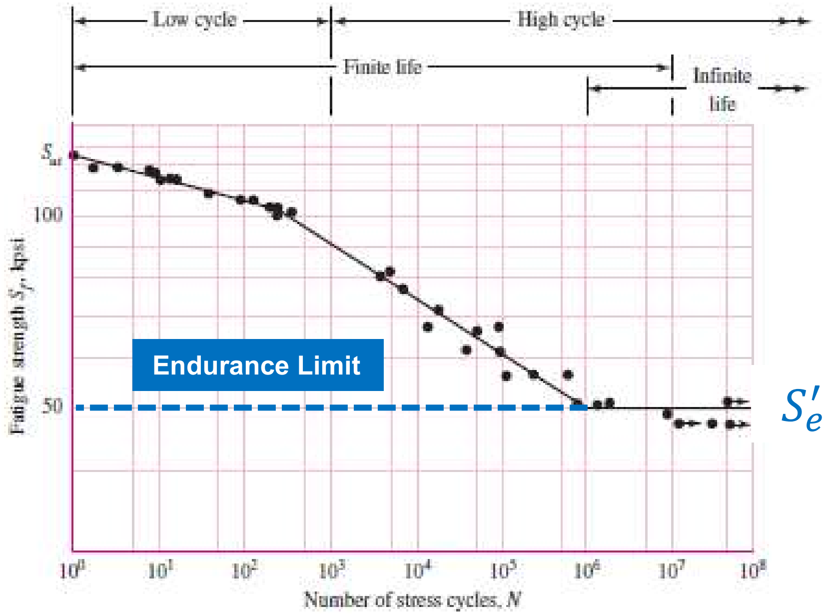


6–9 Endurance Limit Modifying Factors

Note that in this lecture:

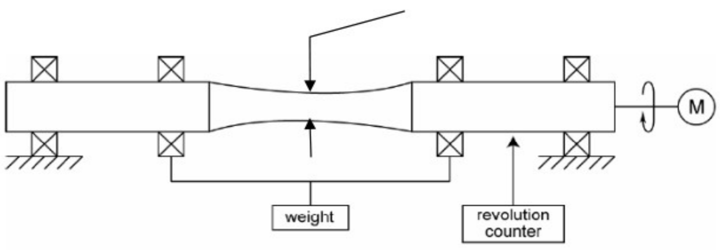
S_e': Unmodified Endurance Limit

S_e : Modified Endurance Limit (used in design)



Why Need the Modifying Factors?

- Rotating-beam specimens used in the laboratory to determine endurance limits are carefully machined and tested under tight-controlled environment.



- It is unrealistic to expect the endurance limit of any mass-produced parts to match the values obtained in the laboratory.
  - Material: batch-to-batch variations
  - Manufacturing: heat treatment, surface conditions, etc.
  - Environment: corrosion, temperature, etc.
  - Design: size, shape, stress state, speed, etc.

Endurance Limit Modifying Factors

S'_e obtained from lab testing must be appropriately modified to account for the physical and environmental differences between the test specimen and the actual part being analyzed:

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

k_a: surface condition

k_b: size

k_c: load

k_d: temperature

k_e: reliability

k_f: miscellaneous

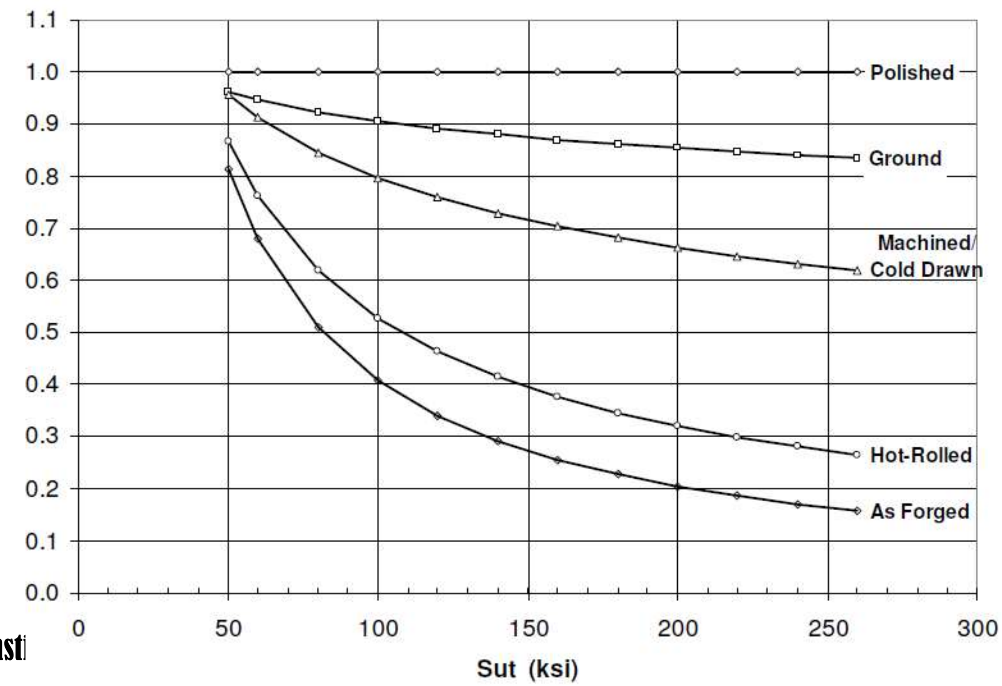
S_e (Corrected Endurance Limit) is the endurance limit used for actual design

Modification Factor: Surface Condition (k_a)

- **Surface (Roughness) Condition (k_a)**

$$k_a = aS_{ut}^b$$

Table 6–2 Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

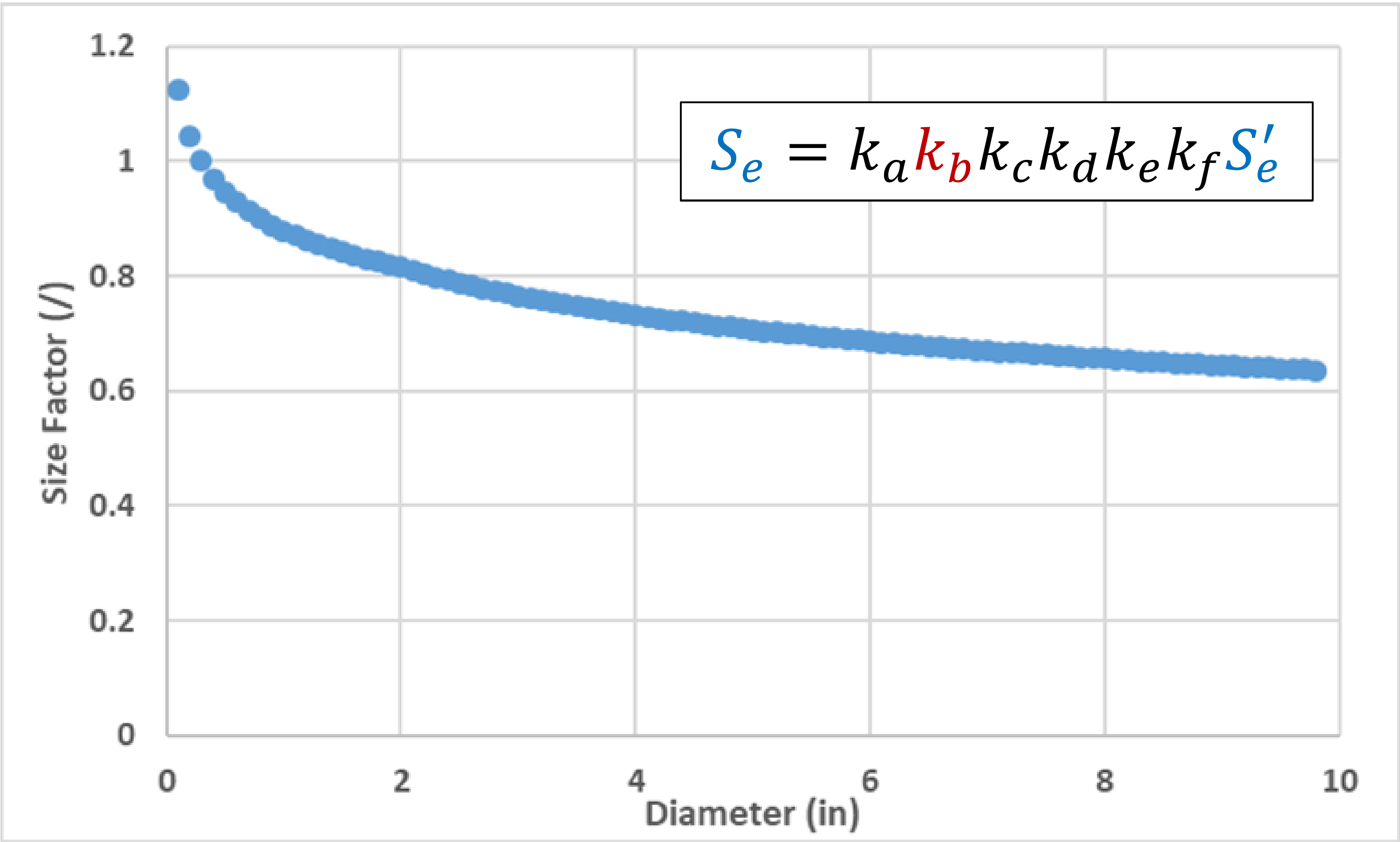


Modification Factor: Size Effect (k_b)

- Case 1: **Rotating** shaft with bending and torsion

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.11 \le d \le 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \le 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 2.79 \le d \le 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < 254 \text{ mm} \end{cases}$$

(d: designed shaft OD)



$$S_e = k_a k_b k_c k_d k_e k_f S_e'$$

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Modification Factor: Size Effect (k_b)

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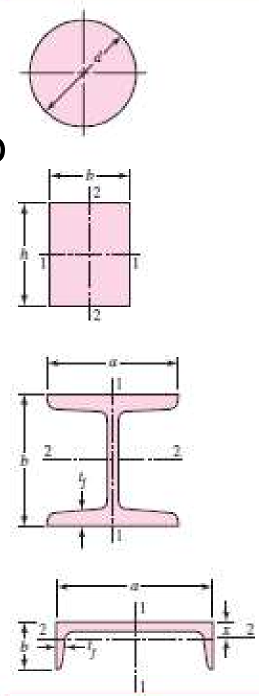
(d: designed shaft OD)

- Case 2: **Non-rotating,** completely reversed bending (Circular or Noncircular parts)
  - Use Table 6–3 to get d_e and substitute into above equation for d.

- For <u>axial loading</u> there is no size effect

$$k_b = 1$$

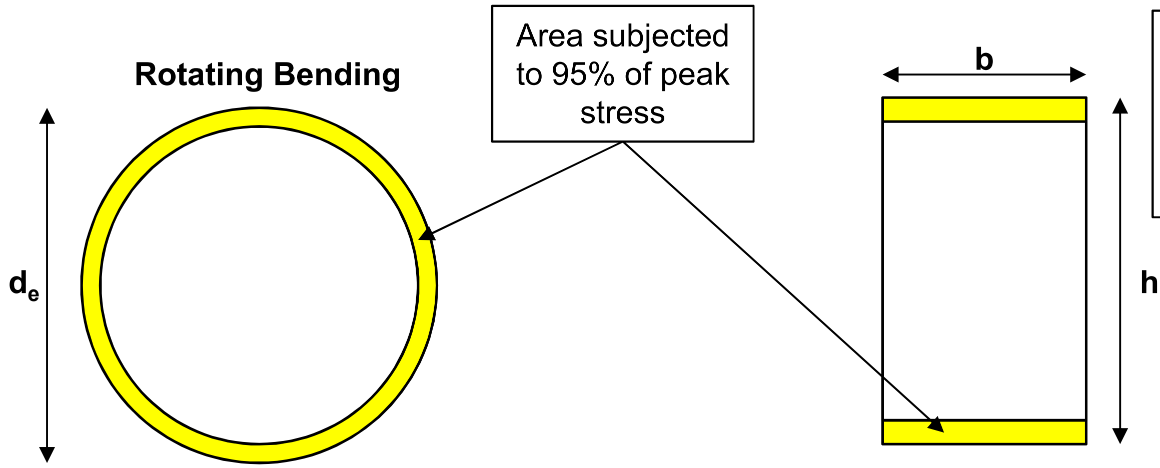
Table 6–3



$A_{0.95\sigma} = 0.01046d^2$, $d_e = 0.370d$	
$A_{0.95\sigma} = 0.05hb$, $d_e = 0.808\sqrt{hb}$	
$A_{0.95\sigma} = 0.10at_f$	axis 1-1
$A_{0.95\sigma} = 0.05ba$, $t_f > 0.025a$	axis 2-2
$A_{0.95\sigma} = 0.05ab$	axis 1-1
$A_{0.95\sigma} = 0.052xa + 0.1t_f(b - x)$	axis 2-2

Size Effect for Non-Rotating, Noncircular Cross Section

Equivalent diameter based on 95 percent stress area rule



95% Stress area: $A_{.95\sigma} = \frac{\pi}{4}[d_e^2 - (0.95d_e)^2] = 0.0766d_e^2 = 0.05bh$

Circular bar under rotating bending beam testing

Noncircular part under non-rotating, reversed bending loading

$$d_e = 0.808\sqrt{bh}$$

Fatigue strength of this rectangular part subjected to fully reversed bending, non-rotating condition is equivalent to a circular specimen with diameter d_e under fully reversed bending, rotating condition.

Example 6-4

A steel shaft loaded in bending is 32 mm in diameter. The shaft material has a mean ultimate tensile strength of 690 MPa. Estimate the size factor k_b if the shaft is used in

(a) rotating mode

(b) nonrotating mode

<u>Solution:</u>

Shaft diameter=32mm

(a) rotating mode: $k_b = \left(\frac{32}{7.62}\right)^{-0.107} = 0.858$

(b) nonrotating mode:

Equivalent diameter: $d_e = 0.37d$

$k_b = \left(\frac{0.37 \cdot 32}{7.62}\right)^{-0.107} = 0.954$ Not as punishing as the rotating mode

Modification Factor: Loading Effect (k_c)

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion}^{17} \end{cases} \; *$$

* Use this only for pure torsional fatigue loading. When torsion is combined with other stresses, such as bending, k_c = 1

Modification Factor: Temperature Effect (k_d)

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

Or Table 6-4

Modification Factor: Reliability Effect (k_e)

Table 6–5

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

95% is commonly used

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EXAMPLE 6–5

A 1035 steel has a tensile strength of 70 ksi and is to be used for a part that sees 450°F in service.

The room-temperature endurance limit by test is $(S_e)_{70^\circ} = 39.0$ ksi.

Estimate the temperature modification factor and $(S_e)_{450^\circ}$

Solution:

Temperature correction factor:

$k_d = 0.975 + 0.432(10^{-3})(450) - 0.115(10^{-5})(450^2) + 0.104(10^{-8})(450^3) - 0.595(10^{-12})(450^4) =$ **1.007**

$(S_e)_{450^\circ} = 1.007 * (S_e)_{70^\circ} = 1.007 * 39.0 = 39.3$ ksi

Calculated correction factor $k_d > 1$

6-10 Stress Concentration and Notch Sensitivity

A notch refers to a deliberately introduced v-shaped, u-shaped or circular defect in a planar material. In structural components, a notch causes a stress concentration which can result in the initiation and growth of fatigue cracks.

3–13 Stress Concentration Factor (K_t or K_{ts})

- Any discontinuity or irregularity in a machine part alters the stress distribution in the neighborhood of the discontinuity so that nominal stresses no longer describe the state of stress at these locations. Such discontinuities are called stress raisers, and the regions in which they occur are called areas of stress concentration.

$$K_t = \frac{\sigma_{\max}}{\sigma_0} \qquad K_{ts} = \frac{\tau_{\max}}{\tau_0}$$

- Determined by geometric parameters, part material is irrelevant.

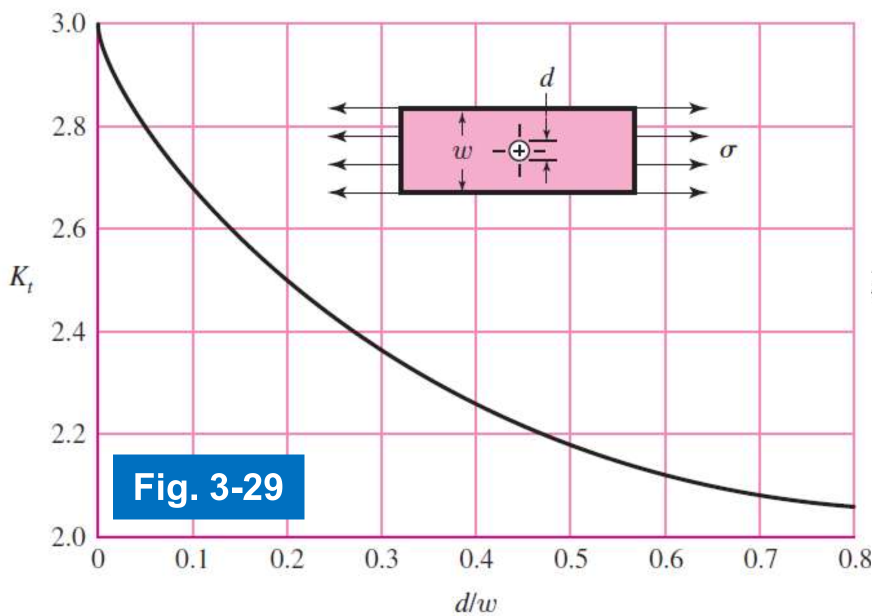
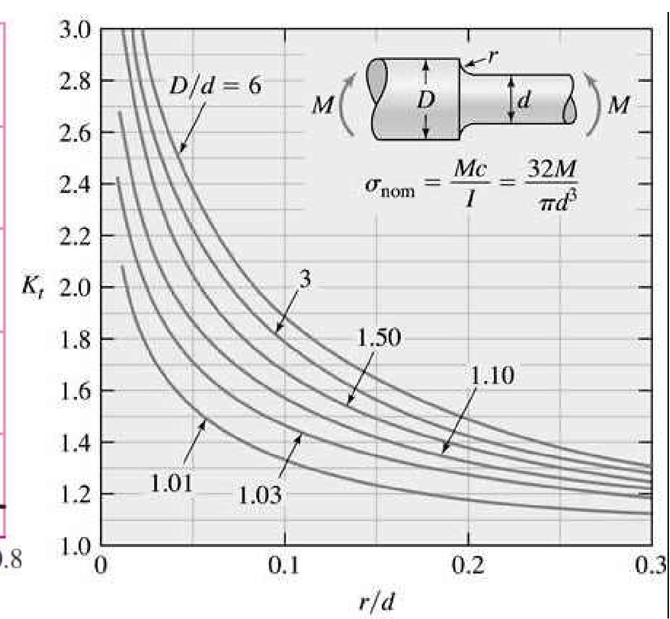


Fig. 3-29



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Fatigue Stress Concentration Factor (K_f or K_{fs})

- Stress concentration factor (K_t or K_{ts})
  - existence of irregularities or discontinuities, such as holes, grooves, or notches, in a part increases the theoretical stresses significantly in the immediate vicinity of the discontinuity.
- What is Notch Sensitivity (q)?
  - sensitivity of the material to developing notches
- Fatigue stress concentration factor (K_f or K_{fs})
  - some materials are not fully sensitive to the presence of notches
  - K_f is a reduced value of K_t

Steps for Fatigue Stress Concentration Calculation

- Calculate stress concentration (K_t or K_{ts}) first
- Find q per Fig. 6-20 for bending/axial loading, or q_{shear} shear per Fig. 6-21 for torsional loading. (For steel material)
- Calculate (K_f or K_{fs})

$$K_f = 1 + q(K_t - 1) \quad \text{or} \quad K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

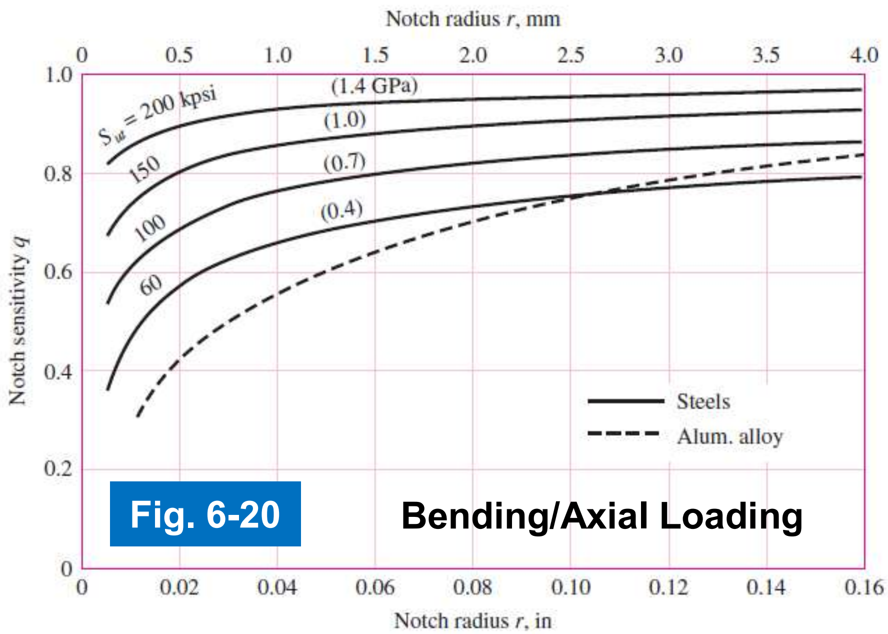


Fig. 6-20

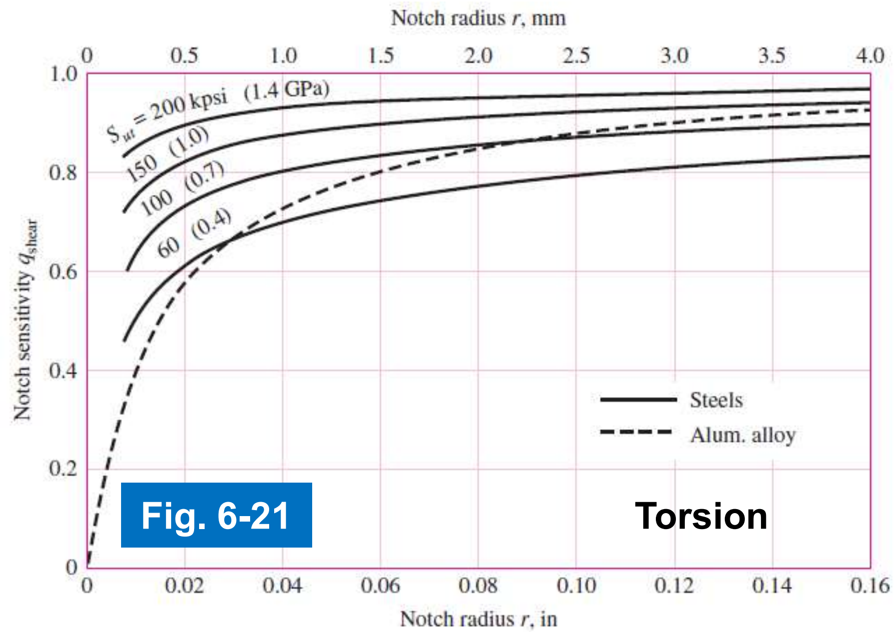


Fig. 6-21

- For conservative approach: use K_f=K_t

Alternative Steps for Fatigue Stress Concentration Calculation

- Bending or axial (S_{ut} in Ksi)

  $$\sqrt{a} = 0.246 - 3.08 \cdot 10^{-3} S_{ut} + 1.51 \cdot 10^{-5} S_{ut}^2 - 2.67 \cdot 10^{-8} S_{ut}^3$$

- Torsion (S_{ut} in Ksi)

  $$\sqrt{a} = 0.190 - 2.51 \cdot 10^{-3} S_{ut} + 1.35 \cdot 10^{-5} S_{ut}^2 - 2.67 \cdot 10^{-8} S_{ut}^3$$

- Fatigue Stress Concentration

  $$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

EXAMPLE 6-6

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32-mm diameter with a 38-mm diameter. Estimate K_f using

(a) Fig 6-20

(b) Equations (6–33) and (6–35)

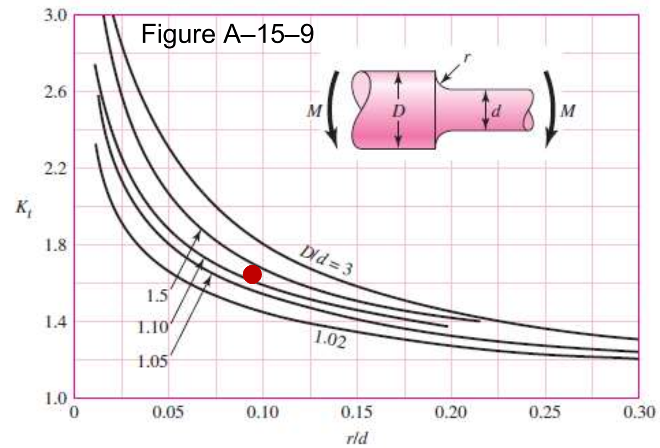


Figure A–15–9

From Fig A-15-9, K_t=1.65

$$\left(\frac{D}{d} = \frac{38}{32} = 1.1875,\ \ \frac{r}{d} = \frac{3}{32} = 0.09375\right);$$

Notch sensitivity *q*=0.84 (r=3mm)

$$K_f = 1 + q(K_t - 1) = 1 + 0.84(1.65 - 1) = 1.55$$

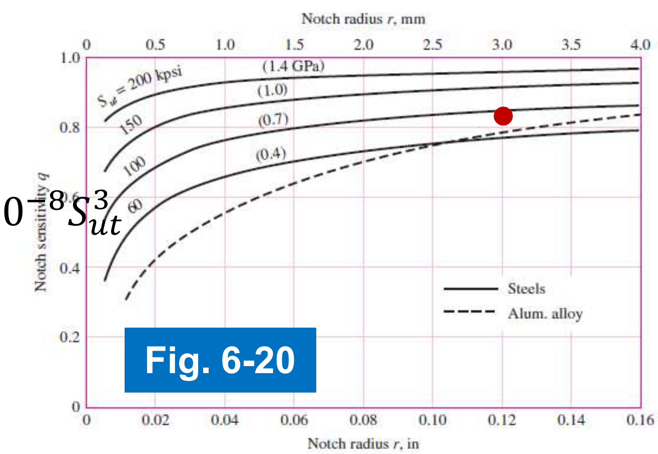


Fig. 6-20

$$S_{ut} = 690\ MPa = 100Ksi$$

$$\sqrt{a} = 0.246 - 3.08 \cdot 10^{-3}100 + 1.51 \cdot 10^{-5}100^2 - 2.67 \cdot 10^{-8}S_{ut}^3$$

$$\sqrt{a} = 0.313\sqrt{mm}$$

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}} = 1 + \frac{1.65 - 1}{1 + \sqrt{0.098/3}} = 1.55$$

Estimation of Endurance Limit (S_e')

Two ways to estimate Endurance Limit (S_e')

- Lab Testing (Sec 6-4, Preferred Approach)
  - Rotating beam testing is a commonly adopted lab method
- Analytical Approach (Sec 6-7 & 6-8)

6-8 Fatigue Strength (Analytical Approach)

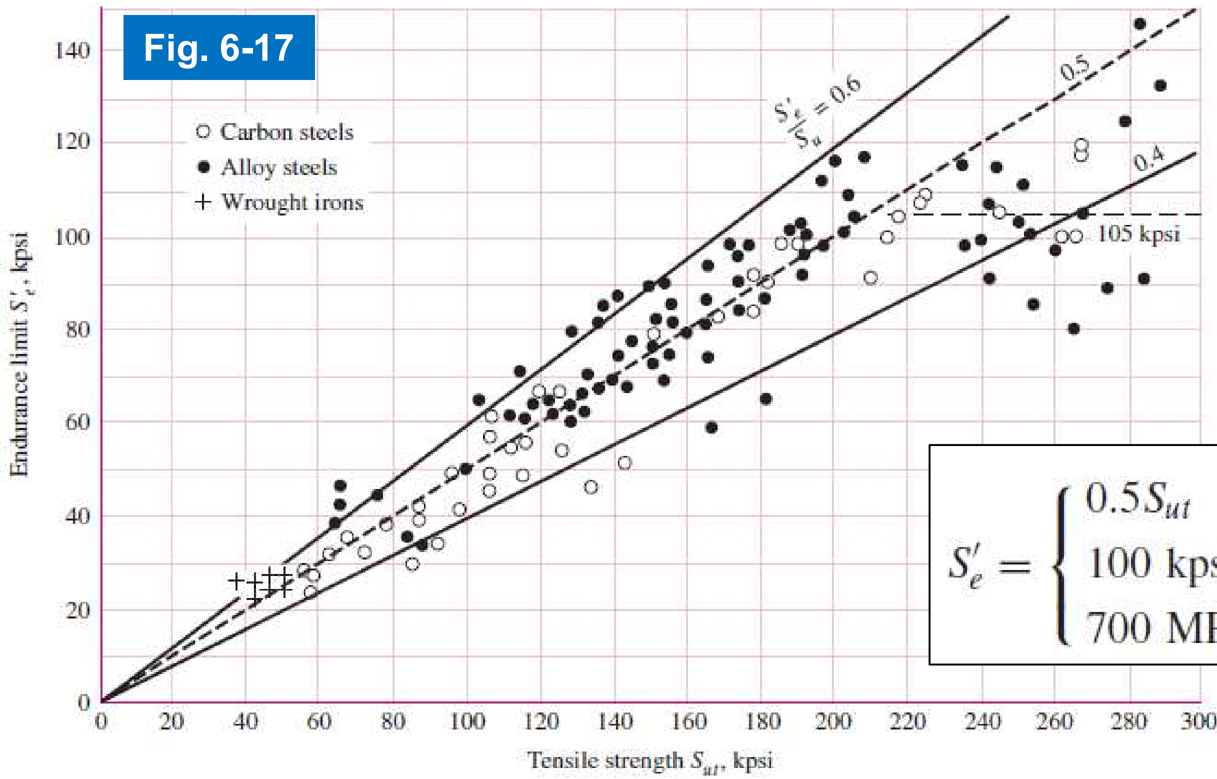
- ► Unmodified Endurance Limit (S_e')
- ► S-N Diagram

Estimate Endurance Limit (S'_e) – Analytical Approach

Analytical approach is also commonly used in engineering practice when lab-tested S'_e is not available

Correlation indicated endurance limit ranges from about 40 to 60 percent of the tensile strength for steels up to about 210 ksi

Fig. 6-17



Step 1

$$S'_e = \begin{cases} 0.5S_{ut} & S_{ut} \le 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Calculate Parameters for S-N Curve Construction

Step 2: Estimate Fatigue Strength Fraction (f)

– f=0.9 (S_{ut}< 70 ksi)

– Use Fig. 6-18 (S_{ut}≥ 70 ksi)

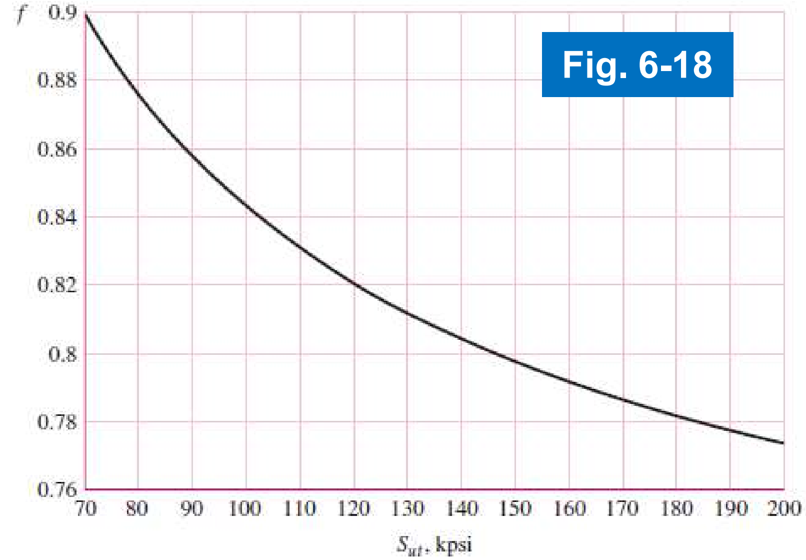
Step 3: Calculate fatigue life constants a and b

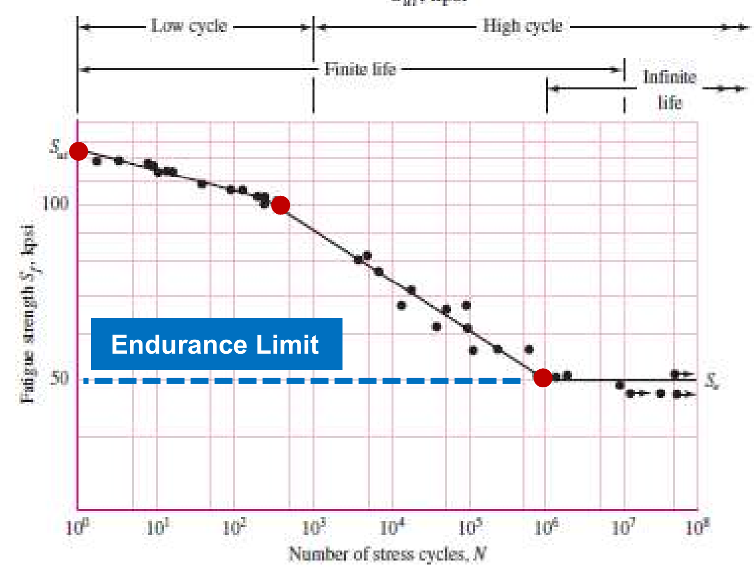
$$a = \frac{(f\,S_{ut})^2}{S_e'} \qquad b = -\frac{1}{3}\log\left(\frac{f\,S_{ut}}{S_e'}\right)$$

Step 4: Draw S-N

- @ Cycle=1, $(S_f)_1$=S_{ut}
- @ Cycle= 10^3, $(S_f)_{1000}$=f S_{ut}
- @ Cycle= 10^6, $(S_f)_{1M}$= S_e'

Cycle Count (N)	S-N Curve (S_f)
1≤ N ≤10^3	$S_f = S_{ut} N^{(\log f)/3}$
10^3≤ N ≤10^6	$S_f = aN^b$
N>10^6	$S_f = S_e'$





Note: this only applies to purely reversing stresses where σ_m = 0.

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Example 6-2

Given a 1050 HR steel.

- Estimate the endurance strength @10^4 cycles
- Estimate the S-N curve.
- Expected life of a polished rotating-beam speci under a completely reversed stress of 55 ksi.

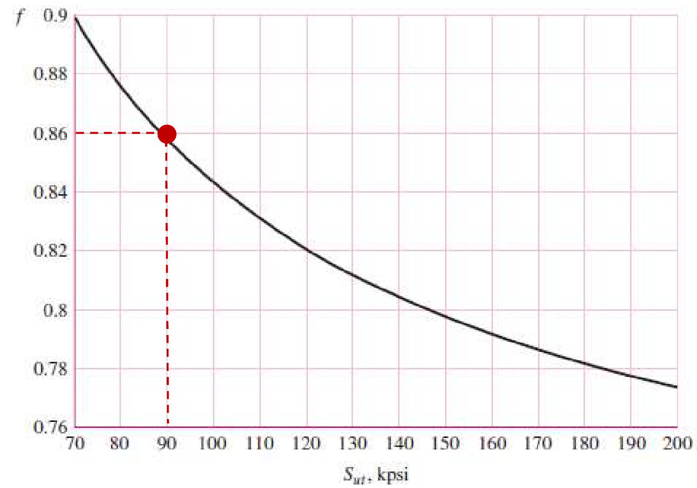
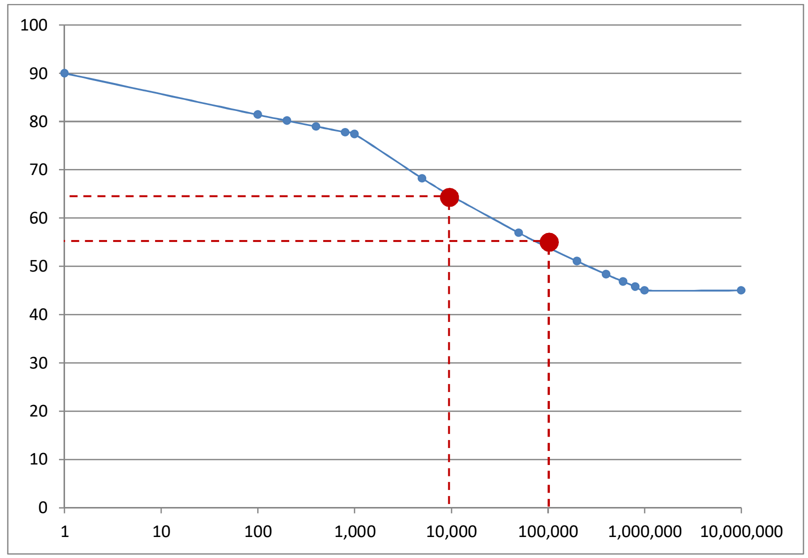


Table A-20: S_{ut}=90Ksi; $S_e' = 0.5 \cdot 90 = 45Ksi$

f=0.86

$$a = \frac{(f\,S_{ut})^2}{S_e} = \frac{(0.86\cdot\quad)^2}{45} = 133.1Ksi$$

$$b = -\frac{1}{3}\log\left(\frac{0.86\cdot 90}{45}\right) = -0.0785$$



$$S_f = aN^b = 133.1(10000)^{-0.0785} = 64.6Ksi$$

Since σ_a= S_f = 55 ksi > S_e'

$$N = \left(\frac{S_f}{a}\right)^{1/b} = \left(\frac{55}{133.1}\right)^{1/-0.0785} = 77500$$

Modification Factor: Temperature Effect (k_d)

- If ultimate strength is known for operating temperature (S_T), then just use that strength. Let k_d = 1 and proceed as usual.
- If ultimate strength is known only at room temperature (S_{RT}), then use Table 6–4 to estimate ultimate strength at operating temperature (S_T). With that strength, let k_d = 1 and proceed as usual.
- Alternatively, use ultimate strength at room temperature and apply temperature factor from Table 6–4 to the endurance limit.

$$S_T = k_d S_{RT}$$

- Use Table 6-4 or the curve-fitted polynomial to get k_d

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

Example 6-8

A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70,000 cycles to failure in an operating environment of 550°F.

Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70,000 cycles.

Material 1015 hot-rolled steel; S_{ut} = 50 ksi at 70°F (A-20)

	(Ksi)	Correction Factor	Parameter
S_{ut} @70°F	50		Table A-20
S_e' @70°F	25.0	0.5	Eq. 6-8
	24.1	$k_a = aS_{ut}^b = 2.7 \cdot 49^{-0.265} = 0.963$	Machined Surface
	24.1	$k_b = 1$	Size
	20.5	$k_c = 0.85$	Loading: Axial
	20.1	$k_d = 0.983$	Temperature 550°F
	16.3	$k_e = 0.814$	Reliability: 99%
S_e @550°F	**16.3**		

Example 6-8

A 1015 hot-rolled steel bar has been machined to a diameter of 1 in. It is to be placed in reversed axial loading for 70,000 cycles to failure in an operating environment of 550°F. Using ASTM minimum properties, and a reliability of 99 percent, estimate the endurance limit and fatigue strength at 70,000 cycles.

Material 1015 hot-rolled steel; S_{ut} = 50 ksi at 70°F (A-20)

	(Ksi)	Correction Factor	Parameter
S_{ut} @70°F	50		Table A-20
S_{ut} @550°F	49	0.983	Temperature 550°F
S_e' @550°F	24.5	0.5	Eq. 6-8
	23.6	$k_a = aS_{ut}^b = 2.7 \cdot 49^{-0.265} = 0.963$	Machined Surface
	23.6	$k_b = 1$	Size
	20.1	$k_c = 0.85$	Loading: Axial
	20.1	$k_d = 1$	Temperature
	16.3	$k_e = 0.814$	Reliability: 99%
S_e @550°F	**16.3**		

Example 6-8 (Cont'd)

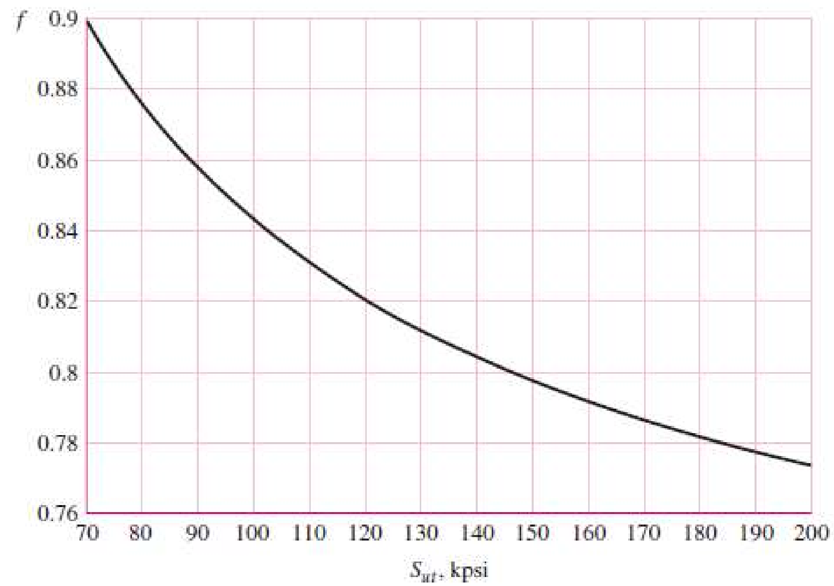
Estimate fatigue strength at 70,000 cycles.

Since S_{ut} = 49 < 70 ksi, then f=0.9

$$a = \frac{(f\, S_{ut})^2}{S_e} = \frac{(0.9\cdot 45)^2}{16.3} = 119.3\ ksi$$

$$b = -\frac{1}{3}\log\left(\frac{0.9\cdot 45}{16.3}\right) = -0.1441$$

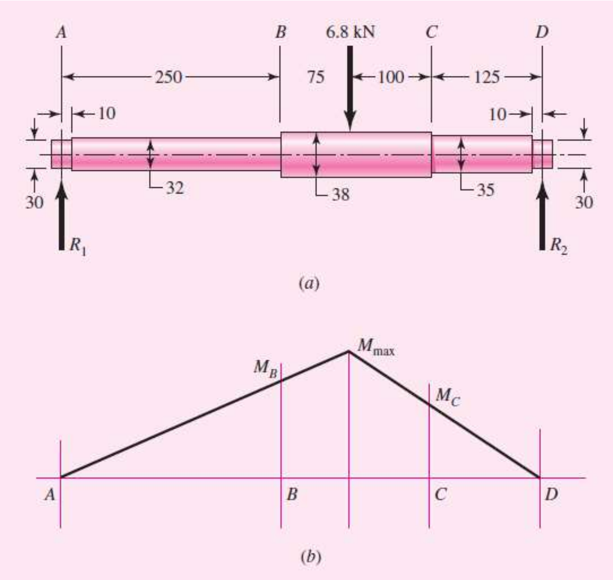
$$S_f = aN^b = 119.3\,(70000)^{-0.1441} = 23.9\ ksi$$



Example 6-9

A rotating shaft simply supported in ball bearings at A and D and loaded by a nonrotating force F of 6.8 kN. Using ASTM "minimum" strengths, estimate the life of the part.

All fillets 3-mm radius. The shaft rotates and the load is stationary; material is machined from AISI 1050 cold-drawn steel.



Material: SAE 1050 CD

Table A-20: S_{ut}=690 MPa; S_y=580 MPa

Critical stress location?

Identify critical stress location: Point B

	(MPa)	Correction Factor	Parameter
S_{ut} @RT	690		Table A-20
S_e' @RT	345	0.5	Uncorrected Endurance Limit
	275.3	$k_a = aS_{ut}^b = 4.51 \cdot 690^{-0.265} = 0.798$	Machined Surface
	236.2	$k_b = (32/7.62)^{-0.107} = 0.858$	Size (d=32mm)
	236.2	$k_c = 1$	Loading: Bending
	236.2	$k_d = 1$	Temperature
	236.2	$k_e = 1$	Reliability: 50%
S_e @RT	**236.2**		Fully Corrected Endurance Limit

SCUPI

Example 6-9 (Cont'd)

From Fig A-15-9, K_t=1.65

$\left(\frac{D}{d} = \frac{38}{32} = 1.1875, \frac{r}{d} = \frac{3}{32} = 0.09375\right)$;

Notch sensitivity q=0.84 (r=3mm)

$K_f = 1 + q(K_t - 1) = 1 + 0.84(1.65 - 1) = 1.55$

Bending moment @Point B:

$$M_B = \frac{6800 \cdot 225 \cdot 250}{550} = 695.5\ N - m$$

Tensile bending stress @Point B:

$$\sigma_B = K_f \frac{M_B \cdot (d/2)}{(\pi d^4/64)} = 1.55 \frac{(695.5 \cdot 1000) \cdot 32}{\pi 32^3} = 335.1 MPa$$

Fig. 6-18: f=0.844

$$a = \frac{(f\ S_{ut})^2}{S_e} = \frac{(0.844 \cdot 690)^2}{236.2} = 1437 MPa$$

$$b = -\frac{1}{3} \log\left(\frac{0.844 \cdot 690}{236.2}\right) = -0.1308$$

Calculate # of cycles under fully reversed stress (σ_B):

$S_f = aN^b = \sigma_B \qquad 335.1 = 1437 \cdot N^{-0.1308}$

N=68,000 cycles

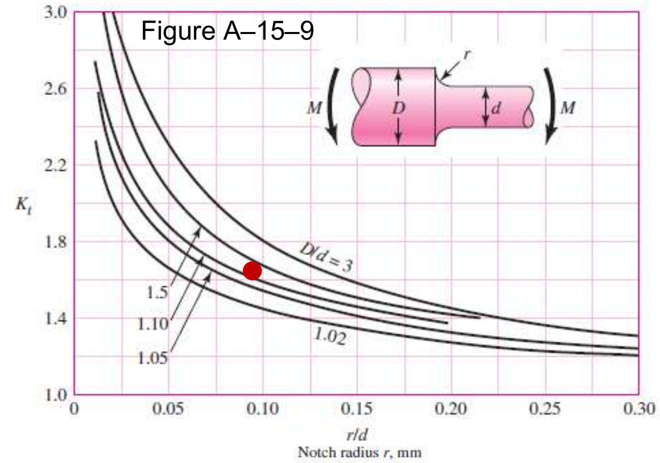
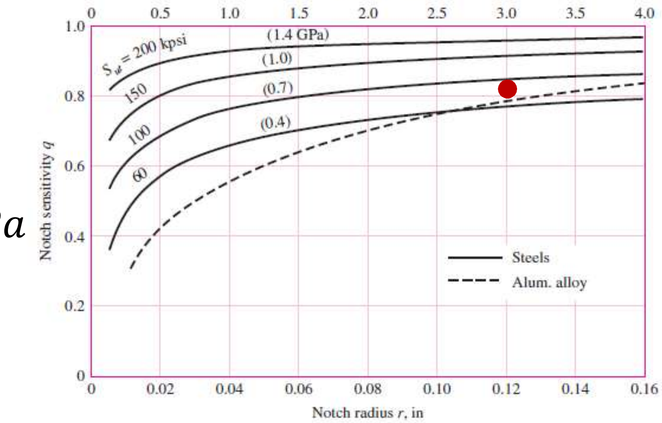


Figure A–15–9



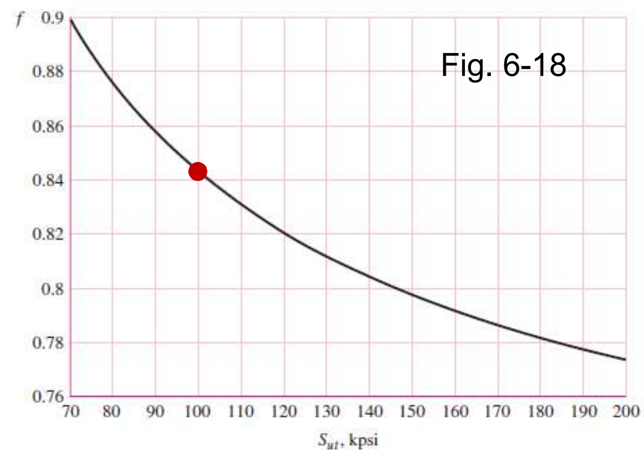
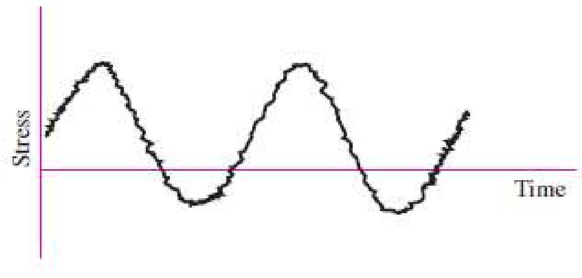
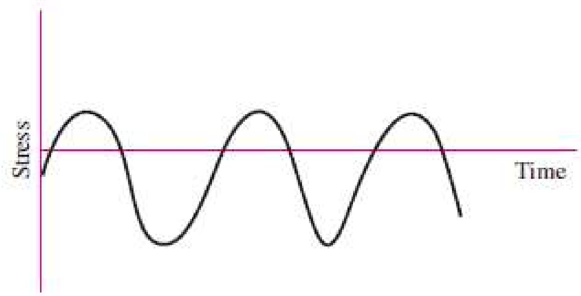


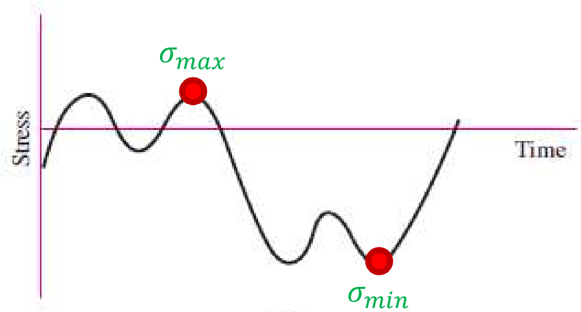
Fig. 6-18

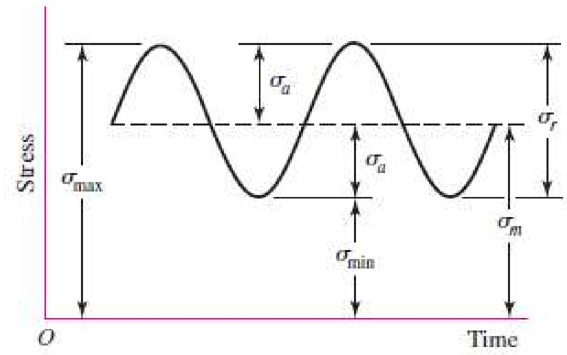
6-11 Characterizing Fluctuating Stresses

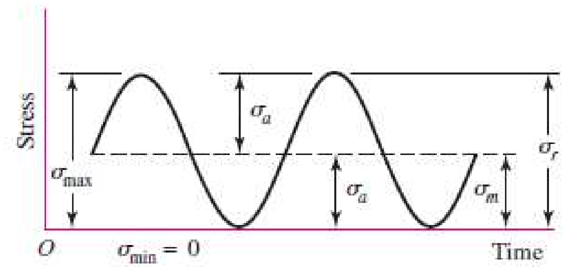
Characterizing Fluctuating Stresses

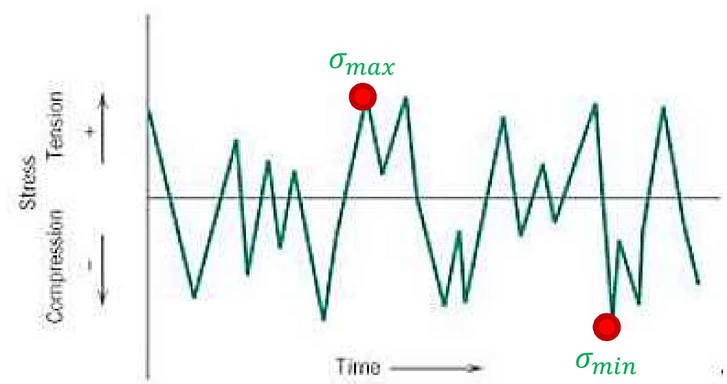












- Mean Stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

- Alternating Stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

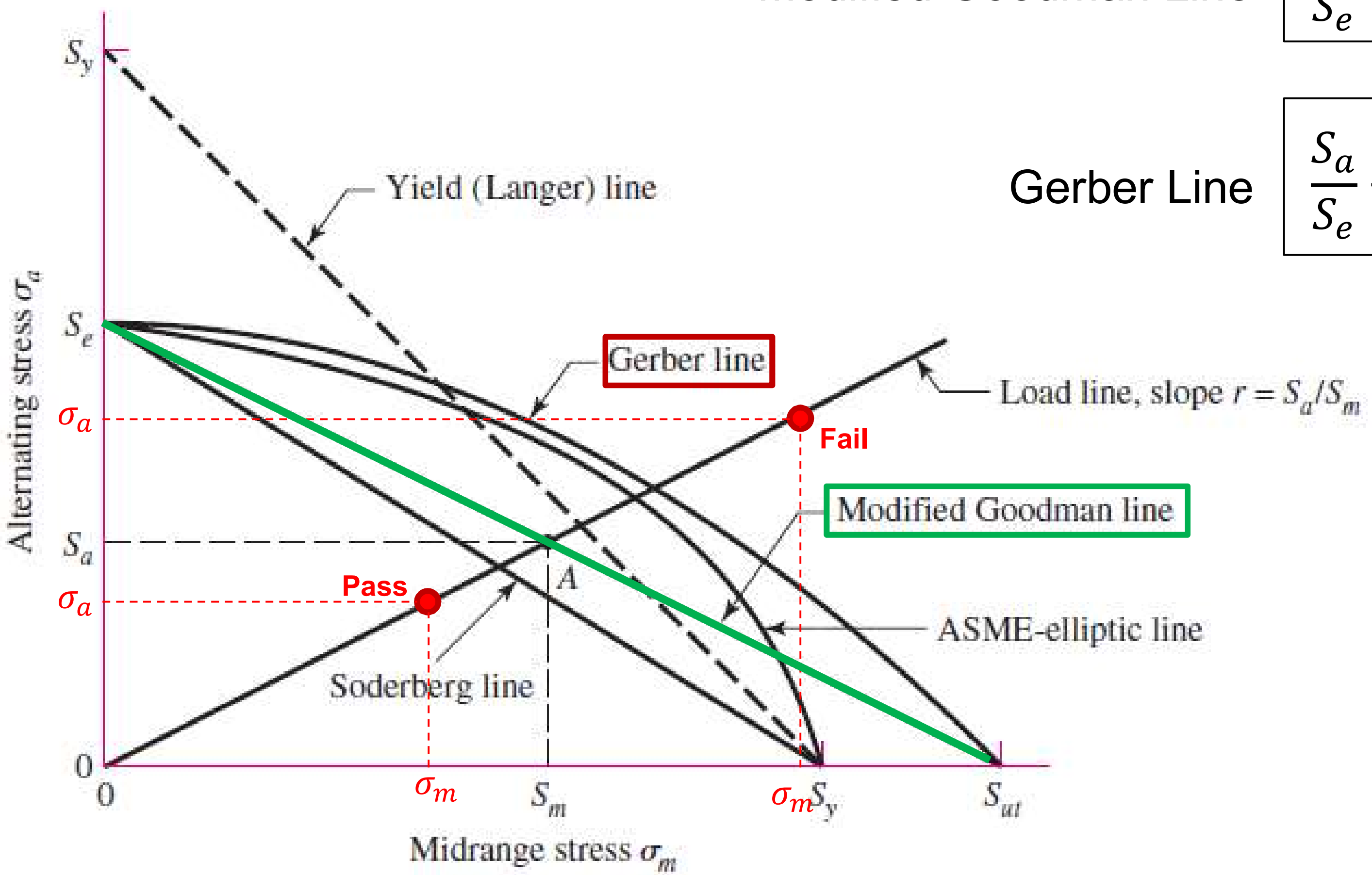
Shape of the wave is not important, but maximum and minimum are important

6-12 Fatigue Failure Criteria for Fluctuating Stress

- Soderberg Line
- Modified Goodman Line
- Gerber Line
- ASME-Elliptic Line

Fatigue Failure Criteria

Fig. 6-27



Modified Goodman Line

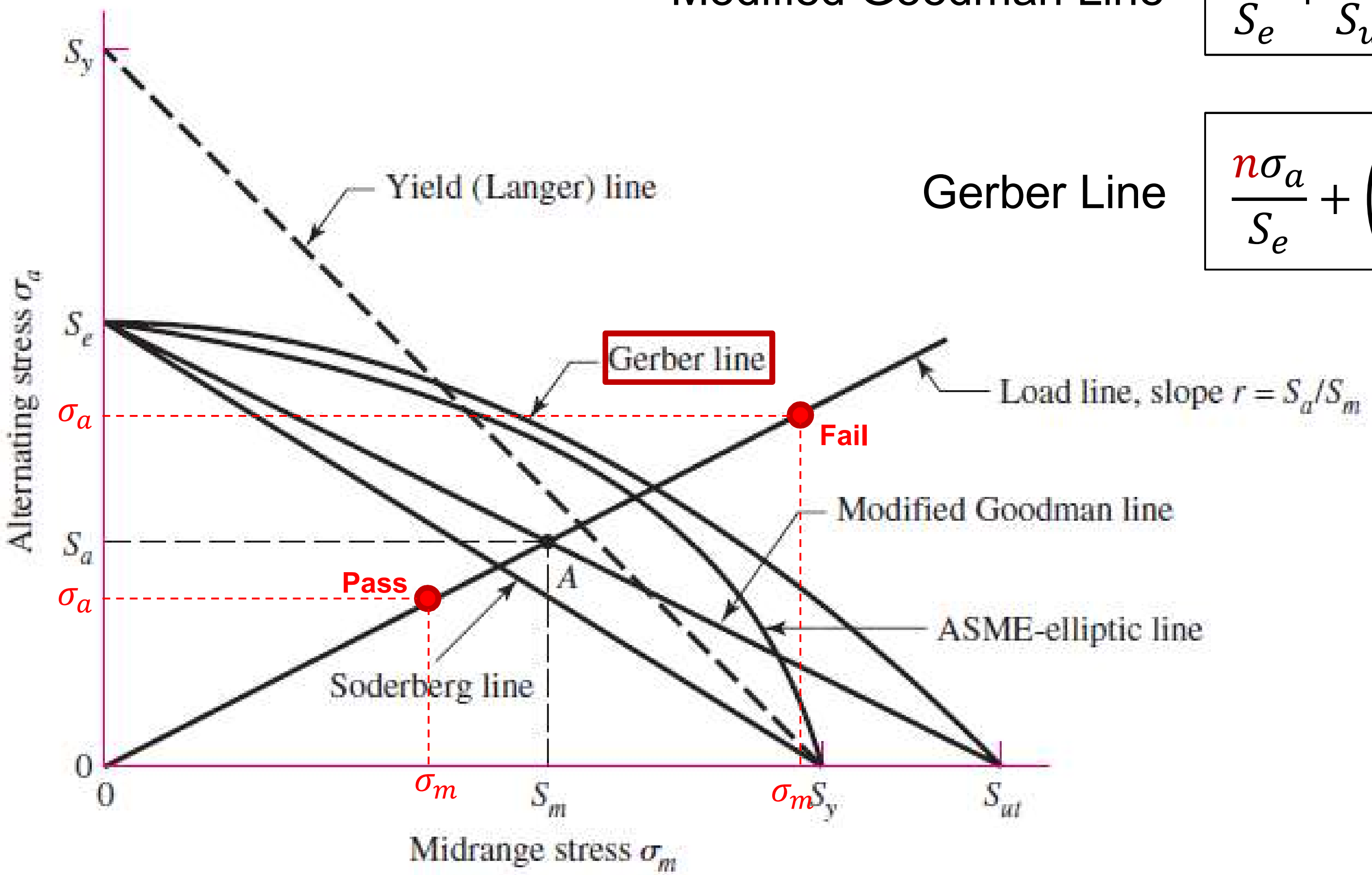
$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

Gerber Line

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

Fatigue Factor of Safety (n)

Fig. 6-27



Modified Goodman Line

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Gerber Line

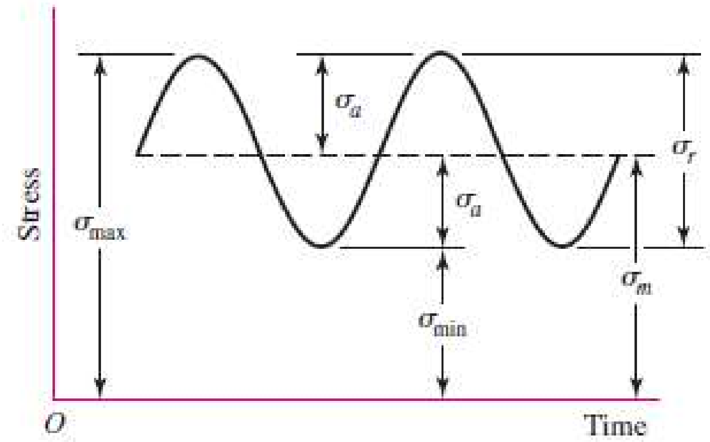
$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

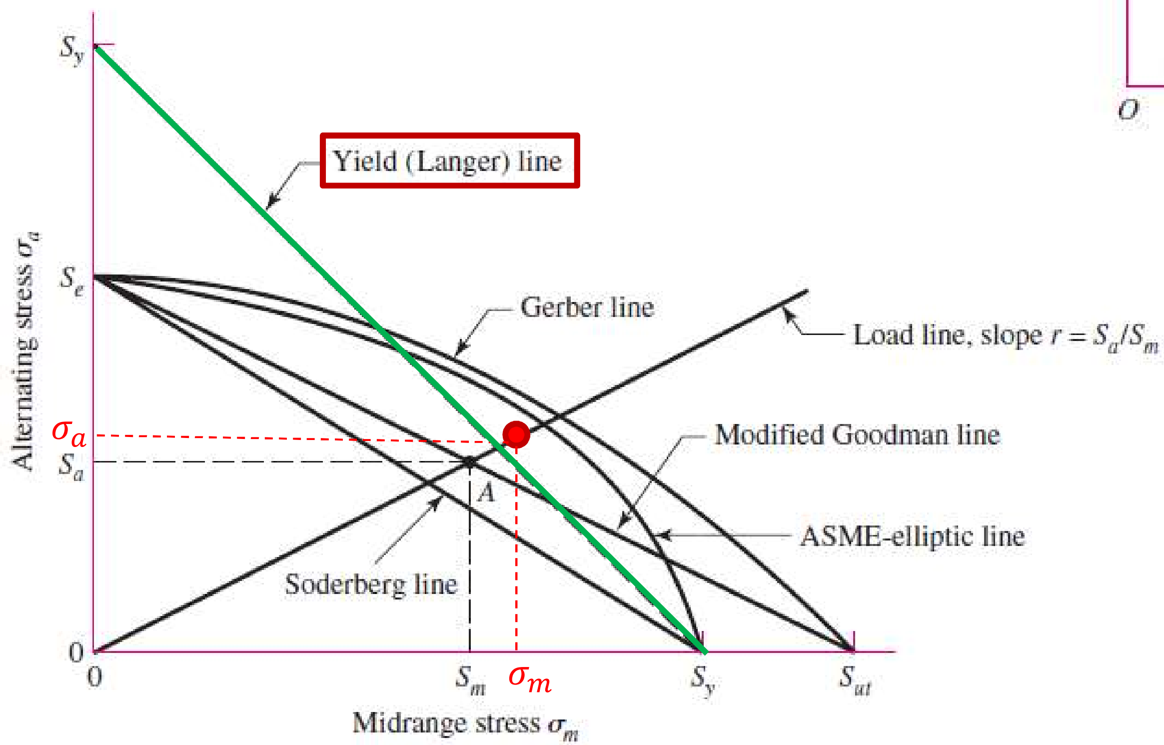
Local Yielding Factor of Safety (n)

Does σ_{max} exceed yield stress (S_y)?

Langer first-cycle-yielding:

$$\sigma_m + \sigma_a = \sigma_{max} = \frac{S_y}{n}$$





Sec 6-18 Road Map for Stress-Life Method

Workflow for Dynamic Fatigue Analysis