Lecture Note 02

Failures Resulting from Static Loading

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ME 1029 Mechanical Design 2

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Sichuan University - Pittsburgh Institute

What is the definition of Static Failure?

- Obviously **fracture** is a form of failure

- In some components **yielding** can also be considered as failure, if yielding distorts the material in such a way that it no longer functions properly

Session Outlines

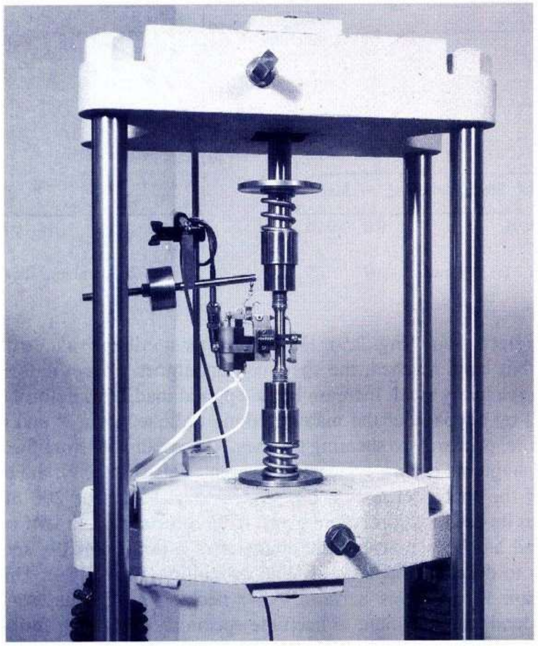
- Review failure criteria for ductile materials under static loading (Sec. 5-3 to 5-5, 5-7)
  - Maximum Shear Stress Theory
  - Distortion Energy Theory
- Calculate critical stresses
- Review characteristic stresses under press-fit
- Apply failure theories for press-fit pass/fail assessment

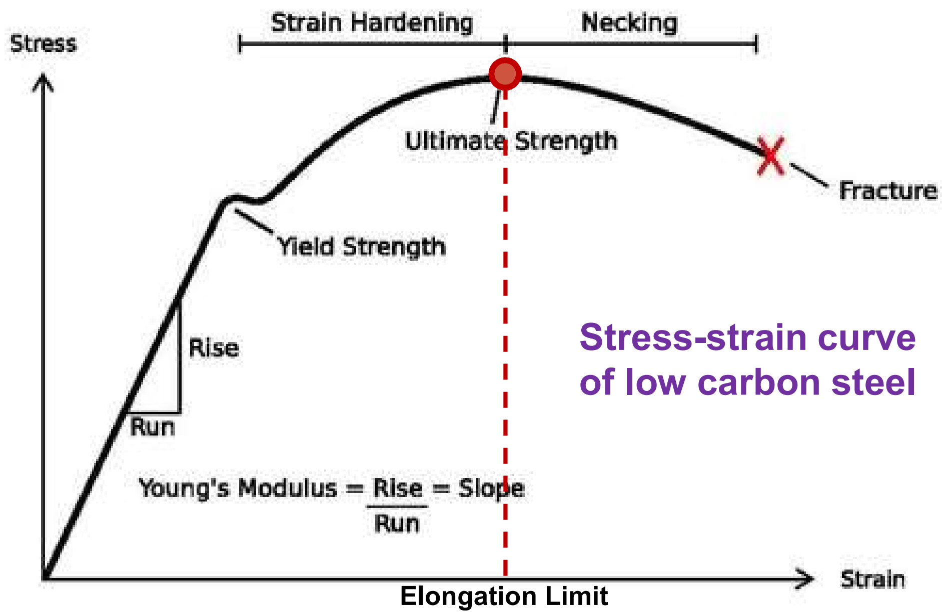
Technical Overview

- A **static load** is a stationary force or couple applied to a member.
  - To be stationary, the force or couple must be unchanging in magnitude, application locations, and direction.
  - A static load can produce axial tension or compression, a shear load, a bending load, a torsional load, or any combination of these.

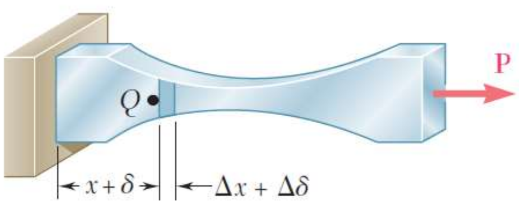
- **Strength** is a property or characteristic of a mechanical element, which is the resistance to failure of a mechanical element.

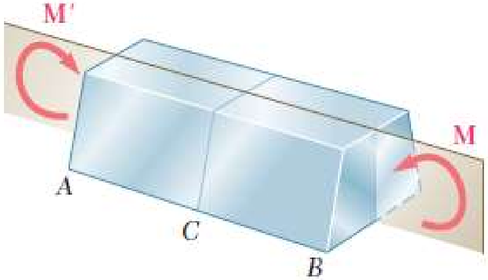
Material Testing (ASTM E8 Uniaxial Tensile Test)

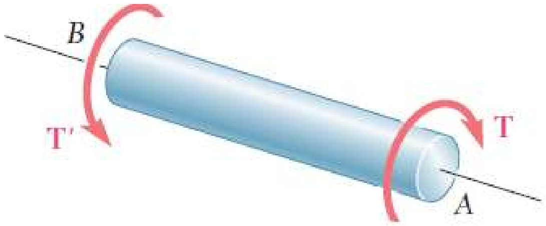




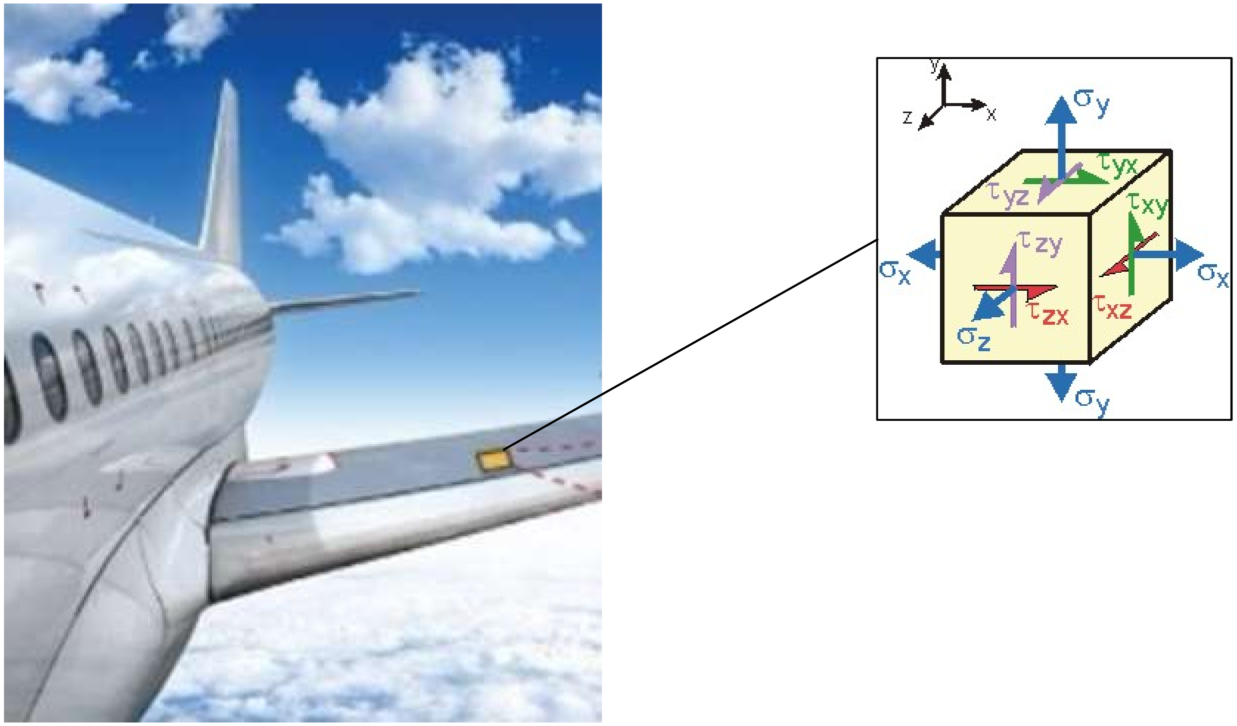
- Uniaxial Loading – System is subjected to one type of loading only: axial tension/compression, bending, or torsion.



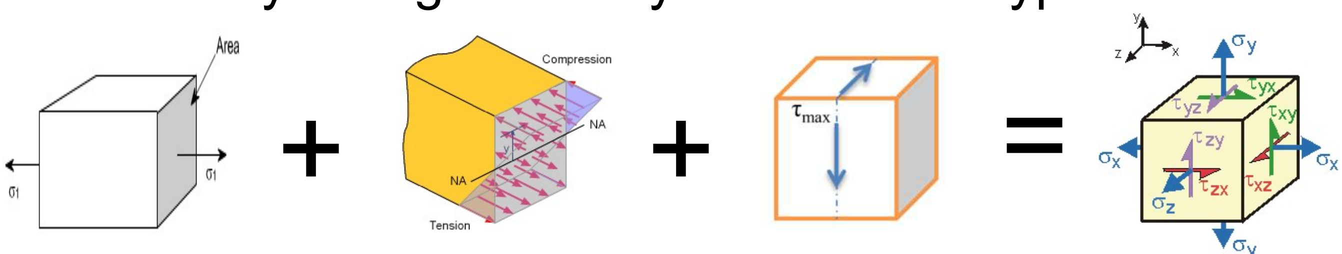




Stress Element in Engineering Systems



- Multiaxial Loading – In a real engineering system, loading condition is usually more complicated than simple loading. Multiple loading types simultaneously acting on the system is more typical.



Failure Prediction vs. Failure Testing

- Ideally, failure prediction should be based on data collected from full-sized prototype testing under application environment.

- In reality, it is usually benchmarked against criteria established from test data collected from simple <u>“coupon” testing</u>.

Failure Predictions Theories

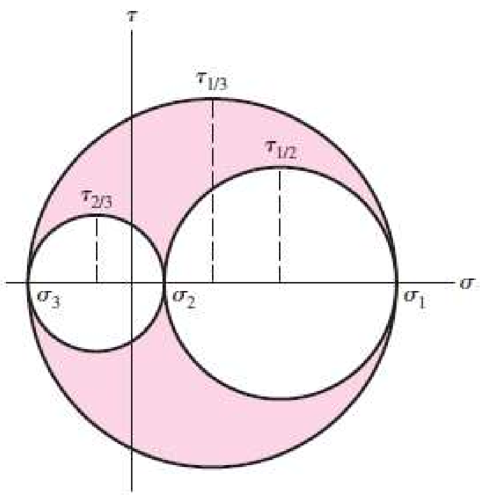
5-4 Maximum-Shear-Stress Theory for Ductile Materials
5-5 Distortion-Energy Theory for Ductile Materials

Failure Prediction Theories

Maximum-Shear-Stress Theory

- Failure occurs whenever the maximum shear stress (τ_{max}) in a design part exceeds the max shear stress at yield point in a tensile test specimen of the same material

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$



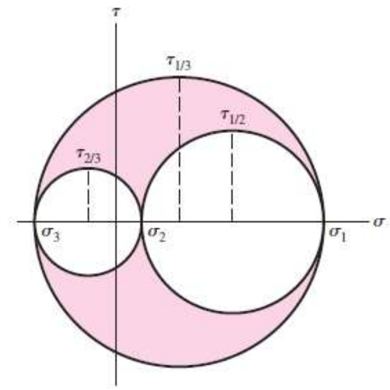
Distortion-Energy Theory (DET)

- Failure is assumed to occur whenever the octahedral shear stress (τ_{oct}) for any stress state equals or exceeds the octahedral shear stress for the simple tension-test specimen at yield point.

$$\tau_{oct} = \frac{\sqrt{2}}{3}\sigma_e$$

$$= \frac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

Maximum-Shear-Stress (Tresca) Theory



Design Part

- Given $\sigma_1 > \sigma_2 > \sigma_3$, maximum shear stress is (Sec. 3-7)

$$(\tau_{max})_{design} = \frac{\sigma_1 - \sigma_3}{2}$$

Simple Tension Test

- For simple tension test (ASTM E8) at yield, $\sigma_1 = S_y$, $\sigma_2 = \sigma_3 = 0$.
- Thus maximum shear stress theory predicts that failure will occur when $(\tau_{max})_{E8} = \frac{S_y}{2}$ *or* $\sigma_1 - \sigma_3 = S_y$
- DET assumed design part reaches failure point when

$$(\tau_{max})_{design} \geq (\tau_{max})_{E8} = \frac{S_y}{2}$$

- To incorporate a factor of safety, n:

$$n = \frac{(\tau_{max})_{E8}}{(\tau_{max})_{design}} = \frac{S_y/2}{(\tau_{max})_{design}} = \frac{S_y}{(\sigma_1 - \sigma_3)_{design}}$$

Example

A hot-rolled steel has a yield strength of S_y= 100 ksi. And is subjected to the stress states of σ_x = 70 ksi, σ_y = 70 ksi, τ_{xy} = 0 kpsi

What is its safety factor per Max Shear Theory?

Example

A hot-rolled steel has a yield strength of S_y= 100 ksi. And is subjected to the stress states of σ_x = 70 ksi, σ_y = 70 ksi, τ_{xy} = 0 kpsi

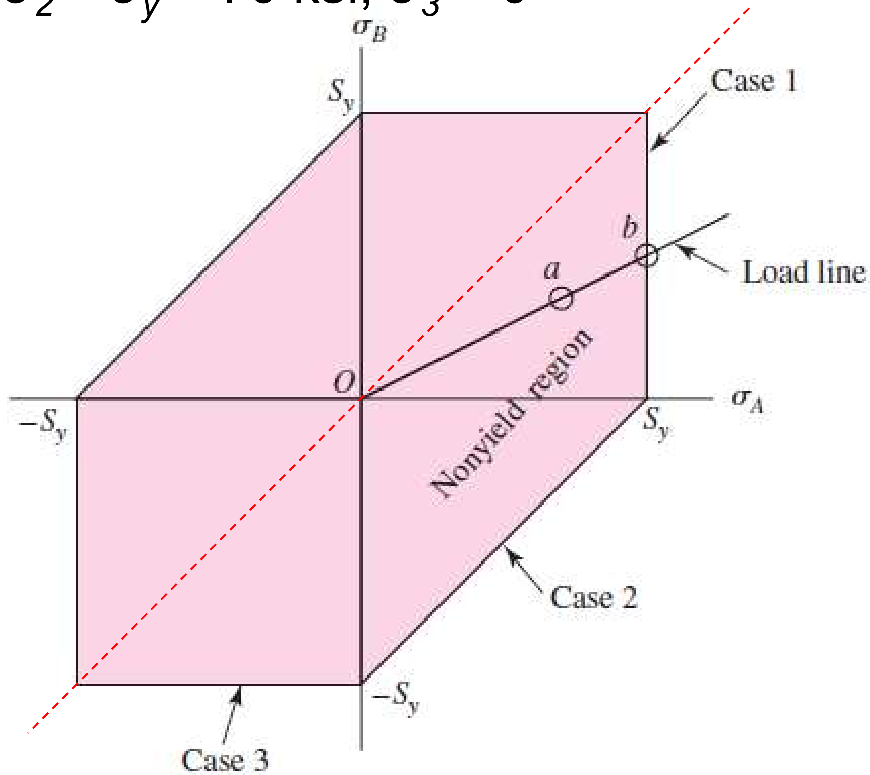
What is its safety factor per Max Shear Theory?

- hot-rolled steel ► Ductile material
- Since τ_{xy}=0; ► $\sigma_1 = \sigma_x$ = 70 ksi ► $\sigma_2 = \sigma_y$ = 70 ksi, σ_3 = 0

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{70-0}{2} = 35\ ksi$$

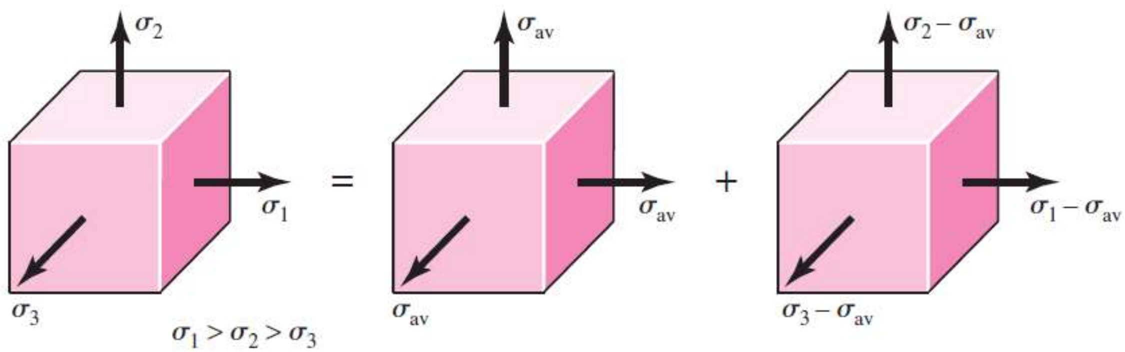
- Safety Factor n

$$n = \frac{S_y/_2}{\tau_{max}} = \frac{100/2}{35} = 1.43$$



Distortion-Energy Theory (DET)

- Materials can take enormous hydrostatic pressures (e.g., rocks deep in the earth that see pressures well above their compressive strength) and not fracture – suggesting that it must be distortion that causes failure.

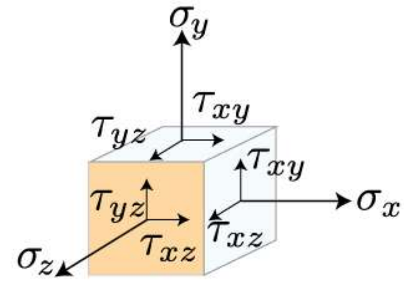


(*a*) Triaxial stresses (*b*) Hydrostatic component (*c*) Distortional component

- DET essentially computes the total strain energy and subtracts the volume change energy to get the distortion energy.
- Refer to Sec. 5-5 for detailed derivation of the von Mises stress using xyz three-dimensional stress

Distortion-Energy Theory (DET)

Failure is assumed to occur whenever the **octahedral shear stress** (τ_{oct}) for any stress state equals or exceeds the octahedral shear stress for the simple tension-test specimen at yield point.



Design Part

- von Mises equivalent stress

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + (\sigma_z - \sigma_x)^2 + 6\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right)\right]^{1/2}$$

or in principal stresses: $\sigma_e = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$

Octahedral shear stresses

$$(\tau_{oct})_{design} = \frac{\sqrt{2}}{3}\sigma_e = \frac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

Simple Tension Test

- (σ_1=S_y , σ_2 = σ_3=0). The τ_{oct} at yield point under simple tension is

$$(\tau_{oct})_{E8} = \frac{\sqrt{2}}{3}S_y$$

Distortion-Energy Theory (DET)

- DET assumed design part reaches failure point when

$$(\tau_{oct})_{design} \geq (\tau_{oct})_{E8} = \frac{\sqrt{2}}{3} S_y$$

- When using von Mises equivalent stress, this also reduces to

$$\sigma_e = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \geq S_y$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2} \geq S_y$$

- Safety Factor n: $n = \frac{S_y}{\sigma_e}$
- Note that, under hydrostatic pressure case (σ_1=σ_2=σ_3), von Mises equivalent stress σ_e=0. DET predicts no failure occurred.

Graphical Illustration of DET Theory Under Plane Stress

Under plane stress state (σ_3=0):

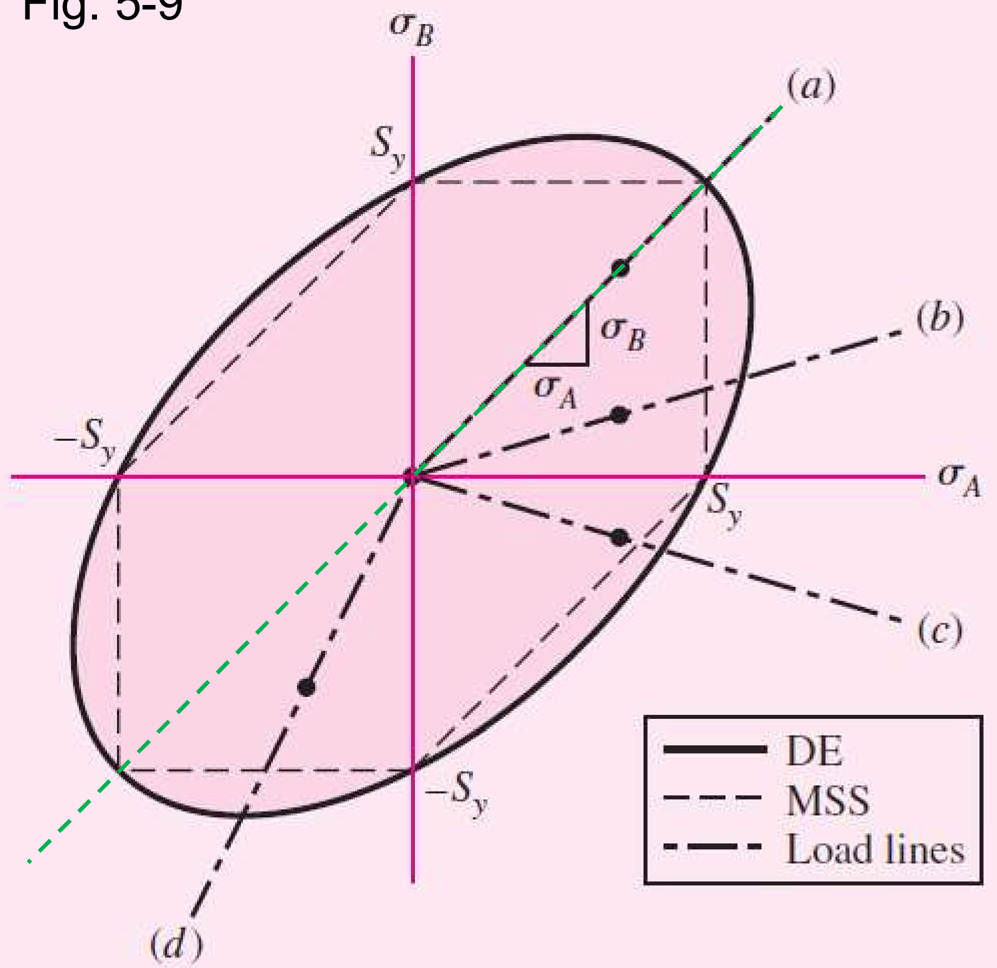
$$\sigma_e = [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2} \geq S_y$$

$$\sigma_e = \left[\sigma_{\mathrm{x}}^2 - \sigma_{\mathrm{x}}\sigma_{\mathrm{y}} + \sigma_{\mathrm{y}}^2 + 3\tau_{\mathrm{xy}}^2\right]^{1/2} \geq S_y$$

- Factor of safety, n: $\mathrm{n} = \frac{S_y}{\sigma_e}$

- MSS Theory provides a lower SF than DET in most cases.
- Case (a) is a special case where SF from MSS is identical to SF from DET

Fig. 5-9

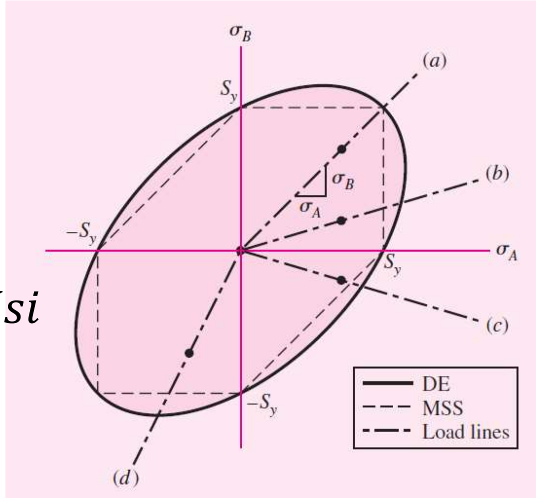


Example

Example: A hot-rolled steel has a yield strength of S_y= 100 ksi. And is subjected to the stress states of

(a) σ_x = 70 ksi, σ_y = 70 ksi, τ_{xy} = 0 kpsi

What is its safety factor per Distortion Energy Theory?



Solution:

$$\sigma_e = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

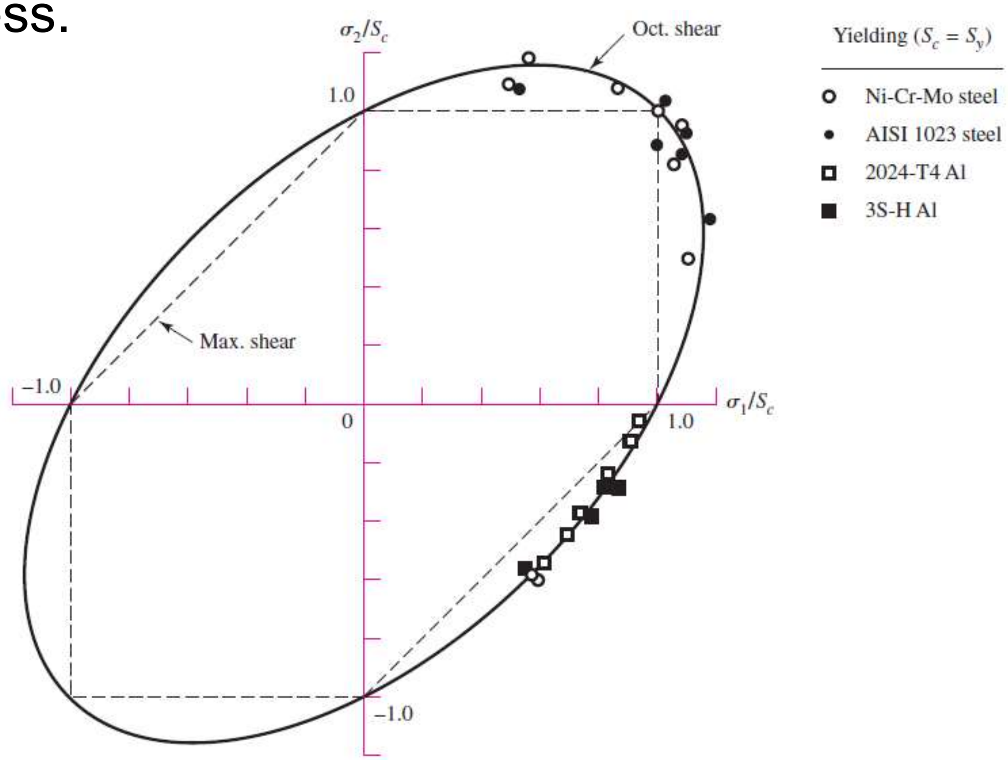
$$\sigma_e = [\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2]^{1/2} = [70^2 - 70 * 70 + 70^2]^{1/2} = 70\ Ksi$$

$$Safety\ Factor = \frac{100}{70} = 1.43$$

Note that, in this case, SF from DET is the same as MSS

MSS versus DET Comparison

- Historical test data indicates that DET is more accurate than MSS in predicting failure point than MSS.
- DET is a prevailing design criteria.
- However, MSS does use when applications warrant greater conservativeness.



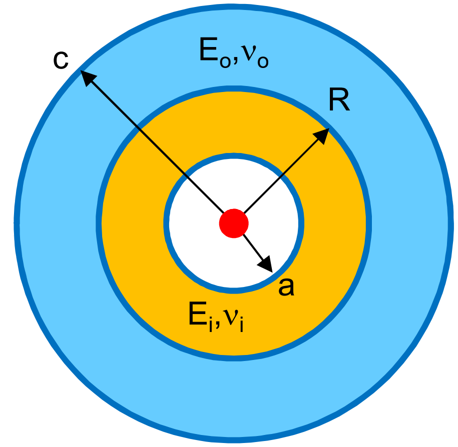
Press-Fit Failure Assessment

- Apply characteristic stresses resulting from press-fit to MSS and DET failure theories

Stresses at Press Fit Interface

- Interfacial Contact Pressure

$$P = \frac{\delta}{b\left[\frac{1}{E_o}\left(\frac{c^2+R^2}{c^2-R^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{R^2+a^2}{R^2-a^2}-\nu_i\right)\right]}$$



- Radial Stress $(\sigma_r)_{b_i} = (\sigma_r)_{b_o} = -P$
- Hoop Stress

$$(\sigma_\theta)_{b_i} = -P\frac{R^2+a^2}{R^2-a^2}$$

$$(\sigma_\theta)_{b_o} = P\frac{c^2+R^2}{c^2-R^2}$$

Note that:

- Critical stress is at the interference-fit interface
- Weak link is the hub ID surface, not the shaft OD surface

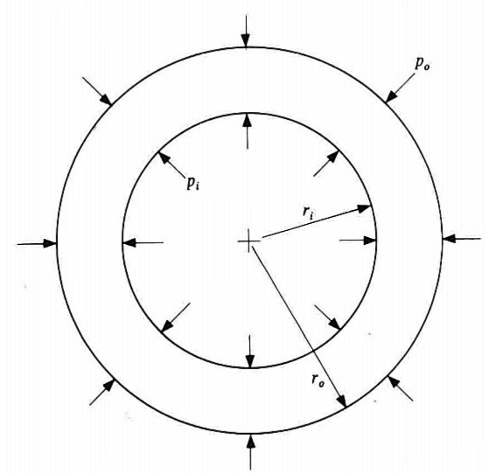
Stress Distribution on Hub ID Surface

- On hub ID surface, principal stresses are:

$$\sigma_r = -P < 0 \qquad \sigma_\theta = P\frac{c^2+R^2}{c^2-R^2} > 0 \qquad \sigma_z = 0 \qquad (\sigma_\theta > \sigma_z > \sigma_r)$$

- Per MSS criterion, press-fit fails @hub ID when:

$$\tau_{max} = \frac{\sigma_\theta - \sigma_r}{2} \geq \frac{S_y}{2} \qquad \frac{2c^2}{c^2-R^2}P \geq S_y$$

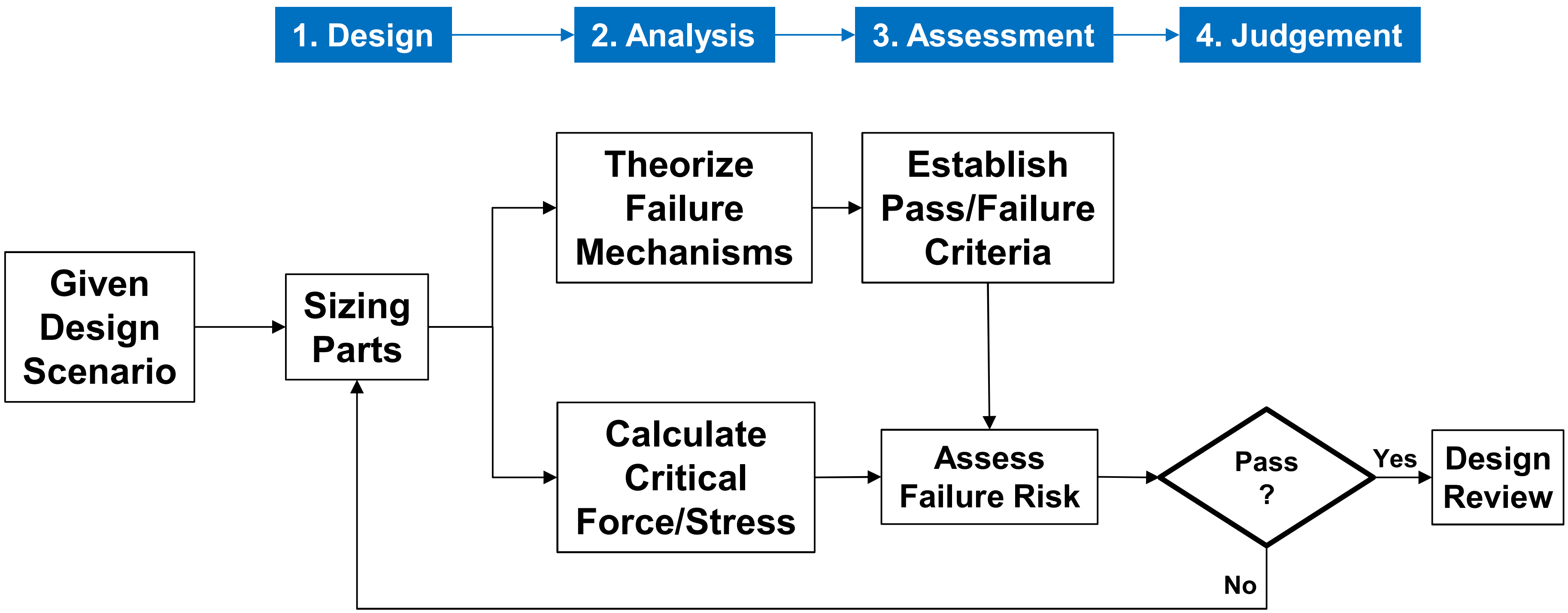


- Per DET criterion, press-fit fails @hub ID when:

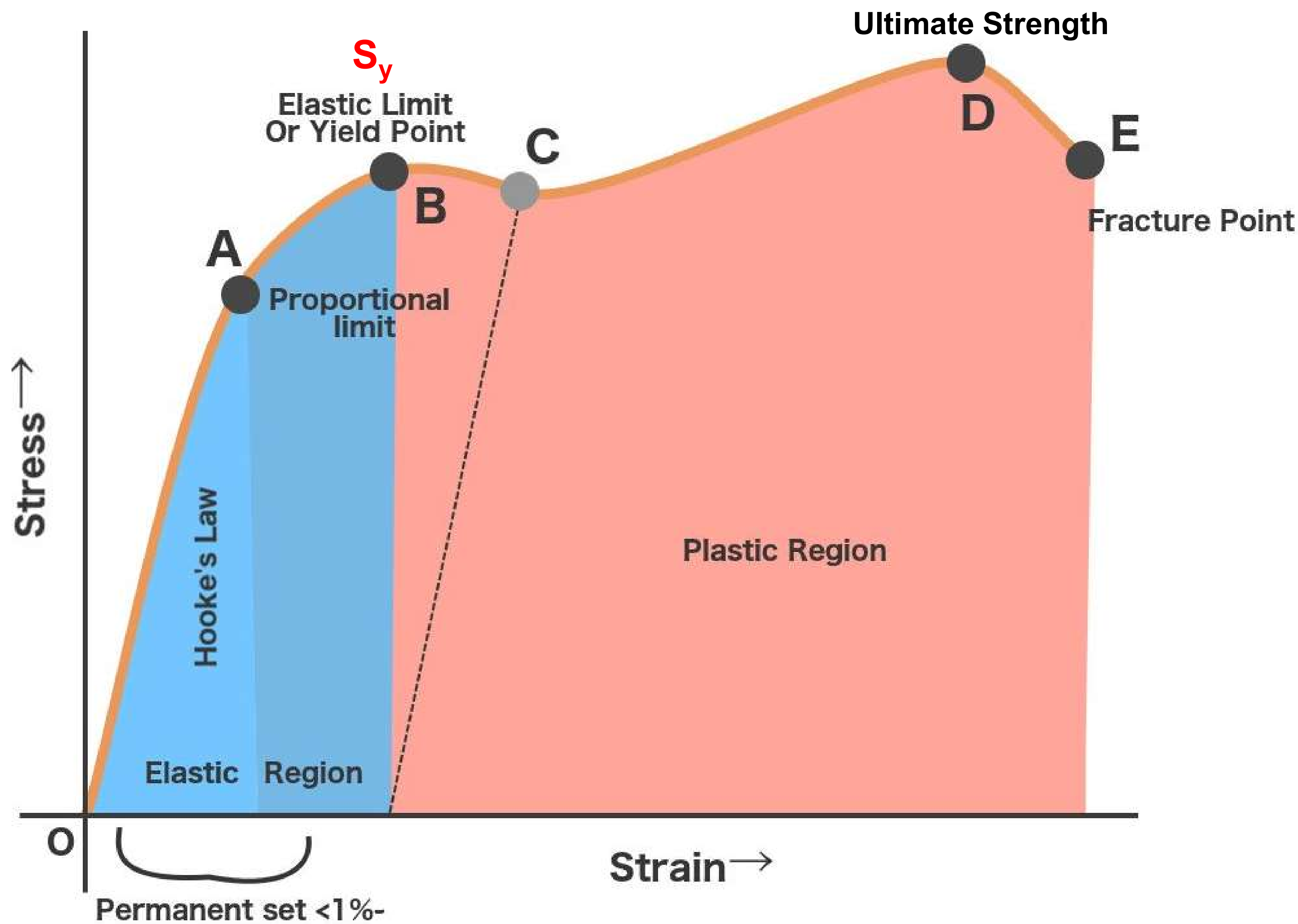
$$\sigma_e = \left[\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2\right]^{1/2} \geq S_y$$

Design Workflow Discipline

Everything we do in this class, we will follow this discipline…..



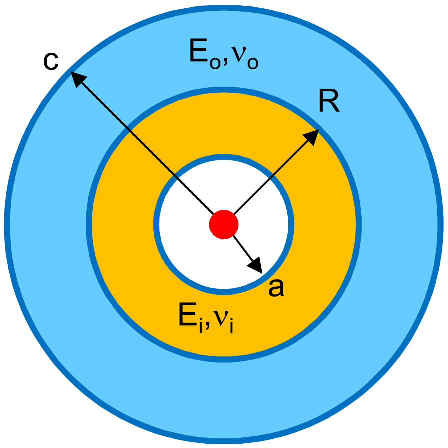
Typical Material Stress-Strain Curve



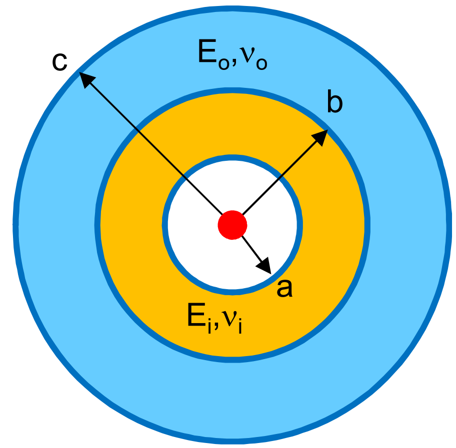
Example

An aluminum cylinder (E=70GPa, ν=0.33) with an outer radius of 150mm and inner radius of 100mm, is press-fitted over a steel cylinder (E=200GPa, ν=0.29) with an outer radius of 100.25 mm and inner radius of 50mm. Calculate safety factor of the design per MSS and DET criteria.

$$P = \frac{\delta}{b\left[\frac{1}{E_o}\left(\frac{c^2+b^2}{c^2-b^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{b^2+a^2}{b^2-a^2}-\nu_i\right)\right]}$$



Example (Cont'd)



Solution:

Hub material: aluminum (E_o=70GPa, ν_o=0.33)

Shaft material: steel (E_i=200GPa, ν_i=0.29)

c=150mm; b=100mm; a=50mm; δ=0.25mm

$$P = \frac{0.25}{100\left[\frac{1}{70000}\left(\frac{150^2+100^2}{150^2-100^2}+0.33\right)+\frac{1}{200000}\left(\frac{100^2+50^2}{100^2-50^2}-0.29\right)\right]} = 51.3\ MPa$$

On hub ID surface, radial stress: $\sigma_r = -P = -51.3\ MPa$

Hoop stress: $\sigma_\theta = P\frac{c^2+b^2}{c^2-b^2} = 51.3\frac{150^2+100^2}{150^2-100^2} = 133.38MPa$

Assume hub material is T6 cast aluminum. Table A-24 shows that its typical yield strength $S_y = 165MPa$

Example (Cont'd)

Solution:

Max shear stress on hub ID surface:

$$\tau_{max} = \frac{\sigma_\theta - \sigma_r}{2} = \frac{133.38-(-51.3)}{2} = 92.34 MPa$$

Safety factor per MSS: $n = \frac{165}{2 \cdot 92.34} = 0.89 < 1$, which indicates hub will fail.

Equivalent stress on hub ID surface:

$$\sigma_e = \left[\sigma_r^2 - \sigma_r\sigma_\theta + \sigma_\theta^2\right]^{1/2} = \sqrt{(-51.3)^2 - (-51.3)(133.38) + 133.38^2} = 165\ MPa$$

Safety factor per DET: $n = \frac{165}{165} = 1$, which indicates hub safety is marginal and leans towards the failure side.