MEMS1028
Mechanical Design 1

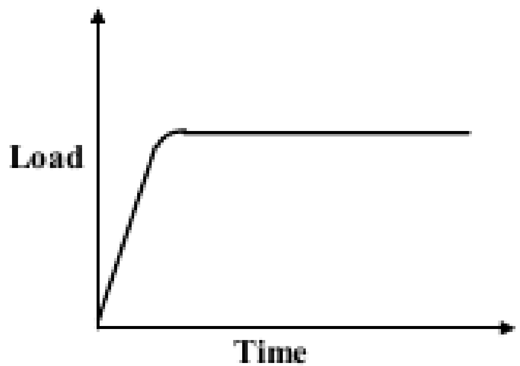
Lecture 9

Failure (Static loading – brittle material)

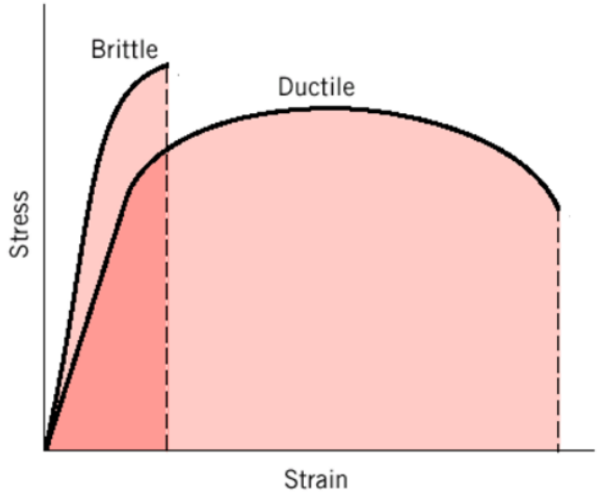
Objectives

- Apply static failure theories in engineering design involving brittle materials
- Explain the concepts of stress intensity factor and fracture toughness in fracture mechanics
- Apply stress intensity factors and fracture toughness in the engineering design analysis

Brittle failure



Static load



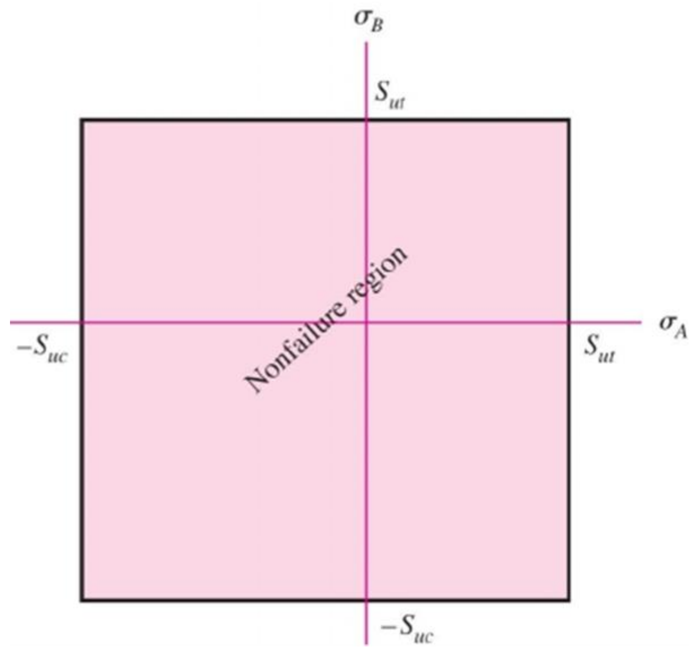
- Brittle Materials have percent elongation < 5%
- Brittle failure gives no warning
- Stress concentration must be included for brittle materials
- Brittle material failures are based on criteria such as
  - ❖ Maximum normal stress (MNS),
  - ❖ Brittle Coulomb-Mohr (BCM),
  - ❖ Modified Mohr (MM)

Maximum normal stress theory

- The maximum-normal-stress theory states that failure occurs whenever one of the three principal stresses equals or exceeds the ultimate strength
- For a general stress state with principal stresses in the ordered form $\sigma_1 \geq \sigma_2 \geq \sigma_3$. this theory predicts that failure occurs whenever

$$\sigma_1 \geq S_{ut} \text{ or } \sigma_3 \leq -S_{uc}$$

- S_{ut} and S_{uc} are the ultimate tensile and compressive strengths, respectively, given as positive quantities
- Maximum-normal-stress theory theory is not very good at predicting failure in the fourth quadrant of the σ_A, σ_B plane. Hence not recommended for use (has been added for historical reason!)



Maximum normal stress theory

- Failure occurs when the maximum principal stress in a stress element exceeds the ultimate strength
- Predicts failure when

$$\sigma_1 \geq S_{ut} \quad \text{or} \quad \sigma_3 \leq -S_{uc}$$

- For plane stress:

$$\sigma_A \geq S_{ut} \quad \text{or} \quad \sigma_B \leq -S_{uc}$$

- Incorporating design factor:

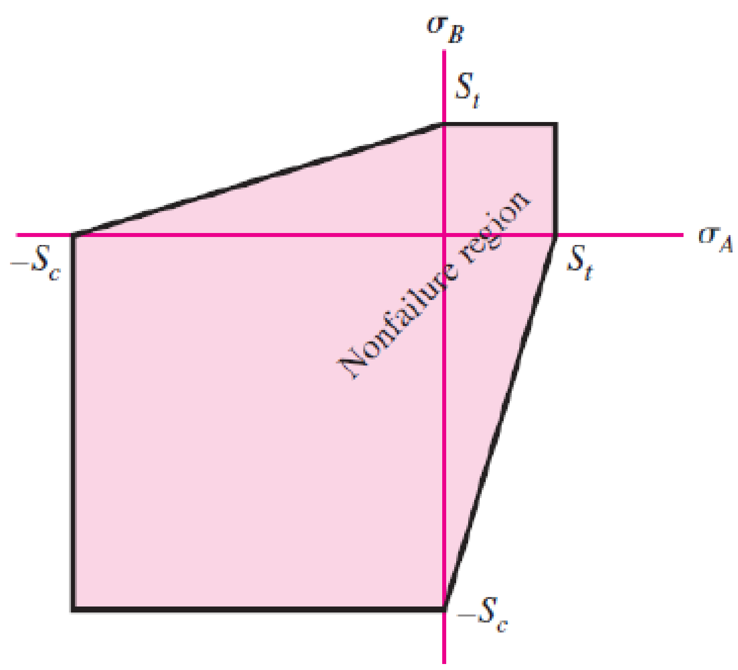
$$\sigma_A = \frac{S_{ut}}{n} \quad \text{or} \quad \sigma_B = -\frac{S_{uc}}{n}$$

- Do not use maximum normal stress theory for ductile materials as it does not fit experimental data

Brittle Coulomb-Mohr theory

- Same as Coulomb-Mohr (for ductile materials), except the brittle materials properties are the ultimate tensile and compressive strengths (remember: use yield tensile & compressive strengths only for ductile materials)
- Failure equations dependent on quadrant

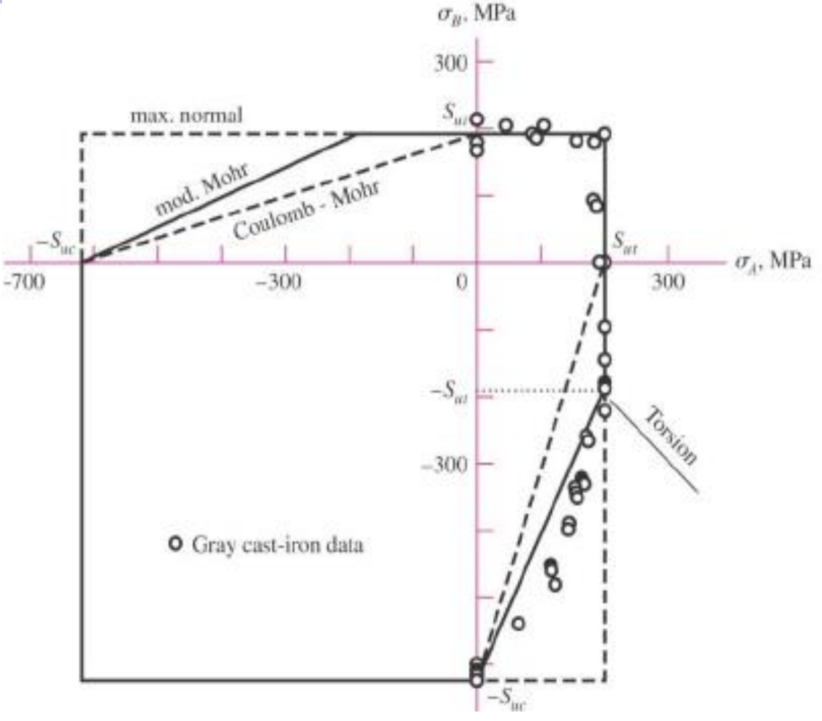
Quadrant condition	Failure criteria
$\sigma_A \geq \sigma_B \geq 0$	$\sigma_A = \frac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$	$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$
$0 \geq \sigma_A \geq \sigma_B$	$\sigma_B = -\frac{S_{uc}}{n}$



Modified Mohr theory

- Coulomb-Mohr is conservative in 4th quadrant
- Modified Mohr criteria adjusts to better fit the data in the 4th quadrant

Quadrant condition	Failure criteria
$\sigma_A \geq \sigma_B \geq 0$	$\sigma_A = \dfrac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$ and $\left\lvert \dfrac{\sigma_B}{\sigma_A} \right\rvert \leq 1$	$\sigma_A = \dfrac{S_{ut}}{n}$
$\sigma_A \geq 0 \geq \sigma_B$ and $\left\lvert \dfrac{\sigma_B}{\sigma_A} \right\rvert > 1$	$\dfrac{(S_{uc} - S_{ut})\sigma_A}{S_{uc}S_{ut}} - \dfrac{\sigma_B}{S_{uc}} = \dfrac{1}{n}$
$0 \geq \sigma_A \geq \sigma_B$	$\sigma_B = -\dfrac{S_{uc}}{n}$



Example 1

A cast pipe of outer diameter D = 100mm and inner diameter d = 60mm is made of an aluminum alloy having ultimate strengths in tension S_{ut} = 200 MPa and compression S_{uc} = 600 MPa. Determine the maximum torque that can be applied without causing rupture using maximum normal stress theory and a safety factor of n = 2. Reworked using Coulomb-Mohr and modified Mohr theories

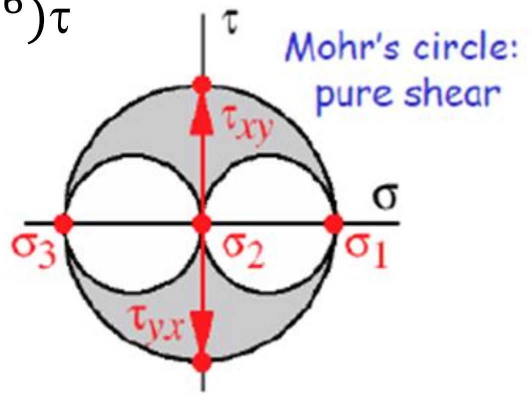
The torque and the maximum shear stress in the pipe are related by the torsion formula:

$$T = \frac{\tau J}{r} = \frac{\pi(0.05^4 - 0.3^4)\tau}{2(0.05)} = 170.9(10^{-6})\tau$$

The principal stress is described by $\sigma_1 = -\sigma_2 = \tau$, $\sigma_3 = 0$

Failure occurs when the maximum principal stress in a stress element first exceeds 200MPa; $\sigma_1 = \frac{S_{ut}}{n} = \tau$;

Hence max torque $T = 170.9(10^{-6})\frac{S_{ut}}{n}$=17.09 kNm



Example 1

$T = 170.9(10^{-6})\tau$ and S_{ut} = 200 MPa while S_{uc} = 600 MPa

- Employing the Coulomb–Mohr Theory, the principal stresses are

$\sigma_A \geq 0 \geq -\sigma_B$ where $\sigma_A = \tau$, $\sigma_B = -\tau$ and $n = 2$

$$\frac{1}{n} = \frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{\tau}{S_{ut}} + \frac{\tau}{S_{uc}} = \tau\left(\frac{1}{S_{ut}} + \frac{1}{S_{uc}}\right)$$

$$\tau = 75\text{MPa}$$

and

$T = 170.9(10^{-6})\tau = 12.82\text{kNm}$

Comparison indicated that on the basis of the maximum normal stress theory, the torque 17.09 kNm that can be applied to the pipe is about 25% larger than 12.82kNm obtained on the basis of the Coulomb–Mohr theory.

Example 1

$T = 170.9(10^{-6})\tau$ and S_{ut} = 200 MPa while S_{uc} = 600 MPa

- Employing the modified Mohr Theory, the principal stresses are

$\sigma_A \geq 0 \geq -\sigma_B$ where $\sigma_A = \tau$, $\sigma_B = -\tau$ and $n = 2$

Note that $\left|\frac{\sigma_B}{\sigma_A}\right| = 1$; hence

$$\sigma_A = \frac{S_{ut}}{n} = \tau$$

$$\tau = 100\text{MPa}$$

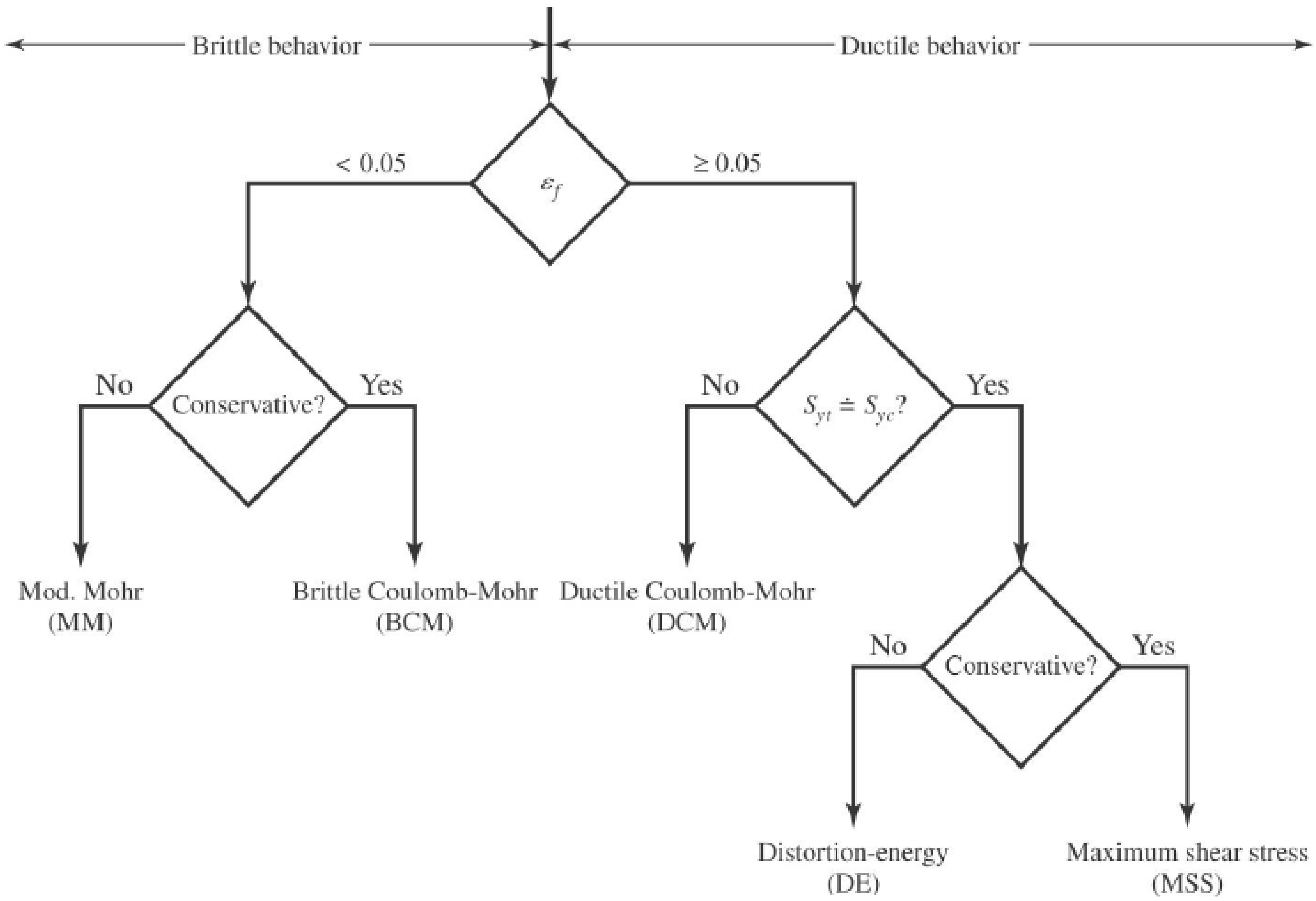
and

$T = 170.9(10^{-6})\tau = 17.09\text{kNm}$

Selection of failure criteria

- First determine if material is ductile or brittle
- For ductile
  - MSS is conservative, often used for design where higher reliability is desired
  - DE is typical, often used for analysis where agreement with experimental data is desired
  - If tensile and compressive strengths differ, use Ductile Coulomb-Mohr
- For brittle
- Brittle Coulomb-Mohr is very conservative in 4th quadrant
- Mohr theory is best, but difficult to use. It is slightly conservative in 4th quadrant, but closer to typical

Selection of failure criteria



Fracture mechanics

- Cracks exist in parts before service begins – "every structure contains small flaws, inclusions, defects, etc. whose size and distribution are dependent upon the material and its processing"
- These cracks can grow during service
- The objective of fracture mechanics analysis is to determine if these small flaws will grow into large enough cracks to cause the component to fail catastrophically
- Linear elastic fracture mechanics (LEFM) is often used to analyse crack growth during service
- Ductile materials can often neglect crack growth due to the plastic deformation that occurs at the crack tip blunting the crack and preventing the crack growth to a certain extent
- Crack growth in brittle materials must always be analysed
- For brittle materials, time is needed to feed the crack energy from the stress field to propagate the crack (e.g. cracking of ice on a frozen pond)

Fracture mechanics

❖ Model of crack of length 2*a*

❖ Maximum stress at (±*a*, 0):

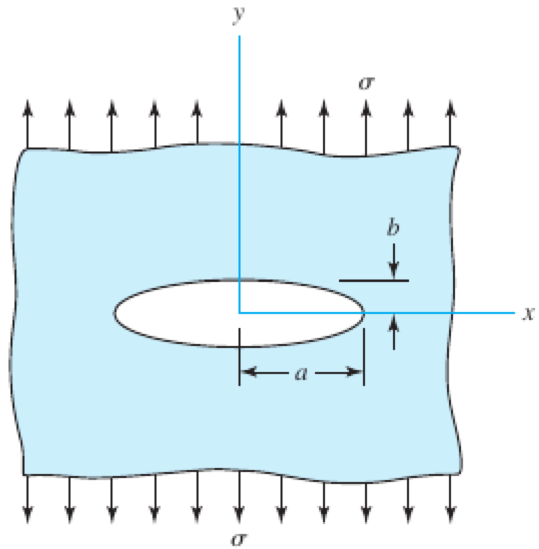
$$\sigma_a = \left(1 + 2\frac{a}{b}\right)\sigma$$

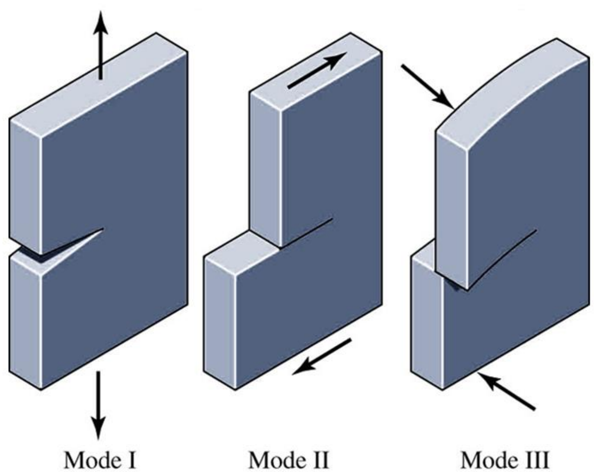
❖ As $b \rightarrow 0$, $\sigma_a \rightarrow \infty$

❖ Three distinct modes of crack propagation:

1) Mode I - Opening crack mode due to tensile stress field
2) Mode II - Sliding mode due to in-plane shear
3) Mode III – Tearing mode due to out-of-plane shear

❖ Combination of modes possible





Stress intensity factor

❖ Model of a mode I crack of length 2*a* in the infinite plate

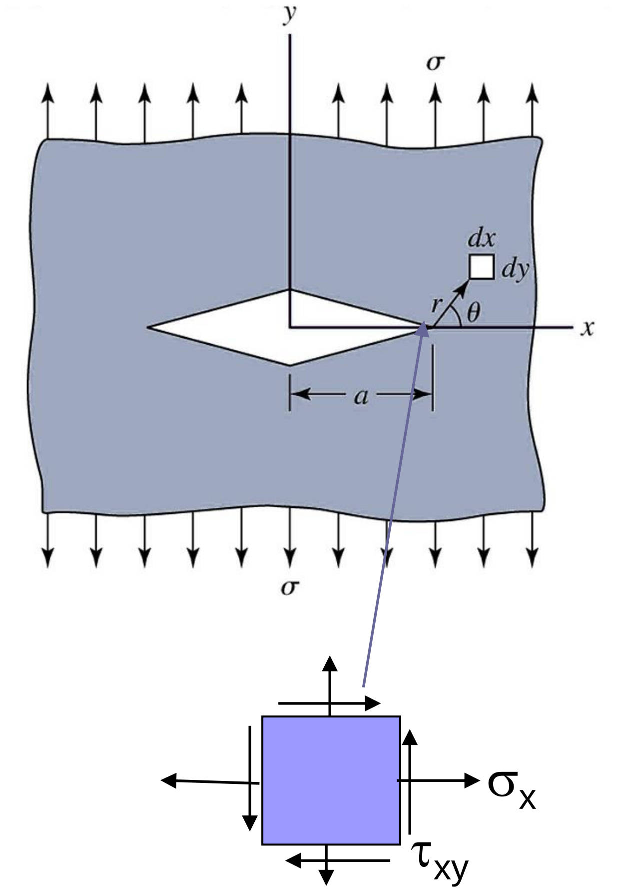
❖ Stress field equation at crack tip:

$$\sigma_x = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \sigma\sqrt{\frac{a}{2r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \sigma\sqrt{\frac{a}{2r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases}$$



As $\theta = 0$, $r \to 0$, $\sigma_y \to \infty$ (practically inappropriate)

❖ Stress concentration factor approach won't work!

Stress intensity factor

- Define stress intensity factor (for mode I):

$$K_I = \sigma\sqrt{\pi a}$$

- Rewrite Stress field equation at crack tip:

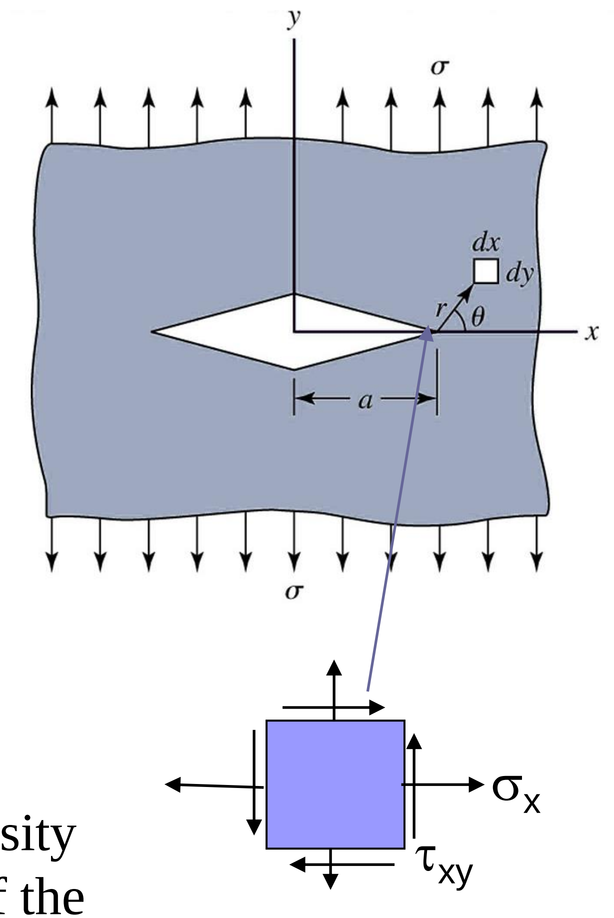
$$\sigma_x = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

$$\sigma_z = \begin{cases} 0 & \text{(for plane stress)} \\ \nu(\sigma_x + \sigma_y) & \text{(for plane strain)} \end{cases}$$

- To overcome $\theta = 0$, $r \rightarrow 0$, $\sigma_y \rightarrow \infty$ the stress intensity factor is a function of geometry, size and shape of the crack, and the type of loading



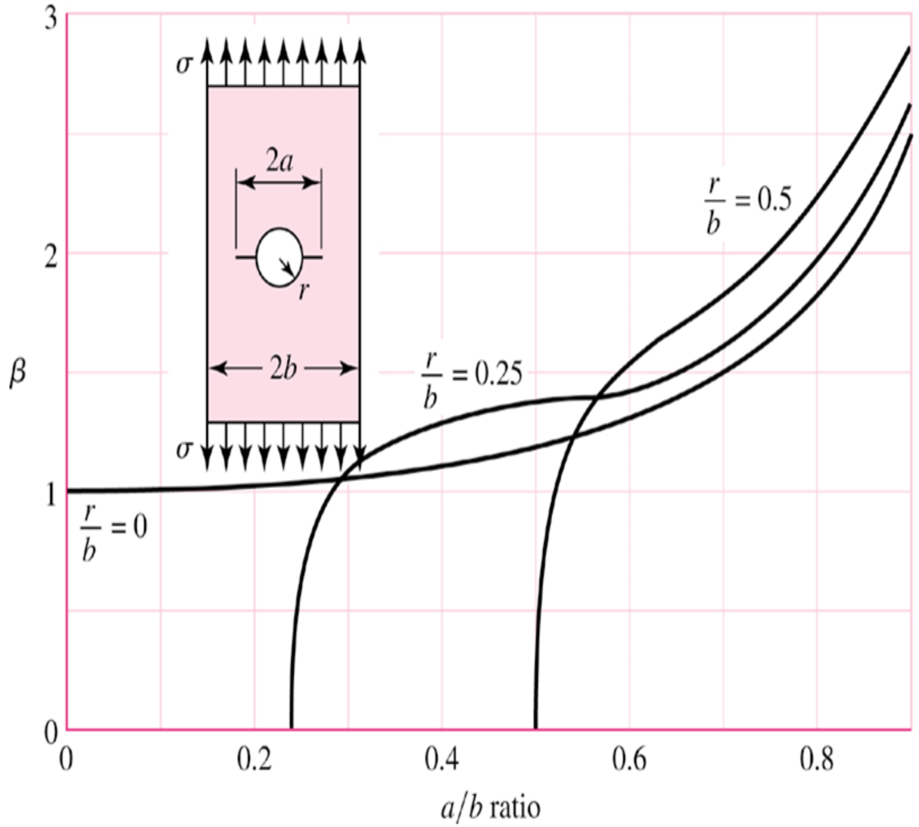
Stress intensity factor

- For various load and geometric configurations, it is more convenient to express the stress intensity factor as

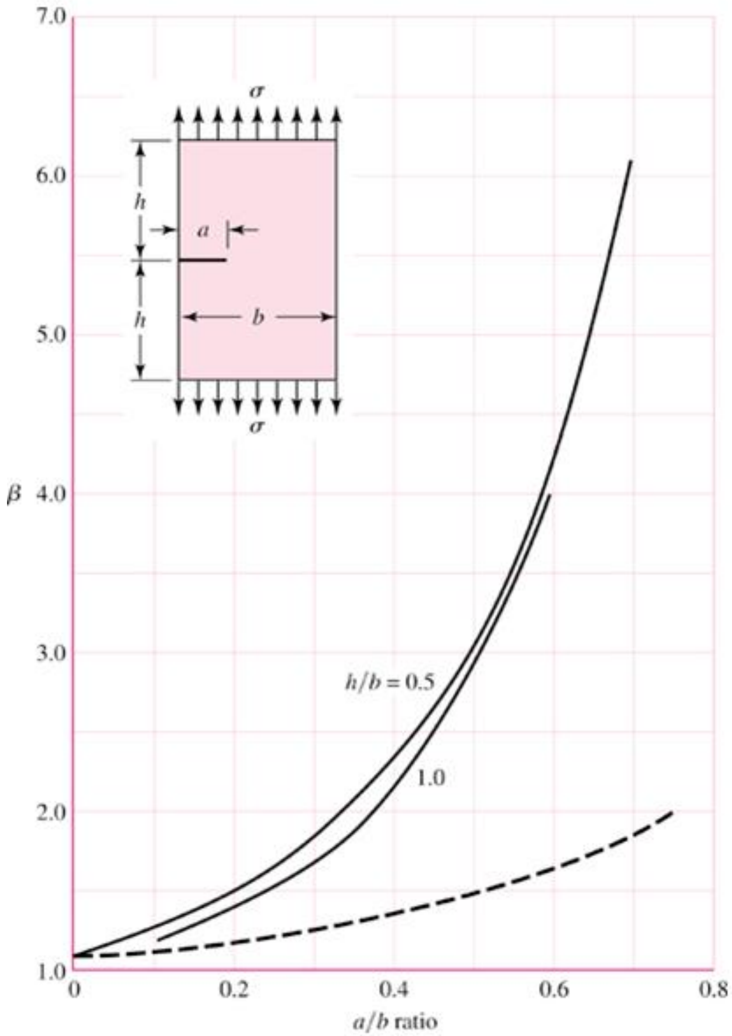
$$K_I = \beta\sigma\sqrt{\pi a}$$

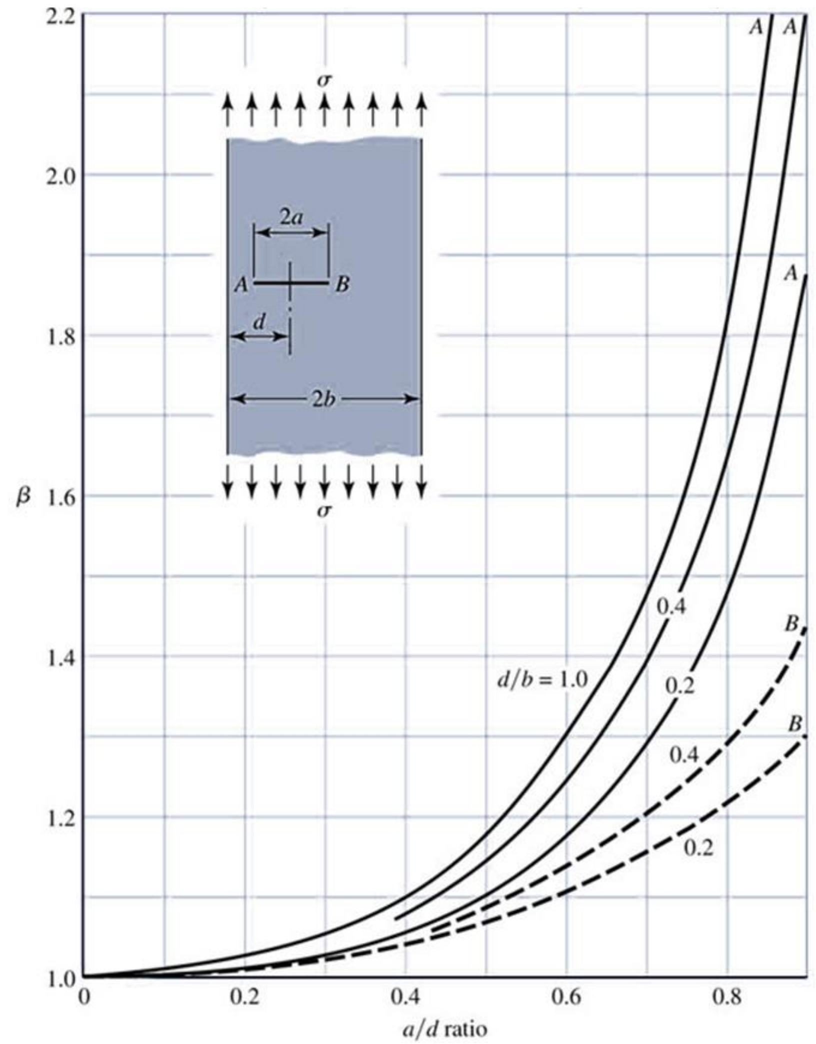
where β is the stress intensity modification factor which is presented either as a Table, Chart, or equation (see Figs. 5.25-5.30)

- Note: σ is stress at the absence of cracks

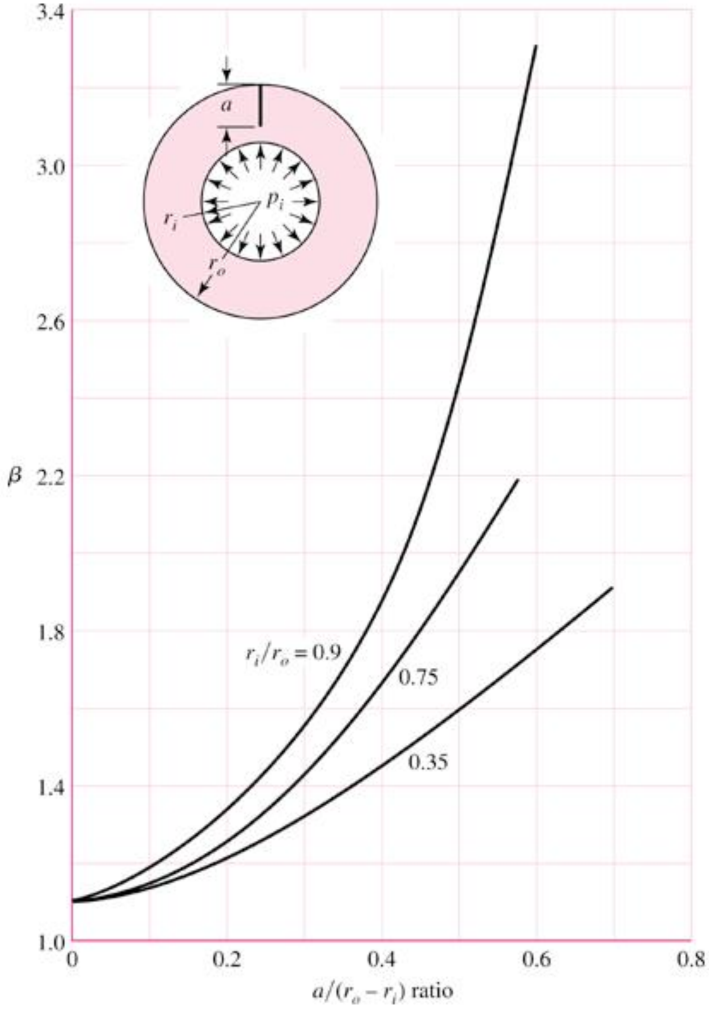


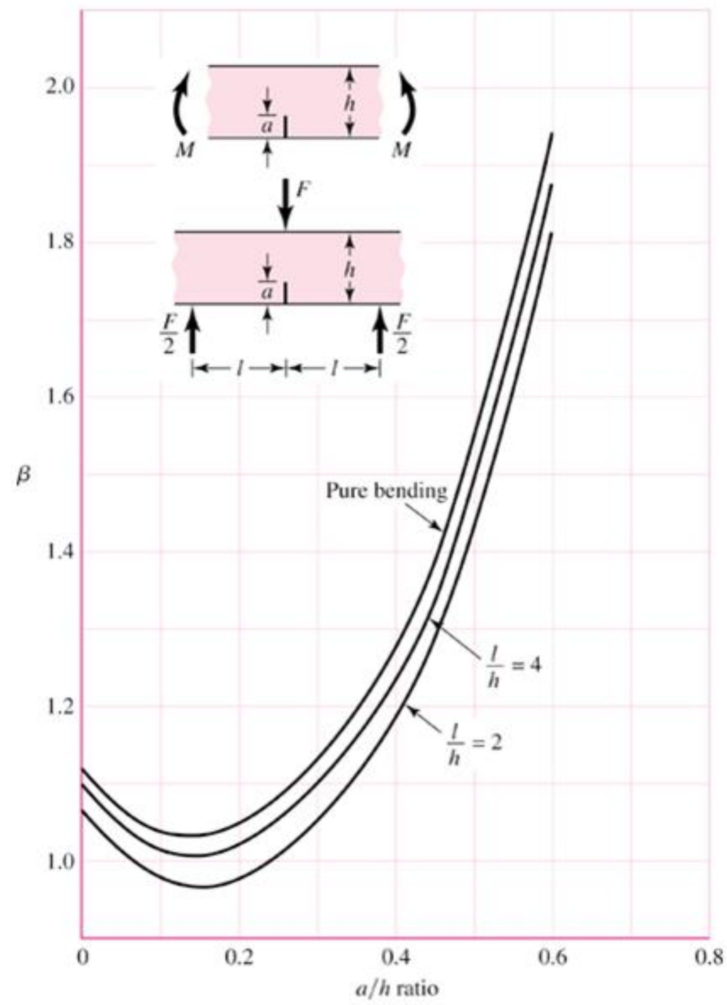
Stress intensity factor





Stress intensity factor





Fracture toughness

- When the magnitude of the mode I stress intensity factor reaches a critical value (i.e. K_{IC}), crack propagation initiates
- The critical stress intensity factor K_{IC} is a material property called fracture toughness (Some K_{IC} values given in Table 5-1)
- As long as the stress intensity factor K_I stays below the critical fracture toughness K_{IC} the crack is considered stable
- If K_I reaches K_{IC}, the crack will propagate and lead to sudden failure. Propagation rates can reach 1mile/sec
- For engineering design purpose:

$$\text{If } K_I < K_{IC} \text{ then no fracture}$$

$$\text{If } K_I \geq K_{IC} \text{ then fracture}$$

- Safety design criterion:

$$\text{Factor of safety } n = \frac{K_{IC}}{K_I}$$

Fracture toughness

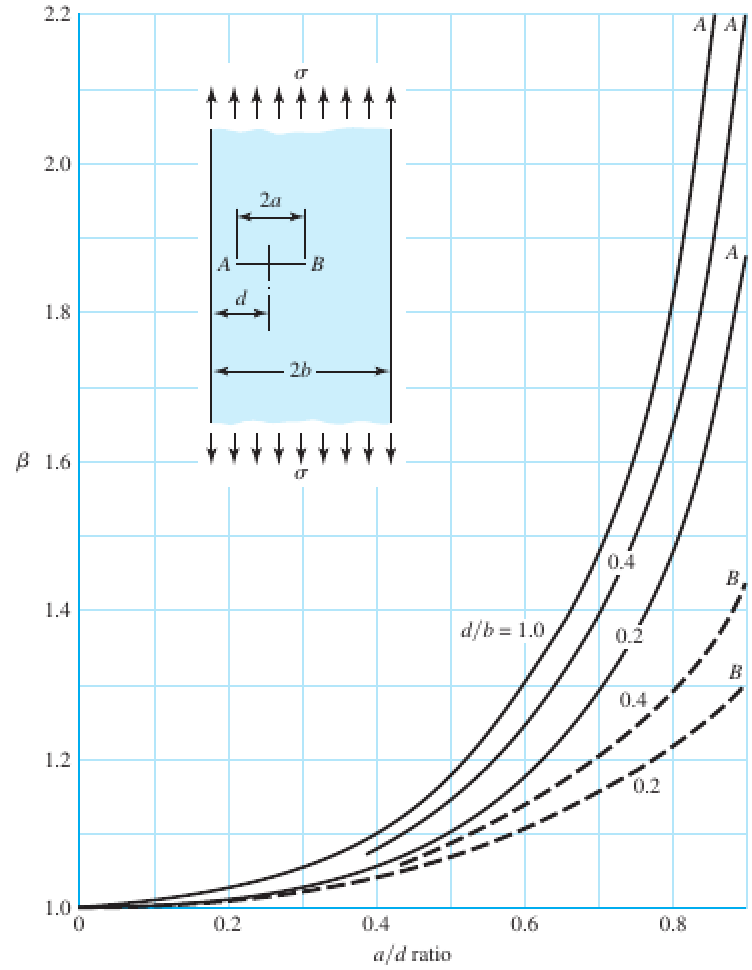
- The fracture toughness is a material property and depends on many factors including material processing, crack mode, temperature, loading rate, and the state of stress at the crack site (Some K_{IC} values given in Table 5-1)

Values of K_{Ic} for Some Engineering Materials at Room Temperature

Material	K_{Ic}, MPa $\sqrt{m}$	S_y, MPa
Aluminum		
2024	26	455
7075	24	495
7178	33	490
Titanium		
Ti-6AL-4V	115	910
Ti-6AL-4V	55	1035
Steel		
4340	99	860
4340	60	1515
52100	14	2070

Example 2

A steel ship deck plate is 30mm thick and 12 m wide. It is loaded with a nominal uniaxial tensile stress of 50 MPa. It is operated below its ductile-to-brittle transition temperature with K_{IC} equal to 28.3MPa$\sqrt{m}$. If a 65-mm-long central transverse crack is present, estimate the tensile stress at which catastrophic failure will occur. Compare this stress with the yield strength of 240MPa for this steel



Central transverse crack $d = b$, $2a$ = 65mm and $2b$ = 12m, so that $d/b = 1$; $a/d = 0.00542$; From fig. $\beta \approx 1$

Example 2

Given $2a$ = 65mm and found $\beta \approx 1$; known K_{IC} = 28.3MPa$\sqrt{\text{m}}$; nominal uniaxial tensile stress σ = 50MPa and yield strength S_y = 240MPa

- ❖ Stress intensity factor (for mode I):

$$K_I = \beta\sigma\sqrt{\pi a} = 16.0\text{MPa}\sqrt{\text{m}}$$

- ❖ Factor of safety $n = \frac{K_{IC}}{K_I} = 1.77$
- ❖ The stress at which catastrophic failure occurs is $\sigma_c = n\sigma = 88.5\text{MPa}$
- ❖ The yield strength is 240 MPa, and catastrophic failure occurs at

$$\frac{88.5}{240} = 0.37 \text{ or at 37 percent of yield}$$

- ❖ Note that with the crack, the factor of safety is NOT $\frac{240}{50} = 4.8$
- ❖ The factor of safety in this circumstance is 1.77

Example 3

Estimate the length of a transverse crack in 2024 Aluminum that would result in catastrophic failure (assuming negligible stress intensity modification)

Material	K_{Ic}, MPa$\sqrt{m}$	S_y, MPa
Aluminum		
2024	26	455
7075	24	495
7178	33	490

Known K_{IC} = 26MPa and S_y = 455MPa; for n = 1, $K_{IC} = K_I$ assuming β=1

$$K_I = K_{IC} = \beta\sigma\sqrt{\pi a} = S_y\sqrt{\pi a}$$

$$a = \frac{1}{\pi}\left(\frac{K_{IC}}{S_y}\right)^2 = 0.001\text{m}$$

The cross section will yield before unstable fracture for any transverse crack less than 2mm in total length (assuming negligible stress intensity modification)