MEMS1028
Mechanical Design 1

Lecture 4

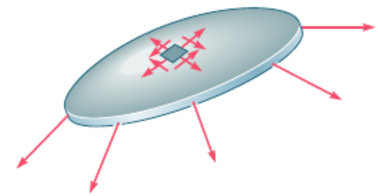
Load & stress analysis (Pressure vessels & others)

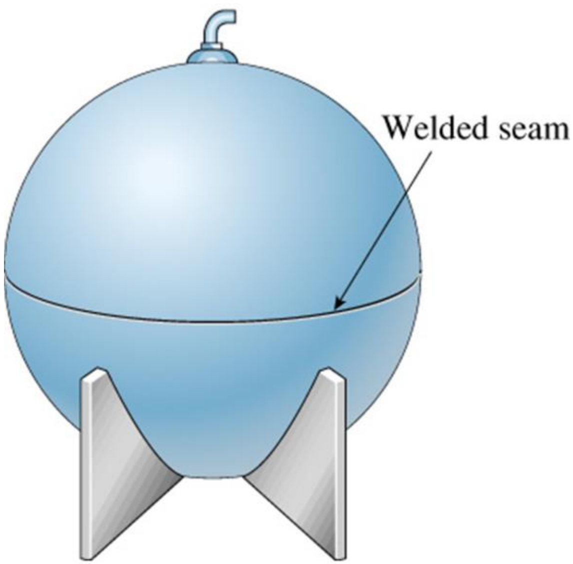
Objectives

- Analyze stresses in the designs of thin-walled vessels and thick-walled pressure cylinders
- Analyze stresses in the designs of rotational rings press and shrink fits and the effects of temperature
- Determine stress concentration factors and analyzing maximum stresses at discontinuities

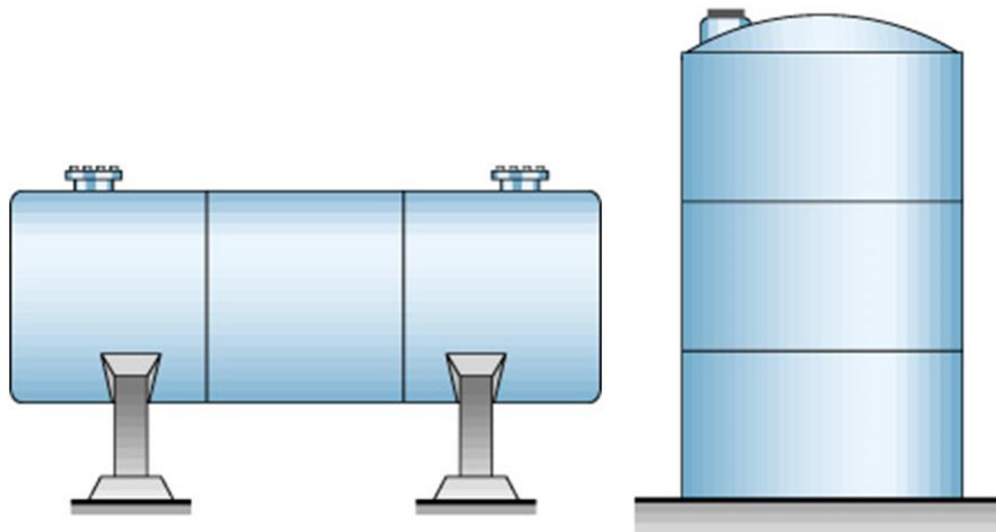
Types of thin-walled vessels

- The thin-walled pressure vessels provide an important application of plane-stress analysis where internal forces are tangential
- Thin-walled ($r/t > 10$)
- Two main types of thin-walled vessels



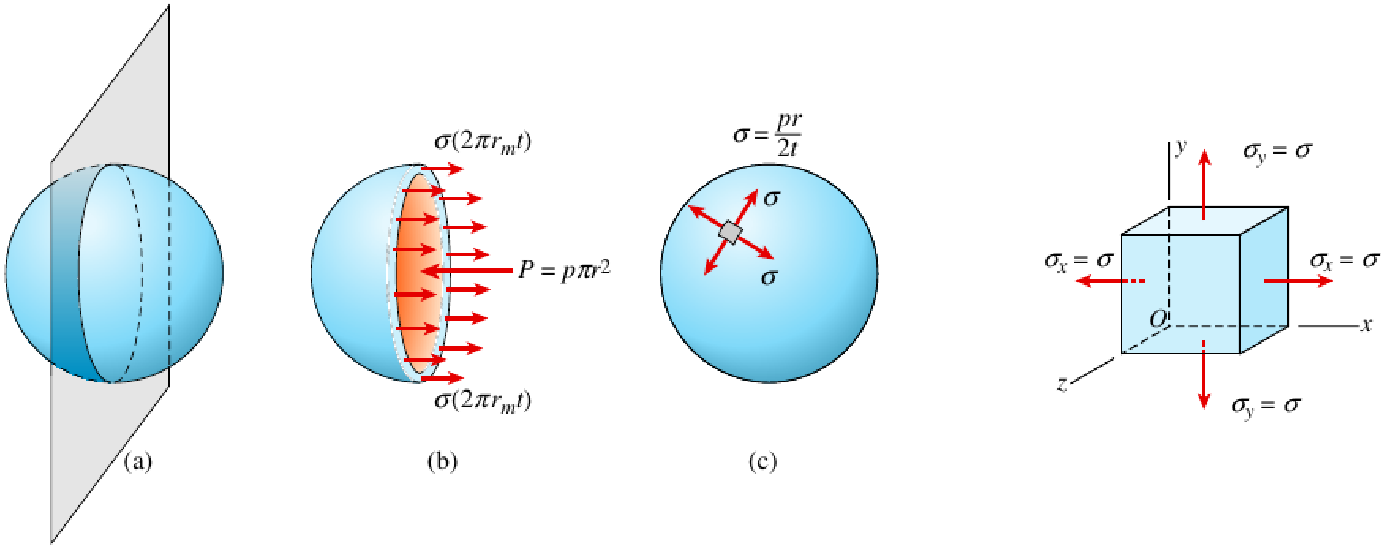


Spherical Pressure Vessels



Cylindrical Pressure Vessels

Spherical pressure vessels

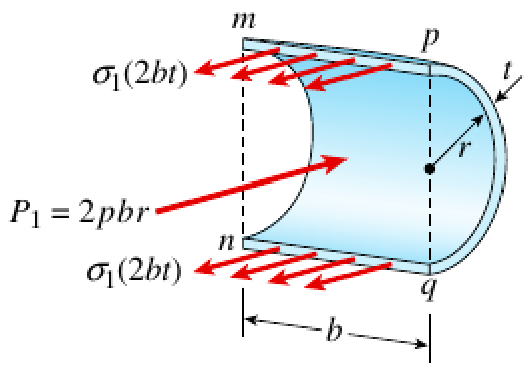


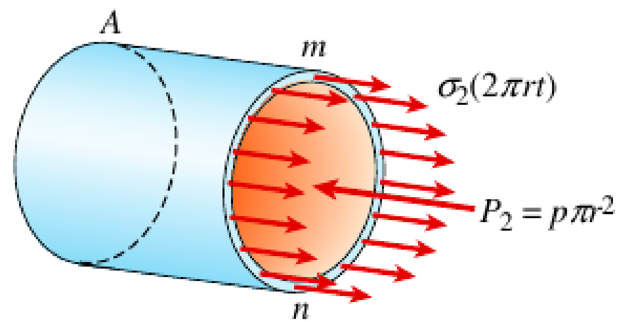
Given inner radius *r*, wall thickness *t*, and internal gage pressure *p*

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

Note: these are principal stresses as there is no load to induce shear stress

Cylindrical pressure vessels

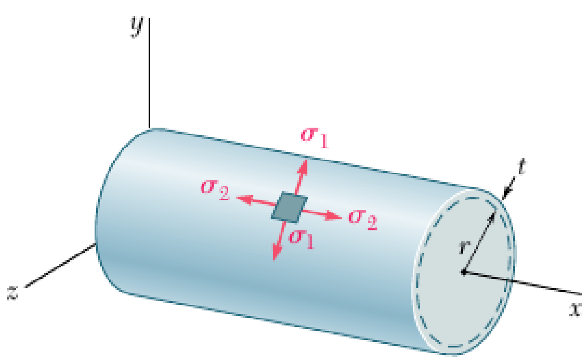


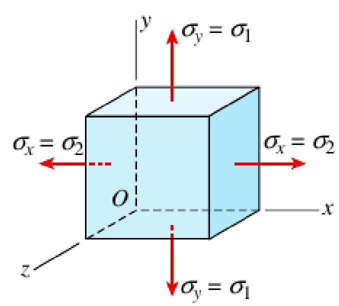


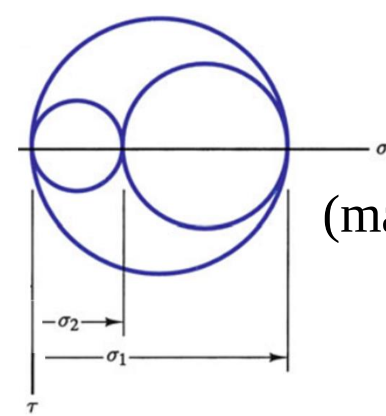
Hoop (or circumferential) stress: $\sigma_1 = \frac{pr}{t}$

Longitudinal stress: $\sigma_2 = \frac{pr}{2t}$

Note: these are principal stresses as there is no load to induce shear stress

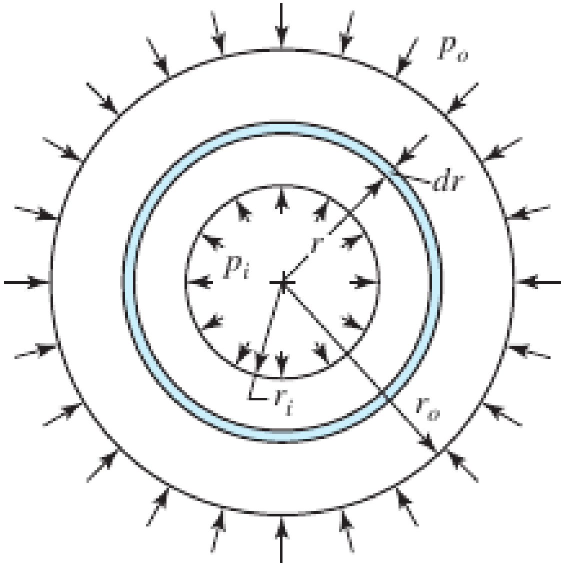






(max $\tau \neq 0$)

Thick-walled cylinders



- Cylindrical pressure vessels, hydraulic cylinders, gun barrels, and pipes subjected to both internal and external pressures
- Both radial σ_r and tangential stresses σ_t are developed

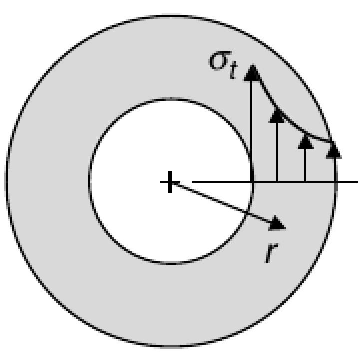
$$\sigma_t = \frac{p_i r_i^2 - p_o r_o^2 - r_i^2 r_o^2 \left(p_o - p_i\right)/r^2}{r_o^2 - r_i^2}$$

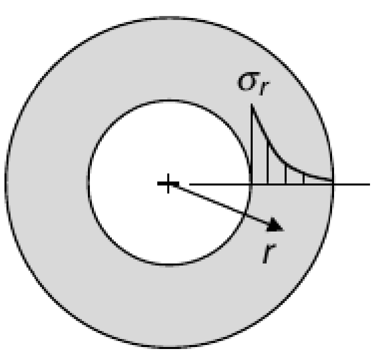
$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2 + r_i^2 r_o^2 \left(p_o - p_i\right)/r^2}{r_o^2 - r_i^2}$$

- Longitudinal stress at ends

$$\sigma_l = \frac{p_i r_i^2}{r_o^2 - r_i^2}$$

Distributions of σ_t and σ_r at p_o=0:



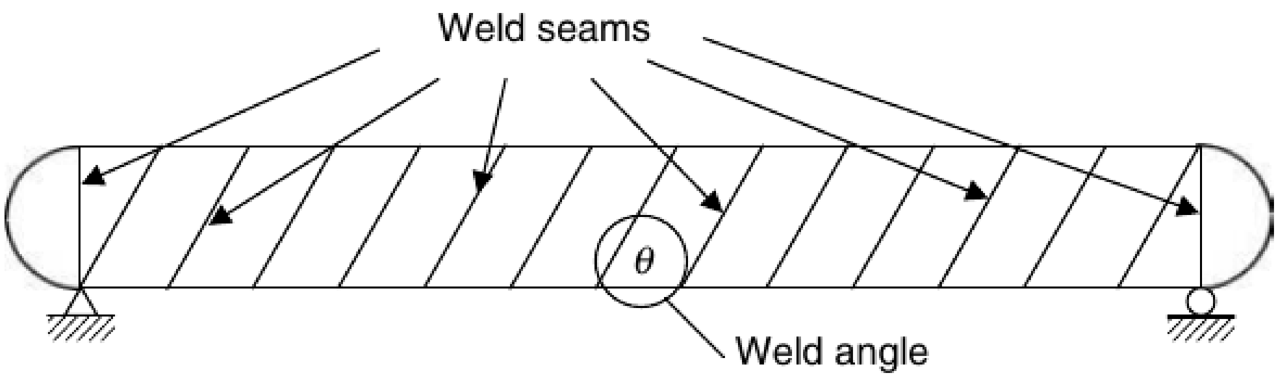


Summary of pressure loadings

Element	Normal stress (σ)	Shear stress (τ)
Thin-wall sphere	$\sigma_{\text{sph}} = \dfrac{p_i\, r_m}{2\,t}$	—
Thin-wall cylinder:		
Axial	$\sigma_{\text{axial}} = \dfrac{p_i\, r_m}{2\,t}$	—
Hoop	$\sigma_{\text{hoop}} = \dfrac{p_i\, r_m}{t}$	—
Thick-wall cylinder: ($p_o = 0$)		
Tangential	$\sigma_t = \dfrac{p_i\, r_i^2}{r_o^2 - r_i^2}\left[1 + \left(\dfrac{r_o}{r}\right)^2\right]$	—
Radial	$\sigma_r = \dfrac{p_i\, r_i^2}{r_o^2 - r_i^2}\left[1 - \left(\dfrac{r_o}{r}\right)^2\right]$	—
Axial	$\sigma_a = \dfrac{p_i\, r_i^2}{r_o^2 - r_i^2}$	—

Example 1

One of the ways thin-walled cylindrical pressure vessels are manufactured is by passing steel plate through a set of compression rollers creating a circular piece of steel that can then be welded along the resulting seams. Such a vessel is shown. Determine the stresses on an element of the cylinder oriented along the welds of the cylindrical tank given the following: θ = 60 degrees CCW, internal pressure P_i = 0.7 MPa, internal diamter D = 1.4 m and thickness t = 0.0065 m



Example 1

Axial stress $\sigma_1 = \frac{P_i}{2t} r = 37.7\text{MPa};$

Hoop stress $\sigma_2 = \frac{P_i}{t} r = 75.4\text{MPa};$

$$\tau_{xy} = 0$$

Using Mohr's circle:

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 56.55\text{MPa}$$

$$R = 18.85\text{MPa}$$

For 2θ = 120 deg: at point D

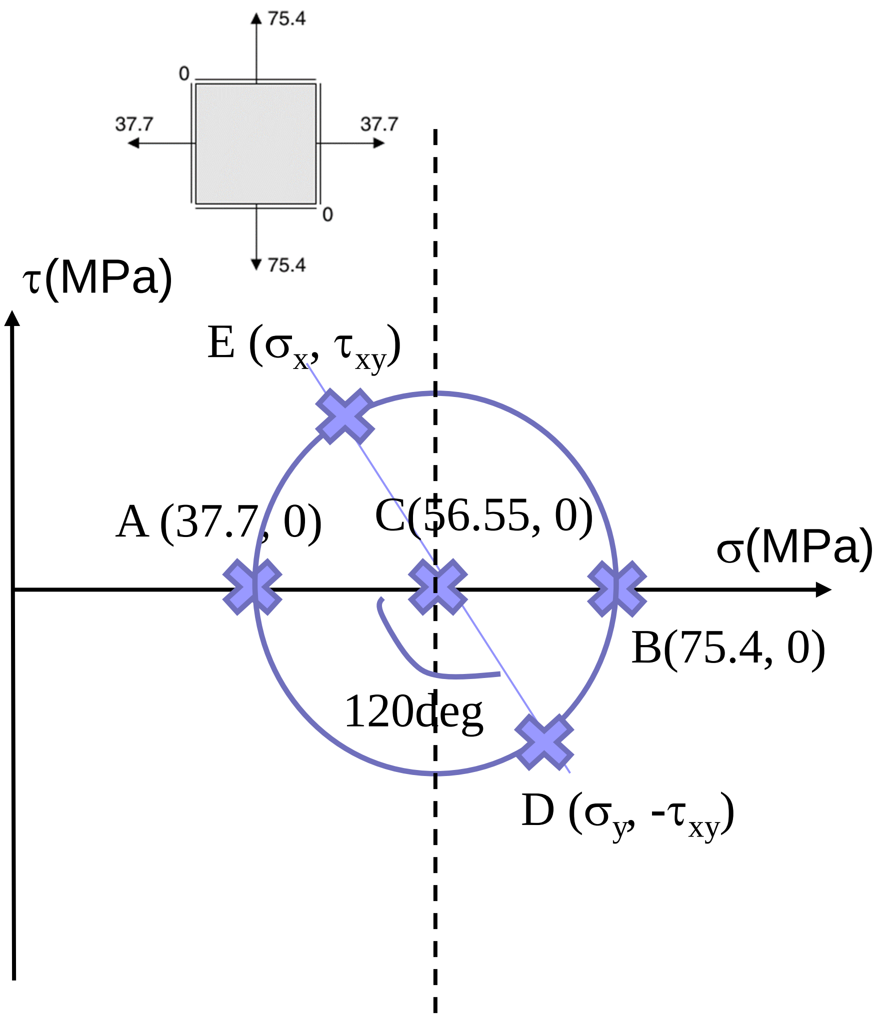
$\sigma_x = 56.55 + R\cos 60 = 66\text{MPa}$

$\tau_{xy} = R\sin 60 = -16.3\text{MPa}$

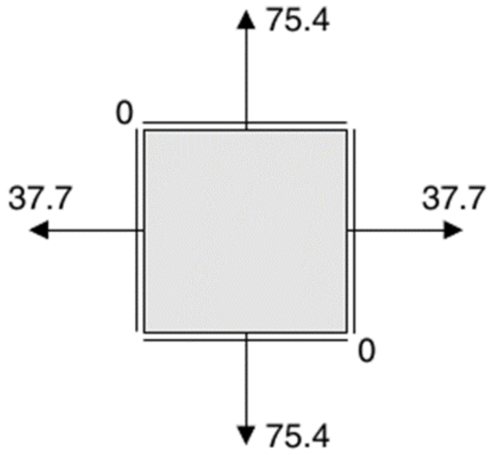
At point E:

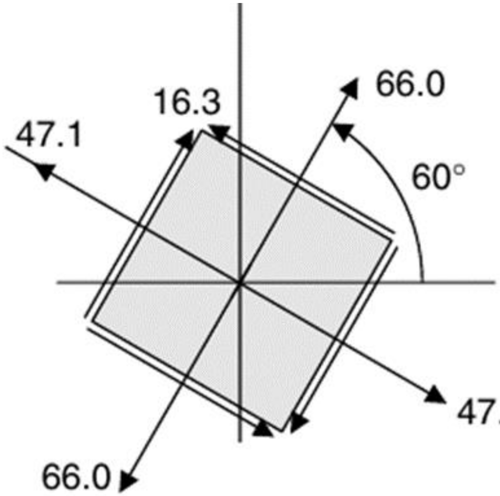
$\sigma_y = 56.55 - R\cos 60 = 47.1\text{MPa}$

$\tau_{xy} = R\sin 60 = 16.3\text{MPa}$



Example 1



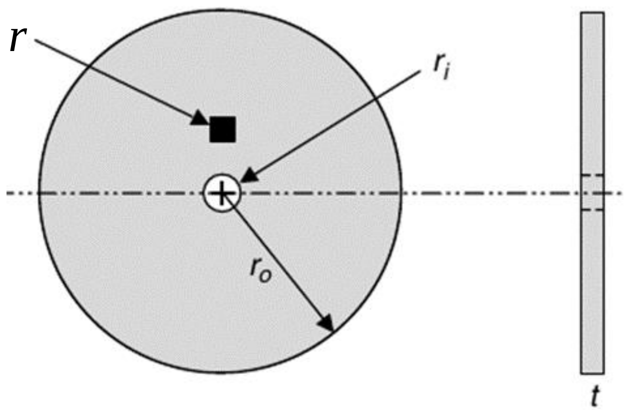


Note:
$\sigma_x + \sigma_y = 75.4 + 37.7 = 113.1\text{MPa}$

Note:
$\sigma_x + \sigma_y = 66 + 47.1 = 113.1\text{MPa}$

Rotational loading in rings

Stress element at radius r



- Examples: flywheel, sawblade, or turbine spinning about a stationary axis at very high speed
- Assumed outside radius of the ring, or disk, is large compared with the thickness $r_o > 10t$;

$$\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r^2} - \frac{1+3\nu}{3+\nu}r^2\right)$$

$$\sigma_r = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 - \frac{r_i^2 r_o^2}{r^2} - r^2\right)$$

Note: ρ = density; ω = angular velocity (rad/s); ν = Poisson's ratio

❖ At $r = r_i$ the σ_t is maximum and radial stress is zero

Illustrative example 1

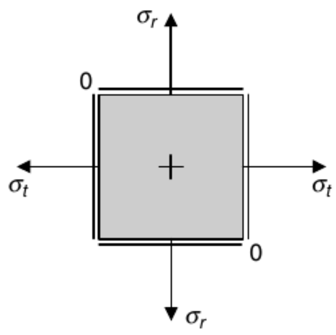
Determine the stress in a thin high-speed saw blade at the inner radius, where ω = 1,000 rpm; r_o = 1m; r_i = 0.025 m; t = 0.006 m; ρ = 7,850 kg/m^3 (steel); S_y = 350MPa (steel); ν = 0.3 (steel)

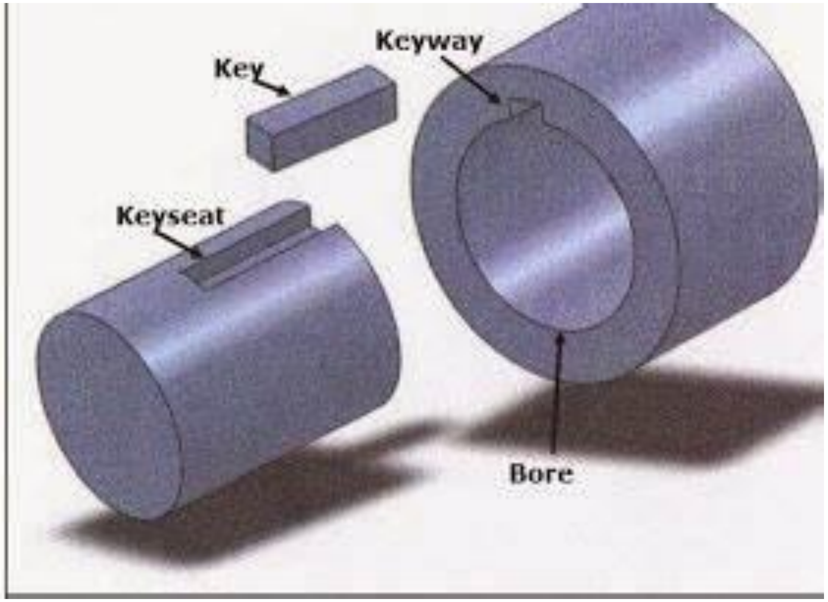
Rotation speed $\omega = 1000\frac{2\pi}{60} = 105$rad/s;

At $r = r_i$ the σ_t is maximum

Max. $\sigma_t = \rho\omega^2\left(\frac{3+\nu}{8}\right)\left(r_i^2 + r_o^2 + \frac{r_i^2 r_o^2}{r_i^2} - \frac{1+3\nu}{3+\nu}r_i^2\right) = 71.4$MPa

Radial stress is zero at the inner radius

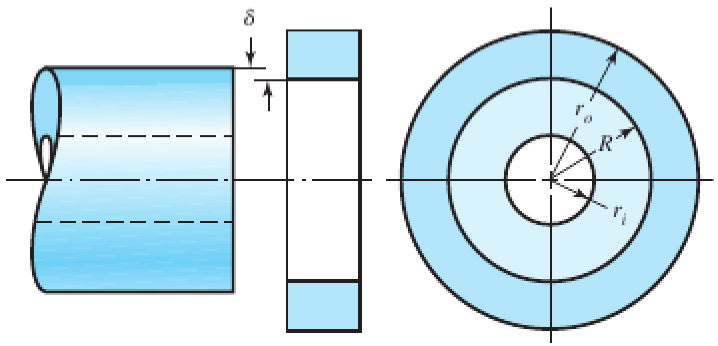




Keys and keyways

Shrink fit

Press & shrink fits



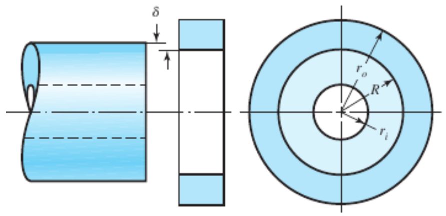
When two cylindrical parts are assembled by shrinking or press fitting one part upon another, a contact pressure is created between the two parts

After assembly, an interference contact pressure *p* develops between the members at the nominal radius *R*, causing stresses at the contacting surfaces

$$p = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2 + R^2}{r_o^2 - R^2} + \nu_0\right) + \frac{1}{E_i}\left(\frac{R^2 + r_i^2}{R^2 - r_i^2} - \nu_i\right)\right]}$$

Note: δ = interference; E =Young's modulus; ν = Poisson's ratio

Press & shrink fits



Interference = δ
Max interference = Biggest shaft − smallest hole
Min interference = Smallest shaft − biggest hole

- For 2 members of the same materials,

$$p = \frac{E\delta}{2R^3}\left[\frac{(r_o^2 - R^2)(R^2 - r_i^2)}{r_o^2 - r_i^2}\right]$$

- For inner member: $p_o = p$ and $p_i = 0$

$$(\sigma_t)_i\Big|_{r=R} = -p\frac{R^2 + r_i^2}{R^2 - r_i^2}$$

- For outer member: $p_o = 0$ and $p_i = p$

$$(\sigma_t)_o\Big|_{r=R} = p\frac{r_o^2 + R^2}{r_o^2 - R^2}$$

Temperature effects

- When heated, an unrestrained body will expand and the normal strain is

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha \Delta T$$

- If a straight bar is restrained at the ends along the *x*-axis so as to prevent lengthwise expansion and is subjected to a uniform increase in temperature ΔT, a compressive stress will develop. The thermal stress is

$$\sigma_x = -\alpha(\Delta T)E$$

Note: α = coefficient of thermal expansion; E = Young's modulus

- If a uniform plate is restrained at the edges along the *x* and *y*-axes and subjected to a uniform increase in temperature ΔT, the compressive stress developed is

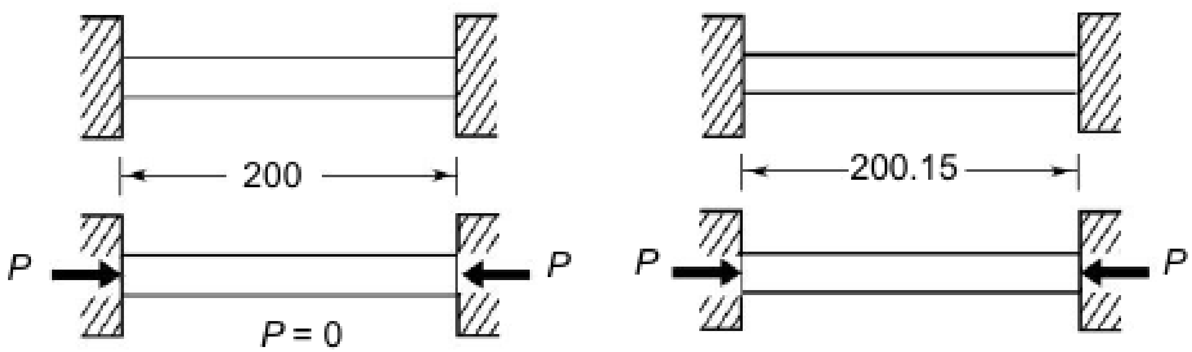
$$\sigma_x = \sigma_y = -\frac{\alpha(\Delta T)E}{1-\nu}$$ where ν = Poisson's ratio

- Similarly, a 3D box restrained to expansion on all 3 sides when subjected to a uniform increase in temperature ΔT,

$$\sigma_x = \sigma_y = \sigma_z = -\frac{\alpha(\Delta T)E}{1-2\nu}$$

Illustrative example 2

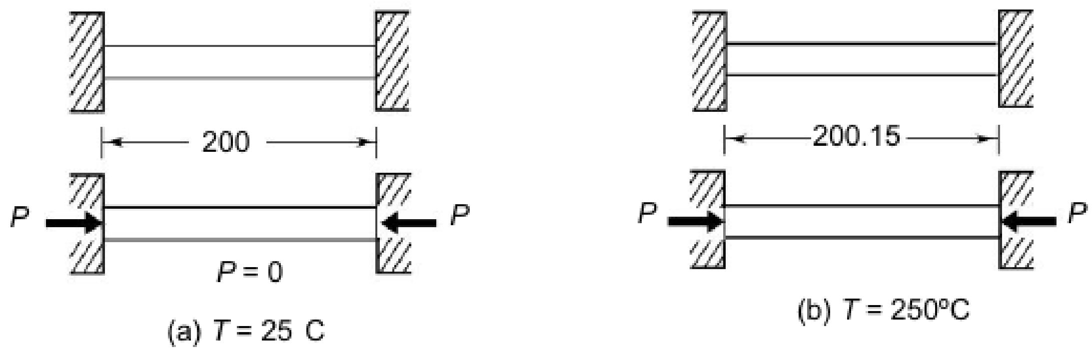
A hollow steel tube is assembled at 25°C with fixed ends as shown in Fig. (a). At this temperature, there is no stress in the tube. The length is 200mm and the cross–sectional area of the tube is 200mm and 300mm^2. During operation, the temperature increases to 250°C. At this temperature, the fixed ends are separated by 0.15 mm as shown in Fig. (b). The modulus of elasticity and coefficient of thermal expansion of steel are 207GPa and 10.8×10^{-6} per °C respectively. Calculate the force acting on the tube and the resultant stress.



(a) T = 25 C

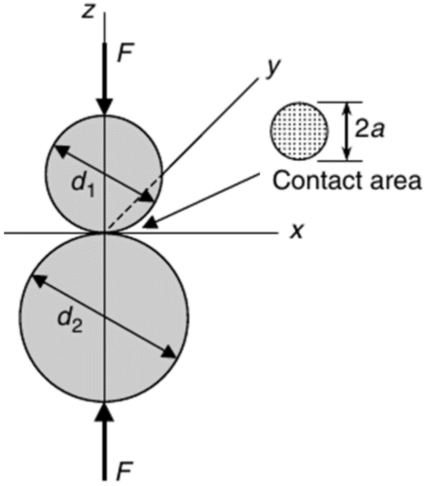
(b) T = 250°C

Illustrative example 2



Given $\Delta T = 250 - 25 = 225°\text{C}$, $\alpha = 10.8 \times 10^{-6}\text{per°C}$, $L = 200\text{mm}$;
Normal strain is $\varepsilon_x = \alpha\Delta T$ and the change in length is $\Delta L = \alpha L \Delta T = 0.486\text{mm}$;
When the tube is unrestrained, the length will increase by 0.486 mm;
However, the fixed ends are separated only by 0.15mm.
The net compression of the tube is $\delta = 0.486 - 0.15 = 0.336\text{mm}$
Restraining force P can be found using $\delta = \frac{PL}{AE}$ where $A = 300\text{mm}^2$, $E = 207\text{GPa}$,
$P = 104328\text{N}$ is the force acting on the tube
The resulting stress is $\sigma = \frac{P}{A} = 347.8\text{MPa}$

Spherical contact



- Common in bearings
- The radius (a) of the area of contact is given by

$$a = \sqrt[3]{\frac{3F}{8}\left\{\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}\right\}}$$

- Maximum pressure at center of contact area:

$$p_{\max} = \frac{3F}{2\pi a^2}$$

E =Young's modulus;
ν = Poisson's ratio

- Principal stresses occurs along z-axis:

$$\sigma_1 = \sigma_2 = \sigma_x = \sigma_y = -p_{\max}\left[\left(1 - \left|\frac{z}{a}\right| \tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{1 + \frac{z^2}{a^2}}$$

Cylindrical contact

The area of contact is a narrow rectangle of width 2b and length L, and the pressure distribution is elliptical. The half-width b is given by

$$b = \sqrt{\frac{2F}{\pi L}\left\{\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}\right\}}$$

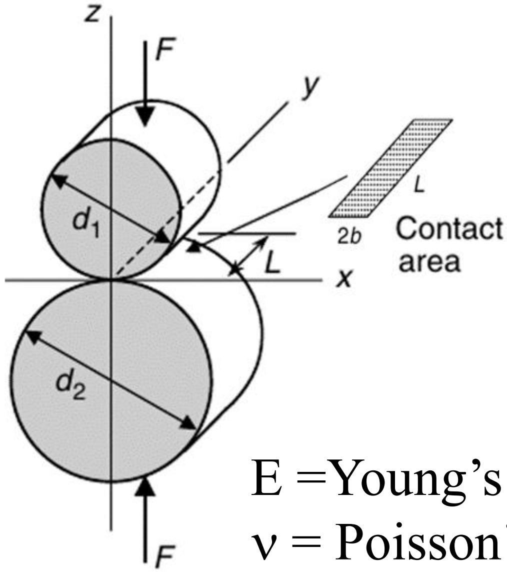
❖ Maximum pressure at center of contact area: $p_{\max} = \dfrac{2F}{\pi b L}$

❖ The stress state along the z-axis:

$$\sigma_x = -2\nu p_{\max}\left[\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right]$$

$$\sigma_y = -p_{\max}\left[\frac{1+2\dfrac{z^2}{b^2}}{\sqrt{1+\dfrac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right]$$

$$\sigma_3 = \sigma_z = \frac{-p_{\max}}{\sqrt{1+z^2/b^2}}$$



E =Young’s modulus;
ν = Poisson’s ratio

Stress concentration

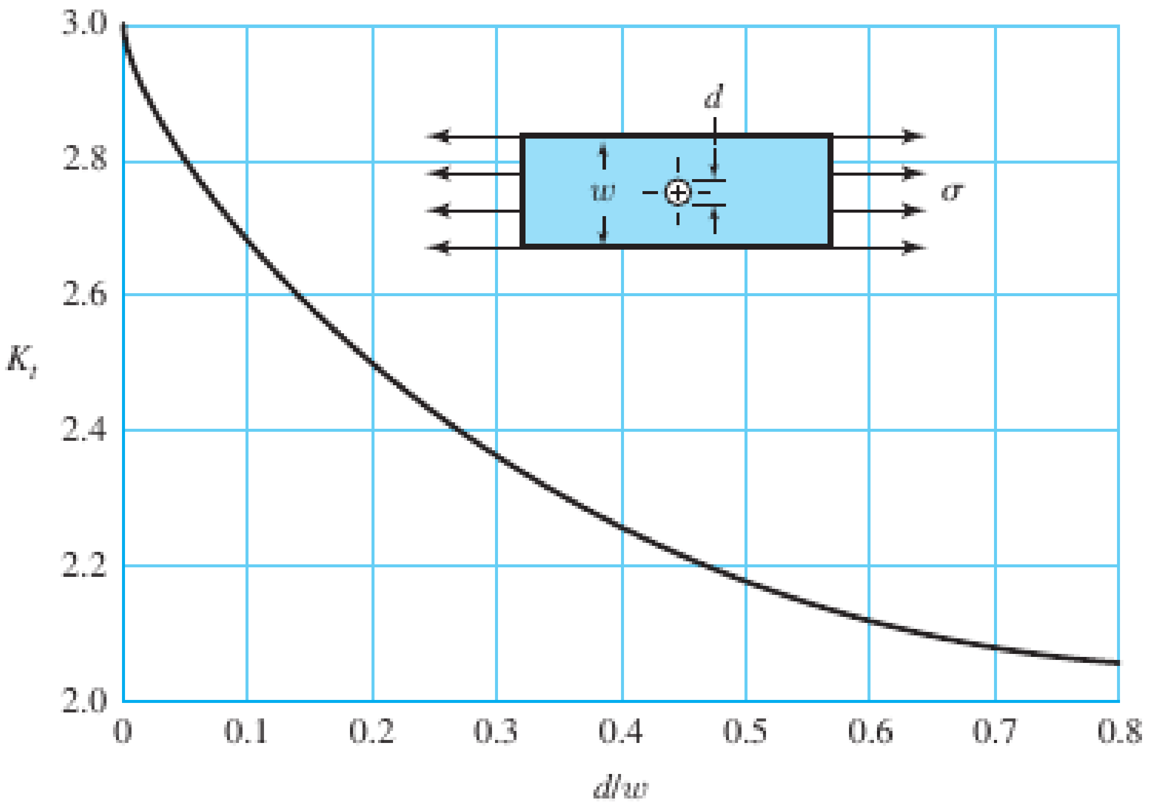
- Geometric irregularities can occur in the member under consideration
- Discontinuity such as holes, notches, fillets, etc. alters the stress distribution and produce stress concentrations in the components
- For ductile materials, stress concentrations are not a problem as the material will deform appropriately to adjust to these stress concentrations
- Brittle materials are very susceptible to stress concentrations, and therefore, stress-concentration factors should always be incorporated
- Theoretical, or geometric, stress-concentration factor K_t or K_{ts} is used to relate the actual maximum stress at the discontinuity to the nominal stress The factors are defined by:

$$\sigma_{max} = K_t \sigma_0 \quad \text{or} \quad \tau_{max} = K_{ts} \tau_0$$

- Nominal stresses σ_0 and τ_0 are calculated by using the elementary stress equations and the net area, or net cross section

Stress concentration factor

❖ (Some stress concentration factors K_t values given in Appendices A-15 & A-16)

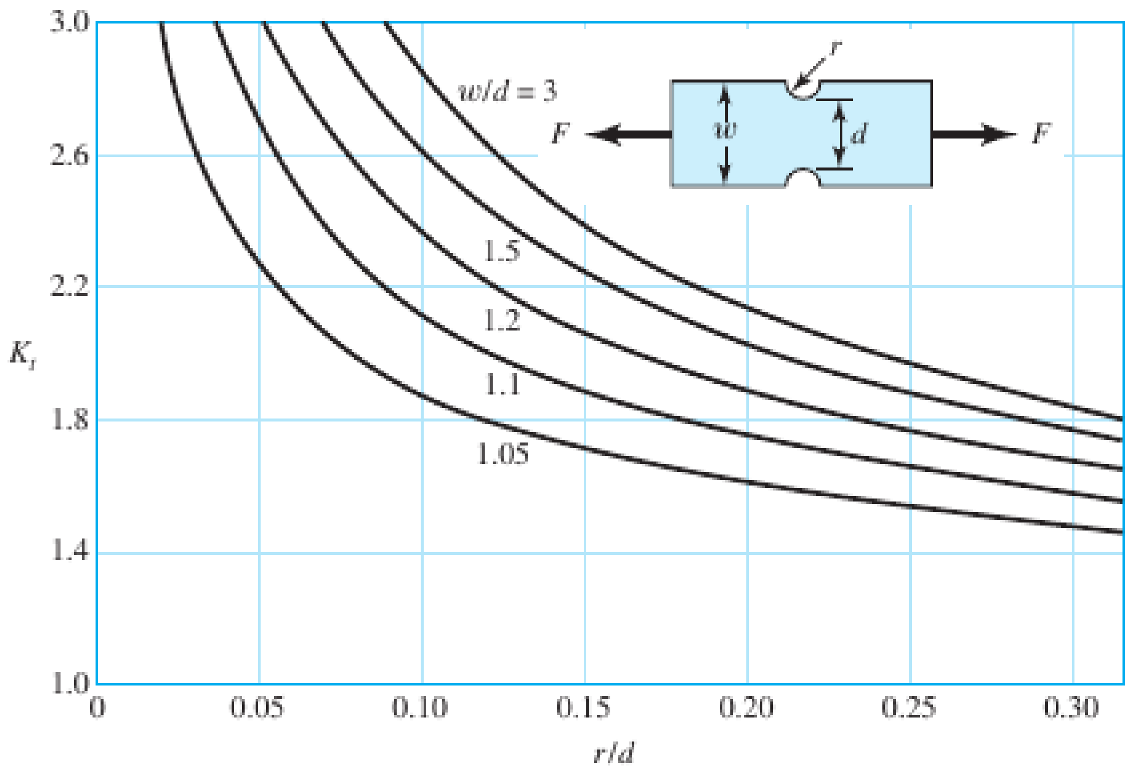


Thin plate in tension or simple compression with a transverse central hole. The net tensile force is $F = \sigma w t$, where t is the thickness of the plate. The nominal stress is given by

$$\sigma_0 = \frac{F}{(w-d)t} = \frac{w}{(w-d)}\sigma$$

Stress concentration factor

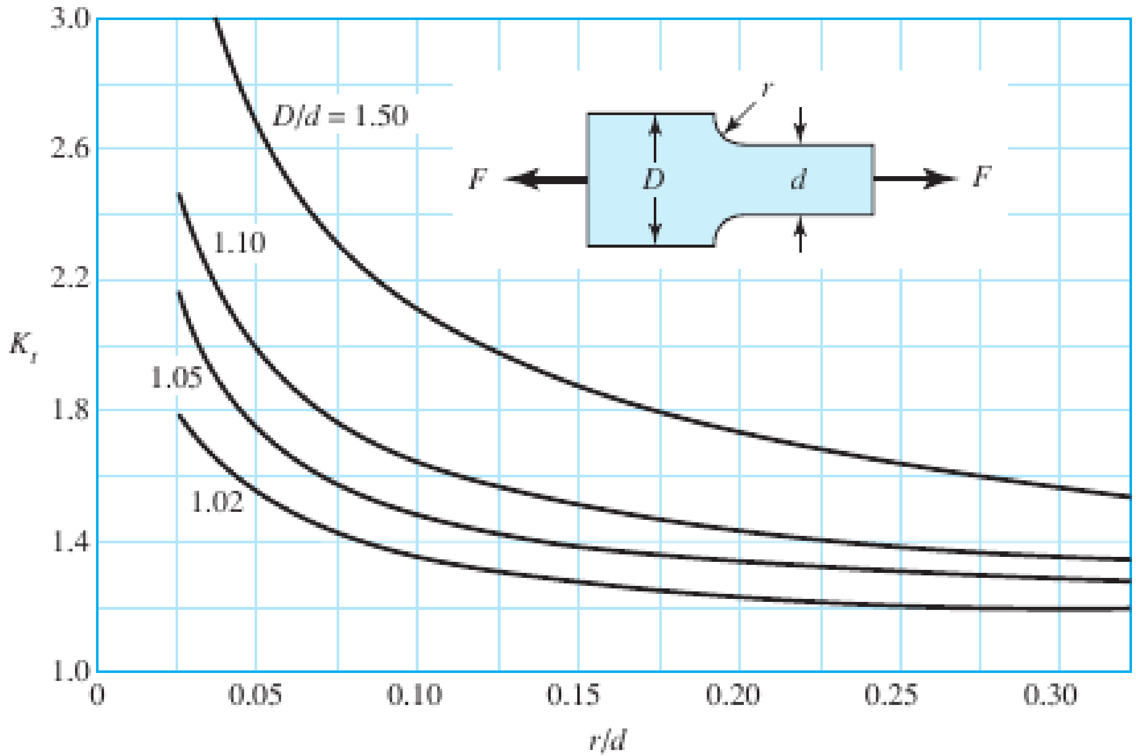
- (Some stress concentration factors K_t values given in Appendices A-15 & A-16)



Notched rectangular bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

Stress concentration factor

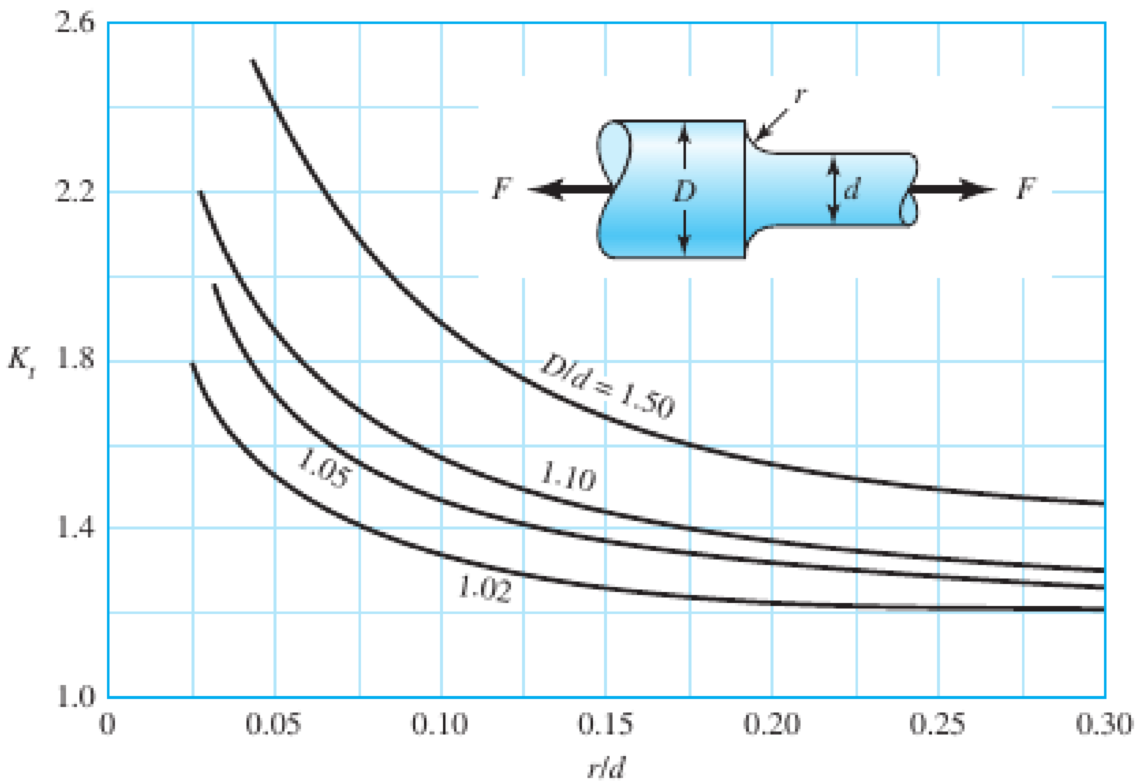
- (Some stress concentration factors K_t values given in Appendices A-15 & A-16)



Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

Stress concentration factor

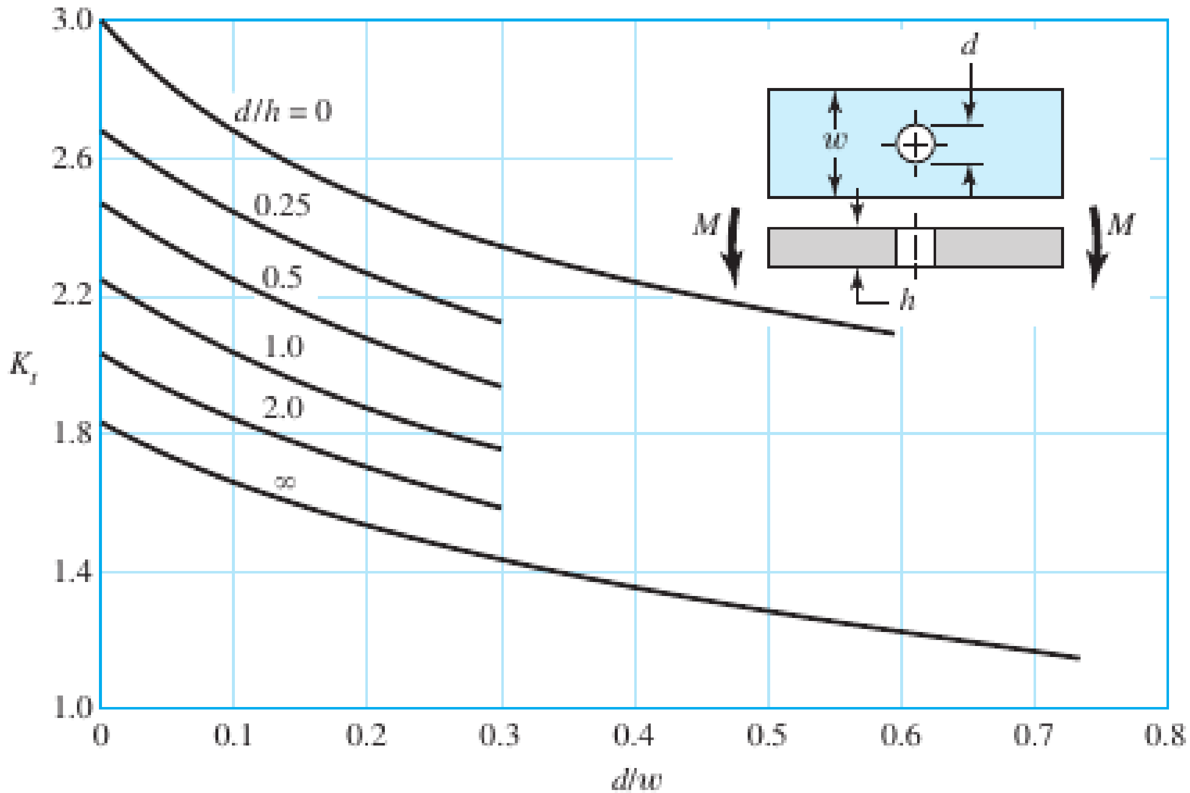
- (Some stress concentration factors K_t values given in Appendices A-15 & A-16)



Round shaft with shoulder fillet in tension. $\sigma_0 = F/A$, where $A = \pi d^2/4$.

Stress concentration factor

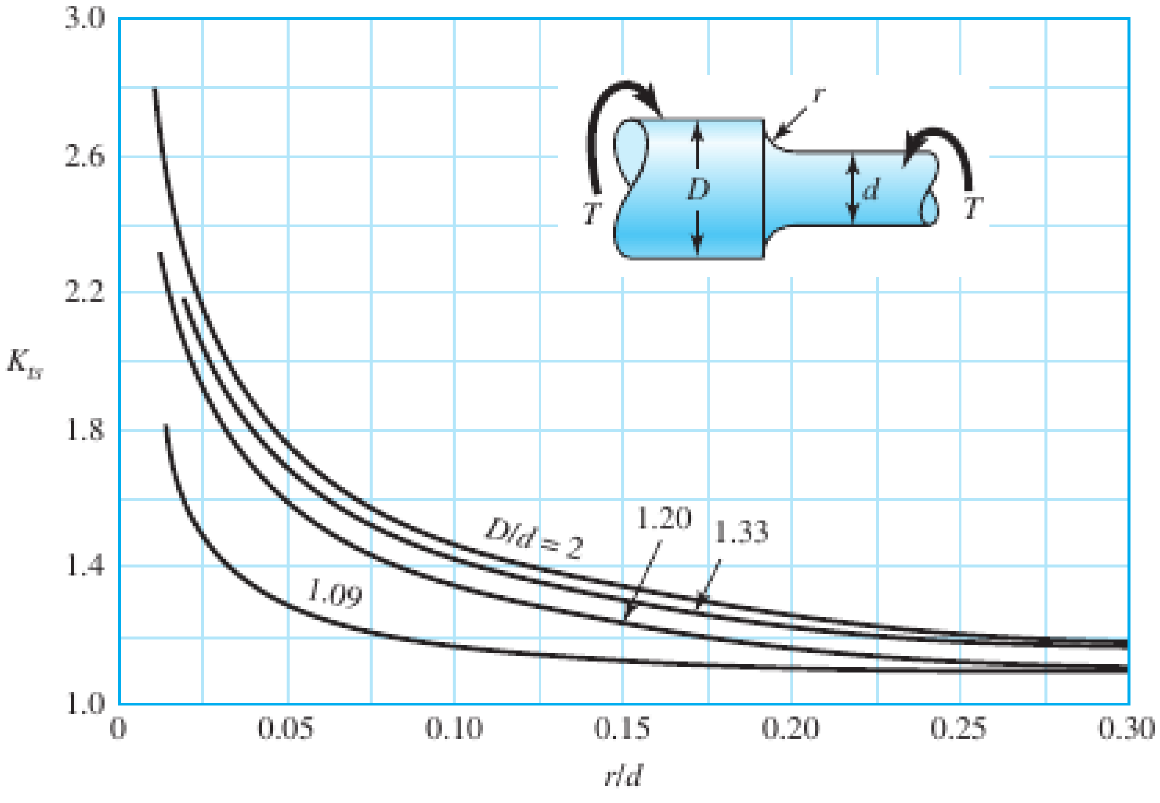
❖ (Some stress concentration factors K_t values given in Appendices A-15 & A-16)



Rectangular bar with a transverse hole in bending. $\sigma_0 = Mc/I$, where $I = (w - d)h^3/12$.

Stress concentration factor

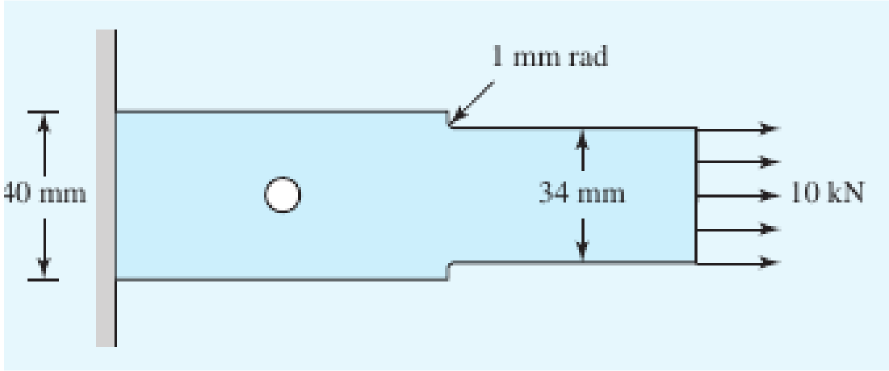
❖ (Some stress concentration factors K_t values given in Appendices A-15 & A-16)



Round shaft with shoulder fillet in torsion. $\tau_0 = Tc/J$, where $c = d/2$ and $J = \pi d^4/32$.

Example 2

The 2-mm-thick bar shown is loaded axially with a constant force of 10 kN. The bar material has been heat treated and quenched to raise its strength, but as a consequence it has lost most of its ductility. It is desired to drill a hole through the center of the 40-mm face of the plate to allow a cable to pass through it. A 4-mm hole is sufficient for the cable to fit, but an 8-mm drill is readily available. Will a crack be more likely to initiate at the larger hole, the smaller hole, or at the fillet?



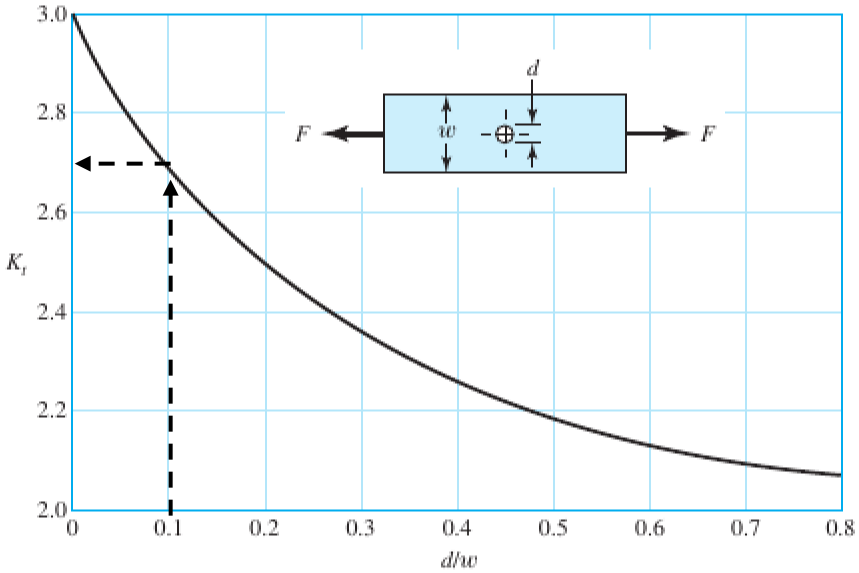
Example 2

Since the material is brittle, the effect of stress concentrations near the discontinuities must be considered. Dealing with the hole first, for a 4-mm hole, the nominal stress is

$$\sigma_0 = \frac{F}{A} = \frac{F}{(w-d)t} = \frac{10000}{(40-4)2} = 139\text{MPa}$$

Using $d/w = 4/40 = 0.1$, stress concentration factor $K_t = 2.7$;

Hence $\sigma_{\max} = K_t\sigma_0 = 2.7(139) = 380\text{MPa}$



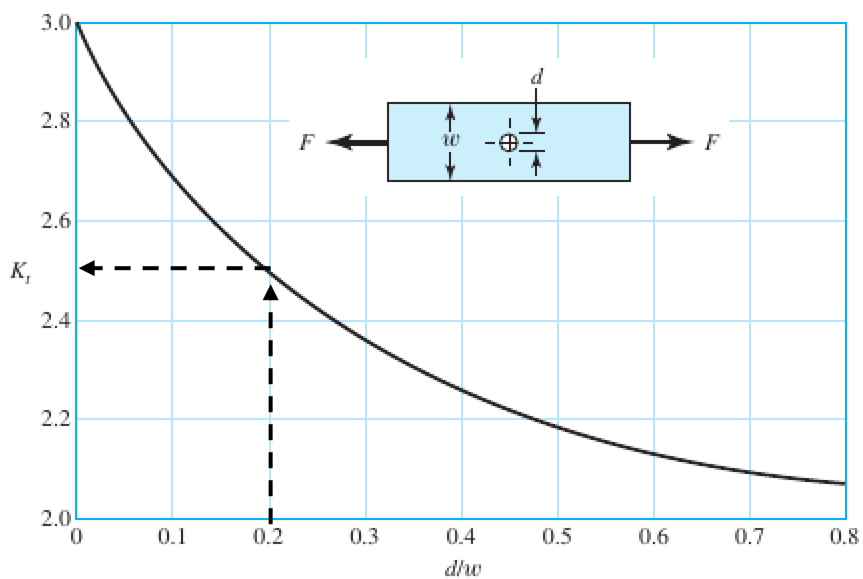
Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

Example 2

For a 8-mm hole, the nominal stress is $\sigma_0 = \frac{F}{A} = \frac{F}{(w-d)t} = \frac{10000}{(40-8)2} = 156\text{MPa}$

Using $d/w = 8/40 = 0.2$, determine the theoretical stress concentration factor $K_t = 2.5$;

Hence $\sigma_{\max} = K_t \sigma_0 = 2.5(156) = 390\text{MPa}$



Bar in tension or simple compression with a transverse hole. $\sigma_0 = F/A$, where $A = (w - d)t$ and t is the thickness.

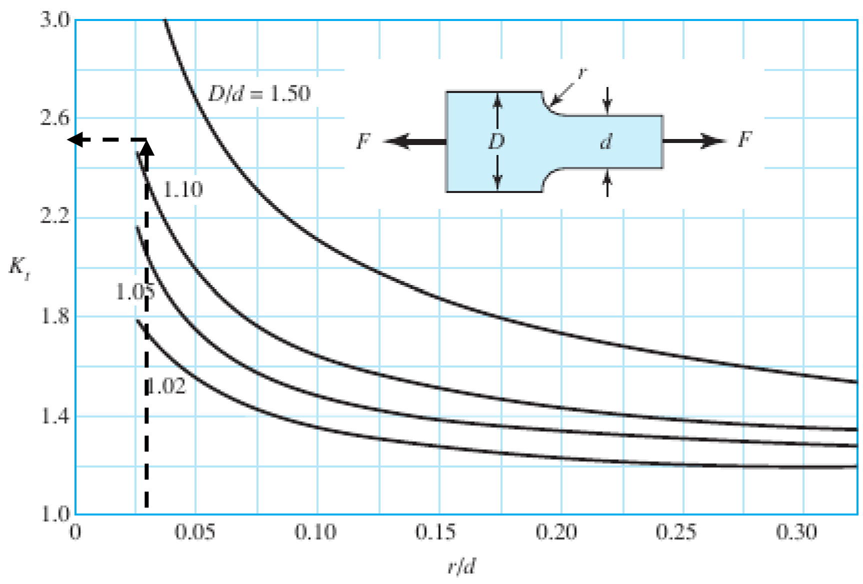
Example 2

For the fillet, the nominal stress is $\sigma_0 = \frac{F}{A} = \frac{F}{(d)t} = \frac{10000}{(34)2} = 147\text{MPa}$

Using $D/d = 40/34 = 1.18$, and $r/d = 1/34 = 0.026$, found $K_t = 2.5$;

Hence $\sigma_{\max} = K_t\sigma_0 = 2.5(147) = 368\text{MPa}$

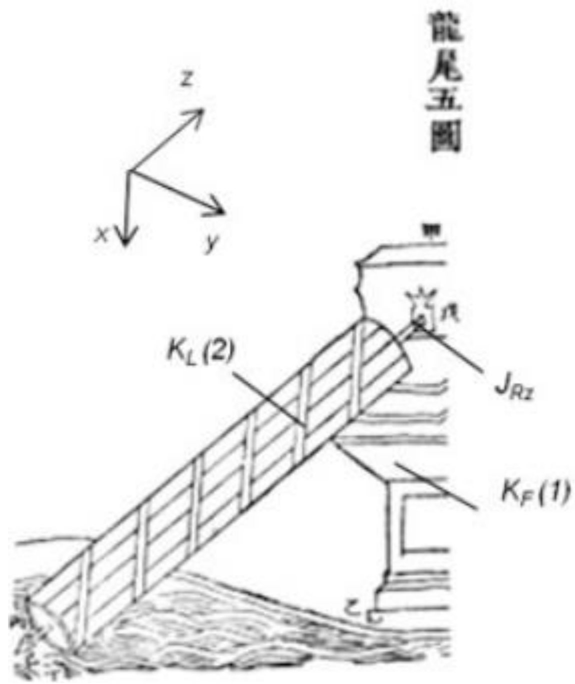
Crack will most likely occur with the 8mm hole, next will be the 4mm hole and the least likely at the fillet



Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.

Ancient Chinese mechanisms

An Archimedean screw (龍尾)



How would you analyse the stress concentrations in the device?