MEMS1028
Mechanical Design 1

Lecture 13

Design codes and misc

Objectives

- Explain the concept of design codes
- Apply ASME BPVC.VIII.1.2019 in the design of simple pressure vessels
- Design leaf springs based on deflections and energy principles
- Design of simple hinges based on angular deflections and stresses
- Explain Miner’s rule in the analysis of varying fluctuating stresses

Introduction

- Safety, reliability and operation efficiency can be achieved by:
  - Proper design and construction
  - Proper maintenance and inspection
  - Proper operation
- Engineering standards ensure safety, reliability and operational efficiency in machine design and mechanical production
- Engineering standards have been established for many industry equipment such as pressure vessels (example: many countries had generated design & construction codes for pressure vessels)

Standards – pressure vessels

Design and Construction Codes for Pressure Vessels

Country	Code	Issuing authority
U.S.	ASME Boiler & Pressure Vessel Code	ASME
U.K.	BS 1515 Fusion Welded Pressure Vessels BS 5500 Unfired Fusion Welded Pressure Vessels	British Standard Institute
Germany	AD Merblatter	Arbeitsgemeinschaft Druckbehalter
Italy	ANCC	Associazione Nationale Per Il Controllo Peula Combustione
Netherlands	Regeis Voor Toestellen	Dienst voor het Stoomvezen
Sweden	Tryckkarls kommissionen	Swedish Pressure Vessel Commission
Australia	AS 1200:SAA Boiler Code AS 1210 Unfired Pressure Vessels	Standards Association of Australia
Belgium	IBN Construction Code for Pressure Vessels	Belgian Standards Institute
Japan	MITI Code	Ministry of International Trade and Industry
France	SNCT Construction Code for Unfired Pressure Vessels	Syndicat National de la Chaudronnerie et de la Tuyauterie Industrielle

Thin cylindrical shells

A simplified equation was developed by the ASME Code, VIII-1 for determining the required thickness of a cylinder for longitudinal joints subjected to internal pressure when the thickness does not exceed one-half of the inside radius, or internal pressure P does not exceed 0.385SE (i.e. $t \le R/2$ or $P \le 0.385SE$):

$$t = \frac{PR}{SE - 0.6P}$$

Where t = required thickness

E = joint efficiency factor (For welded vessels, use the efficiency in UW-12)

P = internal pressure

R = inside radius

S = allowable stress

Note:

$$P = \frac{SEt}{R + 0.6t}$$

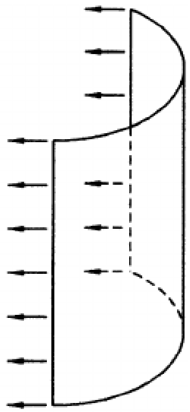


Table UW-12
Maximum Allowable Joint Efficiencies for Welded Joints

Type No.	Joint Description	Limitations	Joint Category	Extent of Radiographic or Ultrasonic Examination [Note (1), Note (2), Note (3)] (a) Full [Note (4)]	(b) Spot [Note (5)]	(c) None
(1)	Butt joints as attained by double-welding or by other means that will obtain the same quality of deposited weld metal on the inside and outside weld surfaces to agree with the requirements of UW-35. Welds using metal backing strips that remain in place are excluded.	None	A, B, C, and D	1.00	0.85	0.70
(2)	Single-welded butt joint with backing strip other than those included under (1)	*(a)* None except as in (b) below	A, B, C, and D	0.90	0.80	0.65
		(b) Circumferential butt joints with one plate offset; see UW-13(b)(4) and Figure UW-13.1, sketch (i)	A, B, and C	0.90	0.80	0.65
(3)	Single-welded butt joint without use of backing strip	Circumferential butt joints only, not over $^5/_8$ in. (16 mm) thick and not over 24 in. (600 mm) outside diameter	A, B, and C	NA	NA	0.60
(4)	Double full fillet lap joint	*(a)* Longitudinal joints not over $^3/_8$ in. (10 mm) thick	A	NA	NA	0.55
		(b) Circumferential joints not over $^5/_8$ in. (16 mm) thick	B and C [Note (6)]	NA	NA	0.55
(5)	Single full fillet lap joints with plug welds conforming to UW-17	*(a)* Circumferential joints [Note (7)] for attachment of heads not over 24 in. (600 mm) outside diameter to shells not over $^1/_2$ in. (13 mm) thick	B	NA	NA	0.50
		(b) Circumferential joints for the attachment to shells of jackets not over $^5/_8$ in. (16 mm) in nominal thickness where the distance from the center of the plug weld to the edge of the plate is not less than 1$^1/_2$ times the diameter of the hole for the plug.	C	NA	NA	0.50
(6)	Single full fillet lap joints without plug welds	*(a)* For the attachment of heads convex to pressure to shells not over $^5/_8$ in. (16 mm) required thickness, only with use of fillet weld on inside of shell; or	A and B	NA	NA	0.45
		(b) for attachment of heads having pressure on either side, to shells not over 24 in. (600 mm) inside diameter and not over $^1/_4$ in. (6 mm) required thickness with fillet weld on outside of head flange only	A and B	NA	NA	0.45
(7)	Corner joints, full penetration, partial penetration, and/or fillet welded	As limited by Figure UW-13.2 and Figure UW-16.1	C and D [Note (8)]	NA	NA	NA
(8)	Angle joints	Design per U-2(g) for Category B and C joints	B, C, and D	NA	NA	NA

GENERAL NOTE: *E* = 1.00 for butt joints in compression.

NOTES:

(1) Some welding processes require ultrasonic examination in addition to radiographic examination, and other processes require ultrasonic examination in lieu of radiographic examination. See UW-11 for some additional requirements and limitations that may apply.

(2) Joint efficiency assignment rules of UW-12(d) and UW-12(e) shall be considered and may further reduce the joint efficiencies to be used in the required thickness calculations.

(3) The rules of UW-12(f) may be used in lieu of the rules of this Table at the Manufacturer's option.

(4) See UW-12(a) and UW-51.

(5) See UW-12(b) and UW-52.

(6) For Type No. 4 Category C joint, limitation not applicable for bolted flange connections.

(7) Joints attaching hemispherical heads to shells are excluded.

(8) There is no joint efficiency *E* in the design equations of this Division for Category C and D corner joints. When needed, a value of *E* not greater than 1.00 may be used.

Thin cylindrical shells

A simplified equation was developed by the ASME Code, VIII-1 for determining the required thickness of a cylinder for circumferential joints subjected to internal pressure when the thickness does not exceed one-half of the inside radius, or internal pressure P does not exceed 1.25SE (i.e. $t \leq R/2$ or $P \leq 1.25SE$):

$$t = \frac{PR}{2SE + 0.4P}$$

Where t = required thickness

E = joint efficiency factor (For welded vessels, use the efficiency in UW-12)

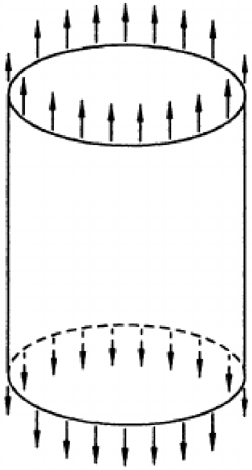
P = internal pressure

R = inside radius

S = allowable stress

Note:

$$P = \frac{2SEt}{R - 0.4t}$$



Thin spherical shells

A simplified equation was developed by the ASME Code, VIII-1 for determining the required thickness of a spherical shells subjected to internal pressure when the thickness does not exceed 0.356 of the inside radius, or internal pressure P does not exceed 1.25SE (i.e. $t \leq 0.356R$ or $P \leq 0.665SE$):

$$t = \frac{PR}{2SE - 0.2P}$$

Where t = required thickness

E = joint efficiency factor (For welded vessels, use the efficiency in UW-12)

P = internal pressure

R = inside radius

S = allowable stress

Note:

$$P = \frac{2SEt}{R + 0.2t}$$

Thick cylindrical shells

When the thickness of the cylindrical shell under internal design pressure exceeds one-half of the inside radius, or when internal pressure P exceeds 0.385SE, the following equations shall apply for longitudinal joints (i.e. $t > R/2$ or $P > 0.385SE$):

$$t = R\left(Z^{1/2} - 1\right) = R_o\left(\frac{Z^{1/2} - 1}{Z^{1/2}}\right)$$

Where $Z^{1/2} = \left(\frac{P}{SE} + 1\right)$

$$P = SE(Z - 1)$$

Where

$$Z = \left(R + \frac{t}{R}\right)^2 = \left(\frac{R_o}{R}\right)^2 = \left(\frac{R_o}{R_o - t}\right)^2$$

Thick cylindrical shells

When the thickness of the cylindrical shell under internal design pressure exceeds one-half of the inside radius, or when internal pressure P exceeds $1.25SE$, the following equations shall apply for circumferential joints (i.e. $t > R/2$ or $P > 1.25SE$):

$$t = R\left(\exp\left\{\frac{P}{SE}\right\} - 1\right) = R_o\left(1 - \exp\left\{\frac{-P}{SE}\right\}\right)$$

Where R_o = outside radius and

$$P = SE\ln\left\{\frac{R+t}{R}\right\} = SE\ln\left\{\frac{R_o}{R_o - t}\right\}$$

Thick spherical shells

When the thickness of the spherical shell under internal design pressure exceeds 0.365 of the inside radius, or when internal pressure *P* exceeds 0.665*SE*, the following equations shall apply (i.e. $t > 0.356R$ or $P > 0.665SE$):

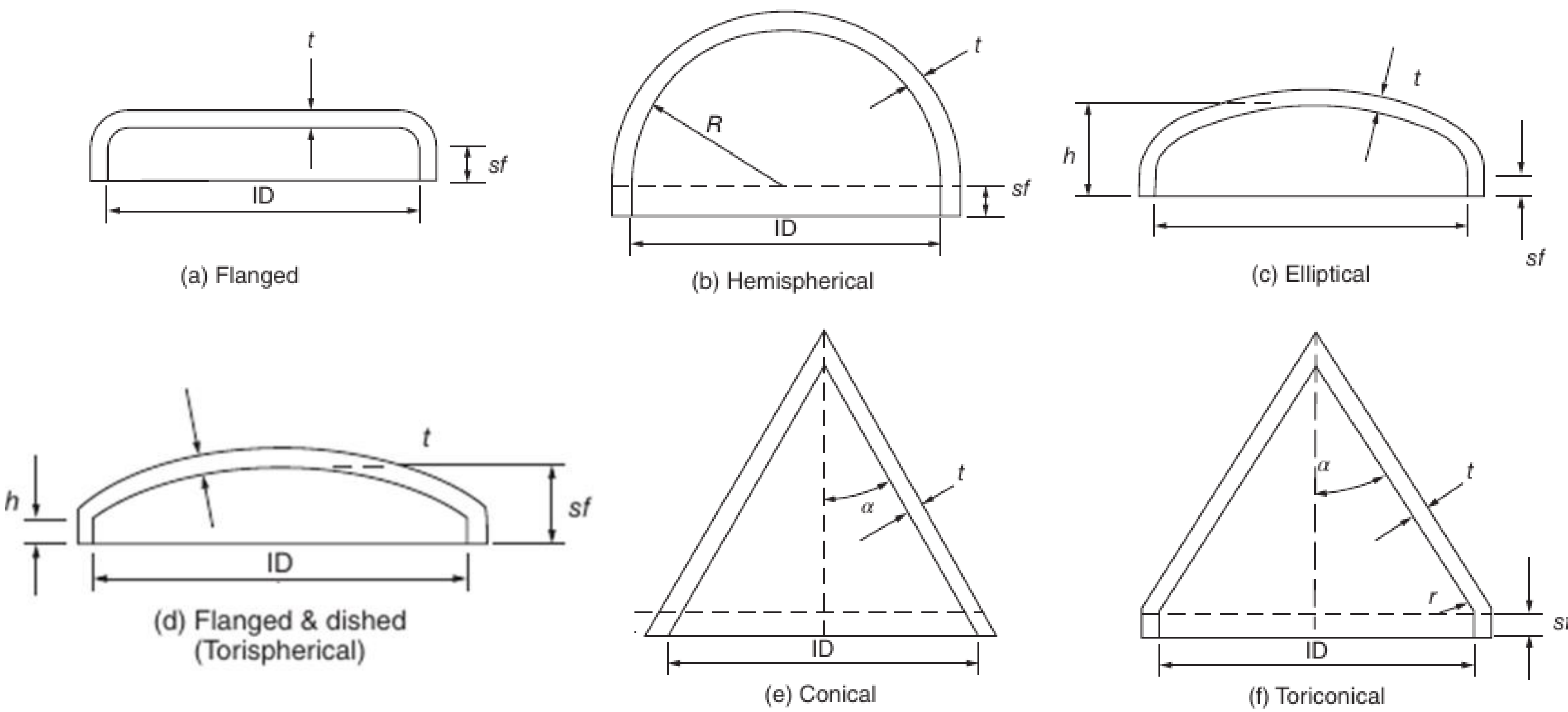
$$t = R\left(\exp\left\{\frac{0.5P}{SE}\right\} - 1\right) = R_o\left(1 - \exp\left\{\frac{-0.5P}{SE}\right\}\right)$$

Where R_o = outside radius and

$$P = 2SE\ln\left\{\frac{R+t}{R}\right\} = 2SE\ln\left\{\frac{R_o}{R_o - t}\right\}$$

Formed heads

- A large variety of end closures and transition sections are available
- Which configuration to use depends on many factors such as method of forming, material cost, and space restrictions

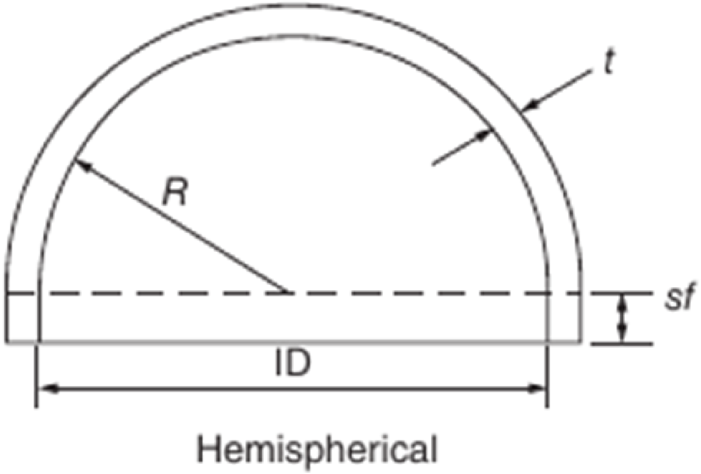


Hemispherical Heads

- Heads are welded to ends of the vessels before operation
- Heads are normally made from the same material as the shell
- The thickness of a hemispherical head is given by:

$$t = \frac{PR}{2SE - 0.2P}$$

Where t = required thickness
E = joint efficiency
P = internal pressure
R = inside radius
S = allowable stress
Note:



$$P = \frac{2SEt}{R + 0.2t}$$

Ellipsoidal heads

❖ The thickness of an ASME ellipsoidal head is given by:

$$t = \frac{PD}{2SE - 0.2P}$$

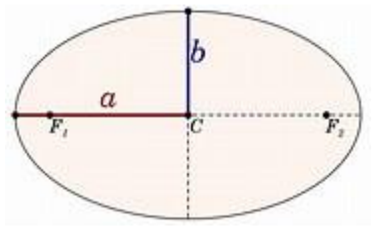
where t = required thickness

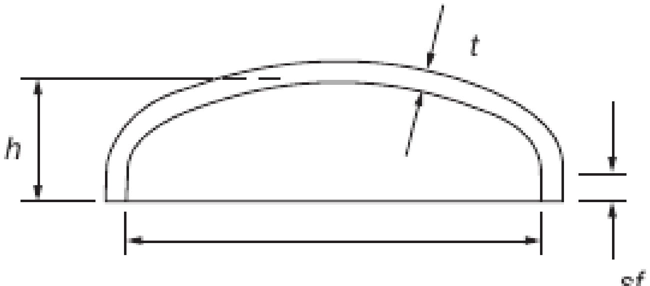
E = joint efficiency

P = internal pressure

D = inside diameter

S = allowable stress





(c) Elliptical

For theASME ellipsoidal head $\left(\frac{a}{b}\right) = \sqrt{2}$

where a and b are the semi-major and semi-minor axes of the ellipse

Note:

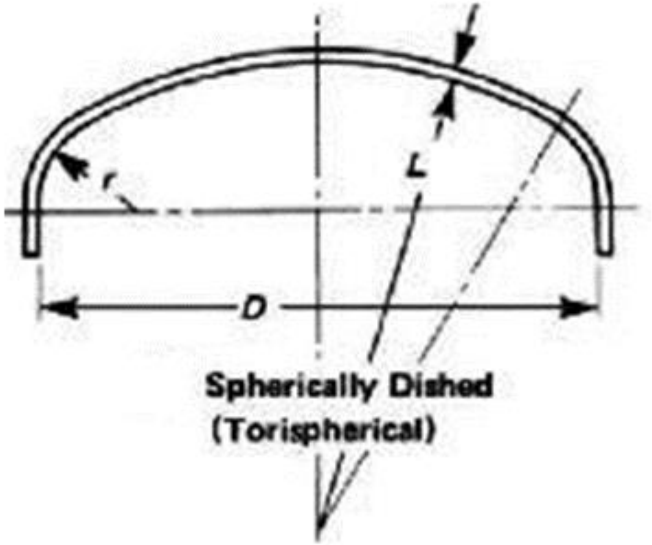
$$P = \frac{2SEt}{D + 0.2t}$$

Torispherical heads

❖ The thickness of an ASME torishperical head is given by:

$$t = \frac{0.885PL}{SE - 0.1P}$$

where t = required thickness
E = joint efficiency
P = internal pressure
S = allowable stress
L = spherical cross radius



For the ASME torispherical head, the knuckle r
Note:

$$P = \frac{SEt}{0.885L + 0.1t}$$

Illustrative example 1

A hemispherical head having an inside radius of 380mm is subjected to an internal pressure of 28MPa. This allowable stress is 160MPa. What is the required thickness using the shell theory and the ASME code (assume joint efficiency, E = 1)?

For spherical vessel, the shell theory for stress: $\sigma_1 = \sigma_2 = \frac{pr}{2t}$

$$t = \frac{pr}{2\sigma_{all}} = \frac{28(10^6)(0{,}38)}{2(160)(10^6)} = 33.25\text{mm}$$

Note: $p = 28 < 0.385SE = 61.6$ MPa;

Using ASME code: $t = \frac{PR}{2SE-0.2P} = \frac{28(10^6)(0{,}38)}{2(160)(10^6)-0.2(28)(10^6)} = 33.8\text{mm}$

The ASME estimate is conservative in this case

Flat heads & covers

- Flat heads and covers are used widely as closures to pressure vessels
- They are either integrally formed with the shell, or may be attached by bolts
- A stayed head has stay rods/bolts attached to the head to prevent it from deforming or failing from either internal or external pressure
- An unstayed head resists forces solely on its own strength
- The minimum required thickness of circular, unstayed flat circular heads and covers without bolting shall be calculated by: $t = d\sqrt{CP/SE}$

where d = effective diameter of head,

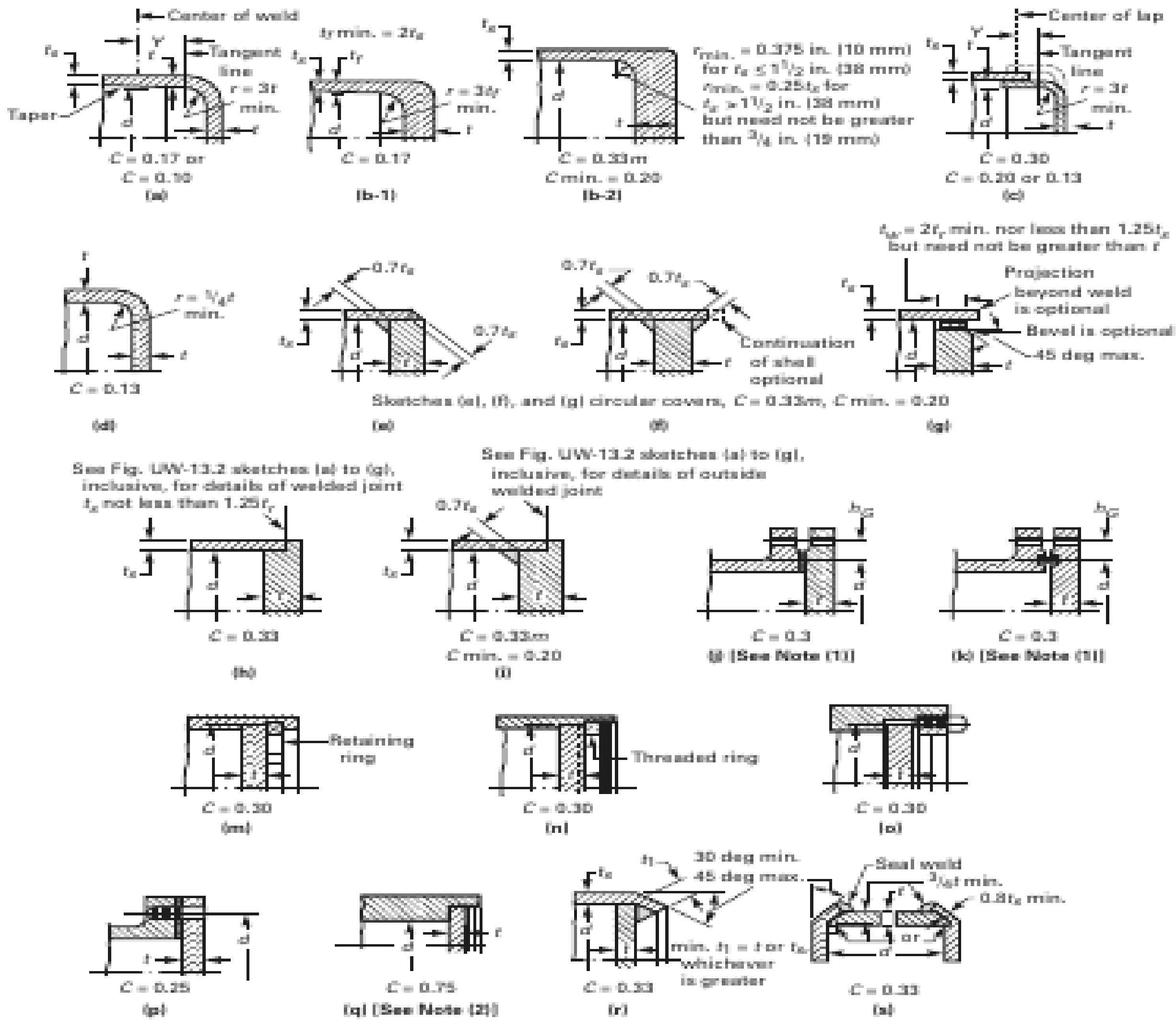
E = joint efficiency;

P = internal pressure;

S = allowable stress;

C = 0.10 through 0.33, depending upon the construction details at the head-to-shell juncture

Figure UG-34
Some Acceptable Types of Unstayed Flat Heads and Covers



GENERAL NOTE: The above sketches are diagrammatic only. Other designs that meet the requirements of UG-34 are acceptable.

NOTES:
(1) Use UG-34(c)(2) eq. (2) or UG-34(c)(3) eq. (5).
(2) When pipe threads are used, see Table UG-43.

Illustrative example 2

A circular plate of diameter 1m, forms the cover for a cylindrical pressure vessel subjected to a pressure of 0.04MPa. Determine the thickness of the head if the allowable stress in the material is limited to 120MPa (with Poisson ratio 0.3) assuming unstayed cover using ASME code with *C*=0.33 and *E*=1. Compare findings with the thickness obtained by assuming the plate is simply supported by the edges. For a simply supported circular plate of radius *R* and thickness *t* subjected to uniform pressure *P*, the maximum stress is given by

$$\sigma_{max} = \frac{3(3+\nu)}{8}\frac{PR^2}{t^2}$$

Assuming simply supported plate:

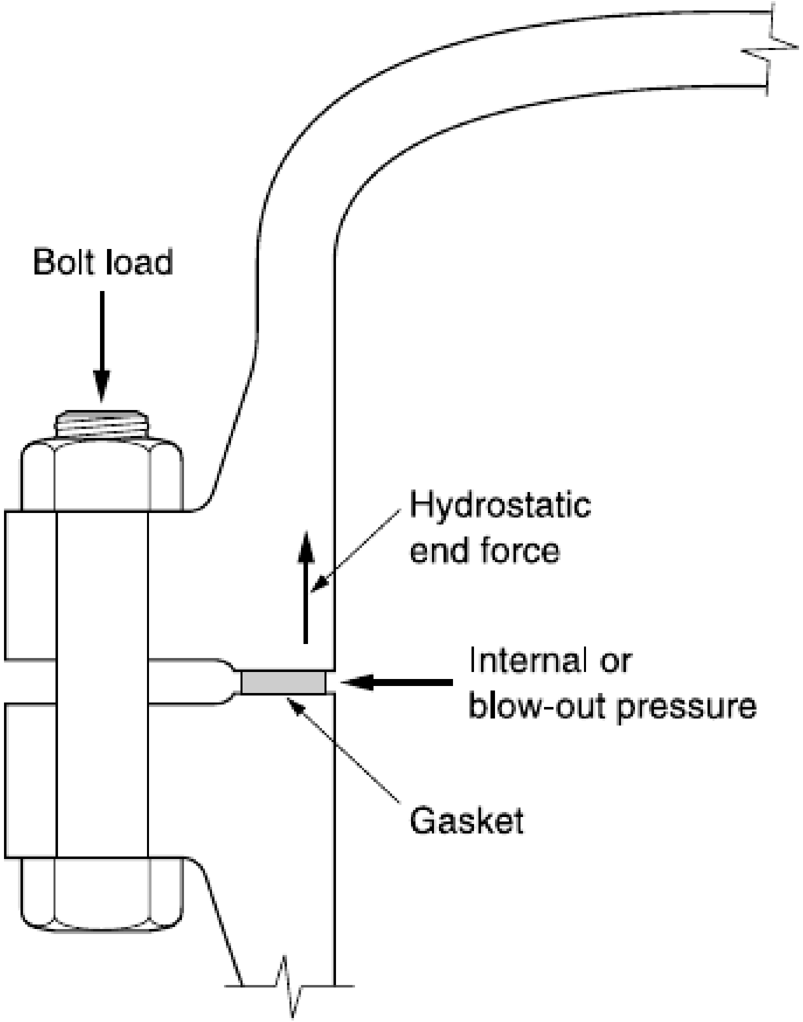
$$\sigma_{max} = \frac{3(3+\nu)}{8}\frac{PR^2}{t^2} = \frac{3(3+0.3)}{8}\frac{0.04(10^6)(0.5^2)}{t^2} = 120(10^6)$$

$$t = 10.16\text{mm}$$

Using *C*=0.33 in ASME code: $t = d\sqrt{CP/SE} = 10.49\text{mm}$

Bolted flange joint

- The flange is a seat for the gasket
- The cover and the gasket is secured to the flange by a number of bolts
- The preload on the bolts must be sufficiently large to seat the gasket but not excessive enough to crush it
- The bolts is designed to contain the preload pressure required to prevent leakage through the gasket
- The flange region is designed to resist bending that occurs in the spacing between the bolt locations



Bolted flange joint

- In the absence of fluid pressure, the required bolt load to seat the gasket is given by

$$W_{m2} = \pi b G y$$

where b = effective gasket seating width;
W_{m2} = gasket seating load (without fluid pressure);
G = minimum gasket diameter;
and y = gasket seating stress (dependent on the gasket type and material);

- In the presence of fluid pressure, the required bolt load to seat the gasket is given by

$$W_{m1} = \frac{\pi}{4} G^2 p + (2b\pi G m p)$$

where p = design pressure;
W_{m1} = gasket seating load (with fluid pressure);
and m = gasket factor (dependent on the gasket type and material);

Bolted flange joint

- The chosen bolt material should be compatible with the flange material
- (i.e. no chemical reaction between the materials)
- The total minimum required cross-sectional area of the bolts should be the greater of the following areas:

$$\frac{W_{m2}}{S_a} \text{ and } \frac{W_{m1}}{S_b}$$

S_a = allowable bolt stress at room temperature
S_b = allowable bolt stress at operating temperature

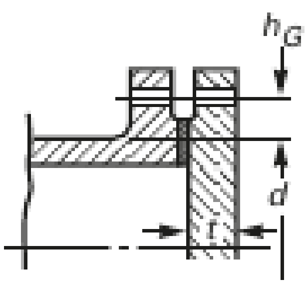
Bolted flange joint

- To determine the thickness of the blind flanges, we must compare the thickness under 2 conditions and select the larger value;
  - The internal pressure equals zero, and the only load is the gasket seating load W_a at ambient temperature with the allowable tensile stress of S_a:

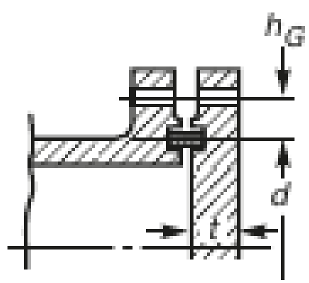
$$t = d\sqrt{1.9(W_a h_G / S_a E d^3)}$$

  - The internal pressure P and gasket loading are applied with W_{m1} at operating temperature with an allowable tensile stress of S_b:

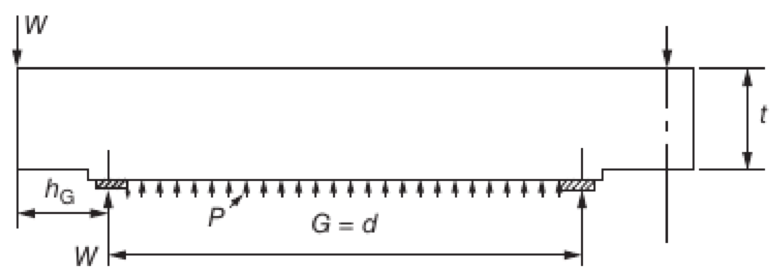
$$t = d\sqrt{0.3(P/S_b E) + 1.9(W_{m1} h_G / S_b E d^3)}$$



C = 0.3
(j) [See Note (1)]



C = 0.3
(k) [See Note (1)]



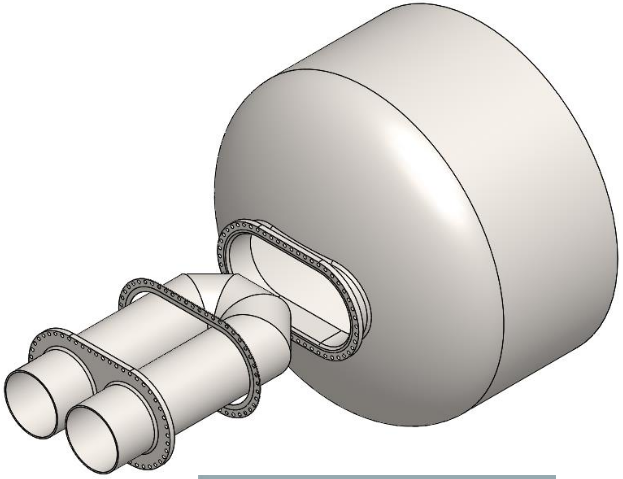
Openings

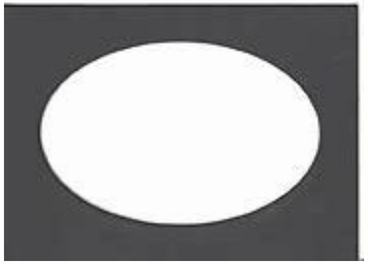
- Three main types:

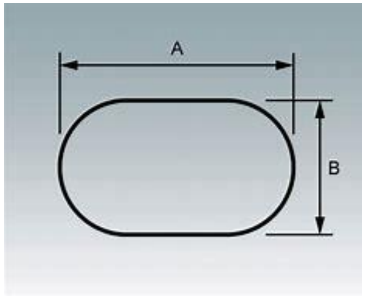
Circular

Elliptical

Obround





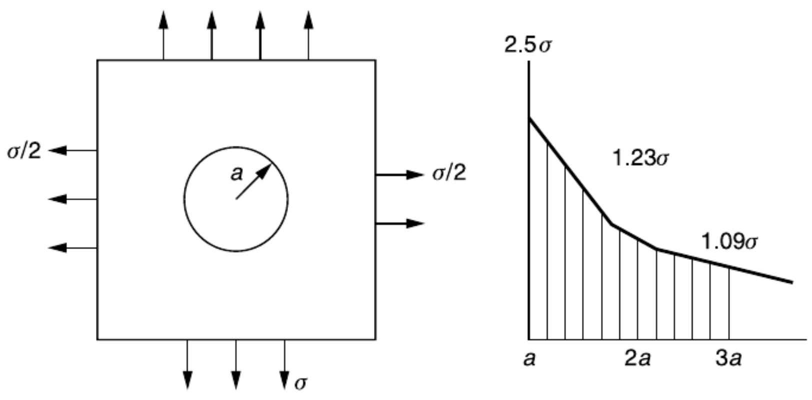


Openings

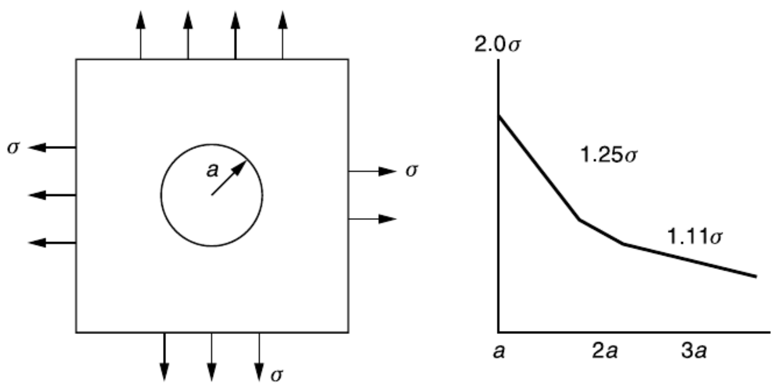
- When the long dimension of an elliptical or obround opening exceeds twice the short dimensions, the reinforcement across the short dimensions shall be increased as necessary to provide against excessive distortion due to twisting moment
- For shells with inside diameter D of 60 in. (1520mm) and less, the opening shall not exceed 0.5D or 20 in. (510 mm)
- For shells with inside diameter D over 60 in. (1520mm), the opening shall not exceed 0.3D or 40 in. (1020 mm)
- When these size limits are exceeded, there are addition rules to be met These additional rules may require some reinforcement to be placed closer to the opening

Reinforcement of openings

- Add material around opening by thickening the shell
- Most reinforcement provided on the outside of the vessel
- The boundary limit for the effective reinforcement is the distance of where stress die out significantly



Hole in cylindrical shell

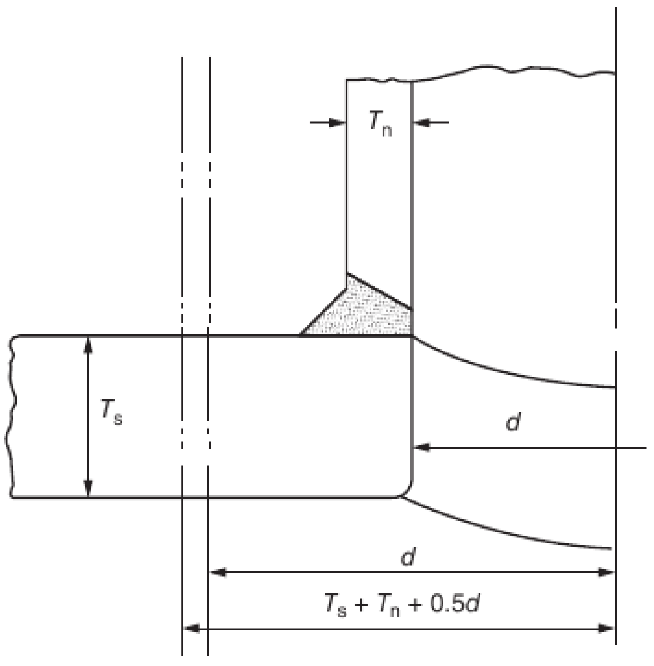


Hole in spherical shell

Reinforcement of openings

When the size of the opening is within the defined limits, the limits of reinforcement parallel to the shell surface measured on each side of the center line is the larger of (1) d or (2) $T_s + T_n + 0.5d$
where "d" is diameter of circular opening; T_s = nominal thickness of shell; T_n = nominal thickness of nozzle wall

The limit of reinforcement perpendicular or normal to the shell measured either inward or outward from the surface of the shell is the smaller of (1) $2.5T_s$ or (2) $2.5T_n$



Area of reinforcement

- For circular opening, total required cross-sectional area of reinforcement in the plane of consideration is $A = dt_r$
- Available Area of Reinforcement for circular opening:

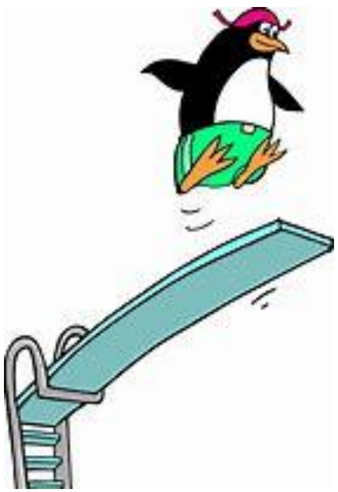
a) Reinforcement area available in the shell wall is $A_1 = (2d - d)(T_s - t_r)$
b) Reinforcement area available in the nozzle wall is $A_2 = (5T_s)(T_n - t_{rn})$
c) Reinforcement area available in the inward nozzle is $A_3 = (5T_s)(T_n)$

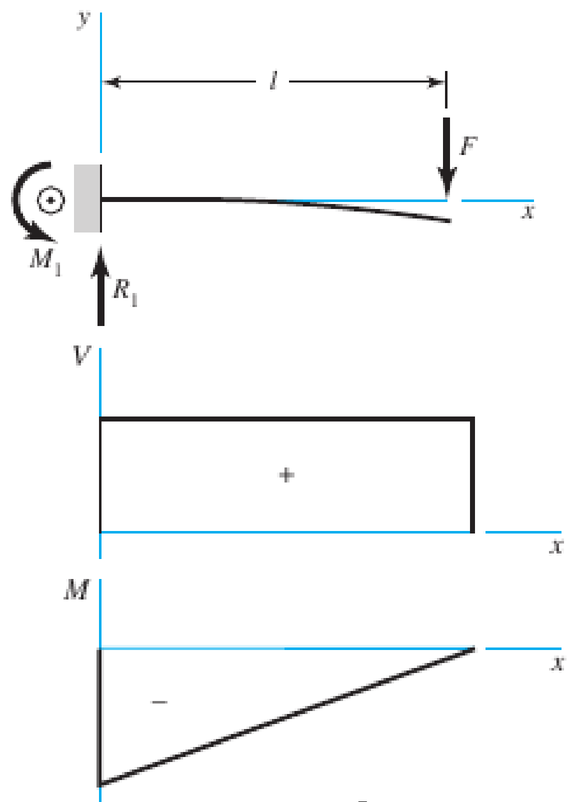
where "d" is diameter of circular opening;
T_s = nominal thickness of shell;
T_n = nominal thickness of nozzle wall;
"t_r" is the minimum required thickness of a seamless shell based on the circumferential stress;
"t_{rn}" is required thickness of a seamless nozzle wall;

- For more complex reinforcement, please refer to the full ASME BPVC.VIII.1.2019

Spring rates

Beam deflections can be used as spring: e.g. diving board





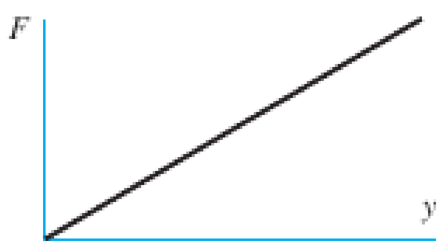
$$R_1 = V = F \qquad M_1 = Fl$$

$$M = F(x - l)$$

$$y = \frac{Fx^2}{6EI}(x - 3l)$$

$$y_{max} = -\frac{Fl^3}{3EI}$$

Assume F-y is linear at L

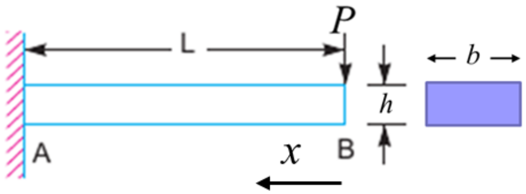


At L: spring rate is

$$k = \frac{F}{y} = \frac{3EI}{L^3}$$

See Appendix A-9

Cantilever beam



Consider a beam with cross-section $b \times h$:

- Along the beam, bending moment varies for $0 \leq x \leq L$:

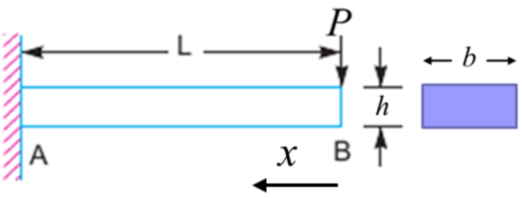
$$M = Px$$

- At each section, max flexure stress occurs at a section top and bottom
- Moment of inertia $I = \frac{1}{12} bh^3$
- Section modulus $Z = \frac{I}{c} = \frac{(bh^3/12)}{(h/2)} = \frac{bh^2}{6}$
- Given distance x of a generic end cross-section, the value of σ_{max} at that section will be

$$\sigma = \frac{Mc}{I} = \frac{M}{Z} = \frac{6Px}{bh^2}$$

- Stress along the beam is not constant

Cantilever beam



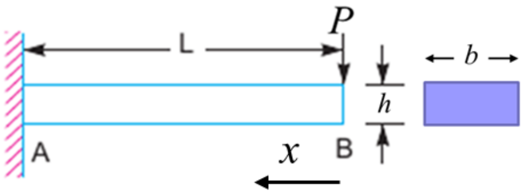
$$\sigma = \frac{6Px}{bh^2}$$

To keep the stress σ constant along the beam, there are two possibilities:

- Keep *h* constant and change *b*
- Keep *b* constant and change *h*

Cantilever beam

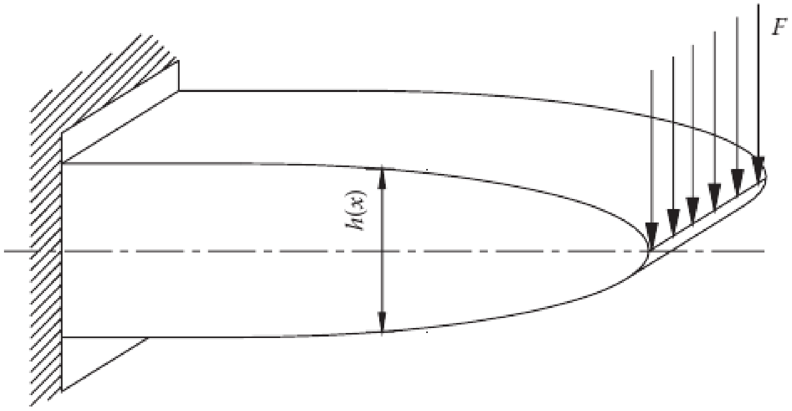
$$\sigma = \frac{6Px}{bh^2}$$



- Investigate keeping *b* constant and change *h* so that σ remain constant

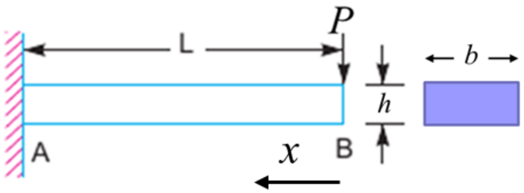
$$h^2 = \frac{6Px}{b\sigma}$$

- Height *h* must vary as follow:



Cantilever beam

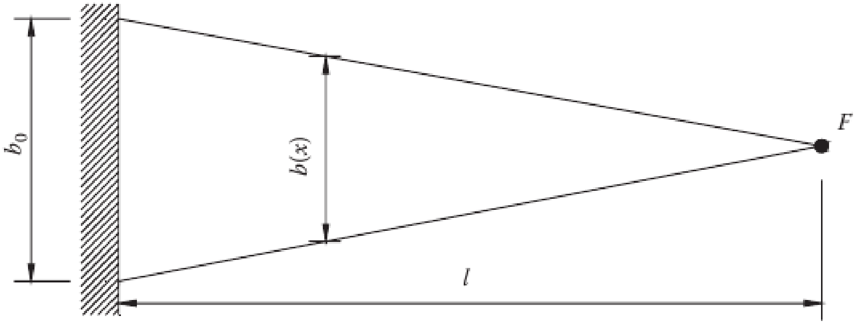
$$\sigma = \frac{6Px}{bh^2}$$



❖ Investigate keeping h constant and change b so that σ remain constant

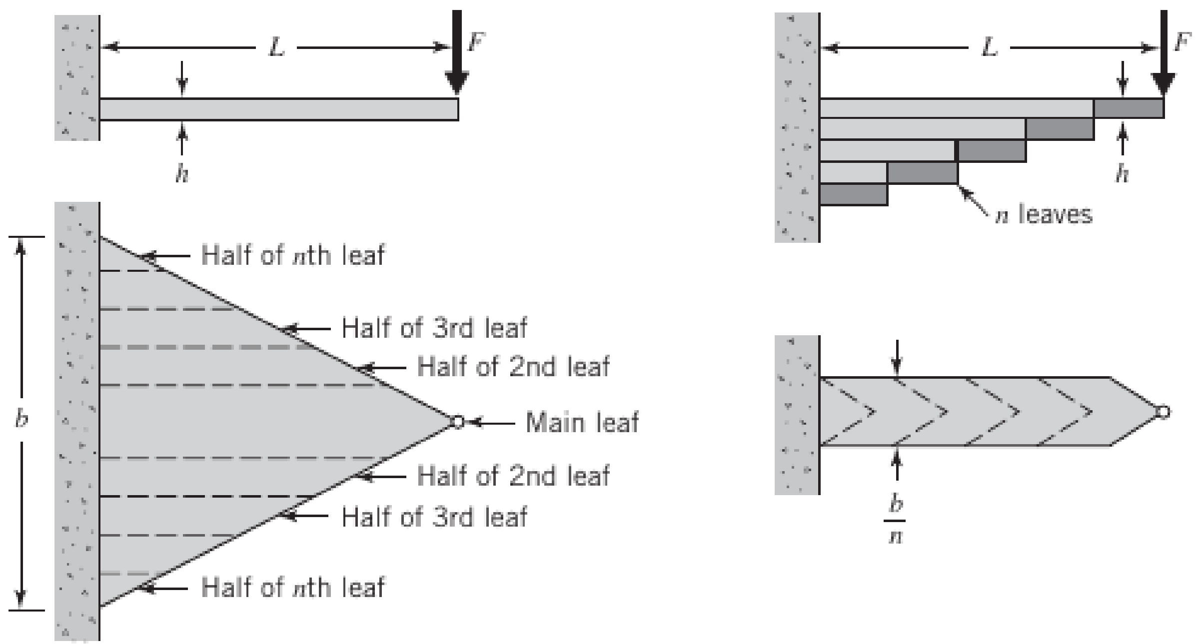
$$b = \frac{6Px}{\sigma h^2}$$

- Note that width b varies linearly with x
- By specifying $b = b_0$ at $x = 0$ and $b = 0$ at $x = L$, we get a triangular profile



Leaf spring

- Constant thickness beam is easier to manufacture
- To make it more compact, it is usually manufactured as an equivalent multiple leaf spring, as shown
- The leaves which are cut from the original triangle are called graduated leaves



Leaf spring

- ❖ Cantilever type leaf springs (or flat springs) are made out of flat plates
- ❖ Advantage of leaf spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device

$$\sigma = \frac{6PL}{nbh^2}$$

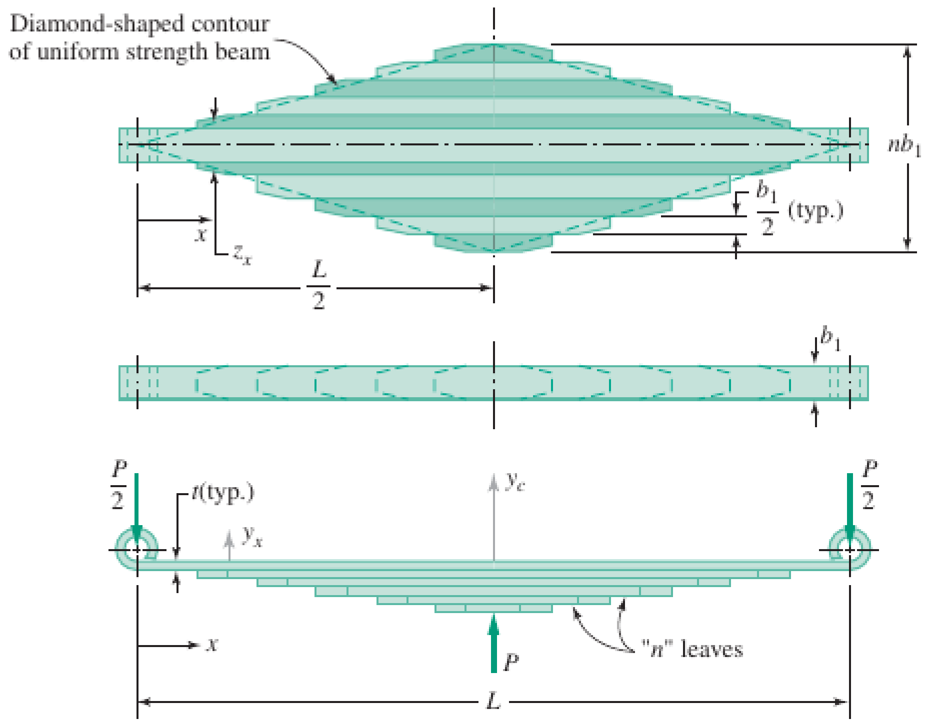
$$\delta = \frac{6PL^3}{Enbh^3}$$

$$k = \frac{Enbh^3}{6L^3}$$

- Number of leaves = n

Leaf spring

- The same concept can also be applied to the simply supported beam



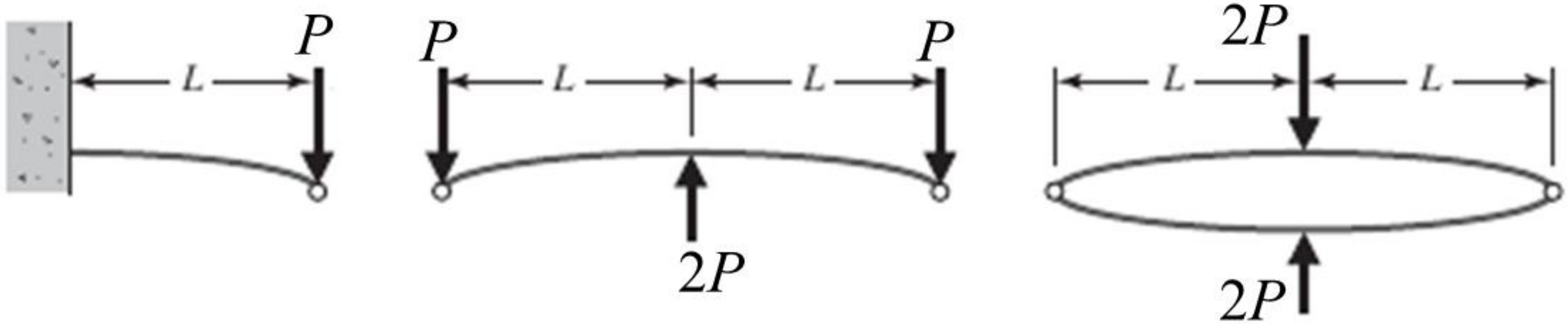
$$\sigma = \frac{3PL}{2nbh^2}$$

$$\delta = \frac{3PL^3}{8Enbh^3}$$

$$k = \frac{8Enbh^3}{3L^3}$$

- Number of leaves = n

Elliptic leaf spring



Quarter-elliptic

- $\sigma = \frac{6PL}{bh^2}$
- $\delta = \frac{6PL^3}{Ebh^3}$

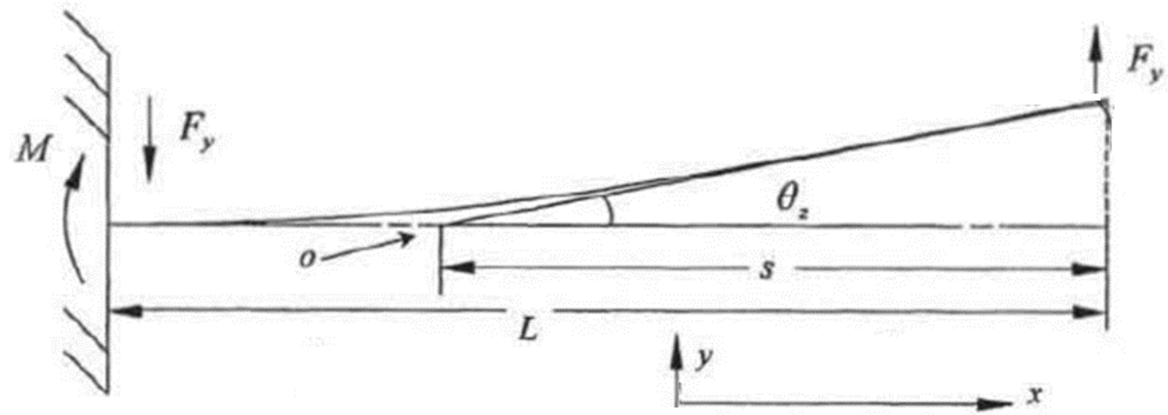
Semi-elliptic

- $\sigma = \frac{6PL}{bh^2}$
- $\delta = \frac{6PL^3}{Ebh^3}$

Full-elliptic

- $\sigma = \frac{6PL}{bh^2}$
- $\delta = \frac{12PL^3}{Ebh^3}$

Rotary hinge



$$s \approx \frac{\delta}{\theta} = \frac{2L}{3}$$

For $0 \le x \le L$: $\quad EI\frac{dy}{dx} = FLx - F\frac{x^2}{2}$ and $EIy = FL\frac{x^2}{2} - F\frac{x^3}{6}$

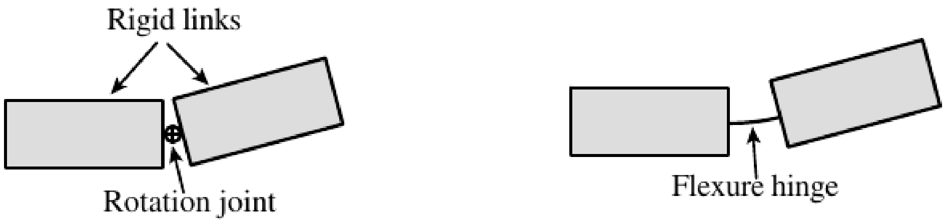
At $x = L$: $\quad EI\frac{dy}{dx} = EI\theta = F\frac{L^2}{2}$ and $EI\delta = F\frac{L^3}{3}$

$$\theta = \frac{FL^2}{2EI} \quad \text{and} \quad \delta = \frac{FL^3}{3EI}$$

View as 2 types of springs:

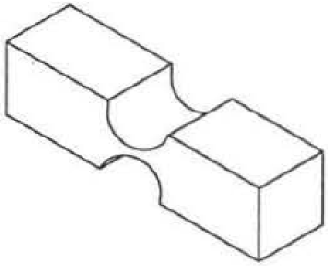
$$k_\theta = \frac{F}{\theta} = \frac{2EI}{L^2} \quad \text{and} \quad k_\delta = \frac{F}{\delta} = \frac{3EI}{L^3}$$

Notch hinge

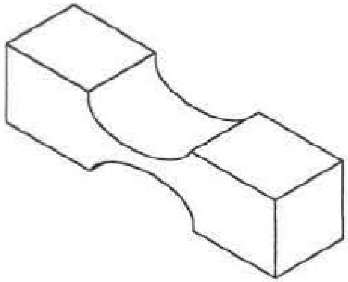


A flexure hinge is a thin member that provides the relative rotation between two adjacent rigid members through flexing (bending), as shown

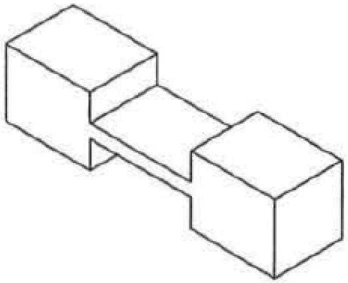
- ❖ Flexure hinges are used in compliant mechanisms for macro- and microscale applications
- ❖ Common notch hinges include the following:



Circular hinge



Elliptic hinge



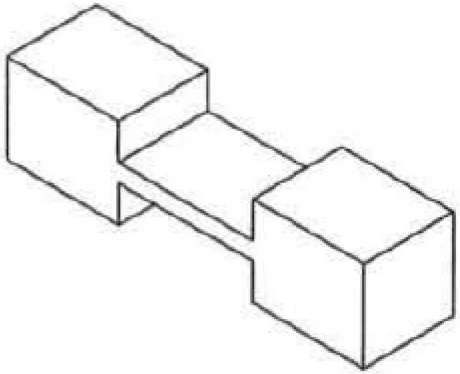
Leaf hinge

Leaf hinge

Angular stiffness $k_\theta = \frac{M}{\theta} = \frac{EI}{2a_x}$

Design thickness $t = \frac{4a_x}{E}\left(\frac{S_y}{\theta_{max}}\right)$

- M = bending moment;
- = angular deflection;
- E = elastic modulus;
- I = second moment of area about neutral axis;
- $2a_x$ = length of the hinge;
- and S_y = yield stress



Leaf hinge

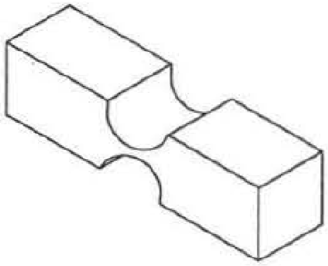
Circular hinge

If web thickness is relatively small in comparison to the radius of the notch, reasonable estimates can be calculated from the approximate equations:

Angular stiffness $k_\theta = \frac{M}{\theta} = \frac{2Ebt^{5/2}}{9\pi R^{1/2}}$

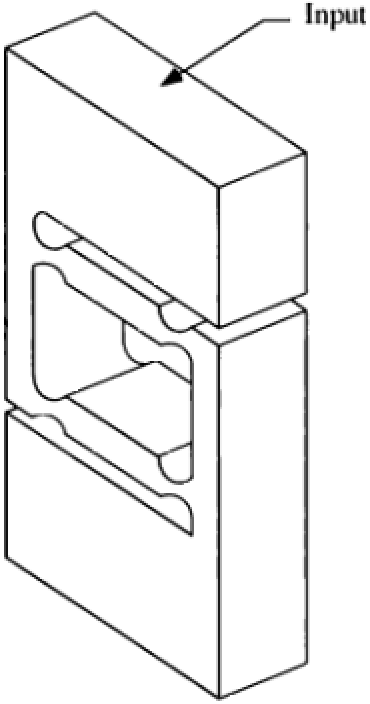
Stress $\sigma = (1 + \beta)^{9/20} \frac{6M}{bt^2}$

- M = bending moment;
- = angular deflection;
- E = elastic modulus;
- b = width of the web;
- R = notch radius; t = web thickness
- and $\beta = \frac{t}{2R}$

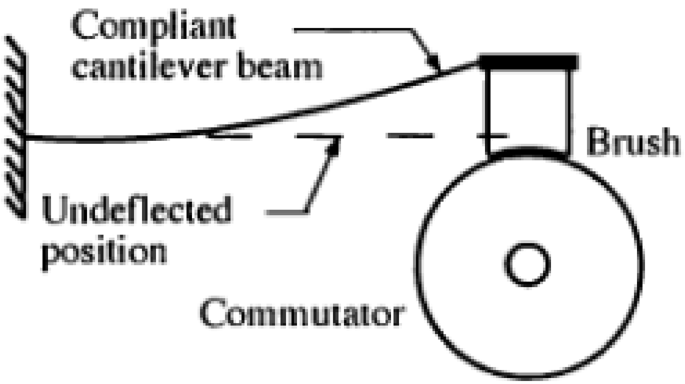


Circular hinge

Applications



Load cell for force measurement



Maintain contact between brush
and commutator in electric motor

Miner's rule

- A common situation is to load at σ_1 for n_1 cycles, then at σ_2 for n_2 cycles, etc.
- The cycles at each stress level contributes to the fatigue damage
- Defining D as the accumulated damage:

$$D = \sum \frac{n_i}{N_i}$$

- where n_i is the number of cycles at stress level σ_i applied to the specimen
- N_i is the life in number of cycles at stress level σ from the S–N curve
- When $D \geq 1$, failure ensues