MEMS1028 Mechanical Design 1

Lecture 12

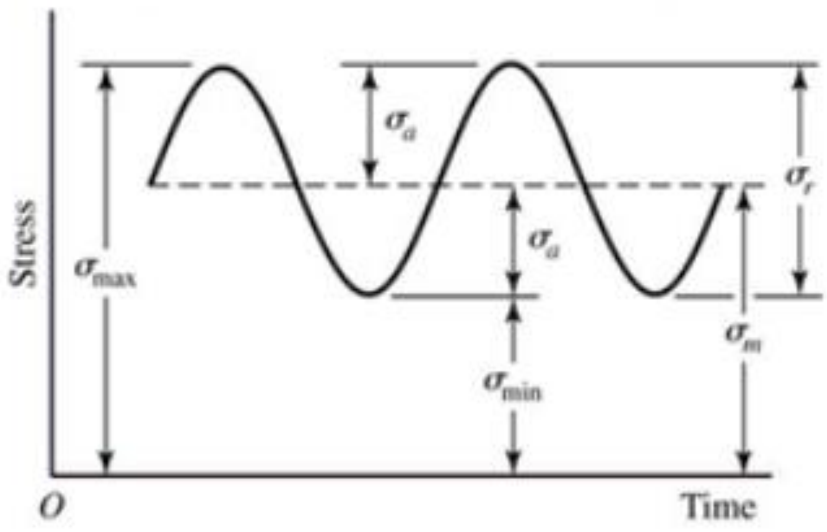
Fatigue failure (Fluctuating simple loads)

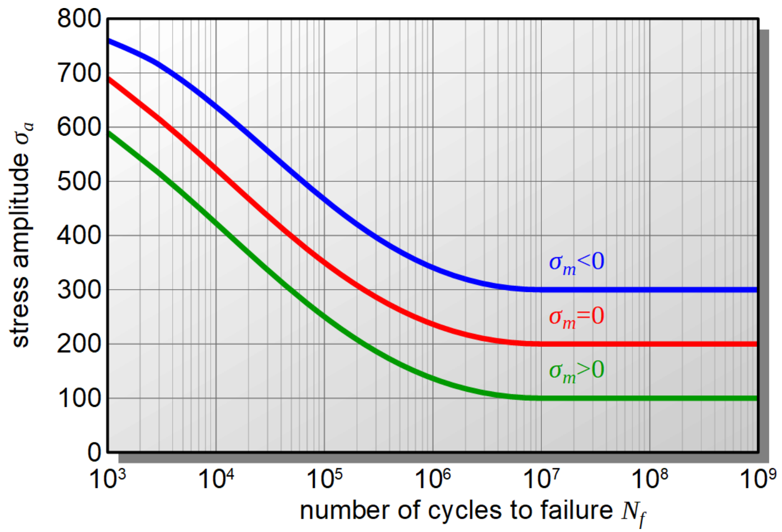
Objectives

- Apply common fatigue failure criteria in engineering design involving fluctuating simple loads
- Analyze the fatigue failure in engineering design involving combination of loading modes

Fluctuating stresses

- In the rotating-beam test, the specimen is subjected to fully reversible stress (i.e. $|\sigma_{\max}| = |\sigma_{\min}|$ with mean stress $\sigma_m = 0$
- A general fluctuating stress can be viewed as a reversing stress with amplitude σ_a about a steady state mean stress σ_m as shown





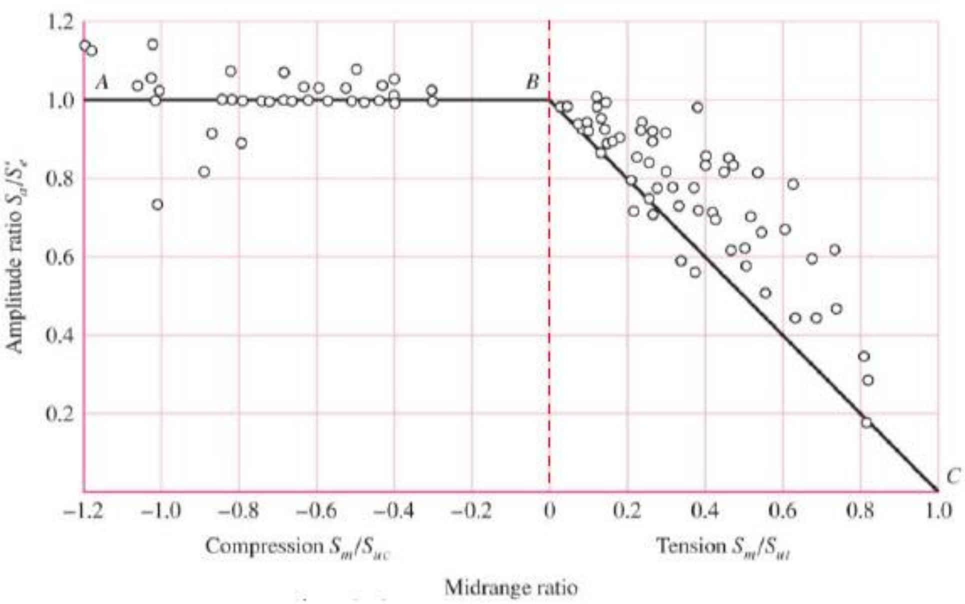
Mean stress: $\sigma_m = \frac{\sigma_{\max}+\sigma_{\min}}{2}$

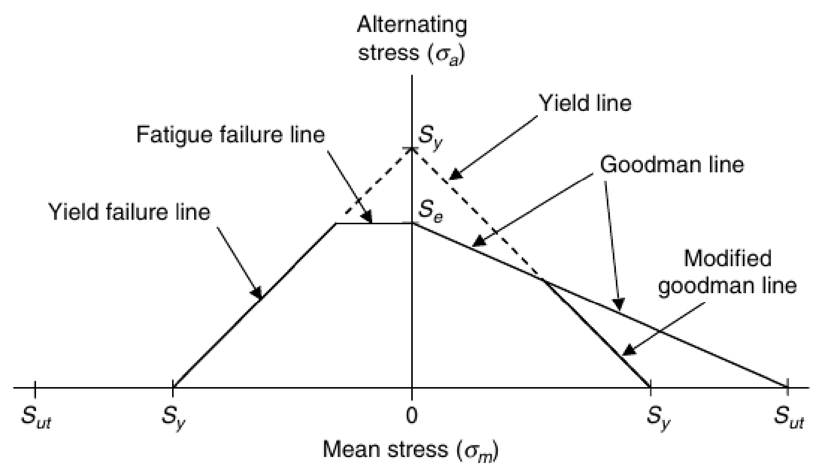
Stress amplitude: $\sigma_a = \left|\frac{\sigma_{\max}-\sigma_{\min}}{2}\right|$

For the same stress amplitude, the mean stress can affect the fatigue life

Fatigue failure

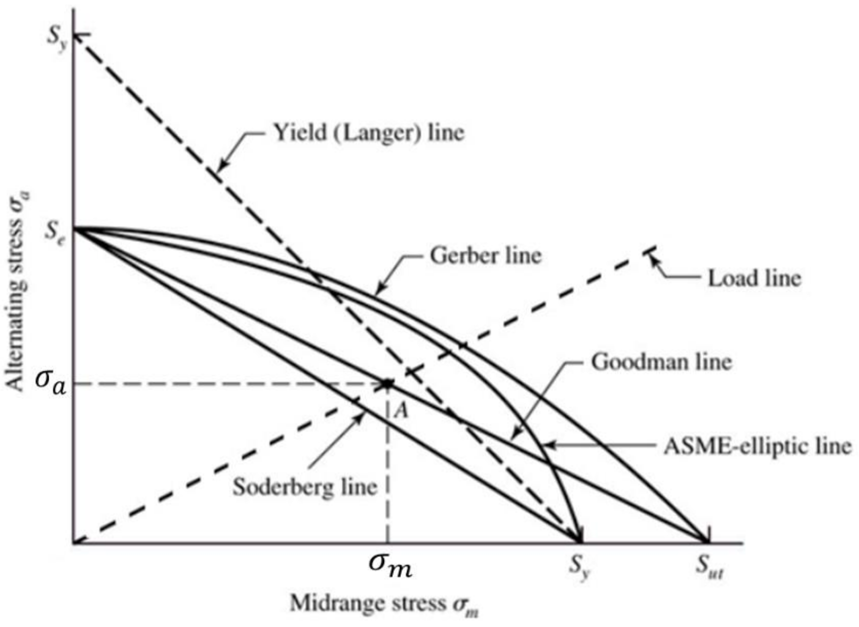
- Experimental data on normalized plot of σ_a vs σ_m
- If is σ_m compressive, then the design is safe if σ_a is less than S_e, as long as the maximum stress $\sigma_m \leq S_{yc}$
- To distinguish the fluctuating stress from the constant stress in the failure criteria, the *y*-axis is used for the σ_a and the *x*-axis for σ_m as shown





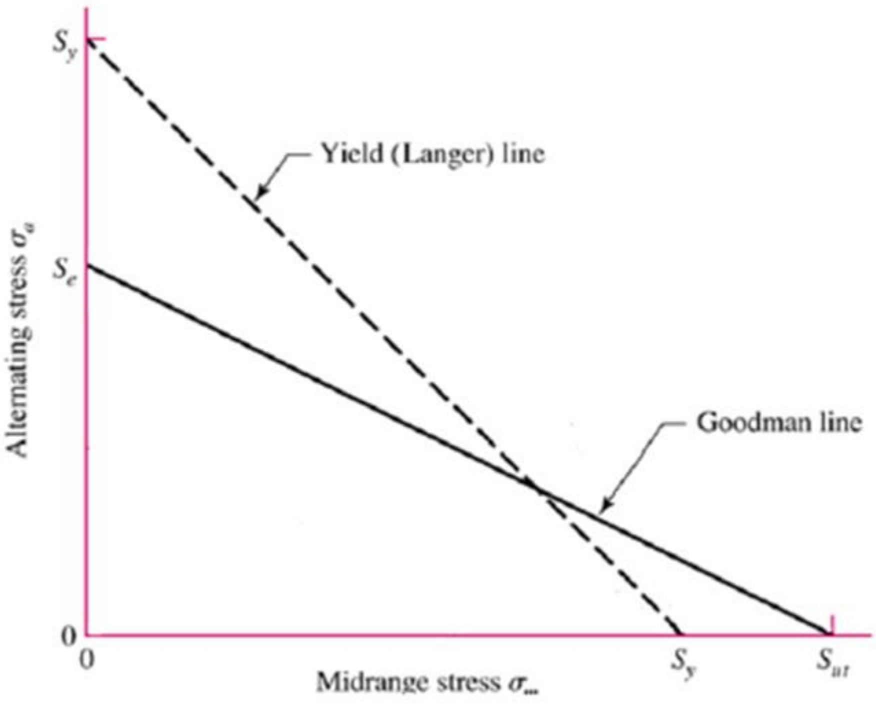
Fatigue failure criteria

- S_e is the modified endurance limit
- σ_a and σ_m should be adjusted by the fatigue stress concentration factor K_f
- The load line represents any combination of σ_a and σ_m
- The intersection of the load line with any of the failure lines give the limiting values S_a and S_m according to the line it intercepts



- Completely reversible fluctuating stress if $\sigma_a \neq 0$ and $\sigma_m = 0$
- Static stress if $\sigma_a = 0$ and $\sigma_m \neq 0$
- Any other combinations of σ_a and σ_m will fall between the two extremes (completely reversed and static)

Fatigue failure criteria



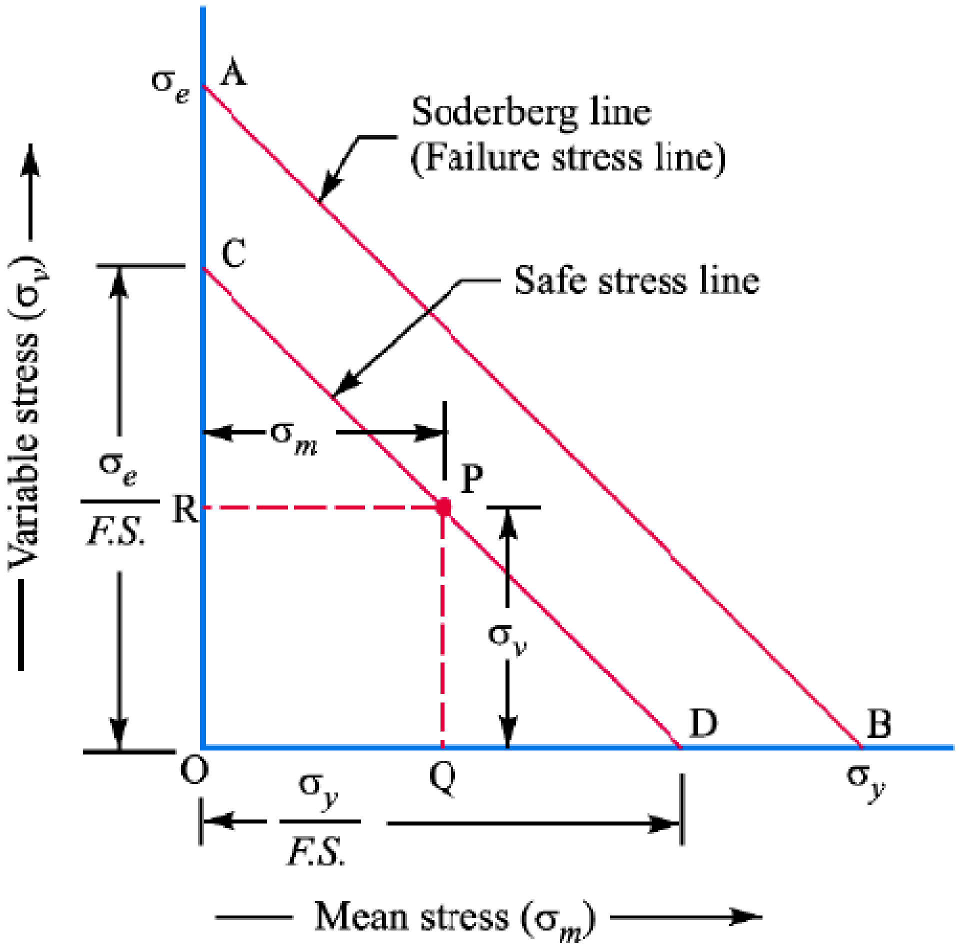
- Langer line connects S_y on the σ_a-axis with S_y on the σ_m-axis but it is not realistic as S_y is usually larger than S_e

$$\frac{\sigma_a}{S_y} + \frac{\sigma_m}{S_y} = 1$$

- Goodman line considers failure due to static loading at S_{ut} rather than S_y; it connects S_e on the σ_a-axis with S_{ut} on the σ_m-axis, i.e. with n = factor of safety:

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

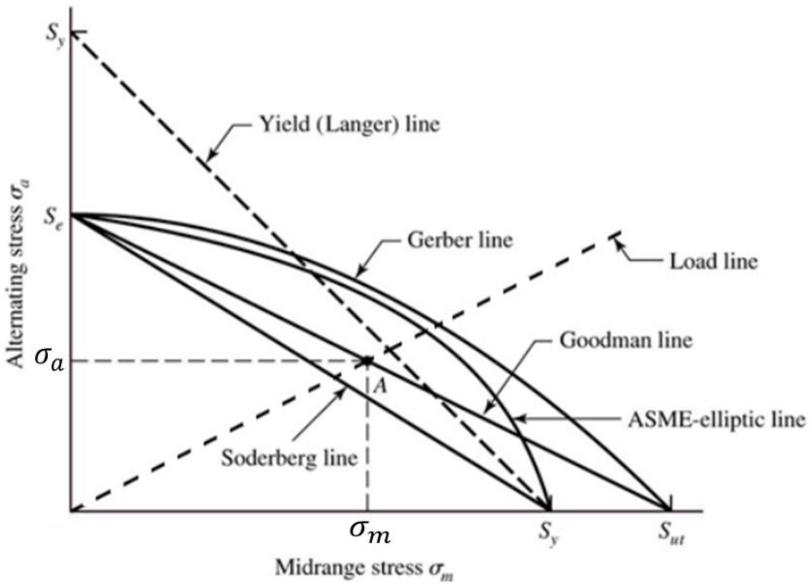
Fatigue failure criteria



Soderberg line is the most conservative and connects S_e on the σ_a-axis with S_y on the σ_m-axis; i.e. with n = factor of safety:

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

Fatigue failure criteria



- Gerber line same as Goodman but uses parabolic instead of a straight line:

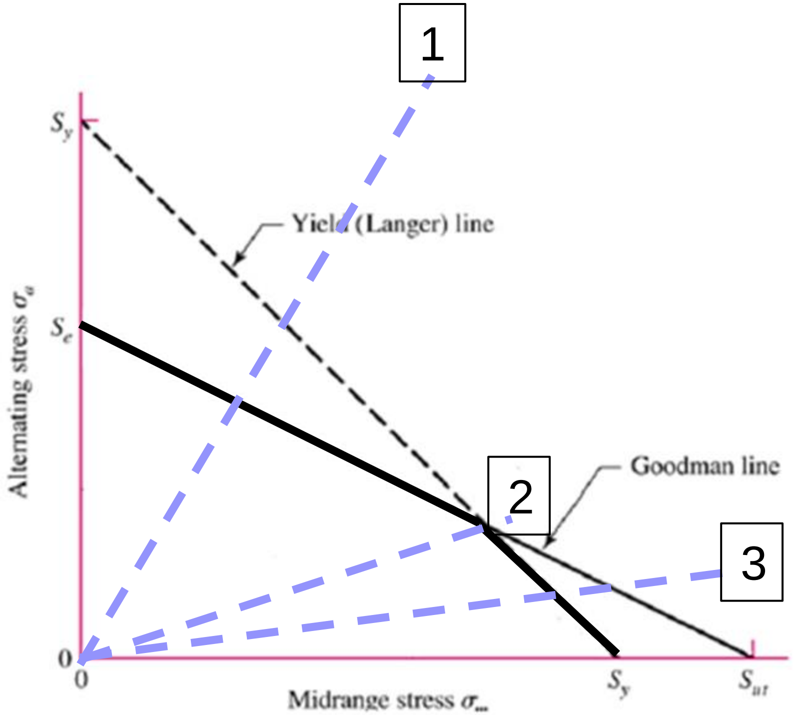
$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

- ASME-elliptic line same as Soderberg but uses an ellipse instead of a straight line:

$$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$$

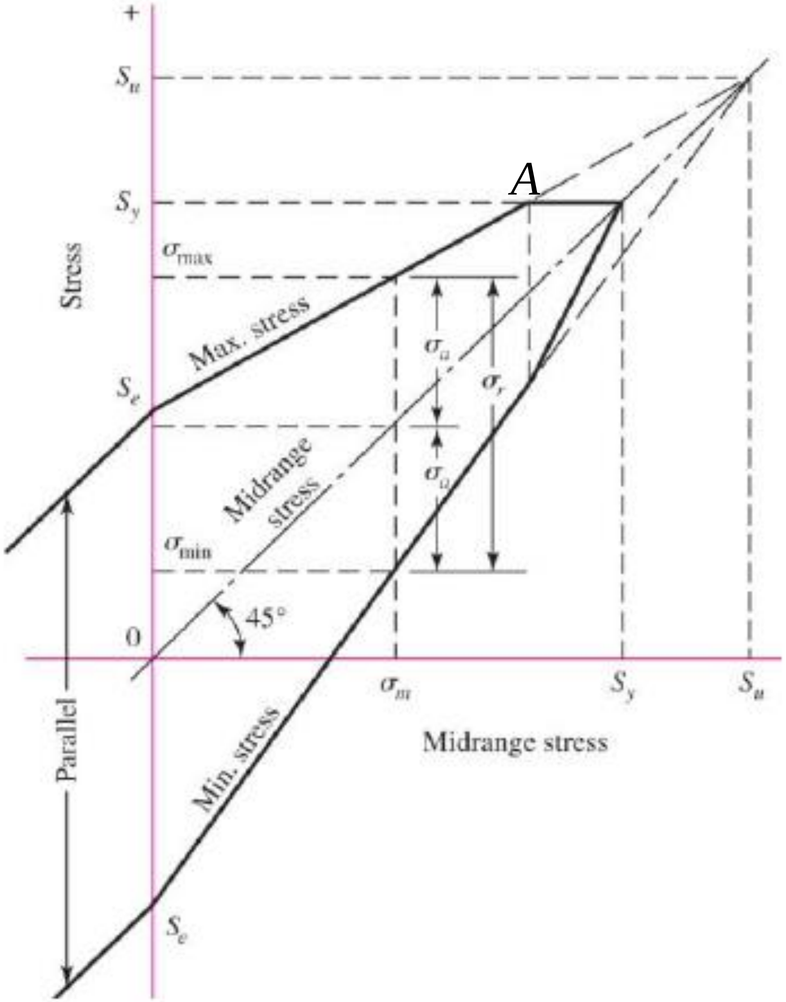
Goodman – Langer

- Modified Goodman combines the Goodman and Langer lines
- The load line $r = \sigma_a/\sigma_m$ can intercept any of these boundary lines or both
- Intercept case 1 gives fatigue criterion
- Intercept case 3 gives yield criterion
- Intercept case 2 gives intersection of the static yield and fatigue criteria



Goodman – Langer

- ❖ The plot of the Modified Goodman stress vs. mean stress axes gives the complete Goodman diagram.
- ❖ The enclosed area is the theoretically safe combinations of mean and alternating stresses that will not cause failure
- ❖ Note: the midrange-stress line is a 45° line from the origin
- ▪ To get the envelope: Connect the endurance limit S_e (above & below the origin) to ultimate strength S_u with straight lines
- ▪ Draw horizontal line from S_y to 45° line
- ▪ Drop vertical line from point A to intersect min. stress line



Goodman – Langer

Table 6–6

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Modified Goodman and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_a = \frac{r S_e S_{ut}}{r S_{ut} + S_e}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1 + r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1 + r}$
$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$	$S_m = \frac{(S_y - S_e) S_{ut}}{S_{ut} - S_e}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a / S_m$

Fatigue factor of safety

$$n_f = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}}$$

Gerber – Langer

Table 6–7

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \frac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\frac{2S_e}{rS_{ut}}\right)^2}\right]$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{rS_y}{1+r}$
Load line $r = \frac{S_a}{S_m}$	$S_m = \frac{S_y}{1+r}$
$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \frac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\frac{2S_e}{S_{ut}}\right)^2\left(1 - \frac{S_y}{S_e}\right)}\right]$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

ASME-elliptic – Langer

Table 6–8

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for ASME-Elliptic and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = \sqrt{\frac{r^2 S_e^2 S_y^2}{S_e^2 + r^2 S_y^2}}$
Load line $r = S_a/S_m$	$S_m = \frac{S_a}{r}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_a = \frac{r S_y}{1+r}$
Load line $r = S_a/S_m$	$S_m = \frac{S_y}{1+r}$
$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_y}\right)^2 = 1$	$S_a = 0, \ \frac{2 S_y S_e^2}{S_e^2 + S_y^2}$
$\frac{S_a}{S_y} + \frac{S_m}{S_y} = 1$	$S_m = S_y - S_a, \ r_{crit} = S_a/S_m$

Fatigue factor of safety

$$n_f = \sqrt{\frac{1}{(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2}}$$

Failure criteria analysis

1) Determine σ_a and σ_m
2) Apply fatigue stress concentration factor K_f to both σ_a and σ_m
3) For $\sigma_m \geq 0$ apply the failure criterion (e.g. Goodman, Gerber, etc.) to find factor of safety "*n*" (Note: *n* > 1 indicates infinite life)

   Otherwise for $\sigma_m < 0$ apply $\sigma_a = \frac{S_e}{n}$ to find factor of safety "*n*";

3) Check for localised yielding by applying Langer failure criterion to find "*n*" (Note: *n* > 1 indicates no yielding)

Torsional fatigue strength

- ❖ Testing has found that the steady-stress component has no effect on the endurance limit for torsional loading if the material is ductile, polished, notch-free, and cylindrical
- ❖ For less than perfect surfaces, the modified Goodman line is suitable
- ❖ The analysis for torsional fatigue is similar:
- Replace σ_m and σ_a with τ_m and τ_a
- Replace S_{ut} with shear ultimate strength $S_{su} = 0.67S_{ut}$
- Replace S_y with shear ultimate yield $S_{sy} = 0.577S_y$; and check for localized yielding by applying

$$\tau_a + \tau_m = \frac{S_{sy}}{n}$$

- Remember for pure torsion cases, normal endurance strength is converted to shear endurance strength with Marin loading factor $k_c = 0.59$ *;*

Fluctuating Simple Loading

For S_e, K_f or K_{fs}, see previous subsection.

1 Calculate σ_m and σ_a. Apply K_f to both stresses.

$$\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 \qquad \sigma_a = |\sigma_{\max} - \sigma_{\min}|/2$$

2 Apply to a fatigue failure criterion,

$\sigma_m \geq 0$

Soderburg	$\sigma_a/S_e + \sigma_m/S_y = 1/n$
mod-Goodman	$\sigma_a/S_e + \sigma_m/S_{ut} = 1/n$
Gerber	$n\sigma_a/S_e + (n\sigma_m/S_{ut})^2 = 1$
ASME-elliptic	$(\sigma_a/S_e)^2 + (\sigma_m/S_y)^2 = 1/n^2$

$\sigma_m < 0$

$$\sigma_a = S_e/n$$

Torsion. Use the same equations as apply for $\sigma_m \geq 0$, except replace σ_m and σ_a with τ_m and τ_a, use $k_c = 0.59$ for S_e, replace S_{ut} with $S_{su} = 0.67S_{ut}$ [Eq. (6–54), p. 325], and replace S_y with $S_{sy} = 0.577S_y$ [Eq. (5–21), p. 239]

3 Check for localized yielding.

$$\sigma_a + \sigma_m = S_y/n$$

or, for torsion, $$\tau_a + \tau_m = 0.577S_y/n$$

Finite-life fatigue strength

If life is finite, then find the fatigue life:

a) Find equivalent completely reversible stress σ_{rev}:

- For modified Goodman

$$\sigma_{rev} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})}$$

- For Gerber

$$\sigma_{rev} = \frac{\sigma_a}{1 - (\sigma_m / S_{ut})^2}$$

b) Determine the finite life N_f (i.e. number of cycles to failure) with a new factor of safety "n" using

$$N_f = \left(\frac{\sigma_{rev}/n}{a}\right)^{1/b}$$

❖ $a = \frac{(f S_{ut})^2}{S_e}$

❖ $b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$

Example 1

A forged steel link with uniform diameter of 30mm is subject to an axial force that varies from 40kN in compression to 160kN in tension. Given the tensile strengths are S_u = 600MPa, S_y = 420MPa, and endurance strength S_e = 240MPa. Determine the factor of safety based on Soderberg criterion

Stress $\sigma_{max} = \frac{160(10^3)}{\pi r^2} = 226\text{MPa};$ and $\sigma_{min} = -\frac{40(10^3)}{\pi r^2} = -56.6\text{MPa};$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2} = 84.7\text{MPa}; \qquad \sigma_a = \left|\frac{\sigma_{max} - \sigma_{min}}{2}\right| = 141.3\text{MPa}$$

Soderberg failure criterion $\frac{1}{n} = \frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{141.3}{240} + \frac{84.7}{420}$

Factor of safety n = 1.26

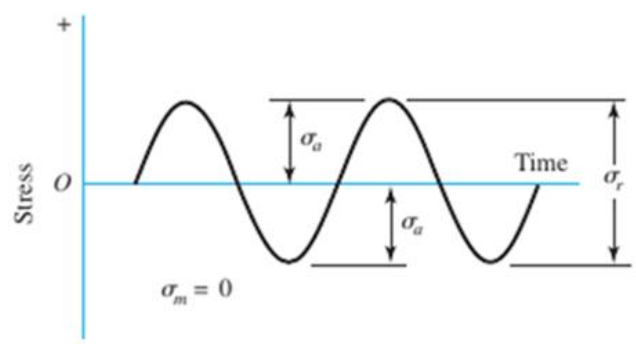
Check for yield using Langer criterion $\sigma_a + \sigma_m = S_y/n$ or n = 1.86

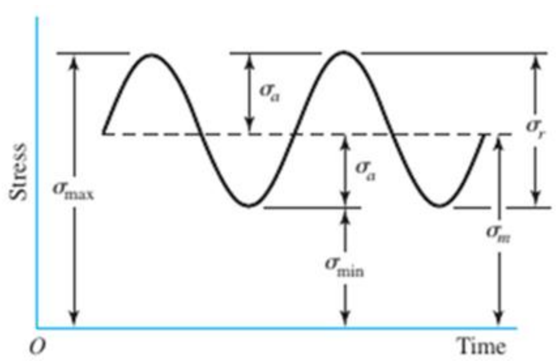
Combination of loading modes

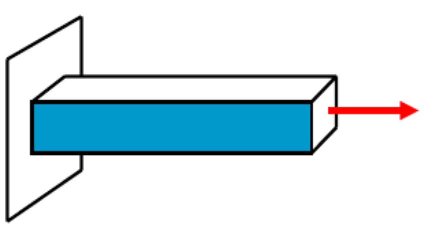
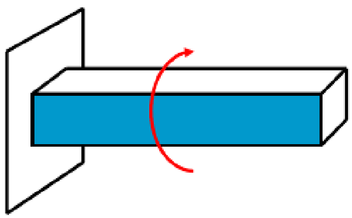
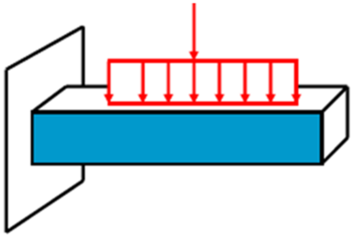
It may be helpful to think of fatigue in 3 categories

1) Completely reversed simple loads
2) Fluctuating simple loads
3) Combination of simple loading modes including axial, torsion & bending

In the first 2 cases, only simple loads are applied





Combination of loading modes

The distortion energy (Von Mises) theory proved to be a satisfactory method for combining static loads and the same approach will be used

1) Generate 2 stress elements: i.e. stress amplitude σ_a and mean stress σ_m. Apply fatigue stress concentration factor K_f to σ_a and σ_m
2) Calculate the equivalent Von Mises stress for each stress element
3) Select the fatigue failure criterion (i.e. Modified Goodman, Soderberg, ASME-elliptic or Gerber):

- For the endurance limit S_e, use the modifiers k_a, k_b and k_c only for bending
- Do not apply the torsional load factor $k_c = 0.59$ as it is accounted for in Von Mises
- The axial load factor k_c should be added in the Von Mises calculation

Equivalent von Mises stresses

The equivalent von Mises stresses σ'_a and σ'_m for the combined bending, torsional shear, and axial stresses can be found using:

$$\sigma'_a = \left\{ \left[(K_f)_{\text{bending}} (\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}} \frac{(\sigma_a)_{\text{axial}}}{0.85} \right]^2 + 3 \left[(K_{fs})_{\text{torsion}} (\tau_a)_{\text{torsion}} \right]^2 \right\}^{1/2}$$

$$\sigma'_m = \left\{ \left[(K_f)_{\text{bending}} (\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}} (\sigma_m)_{\text{axial}} \right]^2 + 3 \left[(K_{fs})_{\text{torsion}} (\tau_m)_{\text{torsion}} \right]^2 \right\}^{1/2}$$

❖ Note: these stresses should be applied to the selected fatigue criterion and used to check for localized yielding using :

$$\sigma'_a + \sigma'_m = \frac{S_y}{n}$$

Example 2

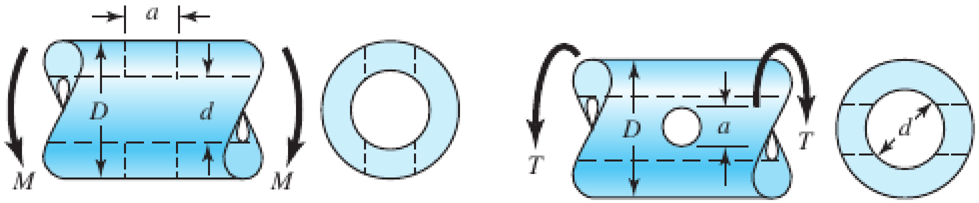
A tubing is made of AISI 1018 cold-drawn steel with external diameter of 42 mm and thickness of 4 mm. It has a 6-mm-diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failures using the Gerber and Langer failure criteria if the shaft is rotating and is subjected to a completely reversed torque of 120Nm in phase with a completely reversed bending moment of 150 Nm.

1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131

$S_{ut} = 440\text{MPa}$, $S_{yt} = 370\text{MPa}$; $S_{ut} < 1400\text{MPa}$ and $S_e' = 0.5S_{ut} = 220\text{MPa}$

Example 2

Given D = 42mm; d = 34mm, t = 4mm, and a = 6-mm, cold drawn



Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Surface factor $k_a = aS_{ut}^b$

From Table: $k_a = 4.51(440)^{-0.265}$

$k_a = 0.899$

Size factor for bending and torsion (applies only for round, rotating diameter):

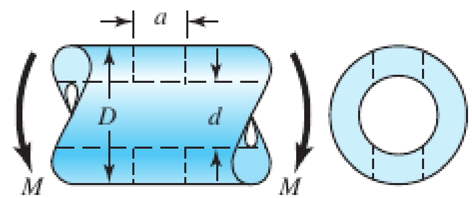
$$k_b = 1.24d^{-0.107} \qquad 2.79 \le d \le 51 \text{ mm}$$

Use external diameter: $k_b = 1.24D^{-0.107} = 0.831$

Example 2

- Loading factor $k_c = 1$ when torsion is combined with other loading, such as bending (combined loading is managed by using the effective von Mises stress)
- No temperature factor $k_d = 1$
- No reliability factor $k_e = 1$
- No miscellaneous-effects factor $k_f = 1$
- $S_e = k_a k_b k_c k_d k_e k_f S_e' = 164.3$MPa
- Bending stress concentration factor

$a/D = 6/42 = 0.143,$
$d/D = 34/42 = 0.810,$
From Table A-16: $A = 0.798$, and $K_t = 2.366$
$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4) = 331(10^3)\text{mm}^3;$

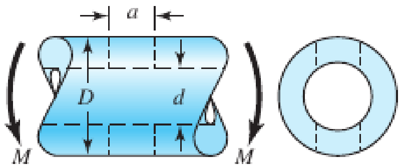


	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Table A–16

Approximate Stress-Concentration Factor K_t of a Round Bar or Tube with a Transverse Round Hole and Loaded in Bending

Source: R. E. Peterson, *Stress-Concentration Factors*, Wiley, New York, 1974, pp. 146, 235.



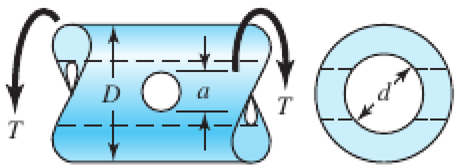
The nominal bending stress is $\sigma_0 = M/Z_{net}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

Values of A are listed in the table. Use $d = 0$ for a solid bar

	d/D					
	0.9		0.6		0	
a/D	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Example 2



Torsion stress concentration factor:

$a/D = 6/42 = 0.143,$

$d/D = 34/42 = 0.810,$

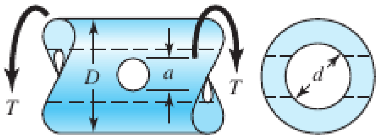
From table: $A = 0.89$, and $K_{ts} = 1.75$

$$J_{net} = \frac{\pi A}{32}(D^4 - d^4) = 155(10^3)\text{mm}^4;$$

	d/D									
	0.9		0.8		0.6		0.4		0	
a/D	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44

Table A–16 (*Continued*)

Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion *Source:* R. E. Peterson, *Stress-Concentration Factors,* Wiley, New York, 1974, pp. 148, 244.



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{net}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{net} = \frac{\pi A(D^4 - d^4)}{32}$$

Values of A are listed in the table. Use $d = 0$ for a solid bar.

	d/D									
	0.9		0.8		0.6		0.4		0	
a/D	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44

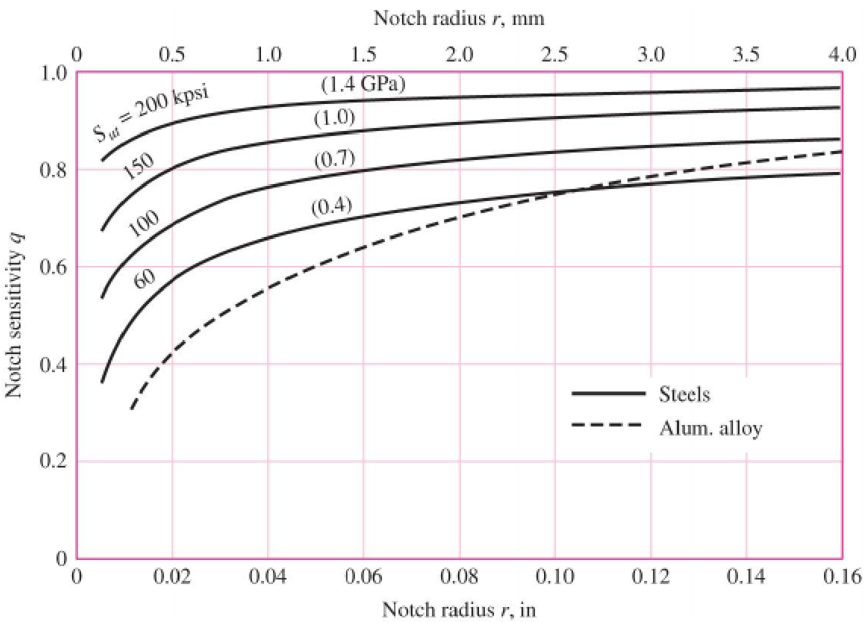
Example 2

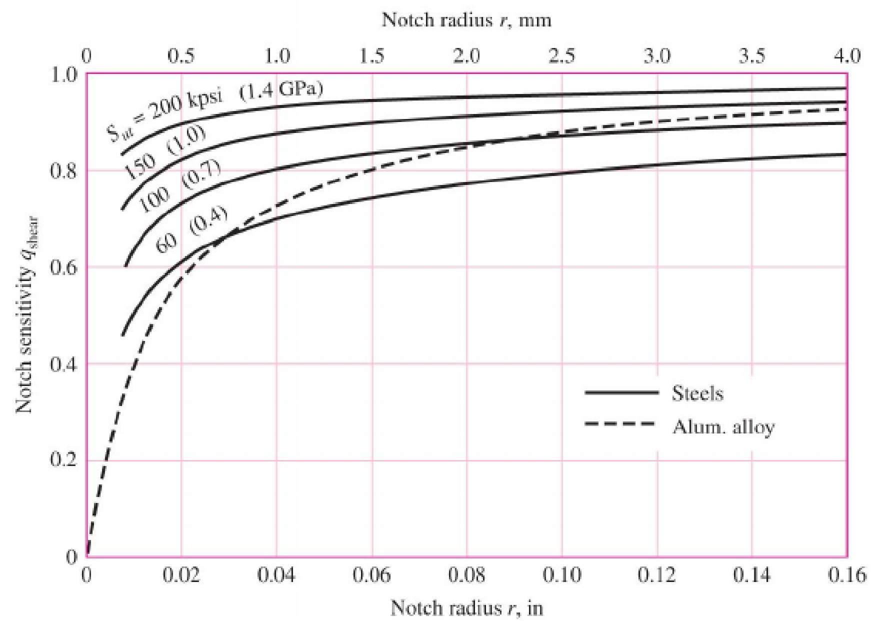
Notch sensitivity for bending: q = 0.78 (based on hole radius r = 3mm)

$$K_f = 1 + q(K_t - 1) = 2.07$$

Notch sensitivity for torsion: q = 0.81 (based on hole radius r = 3mm)

$$K_{fs} = 1 + q(K_{ts} - 1) = 1.61$$





Example 2

- Given completely reversed bending moment M = 150 Nm
- Given completely reversed torque T = 120Nm

Alternating bending stress $K_f\sigma_a = K_f \frac{M}{Z_{net}} = 93.8$MPa

Alternating torsional stress $K_{fs}\tau_a = K_{fs}\frac{T(D/2)}{J_{net}} = 26.2$MPa

Note: both bending and torsion are completely reversible (i.e. $\sigma_m = \tau_m = 0$)

Calculate Von Mises stresses using

$$\sigma'_a = \left\{\left[(K_f)_{\text{bending}}(\sigma_a)_{\text{bending}} + (K_f)_{\text{axial}}\frac{(\sigma_a)_{\text{axial}}}{0.85}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_a)_{\text{torsion}}\right]^2\right\}^{1/2}$$

$$\sigma'_m = \left\{\left[(K_f)_{\text{bending}}(\sigma_m)_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_m)_{\text{axial}}\right]^2 + 3\left[(K_{fs})_{\text{torsion}}(\tau_m)_{\text{torsion}}\right]^2\right\}^{1/2}$$

$$\sigma'_a = \{(93.8)^2 + 3(26.2)^2\}^{1/2} = 104.2\text{MPa; and } \sigma'_m = 0$$

Example 2

Estimate the factor of safety based on Gerber criterion:

$$\frac{n\sigma'_a}{S_e} + \left(\frac{n\sigma'_m}{S_{ut}}\right)^2 = 1$$

$$n = \frac{S_e}{\sigma'_a} = 1.58$$

Check for yielding using Langer failure criterion:

$$\sigma'_a + \sigma'_m = S_y/n$$

$$n = \frac{S_{yt}}{\sigma'_a} = 3.55$$

Other charts

Stress concentration factors (static)

Note: $K_f = 1 + q(K_t - 1)$