MEMS1028
Mechanical Design 1

Lecture 11

Fatigue failure (Reversible simple loads)

Objectives

- Apply stress concentration factor and notch sensitivity in engineering design analysis for simple dynamic loadings
- Apply fatigue strength and endurance limit with modifiers in engineering design analysis for completely reversing simple loading

Endurance limit modifiers

- The rotating-beam endurance limit S'_e is usually known for carefully prepared and tested specimen
- The actual endurance limit S_e in the actual component is often unknown
- When testing of actual parts is not practical, a set of Marin factors are used to adjust the rotating-beam endurance limit S'_e to account for the physical and environmental differences experienced in the actual part, i.e.

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

where

$$k_a = \text{surface condition modification factor}$$
$$k_b = \text{size modification factor}$$
$$k_c = \text{load modification factor}$$
$$k_d = \text{temperature modification factor}$$
$$k_e = \text{reliability modification factor}$$
$$k_f = \text{miscellaneous−effects modification factor}$$

Surface factor (k_a)

- Stresses tend to be high at the surface
- Surface finish has an impact on initiation of cracks at localized stress concentrations (rough surface finish tend to reduce endurance limit
- Surface factor is a function of ultimate strength. Higher strengths are more sensitive to rough surfaces
- These are adjusted by the surface factor $k_a = aS_{ut}^b$

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Size factor (k_b)

- Larger parts have greater surface area at high stress levels
- Likelihood of crack initiation is higher
- Size factor is obtained from experimental data with wide scatter
- For bending and torsion loads, the trend of the size factor data is given by (applies only for round, rotating diameter):

$$k_b = 1.24d^{-0.107} \qquad 2.79 \le d \le 51 \text{ mm}$$

$$k_b = 1.51d^{-0.157} \qquad 51 < d \le 254 \text{ mm}$$

- For axial load, there is no size effect, so $k_b = 1$
- For parts that are not round and rotating, an equivalent round rotating diameter is obtained and used in the above to obtain the size factor

1) For non-rotating round parts, the equivalent diameter is $d_e = 0.37d$
2) For non-rotating rectangular section of height h and breadth b, equivalent diameter is

$$d_e = 0.808(hb)^{1/2}$$

Loading factor (k_c)

- ❖ Accounts for changes in endurance limit for different types of fatigue loading.
- ❖ Only to be used for single load types. Use Combination Loading method when more than one load type is present

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

Temperature factor (k_d)

❖ This relation is summarized in the Table: $k_d = \frac{S_T}{S_{RT}}$

Table 6–4

Effect of Operating Temperature on the Tensile Strength of Steel.* (S_T = tensile strength at operating temperature; S_{RT} = tensile strength at room temperature; $0.099 \leq \hat{\sigma} \leq 0.110$)

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

Reliability factor (k_e)

❖ Simply obtain k_e for desired reliability from Table

Note:

$$k_e = 1 - 0.08 z_a$$

Reliability, %	Transformation Variate z_a	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

Table 6–5

Miscellaneous-effects factor (k_f)

- Reminder to consider other possible factors
  - Residual stresses
  - Directional characteristics from cold working
  - Case hardening
  - Corrosion
  - Surface conditioning, e.g. electrolytic plating and metal spraying
  - Cyclic Frequency
  - Frettage Corrosion
- Limited data is available
- May require research or testing
- The design engineer must carefully think about the environment and the conditions the design will be subject to and identify the appropriate modification factors
- Set $k_f = 1$ if unknown

Stress concentration

- Under fluctuating loading, crack initiation usually starts at notch, which are locations (such as grooves & holes) with high stress concentration
- Stress concentration reduces the fatigue life (and endurance limit)
- The effect of stress concentration on fatigue properties is not the same for different materials
- For materials under static loading, the maximum stress near the notch is

$$\sigma_{max} = K_t \sigma_0 \quad \text{or} \quad \tau_{max} = K_{ts} \tau_0$$

Stress-concentration factors K_t (or K_{ts}) are given in Appendices A-15 & 16

- For materials under fatigue loading, the maximum stress near the notch is

$$\sigma_{max} = K_f \sigma_0 \quad \text{or} \quad \tau_{max} = K_{fs} \tau_0$$

K_f is the fatigue stress concentration factor, which is a reduction of the stress concentration factor (K_t) due to the difference in material sensitivity to the presence of notches

Notch sensitivity

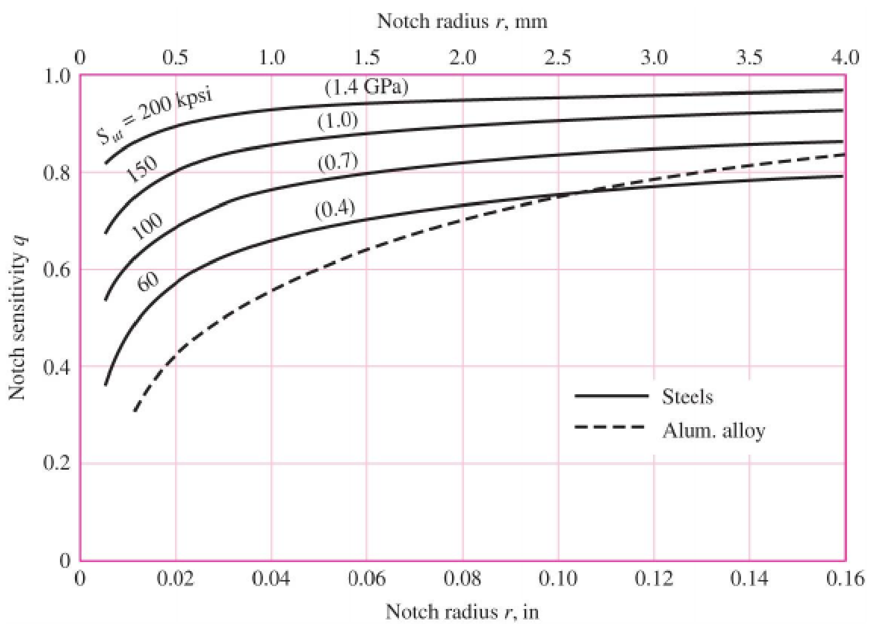
- Notch sensitivity q ranging from 0 (not sensitive) to 1 (fully sensitive) is defined as

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad q = \frac{K_{fs} - 1}{K_{ts} - 1}$$

- For $q = 0$, $K_f = 1$ (material is not sensitive)
- For $q = 1$, $K_f = K_t$ (material is fully sensitive)

- Static stress concentration factor (K_t) can be found from Appendices
- Notch sensitivity q for bending or axial loading can be obtained from Fig. 6–20
- Fatigue stress concentration factor:

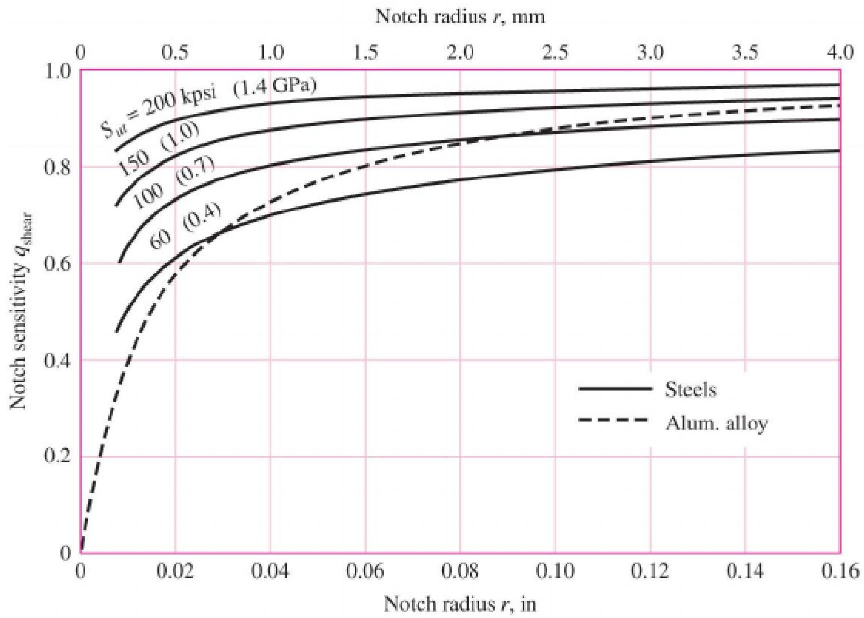
$$K_f = 1 + q(K_t - 1)$$



Notch sensitivity

- Static stress concentration factor (K_{ts}) can be found from Appendices
- Notch sensitivity q for torsional loading can be obtained from Fig. 6–21
- Fatigue stress concentration factor:

$$K_{fs} = 1 + q(K_{ts} - 1)$$



Notch sensitivity

Alternatively, can use curve fit equations for Figs. 6–20 and 6–21 to get the notch sensitivity:

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}}$$

$\sqrt{a}$ is the Neuber constant which is solely a function of the material:

1) For bending or axial loading, find the Neuber constant using:

$$\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

2) For torsional loading, find the Neuber constant using :

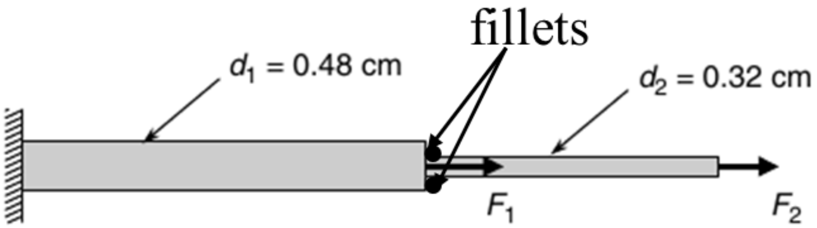
$$\sqrt{a} = 0.190 - 2.51(10^{-3})S_{ut} + 1.35(10^{-5})S_{ut}^2 - 2.67(10^{-8})S_{ut}^3$$

The fatigue stress concentration factor K_f can be then be determined from

$K_f = 1 + \frac{K_t - 1}{1+\sqrt{a/r}}$ where $\sqrt{r}$ is the square root of the notch's radius

Example 1

For the stepped rod shown, which is acted upon by both a fluctuating axial force (F_1) of between − 900N and 3,600N and a constant axial force (F_2) of 2,250N, determine the endurance limit and the stress concentration factor given that the rod is made of high-strength steel ($S_{ut} = 910$ MPa), ground to the dimensions and operates at room temperature with a reliability of 99%



Note: last week lecture endurance limit for steel can be estimated from

$$S'_e \begin{cases} 0.5S_{ut} & S_{ut} \leq 1400 \text{ MPa} \\ 700 \text{ MPa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

Example 1

Given S_{ut} = 910MPa < 1400MPa

$S'_e = 0.5S_{ut} = 455\text{MPa}$

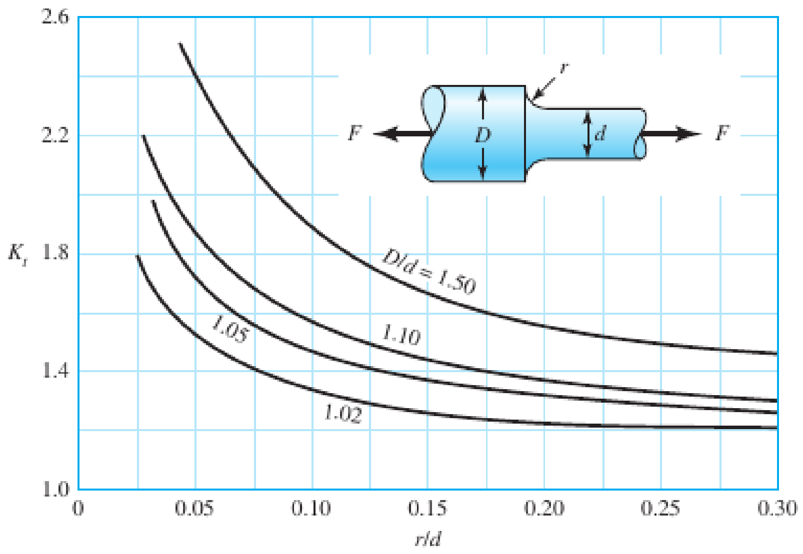
Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

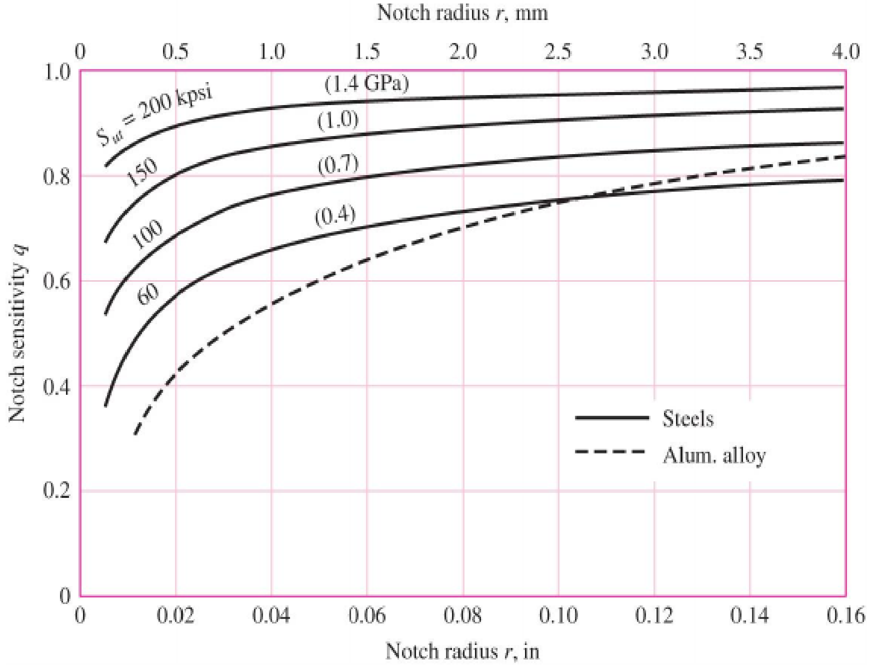
- Step 1: surface factor $k_a = aS_{ut}^b$

  From Table: $k_a = 1.58(910)^{-0.085}$

  $k_a = 0.89$
- Step 2: For axial load, there is no size effect, so $k_b = 1$
- Step 3: For axial, loading factor $k_c = 0.85$
- Step 4: At room temperature, the temperature factor $k_d = 1$
- Step 5: At reliability of 99%, the reliability factor $k_e = 0.814$
- Step 6: No known miscellaneous-effects factor $k_f = 1$
- Actual endurance limit is $S_e = k_a k_b k_c k_d k_e k_f S'_e = 280\text{MPa}$

Example 1





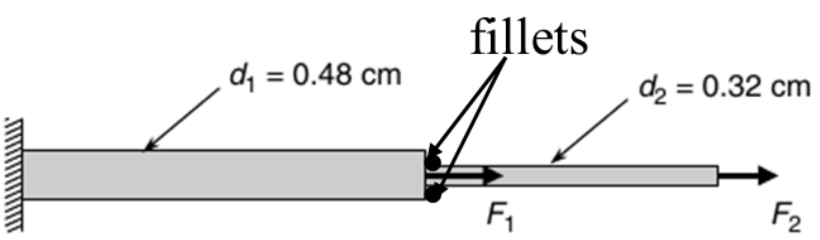
- ❖ $D/d = 1.5, r/d = 0.25; K_t = 1.5$

Notch radius = 0.8mm, $S_{ut} = 910$ MPa

From chart, Notch sensitivity $q = 0.82$

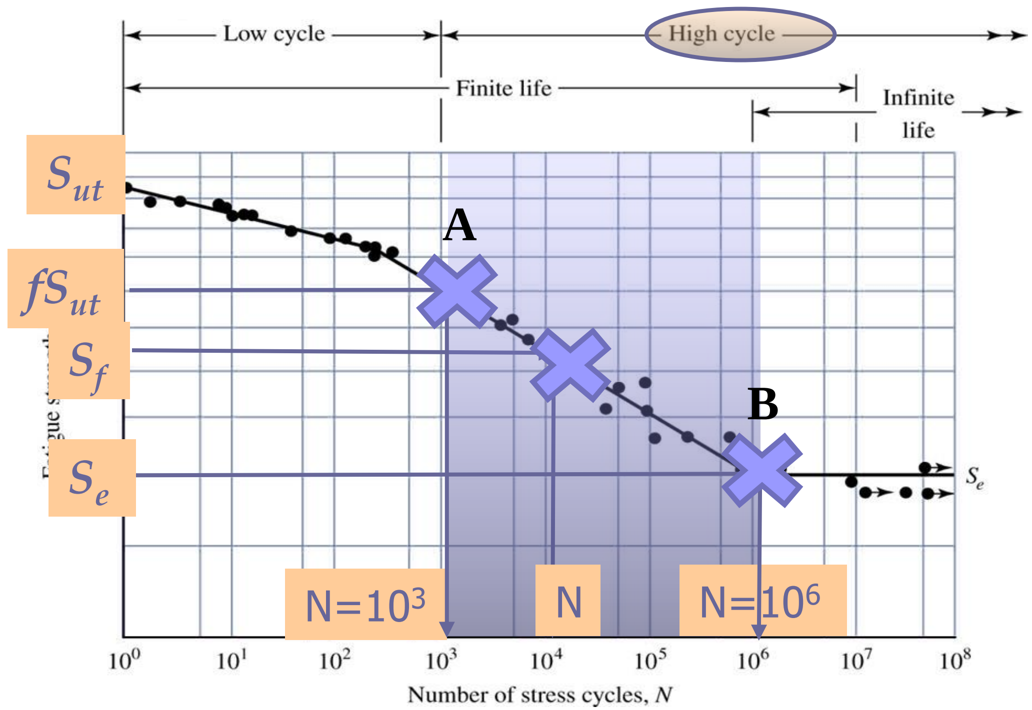
Fatigue stress concentration factor:

$$K_f = 1 + q(K_t - 1) = 1.41$$



Fatigue strength

- In some designs, the number of stress cycles is less than 10^6 and there is no need to design for infinite life using endurance limit
- In these cases, the fatigue strength associated with the desired life has to be found



S-N Diagram for Steel

For an actual mechanical component, S'_e is reduced to S_e

Fatigue strength

Consider the line joining 2 points:

$$A = (10^3, fS_{ut}) \text{ and}$$
$$B = (10^6, S_e)$$

Note: fS_{ut} is a fraction of S_{ut}

- Equation for line:

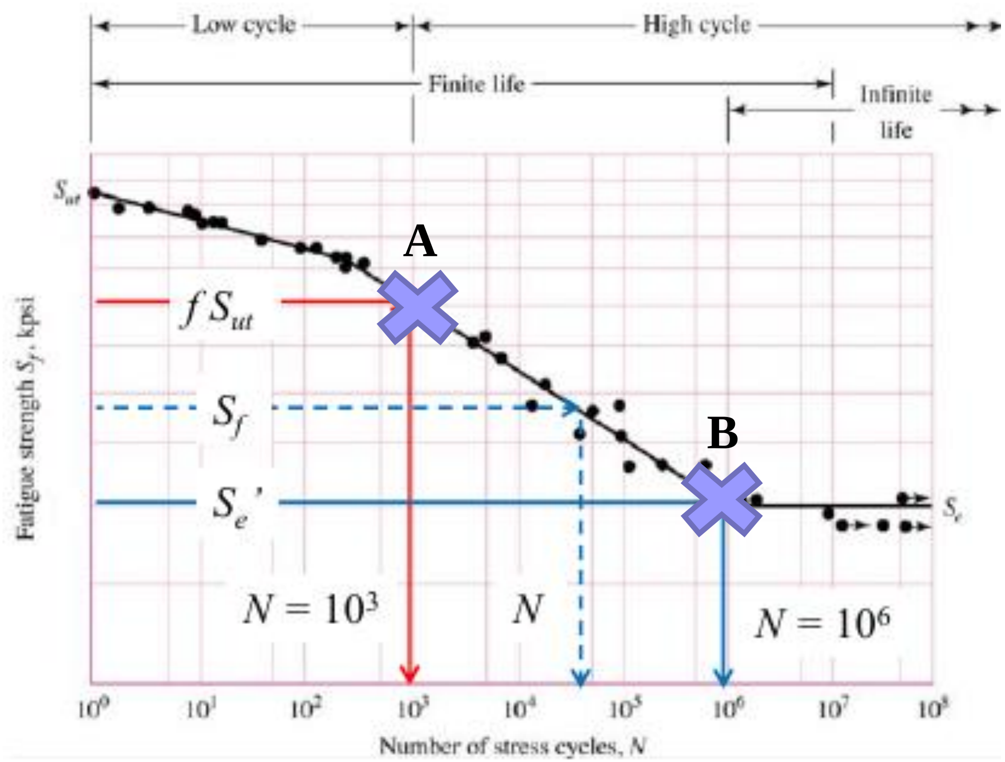
$$S_f = aN^b$$

- Note:

$$a = \frac{(fS_{ut})^2}{S_e}$$

- Slope

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$$

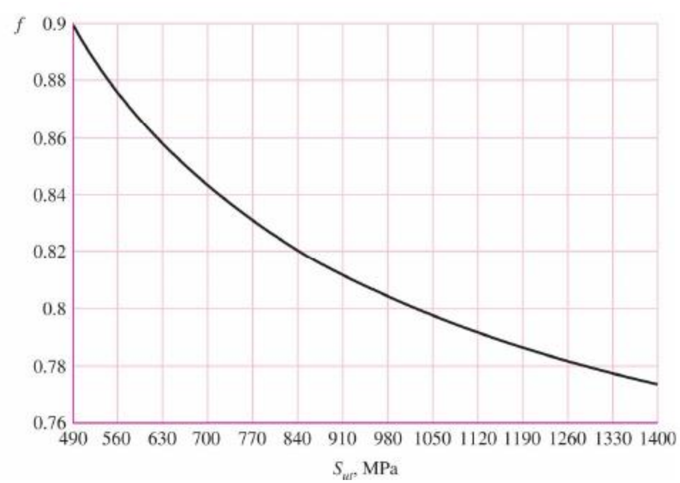


Fatigue strength

Equation for S-N line from 10^3 to 10^6 cycles

$$S_f = aN^b$$

- ❖ $a = \frac{(fS_{ut})^2}{S_e}$
- ❖ $b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$
- ❖ For steel with $H_B \leq 500$, estimated true stress at fracture is $\sigma_f' = S_{ut} + 345\text{MPa}$ and *f* can be found using $f = \frac{\sigma_F'}{S_{ut}}(2 \times 10^3)^b$ or in graphical form:
- ❖ Note: for $S_{ut} \leq 490$ MPa use $f = 0.9$ to be conservative
- ❖ If the value of *f* is known, slope *b* and "y-intercept *a* can be found to solve for the fatigue strength S_f



No. of cycles to failure

- Note that the experimental determination of the S-N diagram is based on completely reversible stresses
- If a completely reversible stress σ_{rev} is given, setting $S_f = \sigma_{rev} = \sigma_a$ (i.e. stress amplitude) in $S_f = aN^b$ and solving for the number of cycles to failure gives

$$N = \left(\frac{\sigma_a}{a}\right)^{1/b}$$

- For general fluctuating loading situations, it is necessary to obtain an equivalent, completely reversing, stress that is considered to be equally as damaging as the actual fluctuating stress
- If low-cycle fatigue failures in the range $1 \leq N \leq 10^3$ is needed, failure is predicted by a straight line between two points $(10^3, f\,S_{ut})$ and $(1, S_{ut})$ by

$$S_f = S_{ut} N^{(\log f)/3} \qquad 1 \leq N \leq 10^3$$

Example 2

Given a 1045 CD steel rotating-beam specimen, estimate
(a) the endurance limit at 10^6 cycles
(b) the fatigue strength corresponding to 5×10^4 cycles to failure
(c) the expected life under a completely reversed stress of 400MPa.

Table A–20

Deterministic ASTM Minimum Tensile and Yield Strengths for Some Hot-Rolled (HR) and Cold-Drawn (CD) Steels [The strengths listed are estimated ASTM minimum values in the size range 18 to 32 mm ($\frac{3}{4}$ to $1\frac{1}{4}$ in). These strengths are suitable for use with the design factor defined in Sec. 1–10, provided the materials conform to ASTM A6 or A568 requirements or are required in the purchase specifications. Remember that a numbering system is not a specification.] *Source:* 1986 SAE Handbook, p. 2.15.

1 UNS No.	2 SAE and/or AISI No.	3 Processing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170
G10450	1045	HR	570 (82)	310 (45)	16	40	163
		CD	630 (91)	530 (77)	12	35	179
G10500	1050	HR	620 (90)	340 (49.5)	15	35	179
		CD	690 (100)	580 (84)	10	30	197

S_{ut}, MPa

S_{ut} = 630MPa < 1400MPa

$S'_e = 0.5S_{ut} = 315$MPa

$\sigma'_F = S_{ut} + 345$=975MPa

f =0.856 (from fig)

$$a = \frac{(fS_{ut})^2}{S_e} = 923.4\text{MPa}$$

$$b = -\frac{1}{3}\log\left(\frac{fS_{ut}}{S_e}\right)$$

$$b = -0.0779,$$

Example 2

$$S_f = aN^b = 925.4N^{-0.0779}$$

- The endurance limit at 10^6 cycles is $S_e' = 315$MPa
- The fatigue strength corresponding to $N = 5\times10^4$ cycles to failure is

$$S_f = aN^b = 925.4N^{-0.0779} = 397.5 \text{ MPa}$$

- The expected life under a completely reversed stress of $\sigma_a = 400$ MPa is

$$N = \left(\frac{\sigma_a}{a}\right)^{1/b} = 46.14(10^3) \text{ cycles}$$

Procedure for analysis

1) Determine S_e' from Tables or from S_{ut}
2) Modify S_e' to S_e with $k_a, k_b, \ldots,$ to get $S_e = k_a k_b k_c k_d k_e k_f S_e'$
3) Determine the fatigue stress concentration factor K_f
4) Determine fatigue life constants a and b
5) Determine fatigue strength S_f at N cycles, or N cycles to failure at a reversing stress $\sigma_{rev} = \sigma_a$ (remember to adjust using K_f)
6) Note the design factor of safety is

$$n_f = \frac{S_e}{\sigma}$$