MEMS1028 Mechanical Design 1

Lecture 10

Fatigue failure (Introduction)

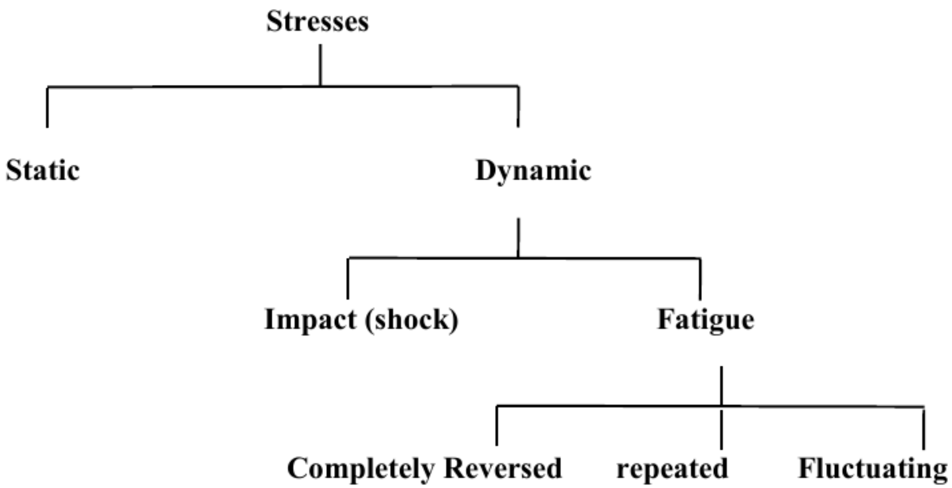
Objectives

- Describe the mechanical failure modes of metals and explain the process and mechanism of fatigue failure
- Characterize stress fluctuations
- Apply fatigue-life models to analyze fatigue failures

Modes of mechanical failure

	Examples of mechanical failure modes of metals
1	Excess deformation – elastic, yielding, or onset of plasticity
2	Ductile fracture – substantial plasticity & high energy absorption
3	Brittle fracture – little plasticity & low energy absorption
4	Impact loading – excess deformation or fracture
5	Creep – excess deformation or fracture
6	Thermal shock – cracking and/or fracture
7	Wear – many possible failure mechanisms
8	Buckling – elastic or plastic
9	Corrosion – hydrogen embrittlement
10	Fatigue – repeated loading

Characterizing stress fluctuation



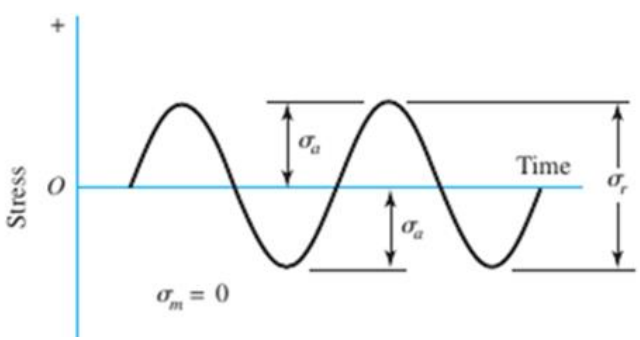
Mean stress: $\sigma_m = \frac{\sigma_{max}+\sigma_{min}}{2}$

Stress amplitude: $\sigma_a = \left|\frac{\sigma_{max}-\sigma_{min}}{2}\right|$

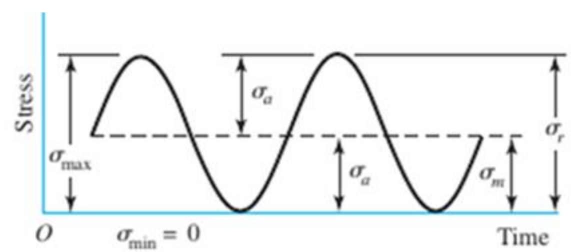
Stress ratio: $R = \frac{\sigma_{min}}{\sigma_{max}}$

Amplitude ratio: $A = \frac{\sigma_a}{\sigma_m}$

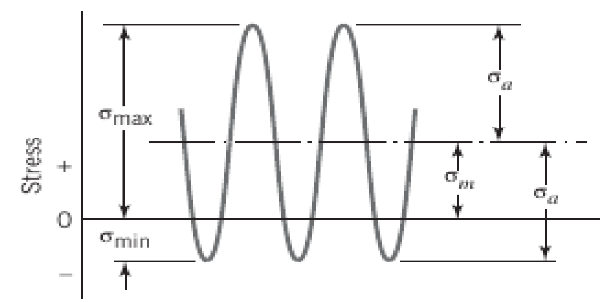
Stress range $\sigma_r = \sigma_{max} - \sigma_{min}$



Completely reversed stress



Repeated stress

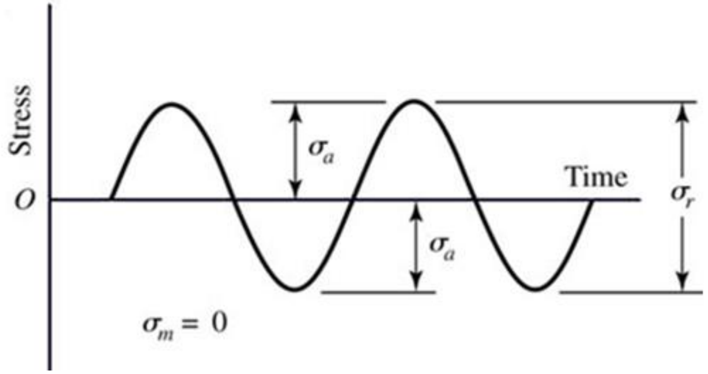


Fluctuating stress

Characterizing stress fluctuation

The stress characteristics for a completely reversing loading stress are:

- $\sigma_{\text{max}} = \sigma_{\text{a}} = -\sigma_{\text{min}}$
- $\sigma_{\text{m}} = 0$
- $\sigma_r = 2\sigma_a$
- $R = -1$
- $A = \infty$



For the fluctuating stress σ_{max} = 400MPa, σ_{min} = -600MPa, and period of 5s. Determine all the stress components, stress ratio, amplitude ratio, and frequency

$\sigma_{\text{m}} = \frac{\sigma_{\text{max}}+\sigma_{\text{min}}}{2} = -100\text{MPa};$ $\quad \sigma_a = \left|\frac{\sigma_{\text{max}}-\sigma_{\text{min}}}{2}\right| = 500\text{MPa}$

$\sigma_{\text{r}} = \sigma_{\text{max}} - \sigma_{\text{min}} = 1000\text{MPa};$ $\quad$ frequency $f = \frac{1}{\tau} = 0.2\text{Hz}$

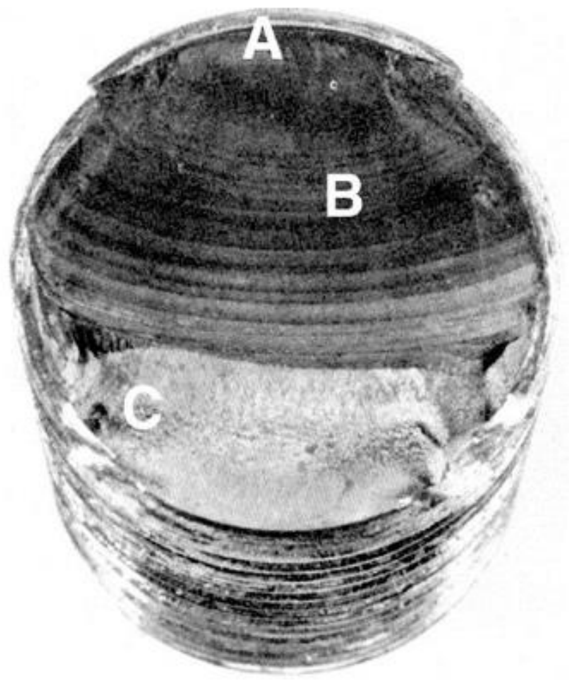
Stress ratio: $R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = -1.5;$ $\quad$ Amplitude ratio: $A = \frac{\sigma_a}{\sigma_m} = -5$

Fatigue failure

- Fatigue is the microstructural degradation of a material due to repeated application of stresses or strains
- Fatigue strength is the resistance of a material to failure under cyclic loading
- It is the most common source of failure in machine components
- Fatigue fracture is dangerous because it is sudden and can occur at stress levels well below the yield strength
- The fatigue fracture surfaces are flat and perpendicular to the stress axis with the absence of necking (look like brittle failure)

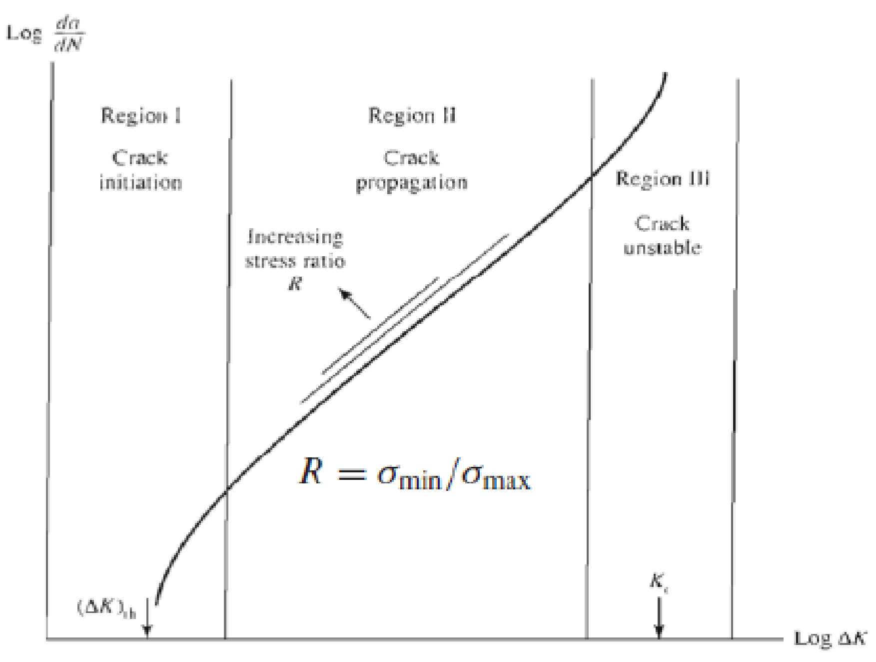
Stages of fatigue failure

- Stage I – Crack Initiation
  - Start at a "notch", or other stress concentration
  - E.g. initiation of micro-crack due to cyclic plastic deformation at the tread root "*A*"
- Stage II – Crack Propagation
  - Each tensile stress cycle causes the crack to grow (10^{-9} to 10^{-5} mm/cycle)
  - E.g. progression to macro-crack that repeatedly opens and closes, creating bands called beach marks at "*B*"
- Stage III – Fracture
  - Sudden failure with no warning
  - E.g. crack propagation such that remaining material cannot carry the load resulting in ultimate failure (starting from "*C*")



E.g. Fatigue failure of a bolt due to repeated unidirectional bending

Linear-Elastic fracture mechanics



This method assumes crack exists and predicts crack growth with respect to stress intensity

- Stress intensity factor $K_I = \beta\sigma\sqrt{\pi a}$ where $2a$ = crack length
- Consider stress fluctuating between the range $\Delta\sigma = \sigma_{max} - \sigma_{min}$
- The stress intensity range per cycle is $\Delta K_I = \beta\Delta\sigma\sqrt{\pi a}$
- At $K_I = K_{IC}$, (i.e. critical toughness value) $a = a_f = \frac{1}{\pi}\left[\frac{K_{ic}}{\beta\sigma_{max}}\right]^2$ and ultimate fracture occurs
- Paris equation estimate crack growth in Region II

Linear-Elastic Fracture Mechanics Method

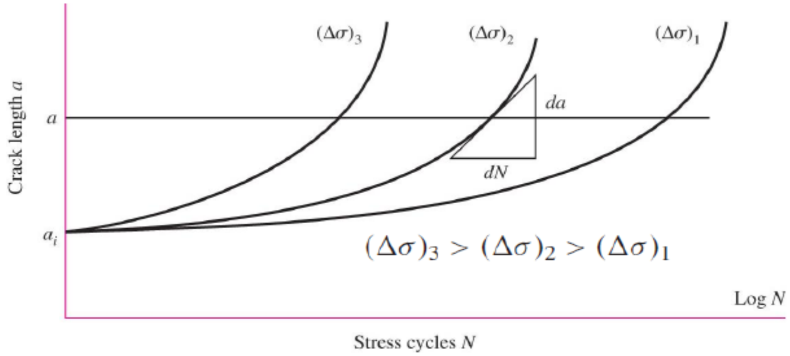
- Paris equation for crack growth from a_i to a_f is

$$\frac{da}{dN} = C(\Delta K_I)^m$$

C and m are material constants (see Table 6.1) and N = number of cycles

- To predict the number of cycles to failure, rewrite Paris equation as

$$dN = \frac{da}{C(\Delta K_I)^m} \text{ and integrate to get } N = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{(\Delta K_I)^m}$$



Illustrative example 1

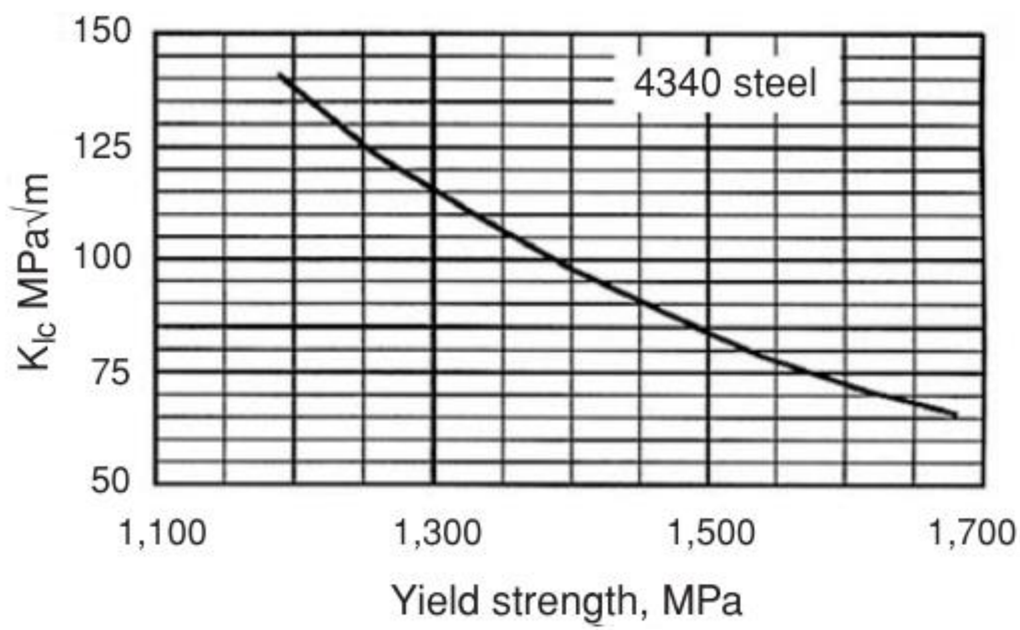
A support is to be made from 4340 steel. The steel may be tempered to different yield strengths. The correlation between strength and toughness is shown in Figure. Assume that $\beta = 1.1$ and a = 2mm. If K_{IC} = 120MPa$\sqrt{\text{m}}$, will the part fail? If it fails, is the cause by yielding or fracture?

From Figure, for given K_{Ic} the S_y = 1275MPa;

At $K_I = K_{IC} = \beta\sigma\sqrt{\pi a}$

$\sigma = \frac{K_{IC}}{\beta\sqrt{\pi a}} = 1376\text{MPa}$

This is higher than the yield strength, so it will fail by yielding when σ first reaches 1,275 MPa.



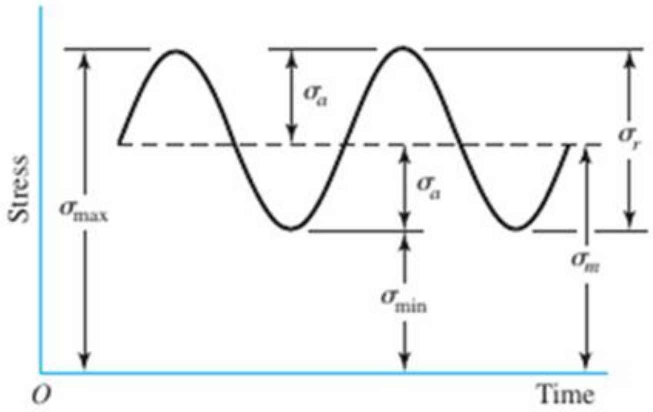
Prevention

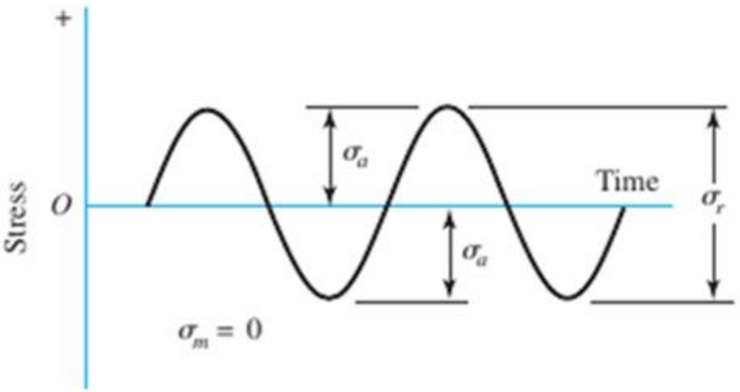
- Crack propagation (growth) is due to tensile stresses and grows along planes normal to the maximum tensile stress
- Cycle stresses that are always compressive will not elicit crack propagation

- Prevention – most effective method is improvements in design:
- Eliminate or reduce stress raisers by streamlining the part
- Avoid sharp surface tears resulting from punching, stamping, shearing, or other processes
- Prevent the development of surface discontinuities during processing
- Reduce or eliminate tensile residual stresses caused by manufacturing
- Improve the details of fabrication and fastening procedures

Stress fluctuation & failure

Compare the 2 fatigue stresses below, which one is expected to fail earlier?





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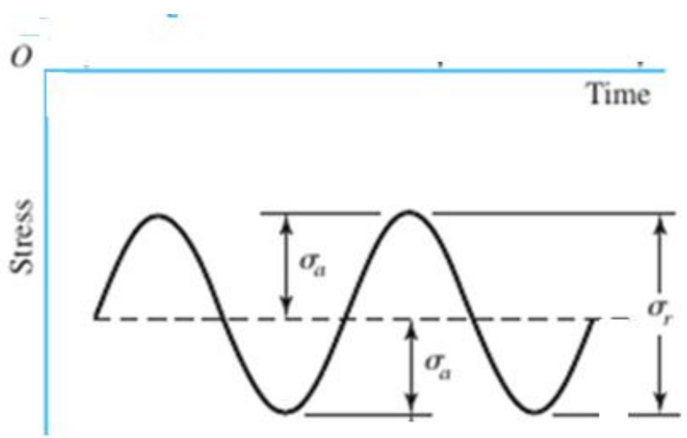
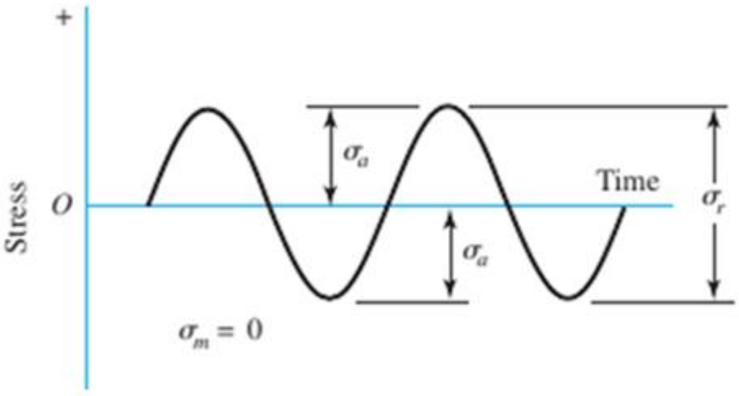
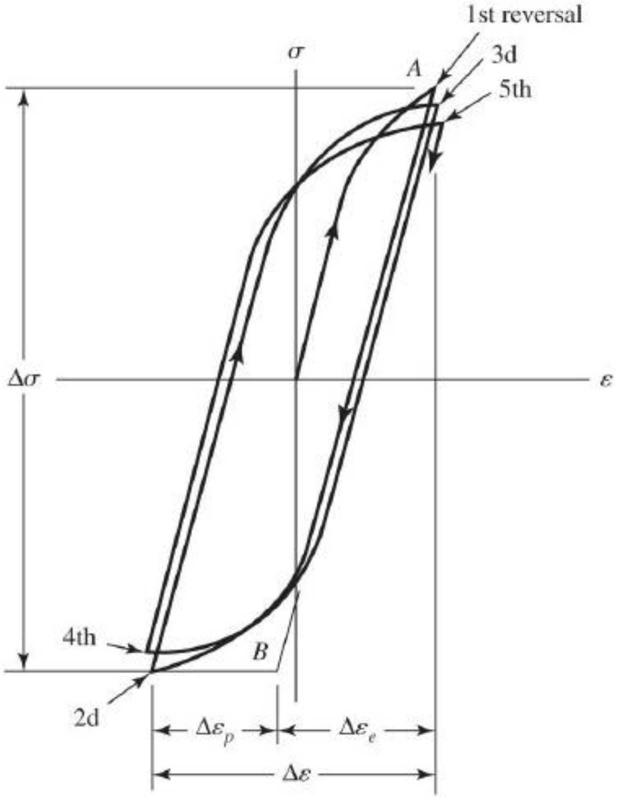




Strain-life method

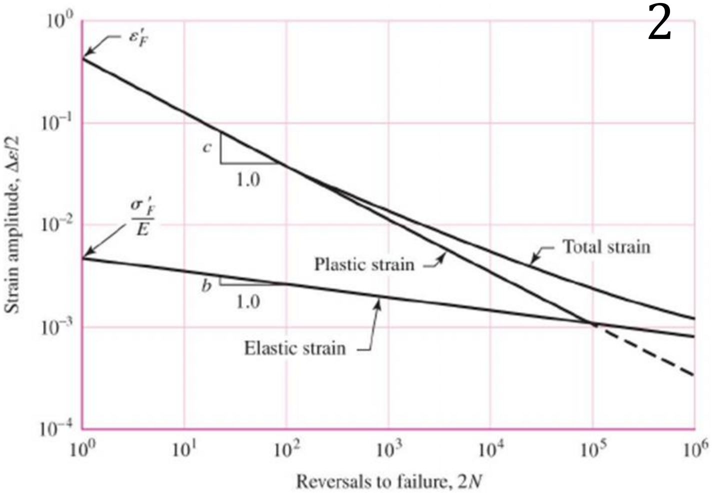
- Fatigue failure almost always begins at a local discontinuity
- When stress at discontinuity exceeds elastic limit, plastic strain occurs
- Cyclic plastic strain can change elastic limit, leading to fatigue
- Fatigue ductility coefficient ε'_F is the true strain corresponding to fracture in one reversal (point A). Plastic-strain line begins at ε'_F
- Fatigue strength coefficient σ'_F is the true stress corresponding to fracture in one reversal (point A). Elastic-strain line begins at σ'_F/E



Strain-life method

- Total strain is sum of elastic and plastic strain $\Delta\epsilon = \Delta\varepsilon_e + \Delta\varepsilon_p$
- Total strain amplitude is half the total strain range $\frac{\Delta\varepsilon}{2} = \frac{\Delta\varepsilon_e}{2} + \frac{\Delta\varepsilon_p}{2}$
- The equation of the elastic strain line is $\frac{\Delta\varepsilon_e}{2} = \frac{\sigma'_F}{E}(2N)^b$
- The equation of the plastic-strain line is $\frac{\Delta\varepsilon_p}{2} = \varepsilon'_F(2N)^c$
- The total-strain amplitude (called Manson-Coffin relationship) is

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_F}{E}(2N)^b + \varepsilon'_F(2N)^c$$



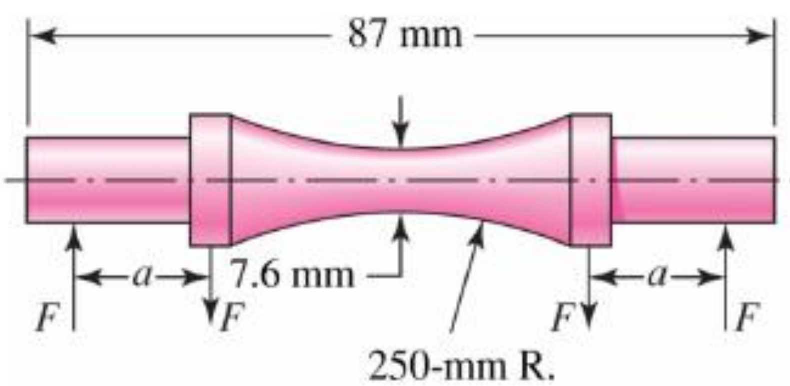
- ❖ N = number of cycles to failure
- ❖ Note: b = fatigue strength exponent or slope of elastic strain line
- ❖ Note: c = fatigue ductility exponent or slope of plastic strain line

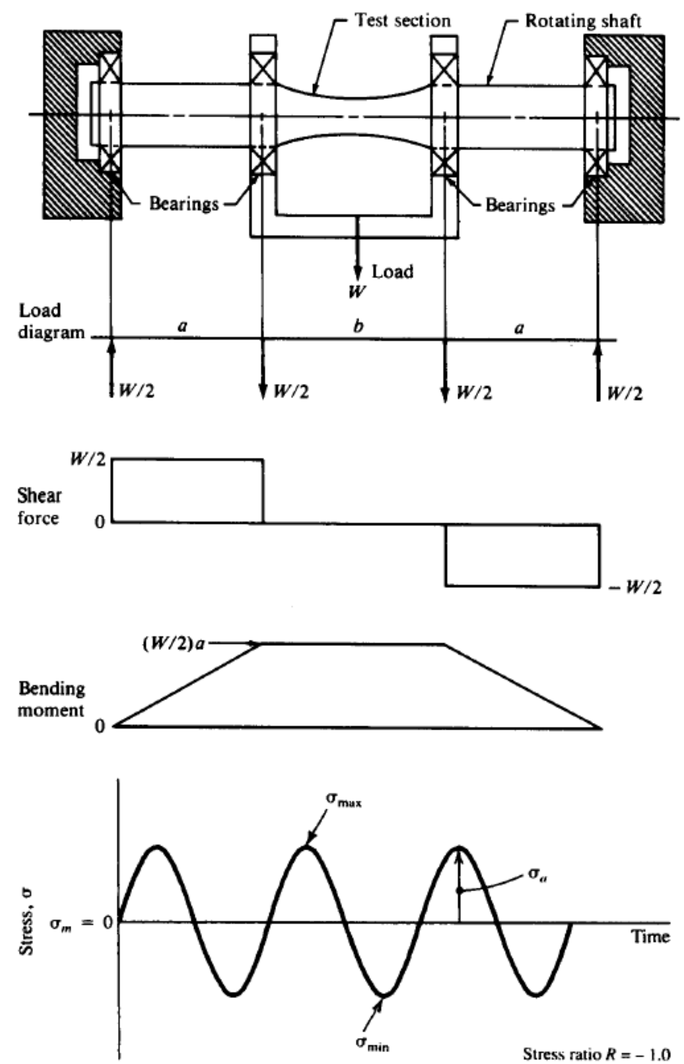
(see Table A-23)

Stress-life method

- Fatigue life is the time (in number of cycles, N) at which a component subjected to a given cyclic load would fail
- Cyclic load can be classified as low cycle fatigue or high cycle fatigue
  a) Low cycle fatigue considers the range from $1 \leq N \leq 10^3$ cycles; yielding usually occurs before fatigue in this zone
  b) High cycle fatigue considers N above 10^3 cycles;
- Stress-life method
  - This method predicts fatigue life N using S-N curves from experimental studies. It does not predict well in low-cycle fatigue regime
  - Specimens are subjected to cycles of fully reversible loads and the number of cycles to failure N are counted
  - The R. R. Moore high-speed rotating-beam machine is the most commonly used equipment

Stress-life method

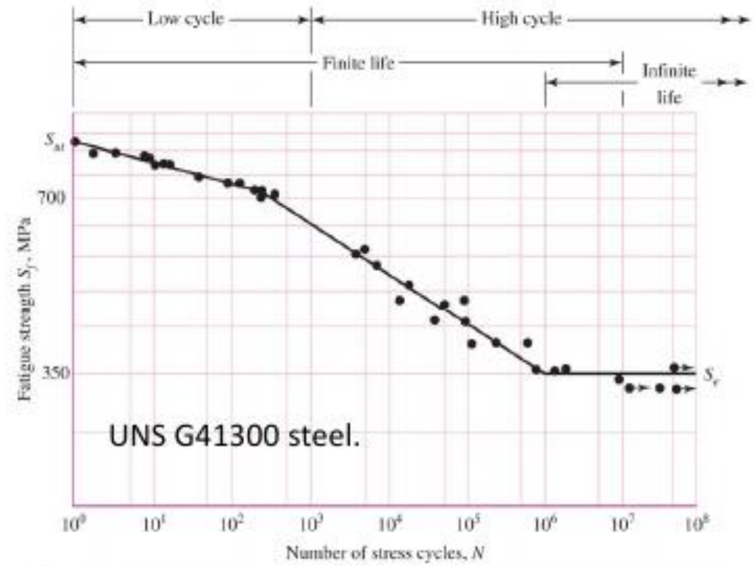


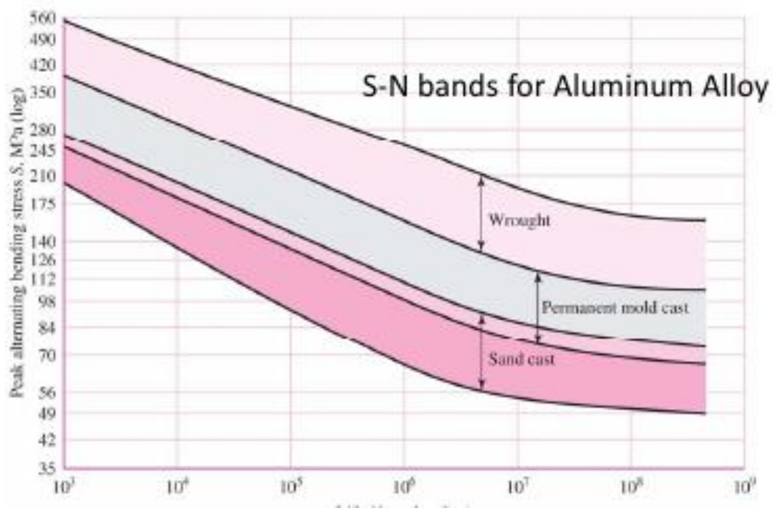


- Specimens are carefully machined and polished
- Many tests are conducted at different (fully reversible) stress amplitudes
- Each test will produce a different number of cycles to failure
- The data collected are the stress levels σ and the number of cycles *N* to failure
- Data is plotted on log-log scale to give the stress-life cycle (S-N) curve

S-N curves

- For steels, a knee occurs near 10^6 cycles with corresponding strength S_e, which is called the endurance limit. Stress amplitude below S_e has infinite life (i.e. no fatigue failure)
- Non ferrous metals and alloys do not have endurance limits. Fatigue strength S_f is reported at a specific number of cycles, e.g. for Al, fatigue strength is reported at $N = 5(10^8)$ cycles





Endurance limit – steels

ASTM Number	Tensile Strength S_{ut}, MPa (kpsi)	Compressive Strength S_{uc}, MPa (kpsi)	Shear Modulus of Rupture S_{su}, MPa (kpsi)	Modulus of Elasticity, Mpsi Tension†	Modulus of Elasticity, Mpsi Torsion	Endurance Limit* S_e, MPa (kpsi)	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
20	152 (22)	572 (83)	179 (26)	9.6–14	3.9–5.6	69 (10)	156	1.00
25	179 (26)	669 (97)	220 (32)	11.5–14.8	4.6–6.0	79 (11.5)	174	1.05
30	214 (31)	752 (109)	276 (40)	13–16.4	5.2–6.6	97 (14)	201	1.10
35	252 (36.5)	855 (124)	334 (48.5)	14.5–17.2	5.8–6.9	110 (16)	212	1.15
40	293 (42.5)	970 (140)	393 (57)	16–20	6.4–7.8	128 (18.5)	235	1.25
50	362 (52.5)	1130 (164)	503 (73)	18.8–22.8	7.2–8.0	148 (21.5)	262	1.35
60	431 (62.5)	1293 (187.5)	610 (88.5)	20.4–23.5	7.8–8.5	169 (24.5)	302	1.50

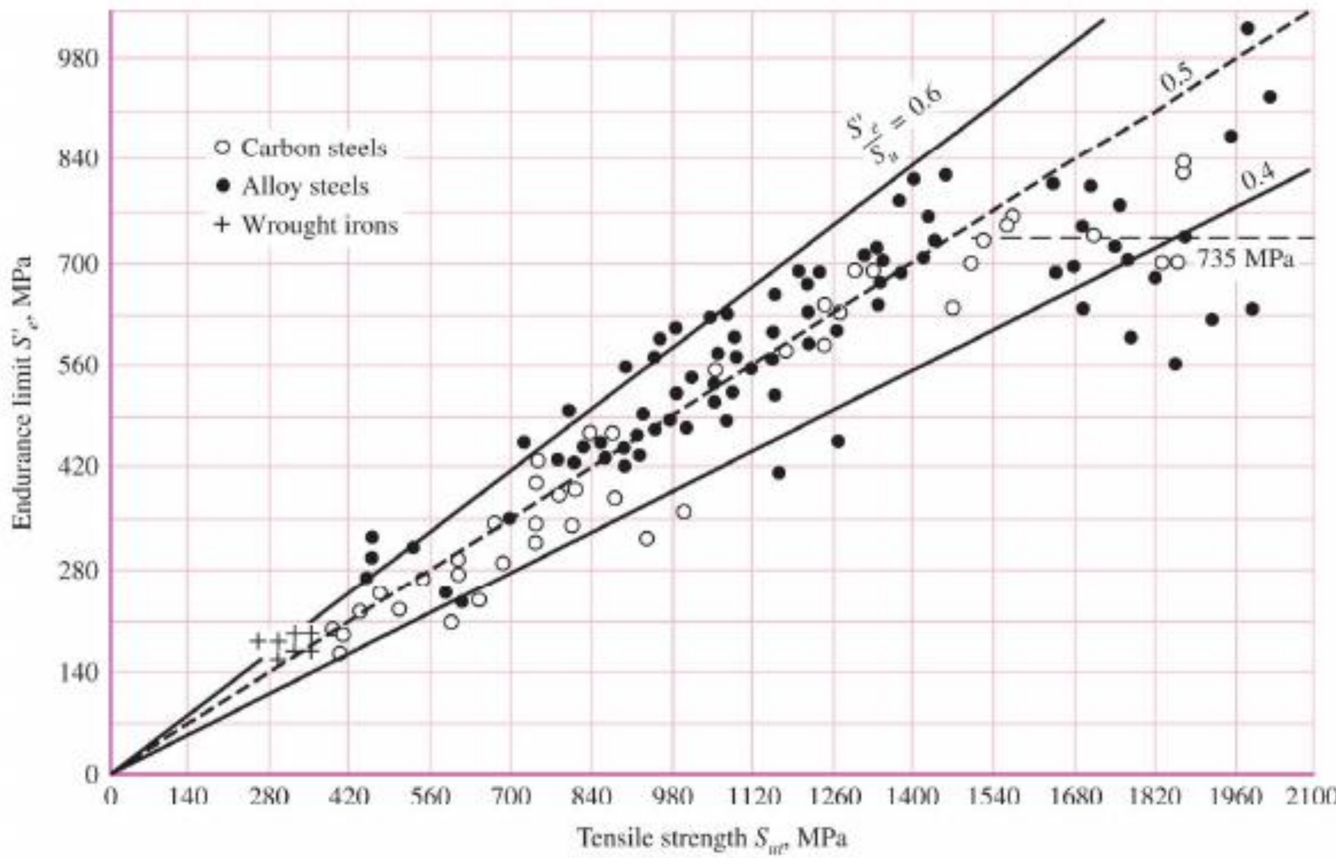
*Polished or machined specimens.

†The modulus of elasticity of cast iron in compression corresponds closely to the upper value in the range given for tension and is a more constant value than that for tension.

- The endurance limit for steels has been experimentally found using the rotating beam to be related to the ultimate strength
- Note: a prime symbol S'_e is associated with the rotating-beam specimens (the unprimed symbol S_e is for the endurance limit of an actual machine element subjected to any kind of loading

Endurance limit – steels

$$S'_e \begin{cases} 0.5S_{ut} & S_{ut} \leq 1400\ \text{MPa} \\ 700\ \text{MPa} & S_{ut} > 1400\ \text{MPa} \end{cases}$$



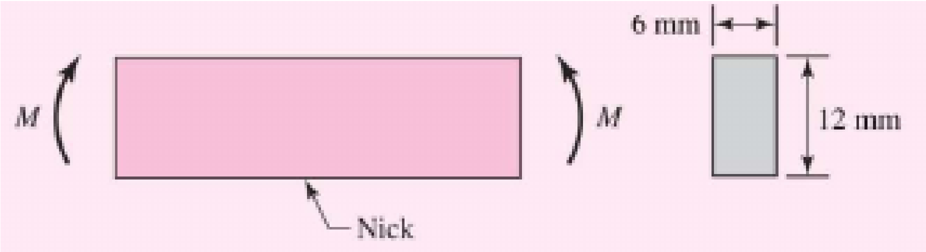
Fatigue strength

Aluminum Association Number	Temper	Yield, S_y, MPa (kpsi)	Strength Tensile, S_u, MPa (kpsi)	Fatigue, S_f, MPa (kpsi)	Elongation in 2 in, %	Brinell Hardness H_B
Wrought:						
2017	O	70 (10)	179 (26)	90 (13)	22	45
2024	O	76 (11)	186 (27)	90 (13)	22	47
	T3	345 (50)	482 (70)	138 (20)	16	120
3003	H12	117 (17)	131 (19)	55 (8)	20	35
	H16	165 (24)	179 (26)	65 (9.5)	14	47
3004	H34	186 (27)	234 (34)	103 (15)	12	63
	H38	234 (34)	276 (40)	110 (16)	6	77
5052	H32	186 (27)	234 (34)	117 (17)	18	62
	H36	234 (34)	269 (39)	124 (18)	10	74
Cast:						
319.0*	T6	165 (24)	248 (36)	69 (10)	2.0	80
333.0†	T5	172 (25)	234 (34)	83 (12)	1.0	100
	T6	207 (30)	289 (42)	103 (15)	1.5	105
335.0*	T6	172 (25)	241 (35)	62 (9)	3.0	80
	T7	248 (36)	262 (38)	62 (9)	0.5	85

The values given for fatigue strength correspond to $50(10^7)$ cycles of completely reversed stress

Example 1

The bar shown is subjected to a repeated moment $0 \leq M \leq 135$ Nm. The bar is AISI4430 steel with $S_{ut} = 1.28$ GPa; $S_y = 1.17$ GPa; $K_{IC} = 81$ MPa$\sqrt{\text{m}}$; Material tests on various specimens of this material with identical heat treatment indicate worst case constants of $C = 114 \times 10^{-15}$ (m/cycle)MPa $\sqrt{\text{m}}$, and $m = 3.0$; As shown, a nick of size 0.1 mm has been discovered at the bottom of the bar. Estimate the number of cycles of life remaining



Given $\Delta M = 135$ Nm;

For pure bending:

$\Delta\sigma = \frac{\Delta M c}{I}$ where

- $c = \frac{h}{2}$ (given h = 12mm)
- $I = \frac{bh^3}{12}$ (given b = 6mm)

$\Delta\sigma = 937.5$ MPa

Example 1

Initial $a_i = 0.1$ mm

At $K_I = K_{IC} = 81\ \text{MPa}\sqrt{\text{m}}$, we get

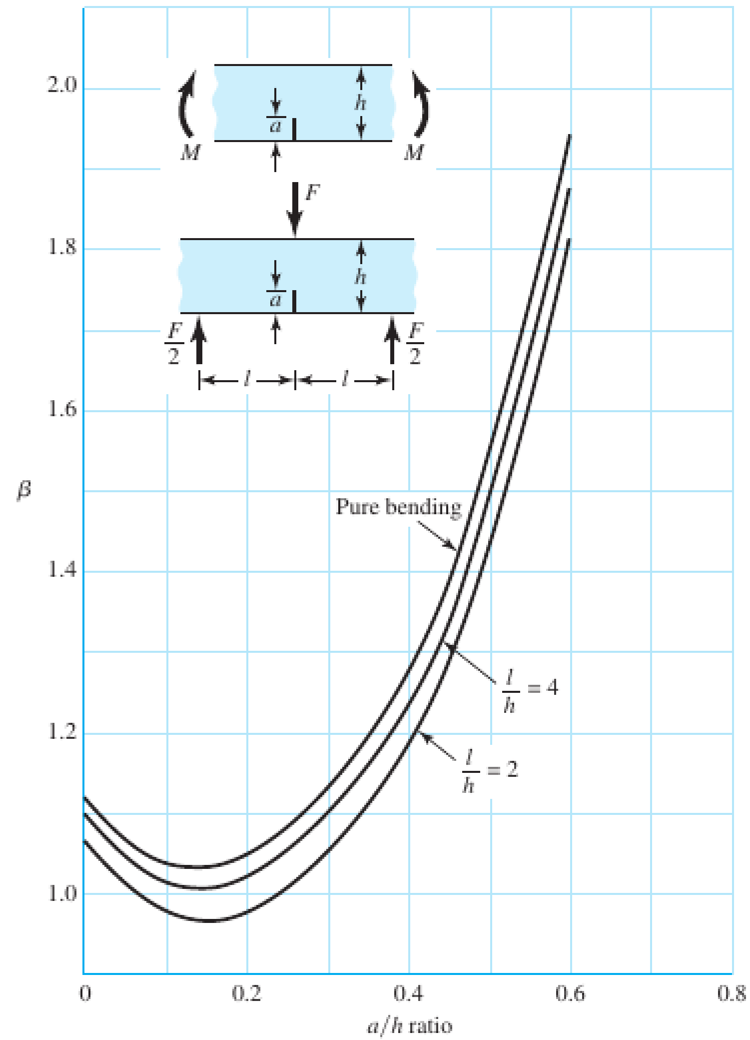
$a = a_f = \frac{1}{\pi}\left[\frac{K_{ic}}{\beta\sigma_{\max}}\right]^2$ and the part will fracture; Let $\beta = 1$ and with $\sigma_{\max} =$ 937.5 MPa:

$$a_f = \frac{1}{\pi}\left[\frac{K_{ic}}{\beta\sigma_{\max}}\right]^2 = 0.0024\ \text{m};$$

$\frac{a_f}{h} = 0.2$ (from figure for pure bending: $\beta = 1.05$; recalculate a_f)

$$a_f = \frac{1}{\pi}\left[\frac{K_{ic}}{\beta\sigma_{\max}}\right]^2 = 0.00216\ \text{m};$$

$$N = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{(\Delta K_I)^m} = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{(\beta\Delta\sigma\sqrt{\pi a})^m}$$



Example 1

$$N = \frac{1}{C}\int_{a_i}^{a_f} \frac{da}{(\beta\Delta\sigma\sqrt{\pi a})^m} = \frac{1}{114 \times 10^{-15}}\int_{0.0001}^{0.00216} \frac{da}{(1.05(937.5)\sqrt{\pi a})^3}$$

$$N = -\frac{825.8}{\sqrt{a}}\Bigg|_{0.0001}^{0.00216}$$

$$N = 64.8(10^3)\ \text{cycles}$$