ME1020
Mechanical vibrations

Lecture 8

Vibration isolation

Objectives

- Explain the criteria for considering the severity of vibration
- Analyze vibration isolation for fixed and moving bases
- Describe the typical applications of industry vibrators in engineering applications

Noise & vibration

- ❖ Vibration is often associated with noise.
- ❖ Noise levels are often described in decibel (dB) scale
- ❖ Decibel is originally defined as a ratio of electric powers (i.e. power P is described relative to some reference value P_0) by:

$$\text{dB} = 10\log_{10}\left(\frac{P}{P_0}\right)$$

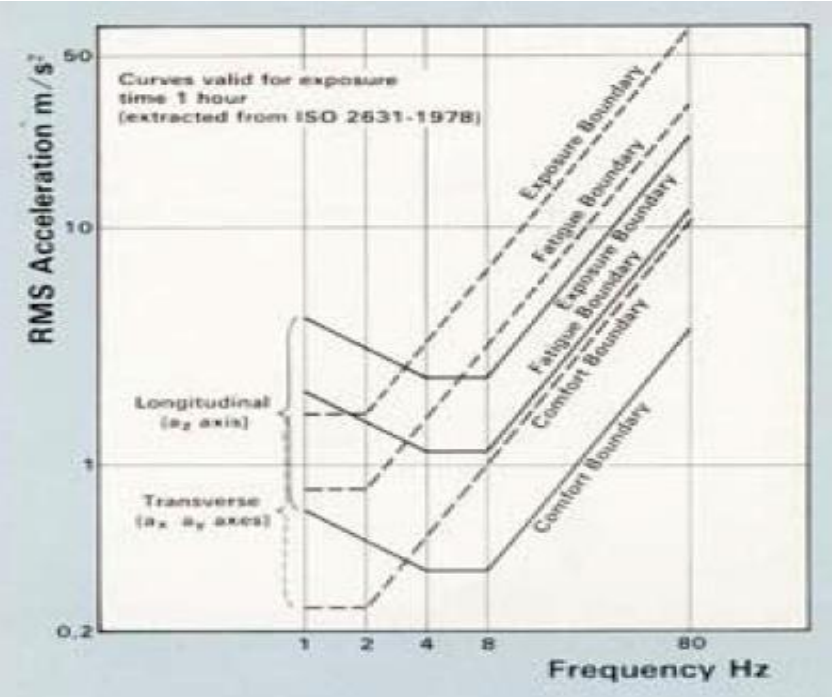
- ❖ It is now often used with various quantities. Example displacement X with respect to X_0

$$\text{dB} = 10\log_{10}\left(\frac{X}{X_0}\right)^2 = 20\log_{10}\left(\frac{X}{X_0}\right)$$

Human comfort

Vibration limits for human comfort (ISO 2631)

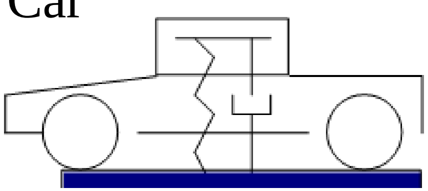
- ❖ Limitation curves for exposure times from 1 minute to 12 hours over the frequency range in which the human body has been found to be most sensitive, namely 1 Hz to 80 Hz.

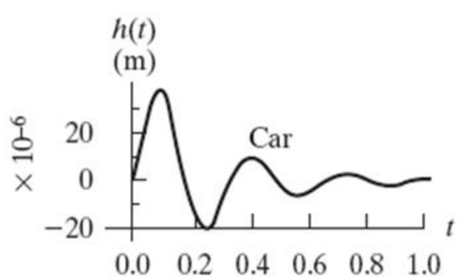


Vibration criteria

- Criteria can be based on amplitudes of displacement, velocity, and acceleration as well as time of exposure
- The severity of the vibration will depend on the system (e.g. consider a car and a CD drive as shown): both have the same natural frequency $\omega_n = 20$ rad/s; damping ratio $\zeta = 0.2$; and damped natural frequency $\omega_d = 19.5959$ rad/s; their responses may have the same form but the magnitudes can be different

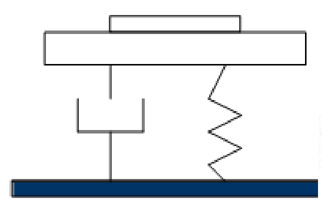
Car

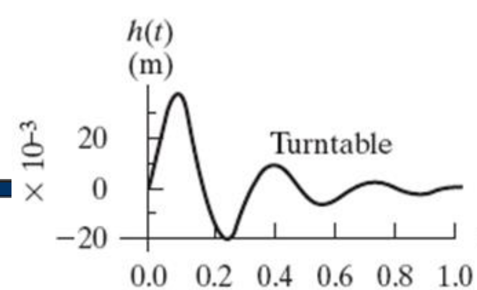




m=1000 kg
k=400,000 N/m
c=8000 Ns/m

Turntable





m=1 kg
k=400 N/m
c=8 Ns/m

Vibration criteria

Consider the responses are simple harmonic motions:

$$x(t) = X \sin \omega t$$
$$v(t) = \dot{x}(t) = \omega X \cos \omega t = 2\pi f X \cos \omega t$$
$$a(t) = \ddot{x}(t) = -\omega^2 X \sin \omega t = -4\pi^2 f^2 X \sin \omega t$$

❖ Amplitude of velocity:

$$v_{\mathrm{max}} = 2\pi f X$$
$$\ln(v_{\mathrm{max}}) = \ln(2\pi f) + \ln X$$

❖ Amplitude of acceleration:

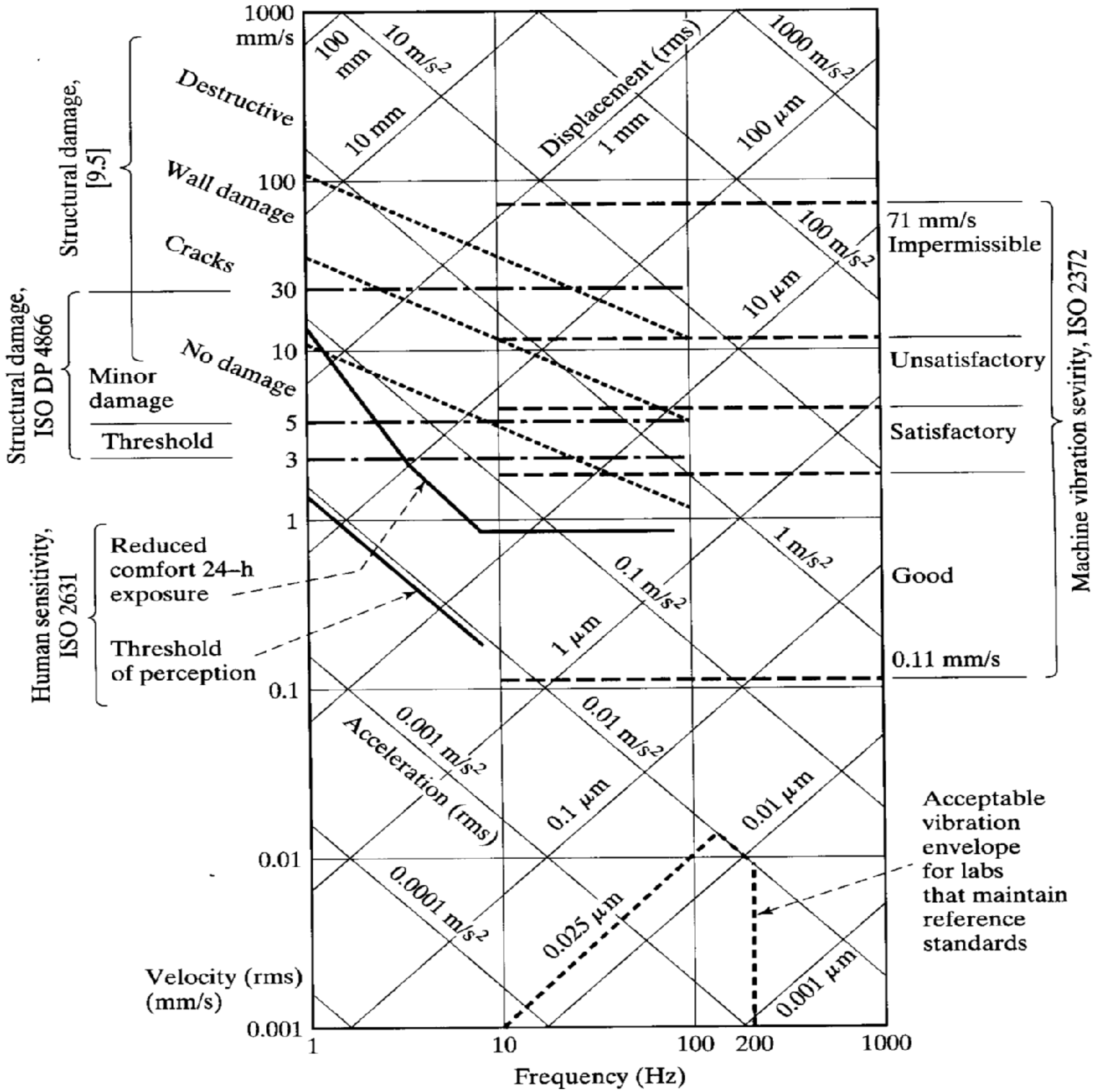
$$a_{\mathrm{max}} = -4\pi^2 f^2 X = -2\pi f v_{\mathrm{max}}$$
$$\ln(a_{\mathrm{max}}) = -\ln(2\pi f) - \ln(v_{\mathrm{max}})$$
$$\ln(v_{\mathrm{max}}) = -\ln(a_{\mathrm{max}}) - \ln(2\pi f)$$

❖ When X is constant, $\ln(v_{\mathrm{max}})$ varies linearly with $\ln(2\pi f)$

❖ When a_{max} is constant, $\ln(v_{\mathrm{max}})$ varies linearly with $\ln(2\pi f)$

❖ These are shown as a nomograph

- Example: vibration severity of whole building vibration (ISO DP 4866)
- Every point denotes a specific sinusoidal vibration (i.e. variations of displacement, velocity and acceleration amplitudes with respect to the frequency



ISO 2372 – ISO Guideline for Machinery Vibration Severity					
Ranges of Vibration severity		Examples of quality judgment for separate classes of machines			
Velocity – in/s – Peak	Velocity – mm/s – rms	Class I	Class II	Class III	Class IV
0.015	0.28				
0.025	0.45				
0.039	0.71				
0.062	1.12				
0.099	1.8				
0.154	2.8				
0.248	4.5				
0.392	7.1				
0.617	11.2				
0.993	18				
1.54	28				
2.48	45				
3.94	71				

A – Good
B – Acceptable
C – Still acceptable
D – Not acceptable

Class I Individual parts of engines and machines integrally connected with a complete machine in its normal operating condition (production electrical motors of up to 15 kW are typical examples of machines in this category).
Class II Medium-sized machines (typically electrical motors with 15–75 kW output) without special foundations, rigidly mounted engines or machines (up to 300 kW) on special foundations.
Class III Large prime movers and other large machines with rotating masses mounted on rigid and heavy foundations, which are relatively stiff in the direction of vibration.
Class IV Large prime movers and other large machines with rotating masses mounted on foundations, which are relatively soft in the direction of vibration measurement (for example – turbogenerator sets, especially those with lightweight substructures).

Vibration isolation

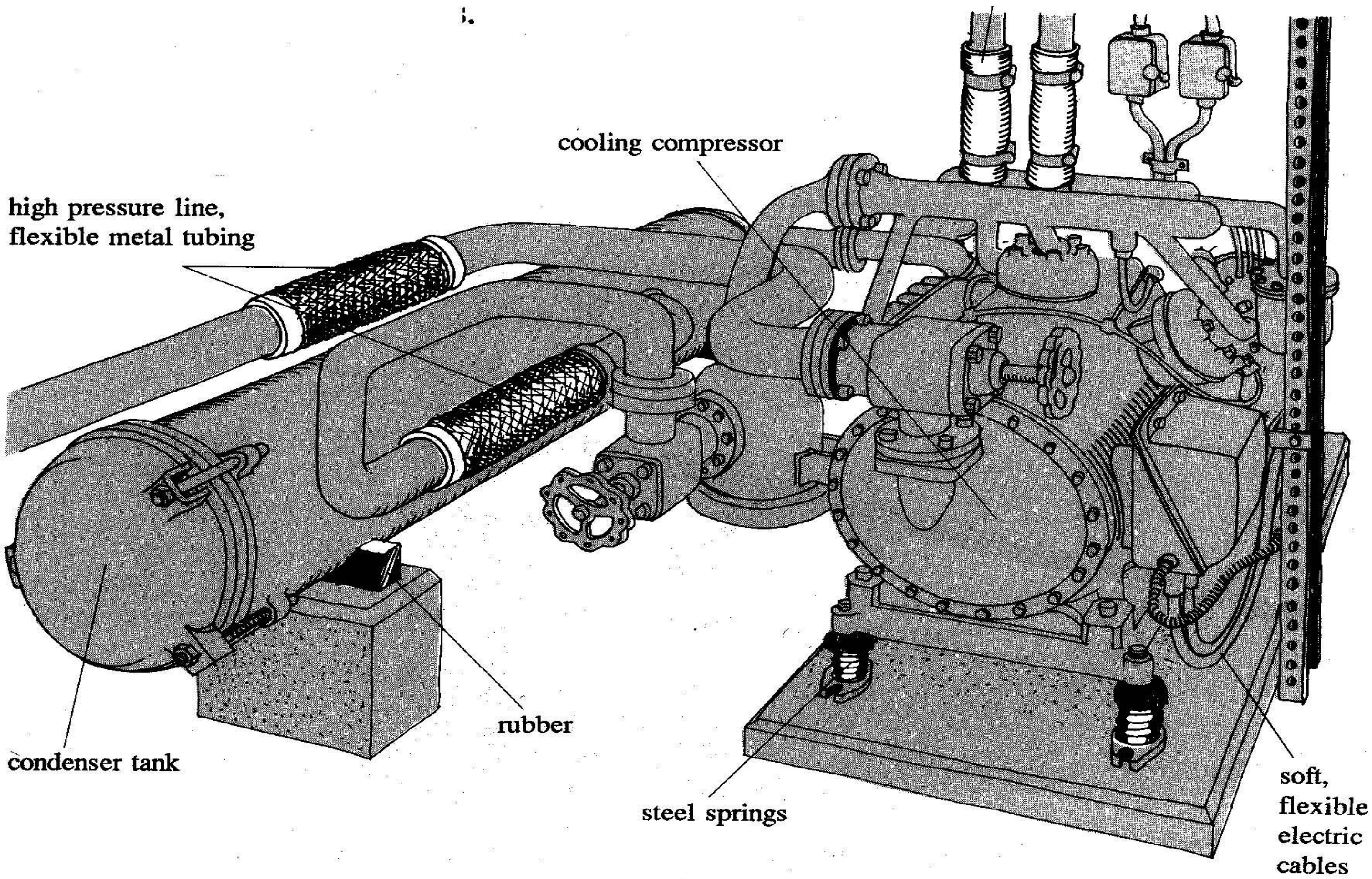
Vibratory forces generated by machines are sometimes unavoidable. However, their effects of the vibration can often be minimized by proper isolator design

An isolation system attempts to:

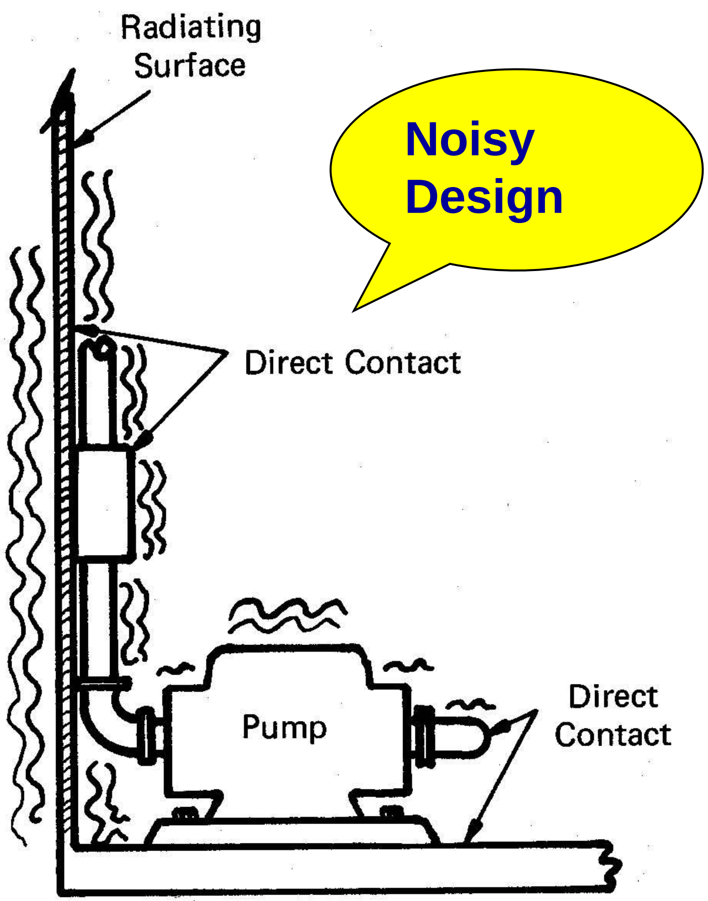
- ➢ Protect an object from excessive vibration transmitted from its supporting structure
- ➢ Prevent transmission of vibratory forces generated by machines to surroundings

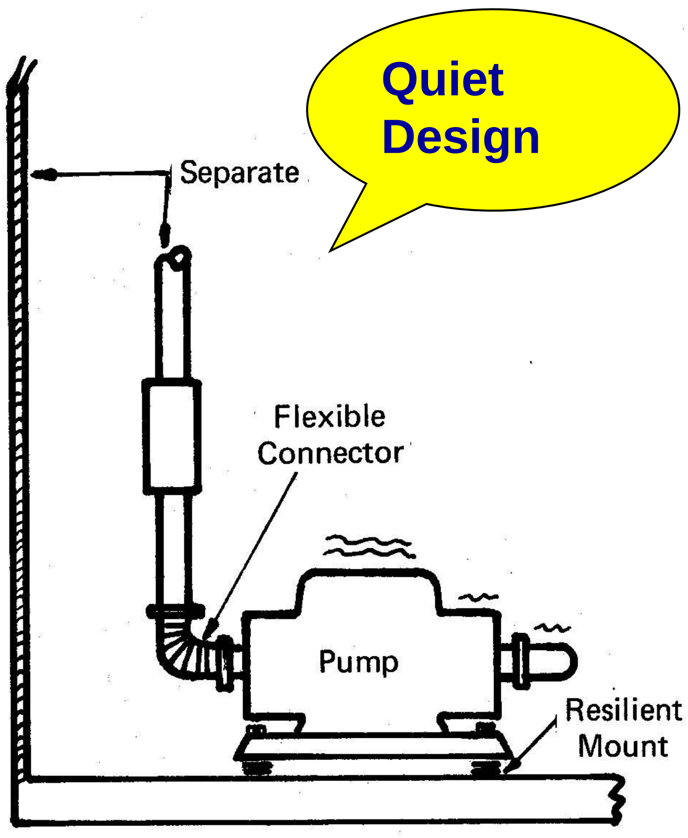
- ❖ Vibration isolators are mainly made up of springs combined with dampers and/or inertia elements
- ❖ Isolator is mainly inserted between the vibrating mass and vibration source to reduce the response

Isolation examples

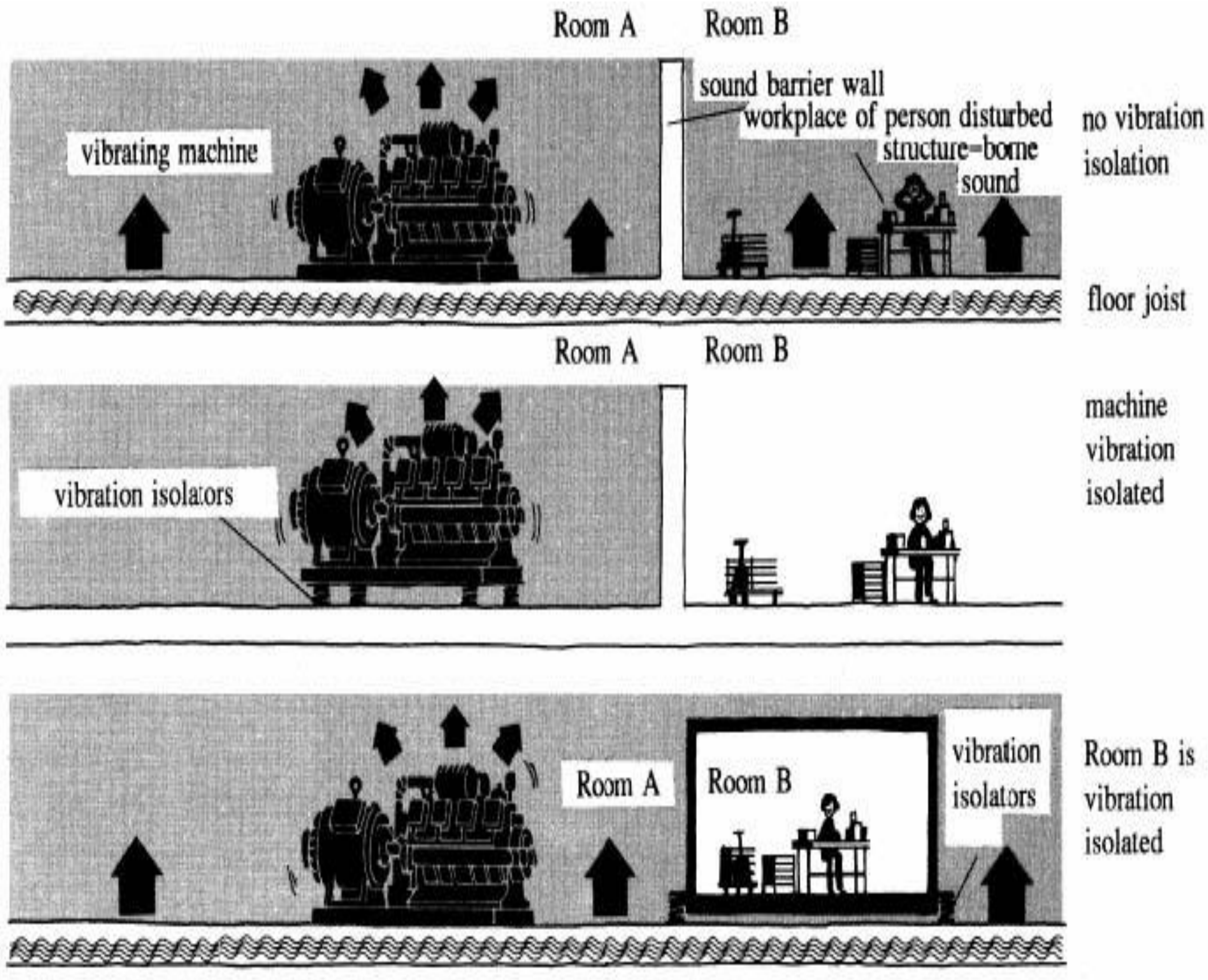


Isolation examples

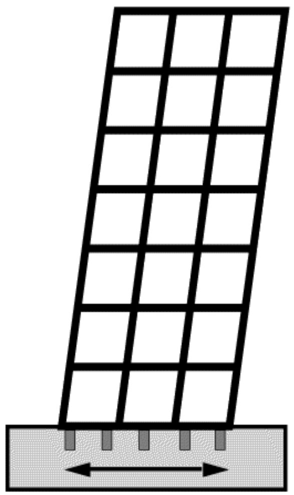


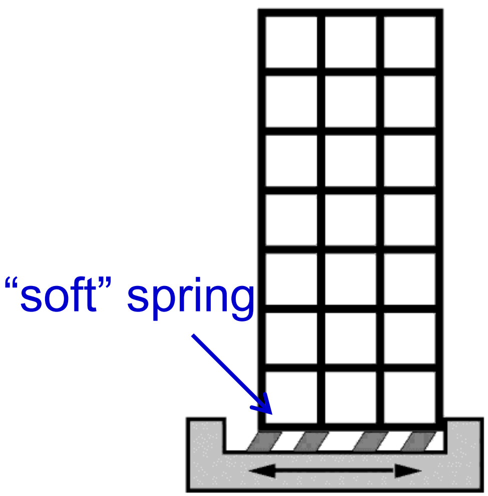


Isolation examples

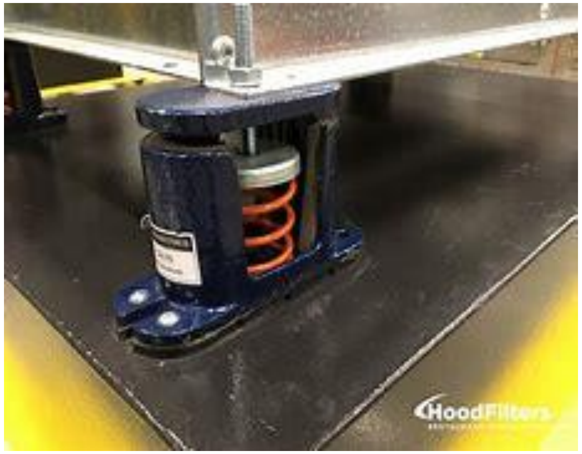


Isolation examples





Industry isolators



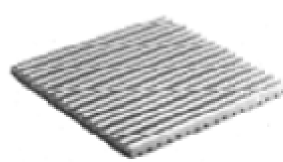
Industry isolators

Selection Guide for Vibration Isolation

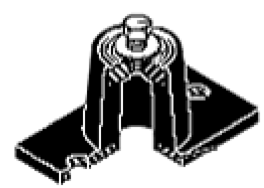
			Equipment Location											
			Slab on Grade			Up to 6 m Floor Span			6- to 9 m Floor Span			9- to 12 m Floor Span		
Equipment Type	Shaft Power, kW and Other	Rpm	Base Type	Iso-lator Type	Min. Defl., in.	Base Type	Iso-lator Type	Min. Defl., in.	Base Type	Iso-lator Type	Min. Defl., in.	Base Type	Iso-lator Type	Min. Defl., in.
Refrigeration Machines and Chillers														
Bare compressors	All	All	A	2	0.25	C	3	0.75	C	3	1.75	C	4	2.50
Reciprocating	All	All	A	2	0.25	A	4	0.75	A	3	1.75	A	4	2.50
Centrifugal	All	All	A	1	0.25	A	4	0.75	A	3	1.75	A	3	1.75
Open centrifugal	All	All	C	1	0.25	C	4	0.75	C	3	1.75	C	3	1.75
Absorption	All	All	A	1	0.25	A	4	0.75	A	3	1.75	A	3	1.75

Base Types:
A. No base, isolators attached directly to equipment
B. Structural steel rails or base
C. Concrete inertia base
D. Curb-mounted base

Isolator Types:
1. Pad, rubber, or glass fiber
2. Rubber floor isolator or hanger
3. Spring floor isolator or hanger
4. Restrained spring isolator
5. Thrust restraint

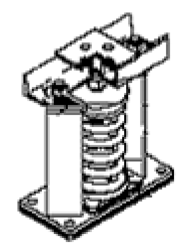


Type 1 Rubber Pad

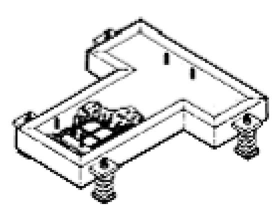


Type 2 Rubber Mount

Type 3 Spring Isolator



Type 4 Restrained Spring Isolator



CONCRETE BASES (Type C)

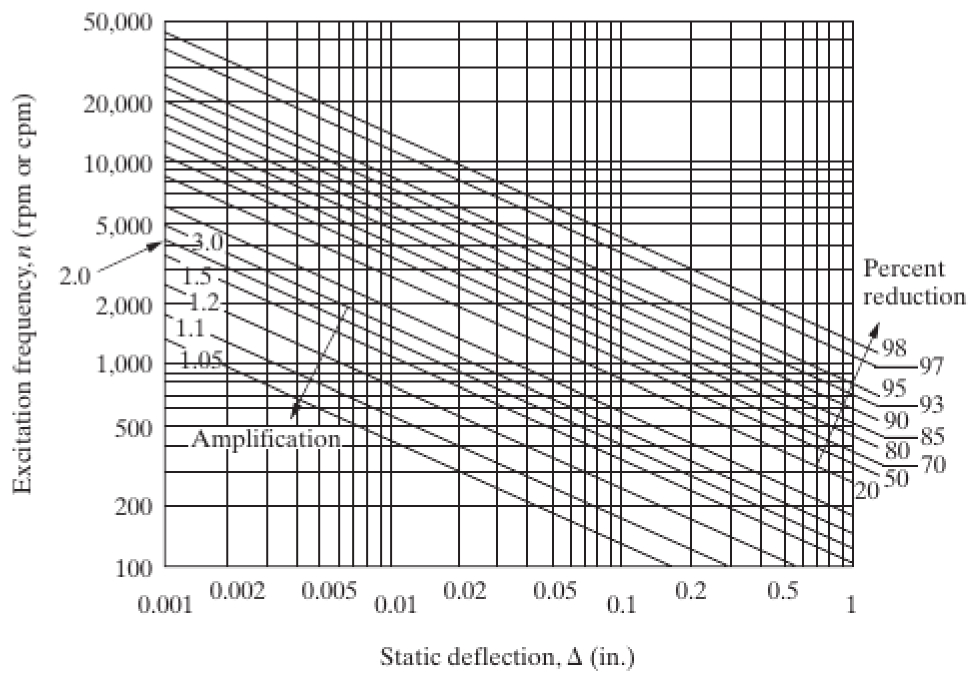


Figure 5.9 Design curves consisting of plots of running speed versus static deflection (or stiffness) for various values of percent reduction in transmitted force.

TABLE 5.3 CATALOG VALUES OF STIFFNESS AND DAMPING PROPERTIES OF VARIOUS OFF-THE-SHELF ISOLATORS

Part No.[a]	R-1	R-2	R-3	R-4	R-5	M-1	M-2	M-3	M-4	M-5
k (10^3 N/m)	250	500	1000	1800	2500	75	150	250	500	750
c (N · s/m)	2000	1800	1500	1000	500	110	115	140	160	200

[a]The "R" in the part number designates that the isolator is made of rubber, and the "M" designates metal. In general, metal isolators are more expensive than rubber isolators.

Isolation problems

Two main types of isolation problems:

1. Fixed base or rigid foundation
2. Moving base

❖ The effectiveness of the isolation is given in terms of the transmissibility, which can be displacement transmission ratio or force transmission ratio:

- Fixed base force transmission ratio

$$\frac{F_T}{F_0} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

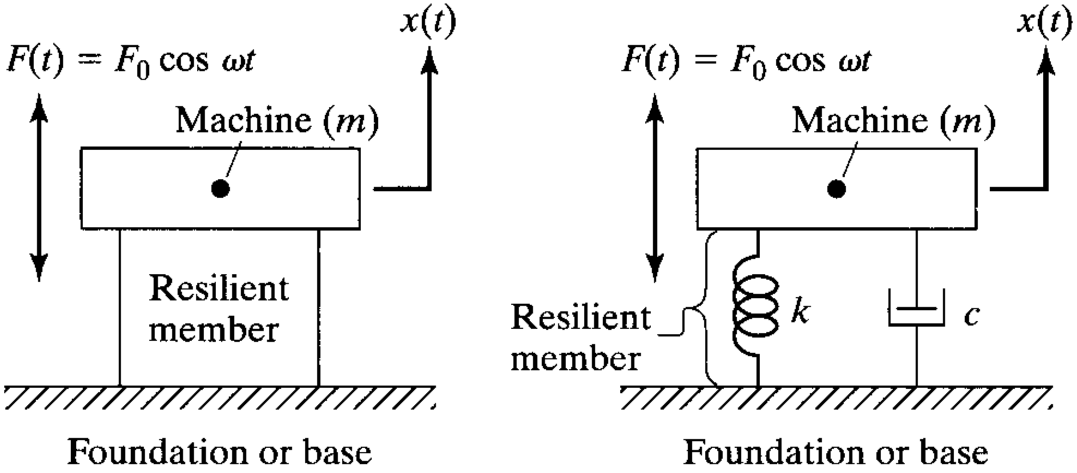
- Moving base force transmission ratio

$$\frac{F_T}{kY} = r^2 \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

- Moving base displacement transmission ratio

$$\frac{X}{Y} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

Fixed base



- A resilient member is placed placed between the vibrating machine and rigid foundation to reduce the vibration and its transmission
- Resilient member is modeled as a spring k and a dashpot c
- The force transmission ratio is

$$\frac{F_T}{F_0} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

Fixed base

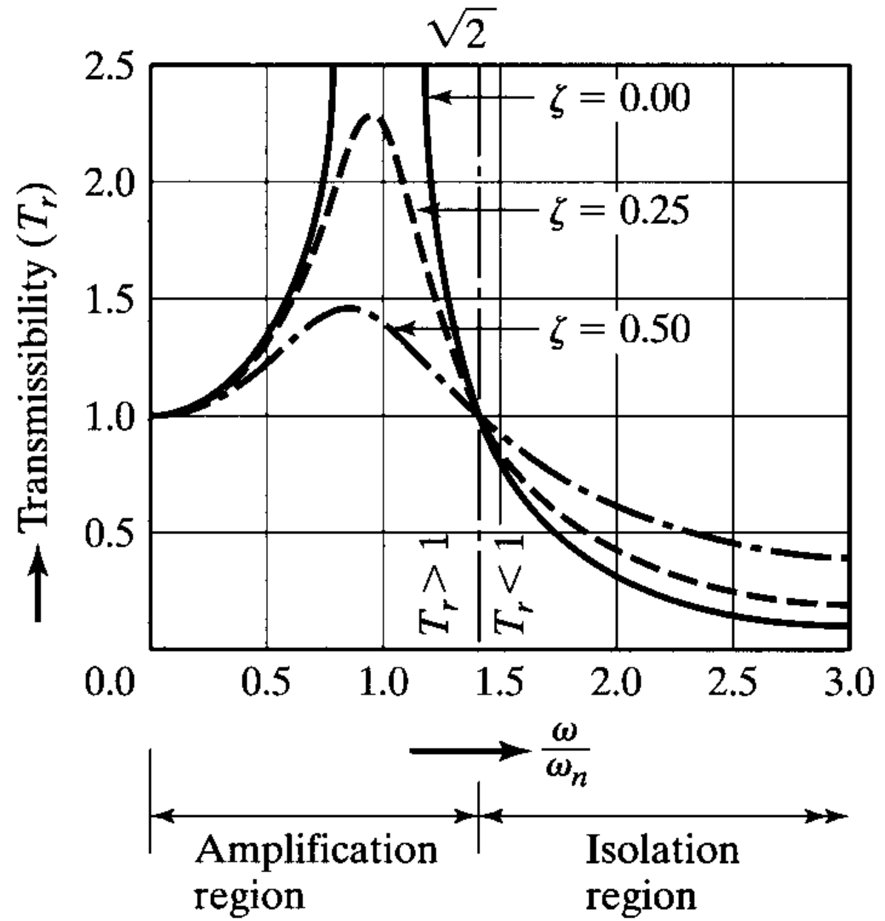
$$T_r = \frac{F_T}{F_0} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

Note: for rotating imbalanced

$$T_r = \frac{F_T}{m_0 e \omega^2} = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

- At $T_r = 1$, $r = \omega/\omega_n = \sqrt{2}$
- For $r > \sqrt{2}$ and small ζ

$$T_r \approx \frac{1}{r^2 - 1}$$



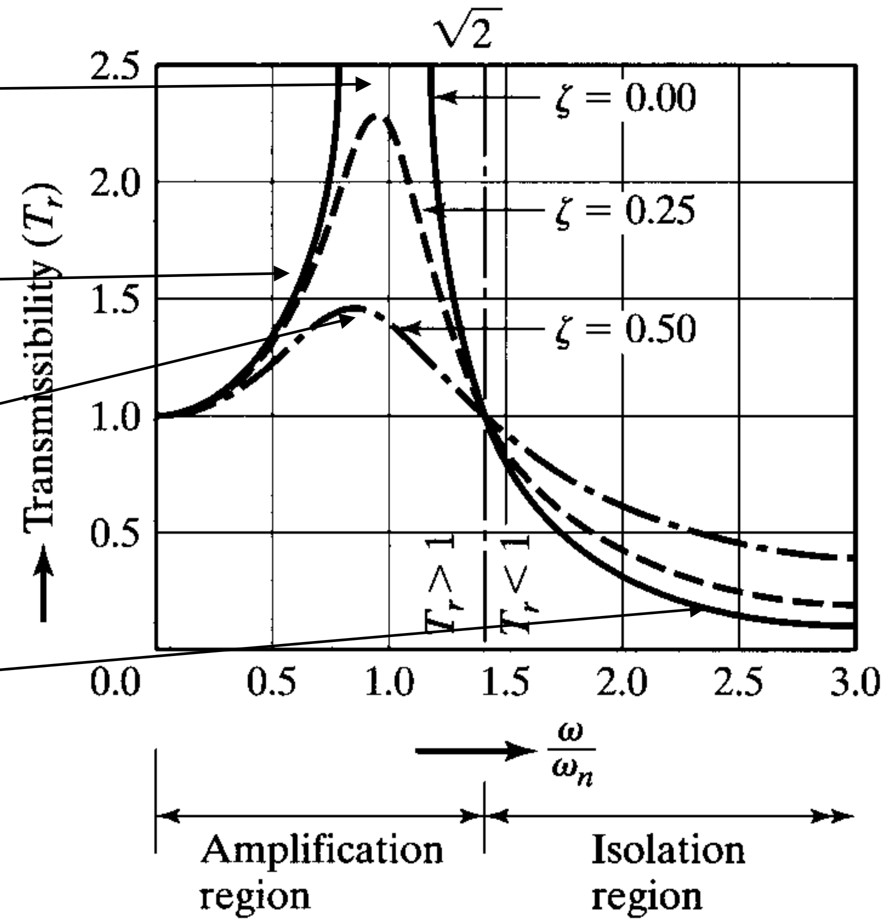
Fixed base

Isolation with little damping:
danger of vibration reinforcement
if $\omega = \omega_n$

Isolation with little damping:
no isolation if $\omega < \omega_n$

Isolation with damping:
no isolation if $\omega = \omega_n$

Isolation with little damping:
good isolation if $\omega > \omega_n$

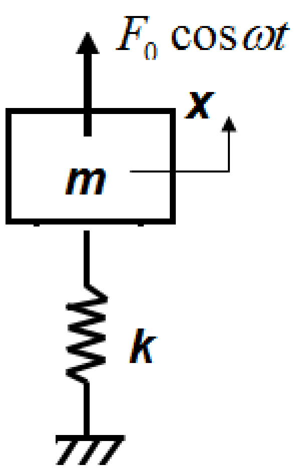


$$T_r = \frac{1}{r^2 - 1} \quad \text{for } r > \sqrt{2} \text{ and small } \zeta$$

Fixed base

Focus on isolation with little (negligible) damping with $\omega > \omega_n$ or $r > \sqrt{2}$ where

$$T_r = \frac{1}{r^2 - 1}$$



❖ For an undamped system:

$$k\delta_{st} = mg$$

where δ_{st} is the static deflection

❖ Hence

$$\frac{k}{m} = \omega_n^2 = \frac{g}{\delta_{st}} \text{ and } r^2 = \frac{\omega^2}{\omega_n^2} = \frac{(2\pi f)^2}{g/\delta_{st}} = \frac{(2\pi f)^2 \delta_{st}}{g}$$

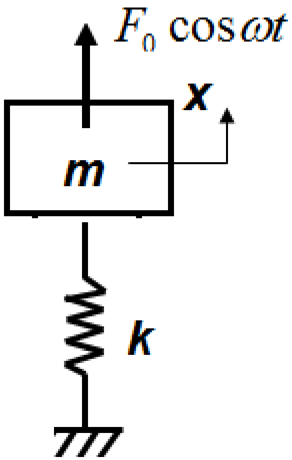
❖ For $r > \sqrt{2}$ and small ζ

$$T_r = \frac{1}{r^2 - 1} = \frac{1}{(2\pi f)^2\, \delta_{st}/g - 1}$$

Fixed base

For the fixed base system, we can specify an isolator with very little damping with frequency ratio $r > \sqrt{2}$ and small ζ such that

$$T_r = \frac{1}{(2\pi f)^2\,\delta_{st}/g - 1}$$



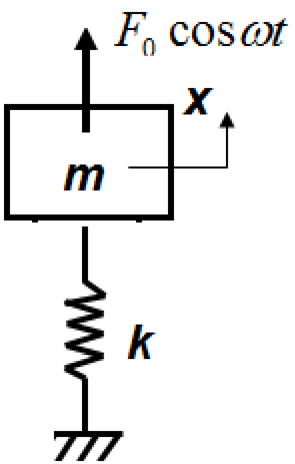
The isolator will reduce the force transmission when $\omega > \omega_n$

We will next examine the steady state amplitude of the isolated mass

Fixed base

The steady state amplitude X of the isolated mass “m” is given by

$$X = \frac{F_o/k}{\sqrt{[1-r^2]^2 + [2\varsigma r]^2}}$$

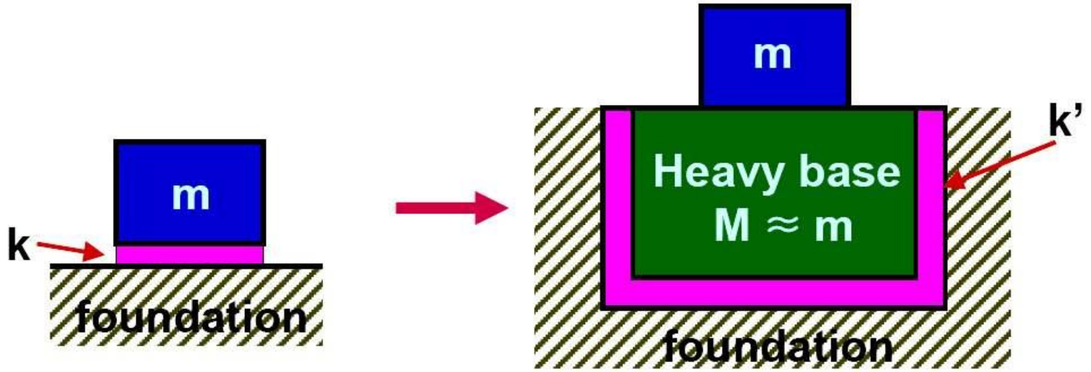


Note that when the mass in increased, the amplitude is reduced. This can be done by placing the isolated mass m on a large mass “M”. However, the stiffness “k” then must be increased to keep the ratio

$$k/(m+M) = \text{constant}$$

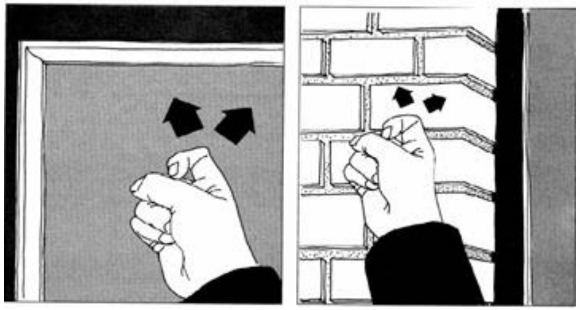
so that the natural frequency and transmission ratio is constant

Fixed base



- The concept of reducing the steady state vibration amplitude by placing the isolated mass *m* on a large mass "*M*" is shown
- Note that the stiffness original "*k*" must be increased to keep the natural frequency and the transmission ratio unchanged

The concept is similar to the situation of knocking on a thin door produces more sound than knocking on thick wall, i.e. noise source should be mounted on heavy or rigid bases



Fixed base

Procedure for fixed base isolation:

- ❖ Select the stiffness of the isolation system such that

$$r = \frac{\omega}{\omega_n} > \sqrt{2}$$

- ❖ Minimize the damping of the isolation system
- ❖ Frequency increases and decreases should be fast enough to avoid transient oscillations at resonance due to low damping
- ❖ Be aware of excessive static deflection and lateral instability
- ❖ Use “inertia blocks” if appropriate

Example 1

A hard disk, of mass 1 kg, generates an excitation force at a frequency of 3Hz. If it is supported on a base through a rubber mount, determine the stiffness of the rubber mount to reduce the vibration displacement transmitted to the base by 80%.

Assume negligible ζ and design for $> \sqrt{2}$

$$T_r = 0.2 = \frac{1}{(2\pi f)^2\,\delta_{st}/g - 1}$$

$$\delta_{st} = \frac{g}{(2\pi f)^2}\left(\frac{1}{0.2} + 1\right) = 0.1656$$

But $\delta_{st} = \frac{mg}{k}$ or $k = \frac{mg}{\delta_{st}} = 59.2\ \mathrm{N/m}$

Example 1

At resonance $r = 1$

$$X = \frac{r^2\,(m_0 e)/m}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{(m_0 e)/m}{2\zeta}$$

Given $\zeta = 0.05$, $m_0 = 0.1m$ and measured deflection at resonance to 0.1 m.

Therefore

$$0.1 = \frac{(0.1me)/m}{2\zeta}$$

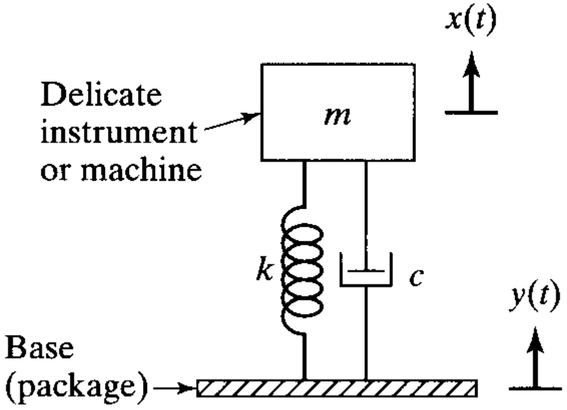
$$e = 2\zeta = 0.1\text{ m;}$$

Desirable to change mass to $m + \Delta m$ so that $X = 0.01$ m; i.e.

$$0.01 = \frac{(0.1me)/(m+\Delta m)}{2\zeta} \quad \text{or} \quad m + \Delta m = 10m$$

$$\Delta m = 9m \ \ \text{(i.e. increase mass by 9 times)}$$

Moving base



- ❖ The isolator is to protect the system against motion of its foundation
- ❖ For base excitation:

- Force transmission is $T_r = \frac{F_T}{kY} = r^2\sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$

- Moving base displacement transmission ratio $T_r = \frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$

Example 2

The seat of a helicopter, with the pilot, weights 1000N and is found to have a static deflection of 10 mm under self-weight. The vibration of the rotor is transmitted to the base of the seat as harmonic motion with frequency 4 Hz and amplitude 0.2 mm (assume no damping).

a) What is the level of vibration felt by the pilot?

b) How can the seat be redesigned to reduce the effect of vibration?

- Mass $m = 1000/9.81 = 101.94$ kg
- Spring constant $k = mg/\delta_{st} = 1000/0.01 = 10000$ N/m
- Natural frequency $\omega_n = \sqrt{k/m} = 31.32$ rad/s or 4.985 Hz
- Frequency ratio $r = \omega/\omega_n = 0.8$
- Amplitude of vibration felt by pilot $X = \frac{Y}{1-r^2} = \frac{0.2}{1-0.8^2} = 0.5616$ mm;
- Max velocity $v_{\text{max}} = 2\pi f X = 14.11$ mm/s;
- Max acceleration $|a_{\text{max}}| = 4\pi^2 f^2 X = 0.355$ mm/s^2;

Example 2

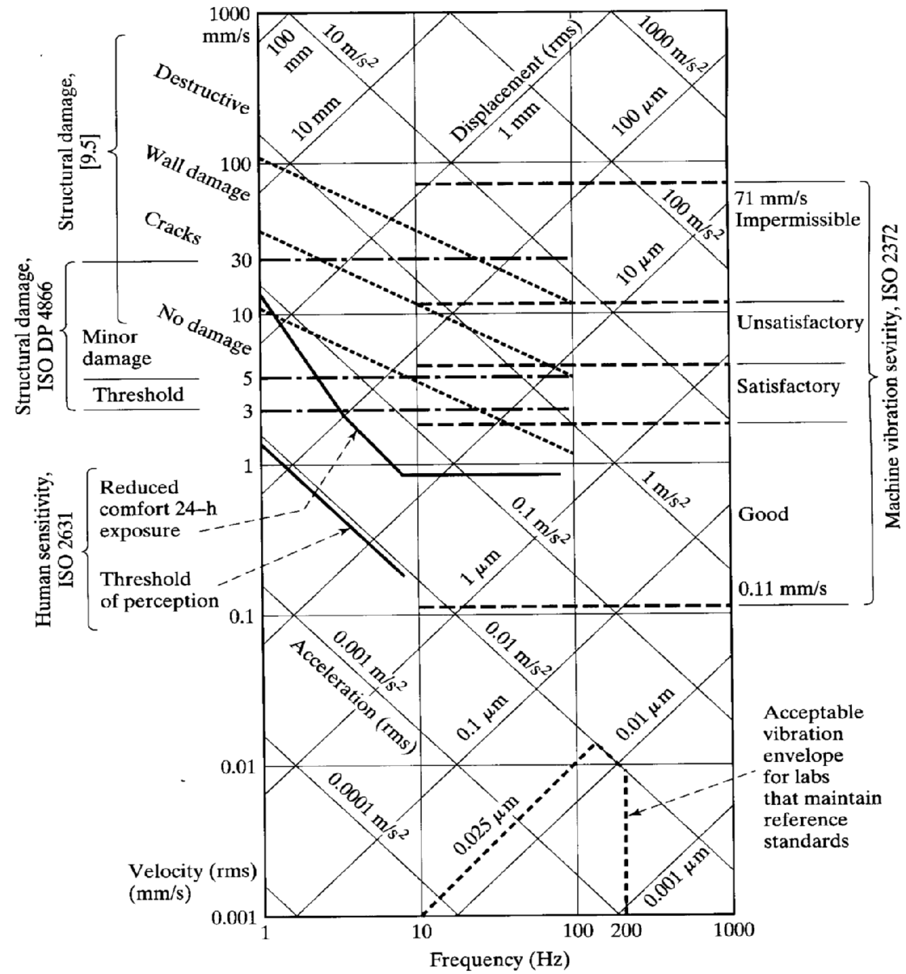
- Max velocity $v_{\max} = 14.11$ mm/s and max acceleration $|a_{\max}| = 0.355$ mm/s^2 at 4 Hz are not acceptable for a comfortable ride.
- Bring $|a_{\max}|$ down to 0.01 m/s^2
- For $a_{\max} = 0.01 = (2\pi f)^2 X$, we need $X = 0.01583$ mm;

$$\frac{X}{Y} = \frac{0.01583}{0.2} = \pm\frac{1}{1-r^2}$$

- We get $r = 3.69$
- $\omega_n = \frac{\omega}{3.69} = \frac{8\pi}{3.6923} = 6.807 = \sqrt{\frac{k}{m}}$
- For $m = 101.94$ kg, we get

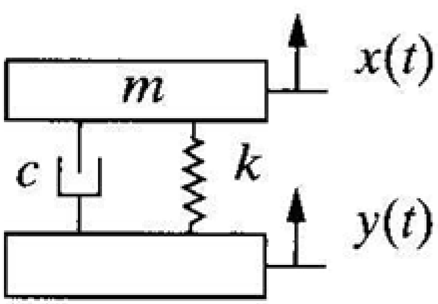
$$k = 4723 \text{ N/m}$$

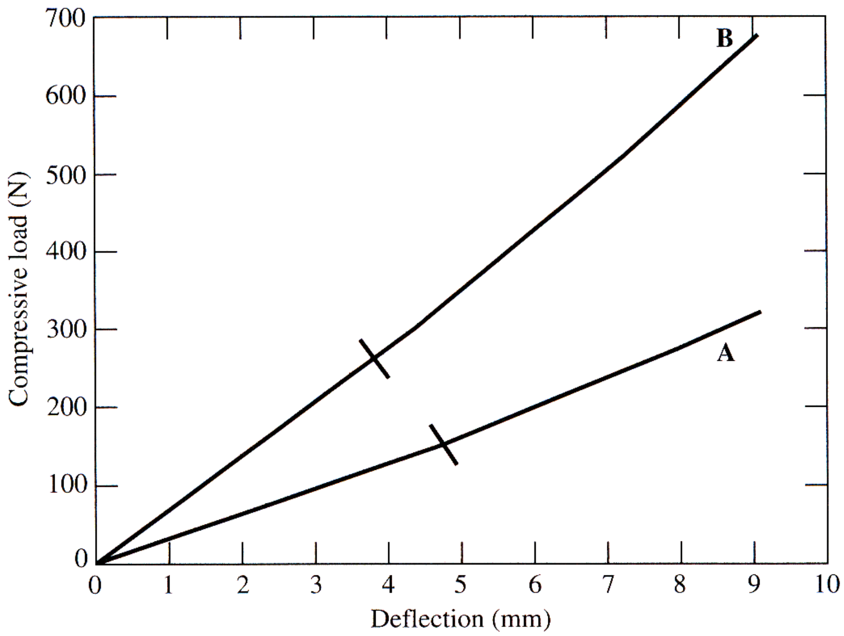
Either use softer material for seat or increase mass of seat



Example 3

A 40 kg instrument is used on a table that vibrates due to nearby machinery. The principle frequency component is 1100 rpm. Determine whether or not the isolator *A* whose load – deflection curve is shown below can used at each of the four corners of the instrument so that no more than 20% of table motion is transmitted to the instrument. The damping ratio for isolator is 0.02.





Example 3

Given displacement transmission ratio $\frac{X}{Y} = \sqrt{\frac{1+(2\varsigma r)^2}{[1-r^2]^2+[2\varsigma r]^2}} = 0.2$

Given damping ratio $\zeta = 0.02$, therefore

$$\sqrt{\frac{1+(0.04r)^2}{[1-r^2]^2+[0.04r]^2}} = 0.2 \text{ which reduces to}$$

$$r^4 - 2.0384r^2 - 24 = 0$$

❖ Solve the equation

$$r^2 = \frac{-(-2.0384) \pm \sqrt{(-2.0384)^2 - 4 \times (-24) \times 1}}{2 \times 1}$$

$$r^2 = 6.023 \quad \text{or} \quad r = 2.454 = \omega/\omega_n$$

❖ The critical input frequency is 1100 rpm

❖ Natural frequency $\omega_n = \omega/r = 2\pi(1100 \div 60)/2.454 = 47$ rad/s

Example 3

The static deflection of the isolator is

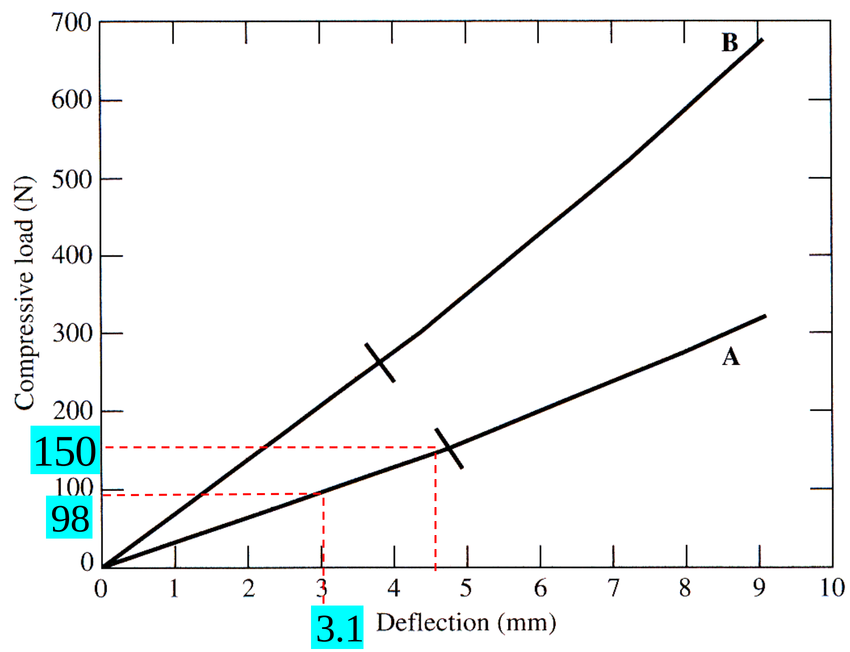
$$\delta_{st} = \frac{g}{\omega_n^2} = \frac{9.81}{47^2} = 4.44 \times 10^{-3}\text{m}$$

The total mass is 40 kg and the load on each of the isolator at the four corners will be $F = \frac{40 \times 9.8}{4} = 98$ N

From the figure, the safe static load for this isolator at 4.44 mm is 150 N

To maintain the T_r the isolator must be mounted on a block weighting 4(150-98) = 208 N and place the isolator between the block and the table

Stiffness of isolators *A* and *B*



Vibration isolation

Vibration isolation