ME1020
Mechanical vibrations

Lecture 7

Balancing rotating machines

Objectives

- Analyze the response of 1DOF vibration system to rotating imbalanced mass including the amplitude ratio, phase shift, and force transmission
- Apply the rotating imbalanced mass analysis to vibration isolation

Introduction

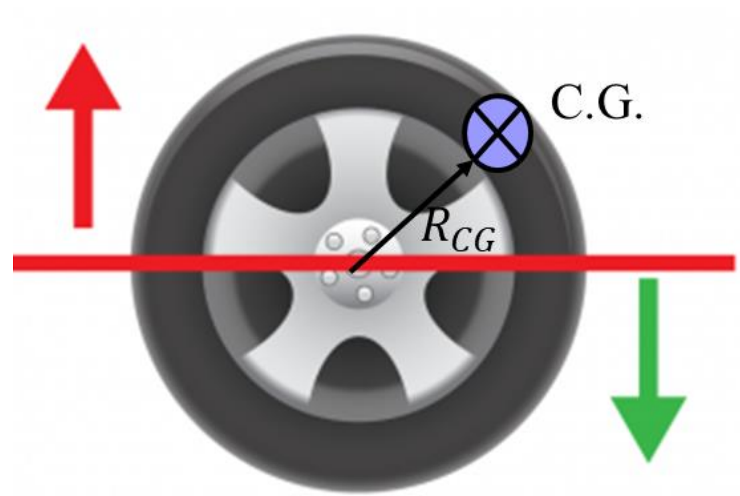
- ❖ Unbalanced rotating machines is a main source of vibration problems
- ❖ Example: Car wheel imbalanced:
- At 60 miles per hour, an average size tire rotates 850 time per minute. At this speed, slight variations in balance, sidewall stiffness or roundness can cause the wheel to literally slam into the pavement 14 times a second. Unchecked, excessive wheel vibration can result in excessive tire wear, as well as damage to suspension and steering components.

Balancing

- ❖ Balancing is the technique of correcting or eliminating unwanted inertia forces and moments in rotating machines
- ❖ Such unwanted inertia forces can
  - Cause vibrations that at times may reach dangerous amplitudes
  - Increase the component stresses and subject bearings to repeated loads that may cause parts to fail prematurely by fatigue
- ❖ In general, it is more economical to produce parts that are not quite exact and then subject them to a balancing procedure than it is to produced such perfect parts that no correction is needed
- ❖ Balancing can be classified as

1) Static balancing
2) Dynamic balancing

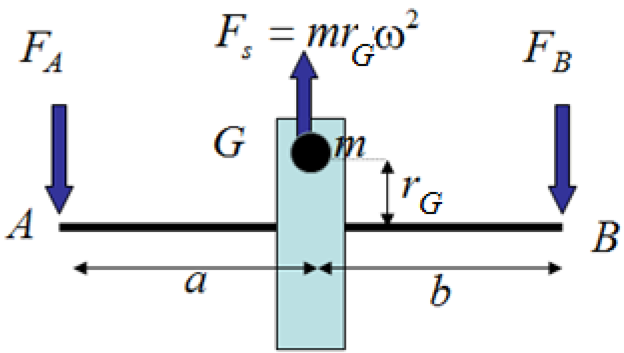
Static balancing



If the wheel is unbalanced, the center of gravity (C.G.) will be offset from the center of the circle. Let the C.G. be located at a radius R_G

- ❖ Rotate the wheel and mark the lowest point when it stops with a chalk
- ❖ Repeat a few times
- ❖ If the chalk marks are coincident, the disk is statically unbalanced. Why?

Static balancing



The amount of unbalance mr_G can be found by rotating the disc at a known speed ω and measuring the reaction F_A (or F_B) at the bearings "*A*" (or "*B*"):

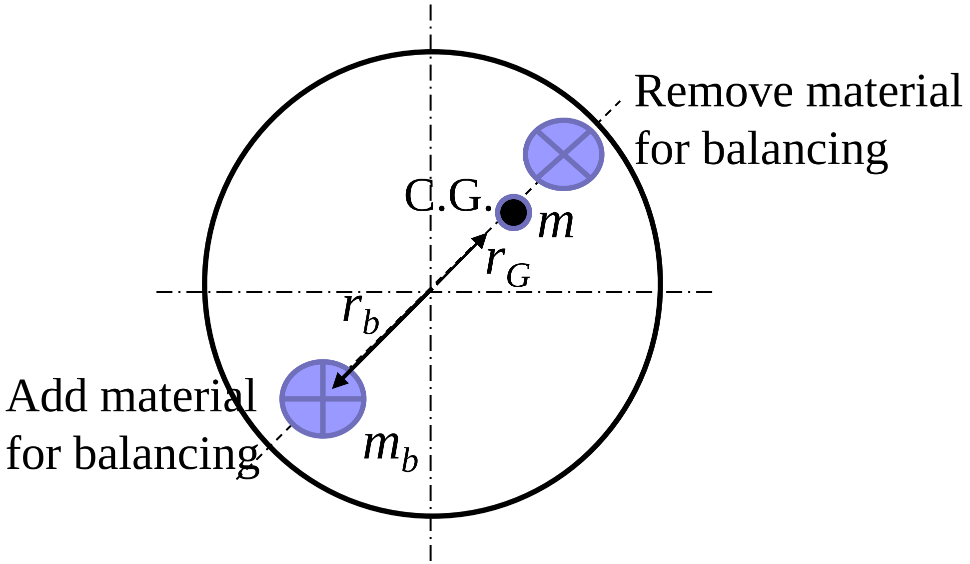
- ❖ Let the centrifugal force for the rotating unbalanced mass be $F_s = mr_G\omega^2$
- ❖ $\sum M_A \Rightarrow F_B(a+b) - F_S a = 0$

$$F_B = \frac{a}{a+b}F_S = \frac{a}{a+b}mr_G\omega^2$$

- ❖ Or $\sum M_B \Rightarrow -F_A(a+b) + F_S b = 0$

$$F_A = \frac{b}{a+b}F_S = \frac{b}{a+b}mr_G\omega^2$$

Static balancing

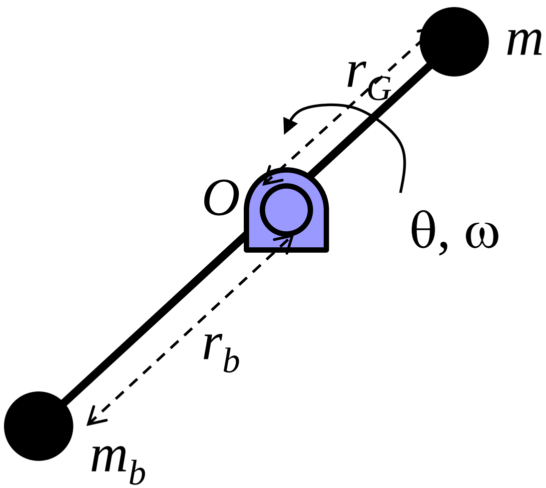


Static unbalance can be corrected by removing material along mr_G or adding material m_b at radius r_b 180^0 from mr_G

In balancing, we are **NOT** concerned absolute values of mass and distance separately. We are concerned with their product $m_b r_b$

The balancing is usually performed at one angular speed ω

Static balancing



To correct the unbalance mass:

- At any time, sum of forces along *x* is

$$mr_G\omega^2\cos(\theta) + m_b r_b \omega^2 \cos(\theta_b) = 0$$

- At any time, sum of forces along *y* is

$$mr_G\omega^2\sin(\theta) + m_b r_b \omega^2 \sin(\theta_b) = 0$$

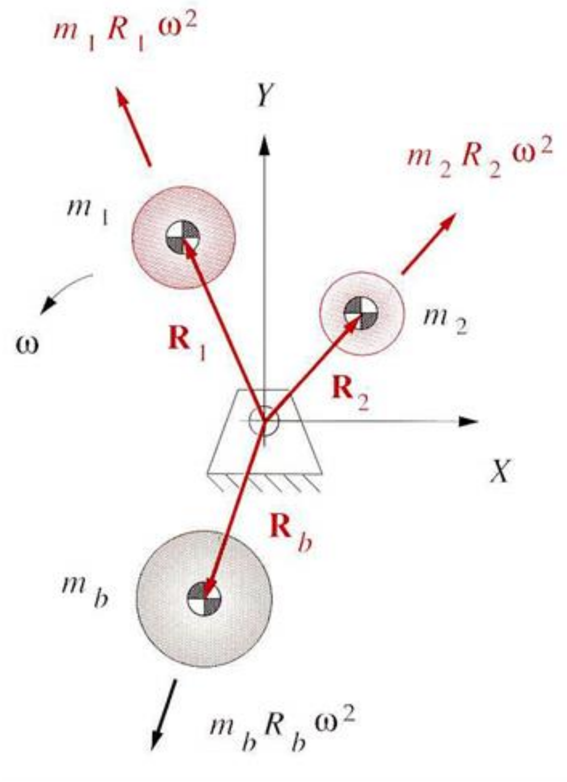
- For $\theta_b = \theta + 180^0$,

$$m_b r_b = mr_G$$

- Note: ω cancels out in the equations

- all angles are measured with respect to the positive *x*-axis with counter-clockwise as positive
- Static balancing will result in zero resultant force in the *x*-*y* plane

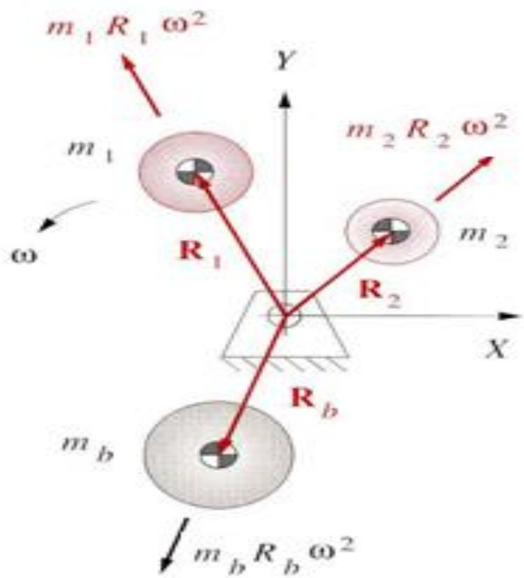
Static balancing

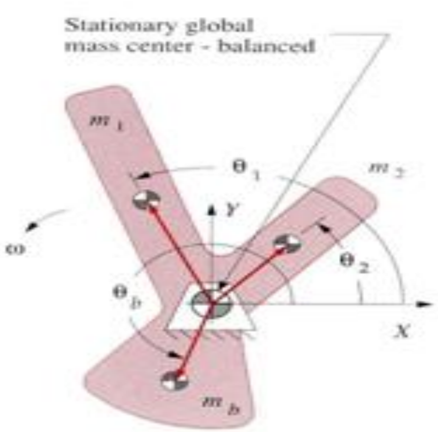


Static balancing can be applied to many unbalanced masses m_i lying in the same plane but at different radii R_i and angles θ_i

The correction is designated as m_b, at R_b and angle θ_b

Static balancing





For static balancing the summation of all inertial forces in the plane equal to zero

- At any time, sum of forces along *x*:

$$m_b R_b \cos\theta_b = -m_1 R_1 \cos\theta_1 - m_2 R_2 \cos\theta_2$$

- At any time, sum of forces along *y*:

$$m_b R_b \sin\theta_b = -m_1 R_1 \sin\theta_1 - m_2 R_2 \sin\theta_2$$

- $\frac{m_b R_b \sin\theta_b}{m_b R_b \cos\theta_b} = \frac{-m_1 R_1 \sin\theta_1 - m_2 R_2 \sin\theta_2}{-m_1 R_1 \cos\theta_1 - m_2 R_2 \cos\theta_2}$

$$\tan\theta_b = \frac{-\sum_1^n m_i R_i \sin\theta_i}{-\sum_1^n m_i R_i \cos\theta_i}$$

Static balancing

- $m_b R_b \cos\theta_b = -m_1 R_1 \cos\theta_1 - m_2 R_2 \cos\theta_2 = -\sum_1^n m_i R_i \cos\theta_i$
- $m_b R_b \sin\theta_b = -m_1 R_1 \sin\theta_1 - m_2 R_2 \sin\theta_2 = -\sum_1^n m_i R_i \sin\theta_i$

$$(m_b R_b \cos\theta_b)^2 + (m_b R_b \sin\theta_b)^2 = \left(-\sum_1^n m_i R_i \cos\theta_i\right)^2 + \left(-\sum_1^n m_i R_i \sin\theta_i\right)^2$$

$$(m_b R_b)^2 = \left(\sum_1^n m_i R_i \cos\theta_i\right)^2 + \left(\sum_1^n m_i R_i \sin\theta_i\right)^2$$

$$m_b R_b = \sqrt{\left(\sum_1^n m_i R_i \cos\theta_i\right)^2 + \left(\sum_1^n m_i R_i \sin\theta_i\right)^2}$$

Example 1

The system shown has the following data: m_1 = 1.2 kg, R_1 = 1.135 m, θ_1 = 113.4^0, : m_2 = 1.8 kg, R_2 = 0.822 m, θ_2 = 48.8^0, and ω = 40 rad/s. Determine the mass m_b, radius R_b and angle θ_b to statically balance the system.

There are 2 unbalanced masses

- ❖ $\sum_1^n m_i R_i \sin\theta_i = 1.2(1.135)\sin 113.4° + 1.8(0.822)\sin 48.8°$

$$\sum_1^2 m_i R_i \sin\theta_i = 1.25 + 1.133 = 2.363$$

- ❖ $\sum_1^n m_i R_i \cos\theta_i = 1.2(1.135)\cos 113.4° + 1.8(0.822)\cos 48.8°$

$$\sum_1^2 m_i R_i \cos\theta_i = -05409 + 0.9746 = 0.4337$$

Example 1

$$\tan\theta_b = \frac{-\sum_1^n m_i R_i \sin\theta_i}{-\sum_1^n m_i R_i \cos\theta_i} = \frac{-2.363}{-0.4337}$$

Note that this is in the third quadrant. $\theta_b = 259.6°$

$$m_b R_b = \sqrt{\left(\sum_1^2 m_i R_i \cos\theta_i\right)^2 + \left(\sum_1^2 m_i R_i \sin\theta_i\right)^2}$$

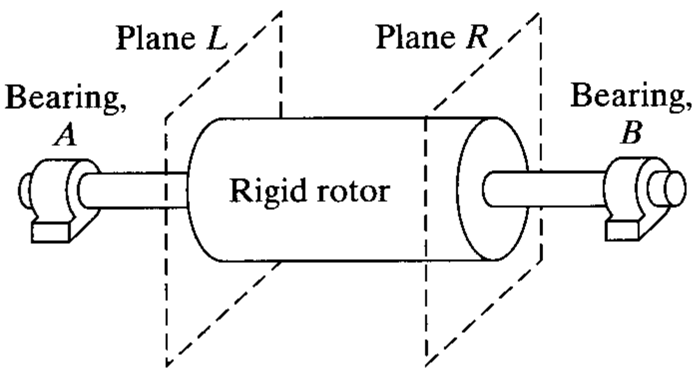
$$m_b R_b = \sqrt{(0.4337)^2 + (\sum_1^n 2.363)^2} = 2.402 \text{ kg-m}$$

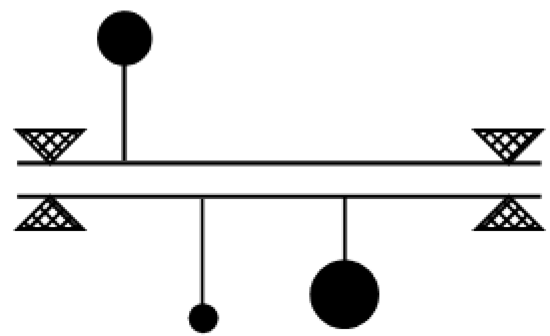
There are many possible combinations of m_b and R_b at $\theta_b = 259.6^0$ that can statically balance the system. One possible solution is to choose $R_b = 0.806$ m and we can then determine the mass m_b needed:

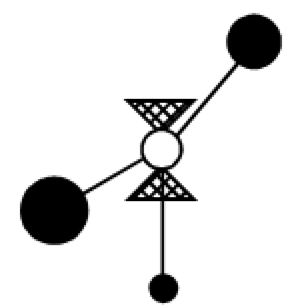
$$m_b = \frac{2.402}{R_b} = \frac{2.402}{0.806} = 2.98 \text{ kg}$$

Dynamic balancing

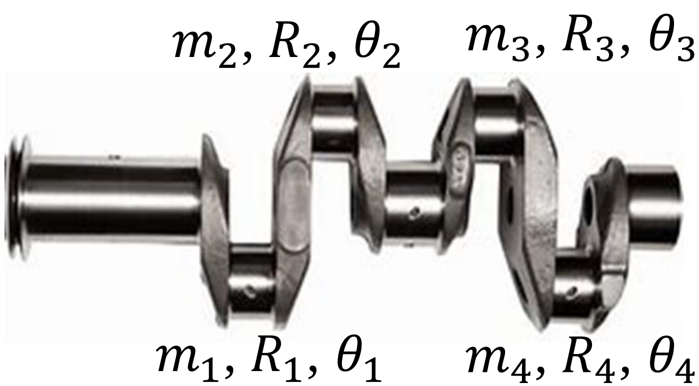
Static balancing assumes the unbalance forces are in one-plane. However in a machine with rotor as shown, unbalance can be anywhere along the length. This will cause forces at the bearings, which is undesirable. Dynamic balancing attempts to reduce the forces at the bearings to zero



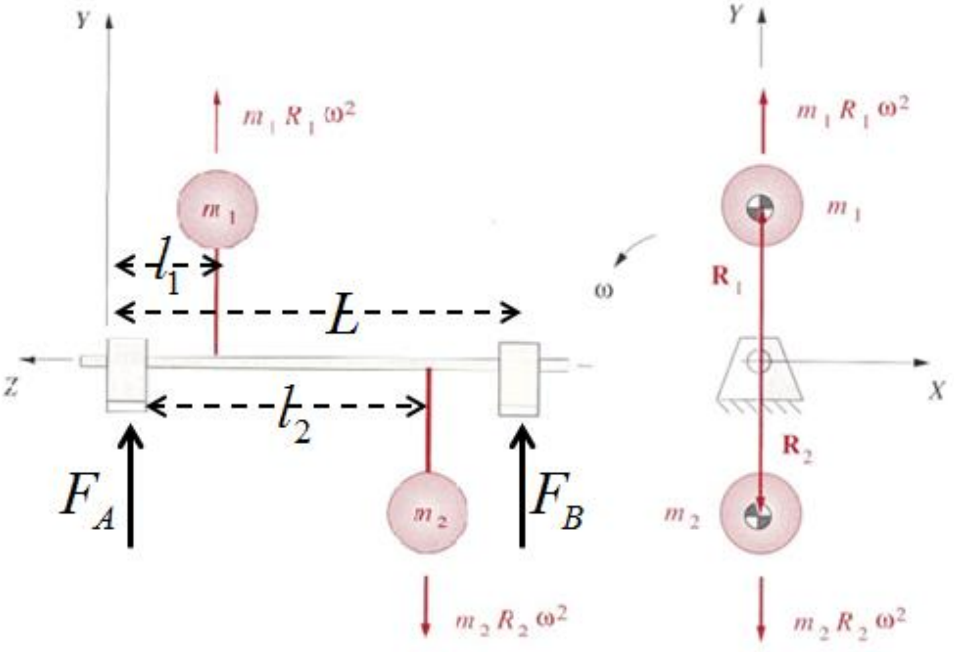




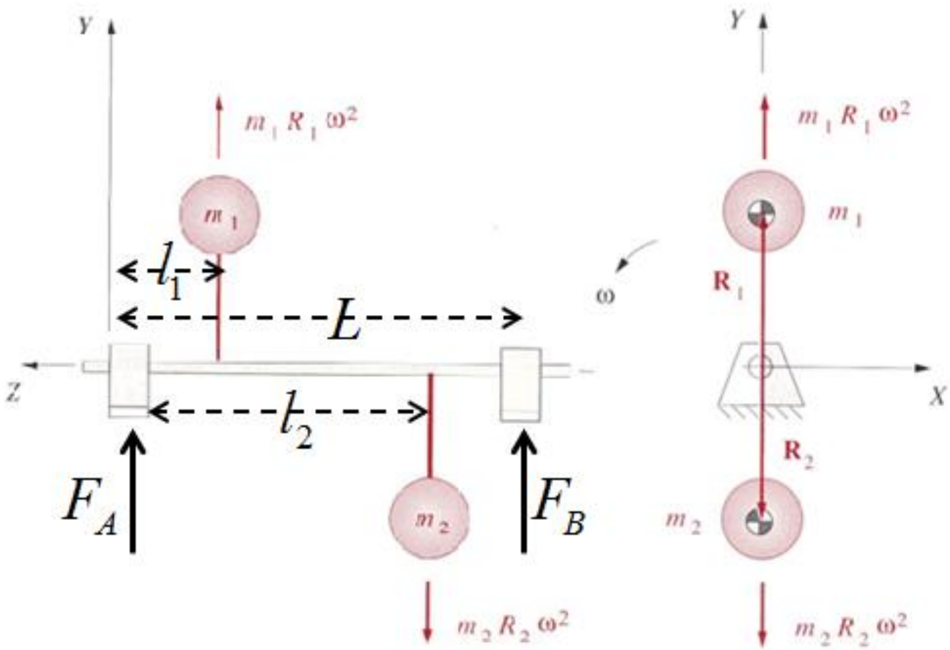
Dynamic balancing



Crankshaft



Dynamic balancing



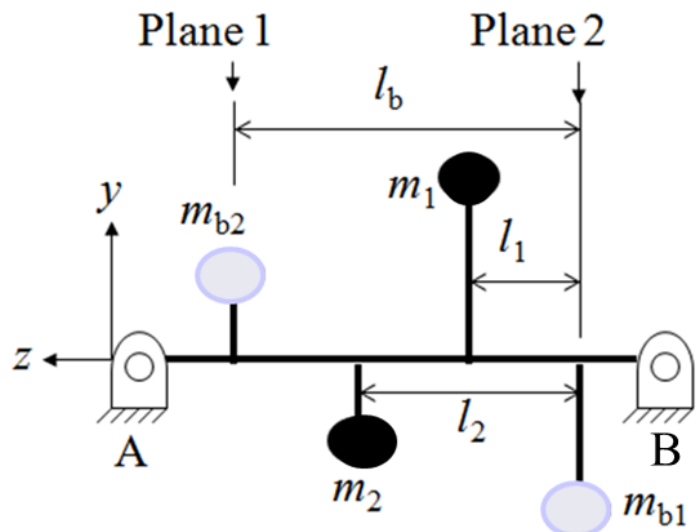
- The system is statically balanced in the *x*-*y* plane with $m_1R_1 = m_2R_2$
- The system is not dynamically balanced, due to unbalanced moments in the *x*-*z* and *y*-*z* planes

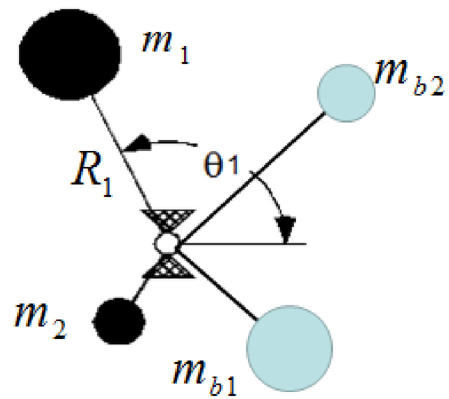
$$\sum M_A \Rightarrow m_1R_1\omega^2 l_1 - m_2R_2\omega^2 l_2 + F_B L = 0$$

$$F_B = \frac{m_2R_2\omega^2 l_2 - m_1R_1\omega^2 l_1}{L}$$

$F_B = 0$ only if $l_1 = l_2$ or the masses are in same plane!

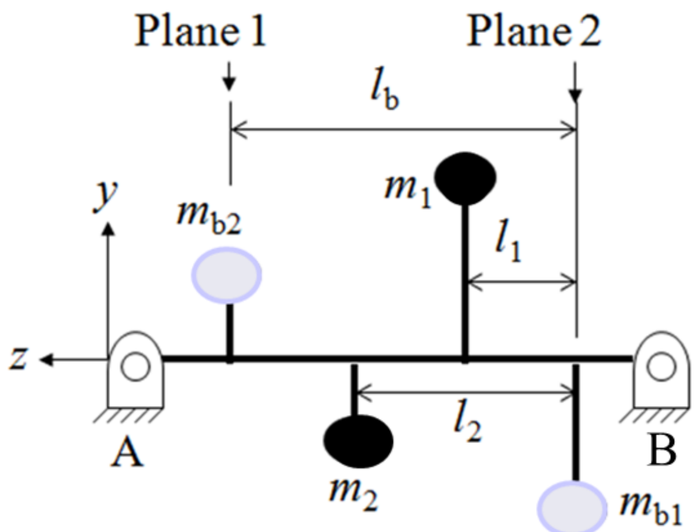
Dynamic balancing

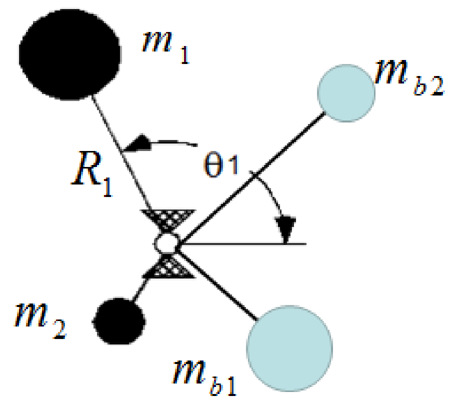




- ❖ A part that is in static balance may still be dynamically unbalanced
- ❖ The centrifugal forces may cause unequal reaction forces at the bearings that change with time and cause a shaking moment
- ❖ For dynamic balancing where unbalance masses are in different planes, we need to put correction masses in two planes (i.e. add two masses (m_{b1}, m_{b2}) located at (r_{b1}, θ_{b1}), (r_{b2}, θ_{b2}) on two different planes "1" and "2" at a distance of l_b apart.

Dynamic balancing





- ❖ Taking moments about plane 2 (with no forces at the bearings):

$$m_1\vec{R}_1\omega^2 l_1 + m_2\overleftarrow{R}_2\omega^2 l_2 + m_{b2}\vec{R}_{b2}\omega^2 l_b = 0$$

$$m_1\vec{R}_1 l_1 + m_2\vec{R}_2 l_2 + m_{b2}\vec{R}_{b2} l_b = 0$$

- ❖ The *x* and *y* components of the above equation are:

$$m_1 R_1 l_1 \cos\theta_1 + m_2 R_2 l_2 \cos\theta_2 + m_{b2} R_{b2} l_b \cos\theta_{b2} = 0$$

$$m_1 R_1 l_1 \sin\theta_1 + m_2 R_2 l_2 \sin\theta_2 + m_{b2} R_{b2} l_b \sin\theta_{b2} = 0$$

Dynamic balancing

- The x and y components of the vector equation can be rewritten as:

$$m_{b2}R_{b2}l_b \cos\theta_{b2} = -m_1R_1l_1\cos\theta_1 - m_2R_2l_2\cos\theta_2 = -\sum_1^n m_iR_i\, l_i\cos\theta_i$$

$$m_{b2}R_{b2}l_b \sin\theta_{b2} = -m_1R_1l_1\sin\theta_1 - m_2R_2l_2\sin\theta_2 = -\sum_1^n m_iR_i\, l_i\sin\theta_i$$

- Note: the equation generalizes to "n" masses
- Combining the 2 equations:

$$\frac{m_{b2}R_{b2}l_b \sin\theta_{b2}}{m_{b2}R_{b2}l_b \cos\theta_{b2}} = \frac{-\sum_1^n m_iR_i\, l_i\sin\theta_i}{-\sum_1^n m_iR_i\, l_i\cos\theta_i}$$

$$\tan\theta_{b2} = \frac{-\sum_1^n m_iR_i\, l_i\sin\theta_i}{-\sum_1^n m_iR_i\, l_i\cos\theta_i}$$

Dynamic balancing

$$m_{b2}R_{b2}l_b\cos\theta_{b2} = -\sum_1^n m_iR_i\, l_i\cos\theta_i$$

$$m_{b2}R_{b2}l_b\sin\theta_{b2} = -\sum_1^n m_iR_i\, l_i\sin\theta_i$$

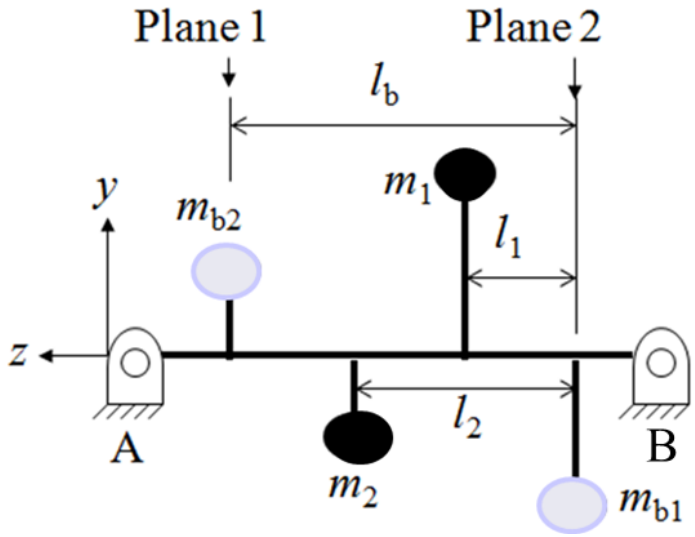
$$(m_{b2}R_{b2}l_b\sin\theta_{b2})^2 + (m_{b2}R_{b2}l_b\cos\theta_{b2})^2 = \left(-\sum_1^n m_iR_i\, l_i\sin\theta_i\right)^2 + \left(-\sum_1^n m_iR_i\, l_i\cos\theta_i\right)^2$$

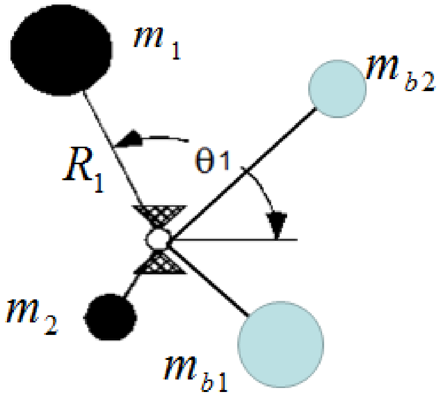
$$(m_{b2}R_{b2}l_b)^2 = \left(\sum_1^n m_iR_i\, l_i\sin\theta_i\right)^2 + \left(\sum_1^n m_iR_i\, l_i\cos\theta_i\right)^2$$

$$m_{b2}R_{b2}l_b = \sqrt{\left(\sum_1^n m_iR_i\, l_i\sin\theta_i\right)^2 + \left(\sum_1^n m_iR_i\, l_i\cos\theta_i\right)^2}$$

- Note: We can specify the plane distance l_b. Then $m_{b2}R_{b2}$ can be found. There are many possible solutions. R_{b2} needs to be specified to determine m_{b2}.

Dynamic balancing





So far we have only determined the corrections at plane 2, i.e. $m_{b2}R_{b2}$ and θ_{b2}
We can use these to find the corrections in plane 1 (i.e. $m_{b1}R_{b1}$ and θ_{b1}) by static balancing, where the x and y components are:

$$m_1R_1\cos\theta_1 + m_2R_2\cos\theta_2 + m_{b1}R_{b1}\cos\theta_{b1} + m_{b2}R_{b2}\cos\theta_{b2} = 0$$
$$m_1R_1\sin\theta_1 + m_2R_2\sin\theta_2 + m_{b1}R_{b1}\sin\theta_{b1} + m_{b2}R_{b2}\sin\theta_{b2} = 0$$

Dynamic balancing

- Rearrange the 2 equations:

$$m_{b1}R_{b1}\cos\theta_{b1} = -m_1R_1\cos\theta_1 - m_2R_2\cos\theta_2 - m_{b2}R_{b2}\cos\theta_{b2}$$

$$m_{b1}R_{b1}\sin\theta_{b1} = -m_1R_1\sin\theta_1 - m_2R_2\sin\theta_2 - m_{b2}R_{b2}\sin\theta_{b2}$$

- These can be simplified to

$$m_{b1}R_{b1}\cos\theta_{b1} = -m_{b2}R_{b2}\cos\theta_{b2} - \sum_1^n m_iR_i\cos\theta_i$$

$$m_{b1}R_{b1}\sin\theta_{b1} = -m_{b2}R_{b2}\sin\theta_{b2} - \sum_1^n m_iR_i\sin\theta_i$$

- Combining the equations:

$$\frac{m_{b1}R_{b1}\sin\theta_{b1}}{m_{b1}R_{b1}\cos\theta_{b1}} = \frac{-m_{b2}R_{b2}\sin\theta_{b2} - \sum_1^n m_iR_i\sin\theta_i}{-m_{b2}R_{b2}\cos\theta_{b2} - \sum_1^n m_iR_i\cos\theta_i}$$

$$\tan\theta_{b1} = \frac{-(m_{b2}R_{b2}\sin\theta_{b2} + \sum_1^n m_iR_i\sin\theta_i)}{-(m_{b2}R_{b2}\cos\theta_{b2} + \sum_1^n m_iR_i\cos\theta_i)}$$

Dynamic balancing

$$m_{b1}R_{b1}\cos\theta_{b1} = -m_{b2}R_{b2}\cos\theta_{b2} - \sum_{1}^{n} m_i R_i \cos\theta_i$$

$$m_{b1}R_{b1}\sin\theta_{b1} = -m_{b2}R_{b2}\sin\theta_{b2} - \sum_{1}^{n} m_i R_i \sin\theta_i$$

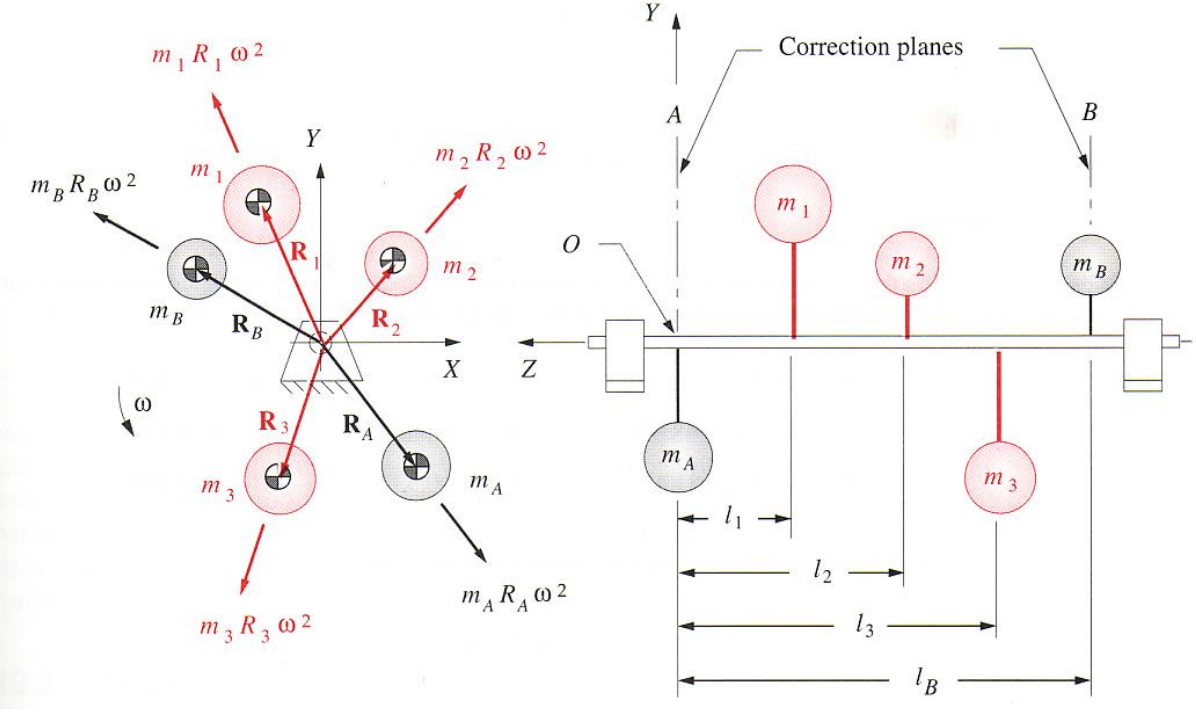
- Combining the equations:

$$(m_{b1}R_{b1}\sin\theta_{b1})^2 + (m_{b1}R_{b1}\cos\theta_{b1})^2$$
$$= \left(-m_{b2}R_{b2}\sin\theta_{b2} - \sum_{1}^{n} m_i R_i \sin\theta_i\right)^2$$
$$+ \left(-m_{b2}R_{b2}\cos\theta_{b2} - \sum_{1}^{n} m_i R_i \cos\theta_i\right)^2$$

$$m_{b1}R_{b1} = \sqrt{\left(m_{b2}R_{b2}\sin\theta_{b2} + \sum_{1}^{n} m_i R_i \sin\theta_i\right)^2 + \left(m_{b2}R_{b2}\cos\theta_{b2} + \sum_{1}^{n} m_i R_i \cos\theta_i\right)^2}$$

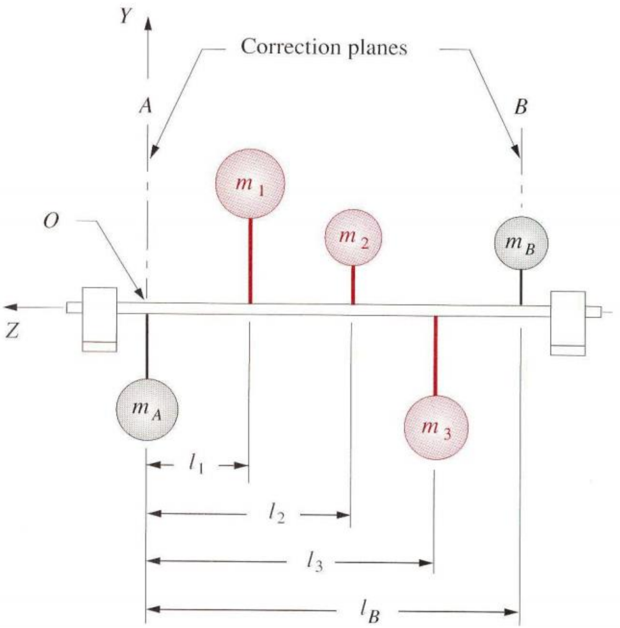
Example 2

The system shown has m_1 = 1.2 kg, m_2= 1.8 kg, m_3 = 2.4 kg, R_1 = 1.135 m at 113.4 deg, R_2 = 0.822 m at 48.8 deg, and R_3 = 1.04 m at 251.4 deg, l_1 = 0.854 m, l_2 = 1.7 m, l_3 = 2.39 m, and l_b = 3.097 m. Perform a dynamical balance on the system.



Example 2

m_1 = 1.2 kg, m_2= 1.8 kg, m_3 = 2.4 kg,
R_1 = 1.135 m at 113.4 deg,
R_2 = 0.822 m at 48.8 deg, and
R_3 = 1.04 m at 251.4 deg,
l_1 = 0.854 m, l_2 = 1.7 m, l_3 = 2.39 m, and
l_b = 3.097 m.



Take moment about plane "*A*" to find m_B, θ_B, R_B:

- $\sum_1^3 m_i R_i\, l_i \cos\theta_i = m_1 R_1 l_1 \cos\theta_1 + m_2 R_2 l_2 \cos\theta_2 + R_3 l_3 \cos\theta_3$

$$\sum\nolimits_1^3 m_i R_i\, l_i \cos\theta_i = -0.4619 + 1.6568 - 1.9027 = -0.7078$$

- $\sum_1^3 m_i R_i\, l_i \sin\theta_i = m_1 R_1 l_1 \sin\theta_1 + m_2 R_2 l_2 \sin\theta_2 + m_3 R_3 l_3 \sin\theta_3$

$$\sum\nolimits_1^3 m_i R_i\, l_i \sin\theta_i = 1.0675 + 1.8926 - 5.6359 = -2.6938$$

Example 2

$$\tan\theta_B = \frac{-\sum_1^3 m_i R_i\, l_i \sin\theta_i}{-\sum_1^3 m_i R_i\, l_i \cos\theta_i} = \frac{2.6938}{0.7078}$$

The angle is in the first quadrant and

$$\theta_B = \tan^{-1}\frac{2.6938}{0.7079} = 75.27^0$$

$$m_B R_B l_B = \sqrt{\left(\sum_1^3 m_i R_i\, l_i \sin\theta_i\right)^2 + \left(\sum_1^3 m_i R_i\, l_i \cos\theta_i\right)^2}$$

$$m_B R_B l_B = \sqrt{(2.6938)^2 + (0.7078)^2} = 2.7853$$

$$m_B R_B = \frac{2.7853}{l_B} = \frac{2.7853}{3.097} = 0.8993 \text{ kg-m}$$

For $m_B = 1$ kg, $R_b = 0.8993$ m (many possible answers)

Example 2

Perform static balancing to find m_A, θ_A, R_A:

$$m_B R_B \sin\theta_B + \sum_1^3 m_i R_i \sin\theta_i = m_B R_B \sin\theta_B + m_1 R_1 \sin\theta_1 + m_2 R_2 \sin\theta_2 + m_3 R_3 \sin\theta_3$$

$$m_B R_B \sin\theta_B + \sum_1^3 m_i R_i \sin\theta_i = 0.8698 + 1.25 + 1.1133 - 2.3656 = 0.8675$$

$$m_B R_B \cos\theta_B + \sum_1^3 m_i R_i \cos\theta_i = m_B R_B \cos\theta_B + m_1 R_1 \cos\theta_1 + m_2 R_2 \cos\theta_2 + m_3 R_3 \cos\theta_3$$

$$m_B R_B \cos\theta_B + \sum_1^3 m_i R_i \cos\theta_i = 0.2286 - 0.5409 + 0.9756 - 0.7961 = -0.1328$$

$$\tan\theta_A = \frac{-(m_B R_B \sin\theta_B + \sum_1^n m_i R_i \sin\theta_i)}{-(m_B R_B \cos\theta_B + \sum_1^n m_i R_i \cos\theta_i)} = \frac{-0.8675}{0.1328}$$

The angle is in the fourth quadrant and

$$\theta_A = \tan^{-1}\frac{-0.8675}{0.1328} = -81.23^0$$

Example 2

$$m_B R_B \sin\theta_B + \sum_1^3 m_i R_i \sin\theta_i = 0.8698 + 1.25 + 1.1133 - 2.3656 = 0.8675$$

$$m_B R_B \cos\theta_B + \sum_1^3 m_i R_i \cos\theta_i = 0.2286 - 0.5409 + 0.9756 - 0.7961 = -0.1328$$

$$m_A R_A = \sqrt{\left(m_B R_B \sin\theta_B + \sum_1^3 m_i R_i \sin\theta_i\right)^2 + \left(m_B R_B \cos\theta_B + \sum_1^3 m_i R_i \cos\theta_i\right)^2}$$

$$m_A R_A = \sqrt{(0.8675)^2 + (-0.1328)^2} = 0.8777 \text{ kg-m;}$$

For $m_A = 1$ kg, $R_A = 0.8777$ m; (many possible solutions)

Static vs dynamic balance

When a shaft is not balanced, a resultant unbalanced force and unbalanced moment exist in the system and rotate with it. For complete balance, both the resultant force and resultant moment must be zero. These two conditions can be considered separately:

- Static balance means that the shaft will be in equilibrium in all positions when at rest under gravity (i.e. resultant force is zero but resultant moment may not be zero)
- Dynamic balance is achieved if both the resultant force and resultant moment are zero. In order to balance dynamically any system of rotating masses, we need to provide two corrections in two different planes

Limits of unbalance

There are many different types of balancing problems as there are many different types of rotating machinery:

- Rotors may be small mass, high speed such as in a dentist’s drill
- Rotors may also have large mass, low speed such as the winding drum in a mine hoist
- “What amount of unbalance is acceptable for the particular application?”
- Guides are available to answer this question; e.g. ISO – 1940 Balancing Quality of Rotating Rigid Bodies

Balancing quality examples

Quality Grade G (ISO-1940)	Example with service speed (RPM)		Equivalent CG eccentricity (μm)
0.4	Reference Gyro	24000	0.15
1	Precision electric motors	6000	1.6
2.5	Fractional HP electric motors	3000	8
6.3	Large electric motors	1500	40
40	Vehicle engine crankshaft	6000	60

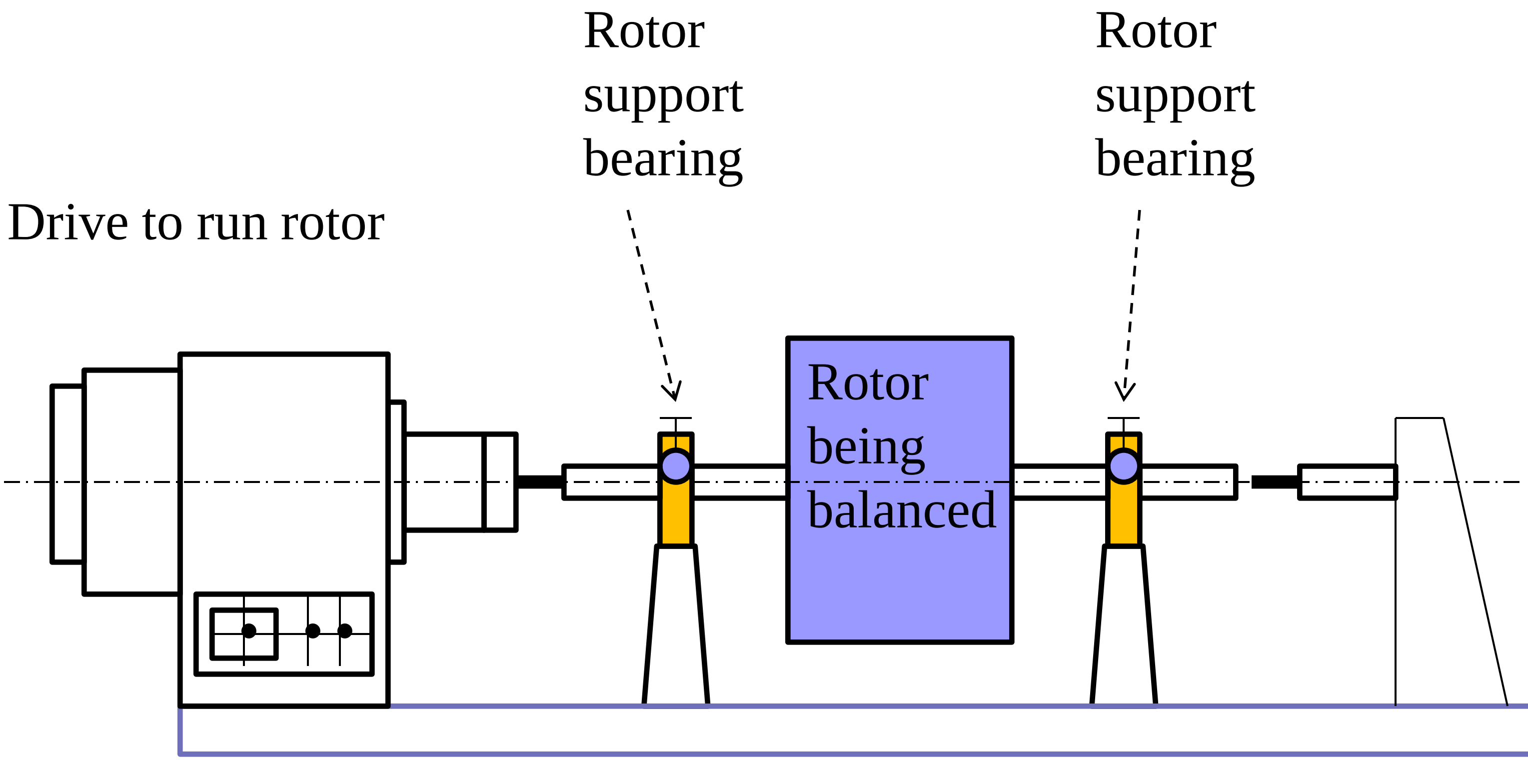
Practical balancing techniques

How to measure the unbalance and then add mass to, or subtract mass from, appropriate points on the rotor to bring the unbalance within limits?

There are many types of balancing machines available. Basically:

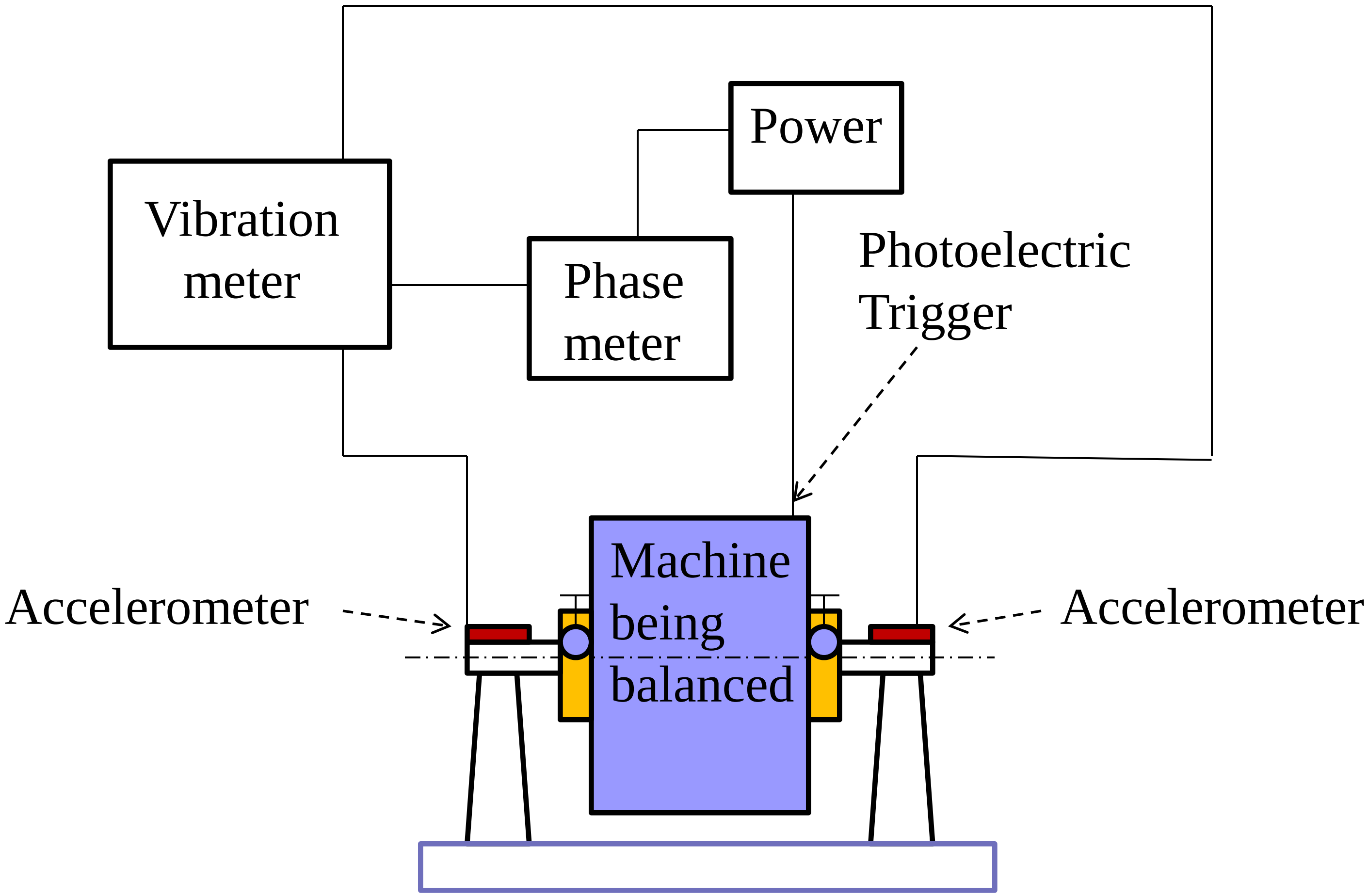
- ❖ The rotor is supported in bearings
- ❖ The rotor is rotated and the dynamic reactions at the bearings are measured
- ❖ From the dynamic reactions, the computer calculates the unbalance
- ❖ The size and location of the balancing mass that must be added to correct the unbalance are then determined

Practical balancing techniques



Practical balancing techniques

The bearing vibration is measured to determine the correction mass and location



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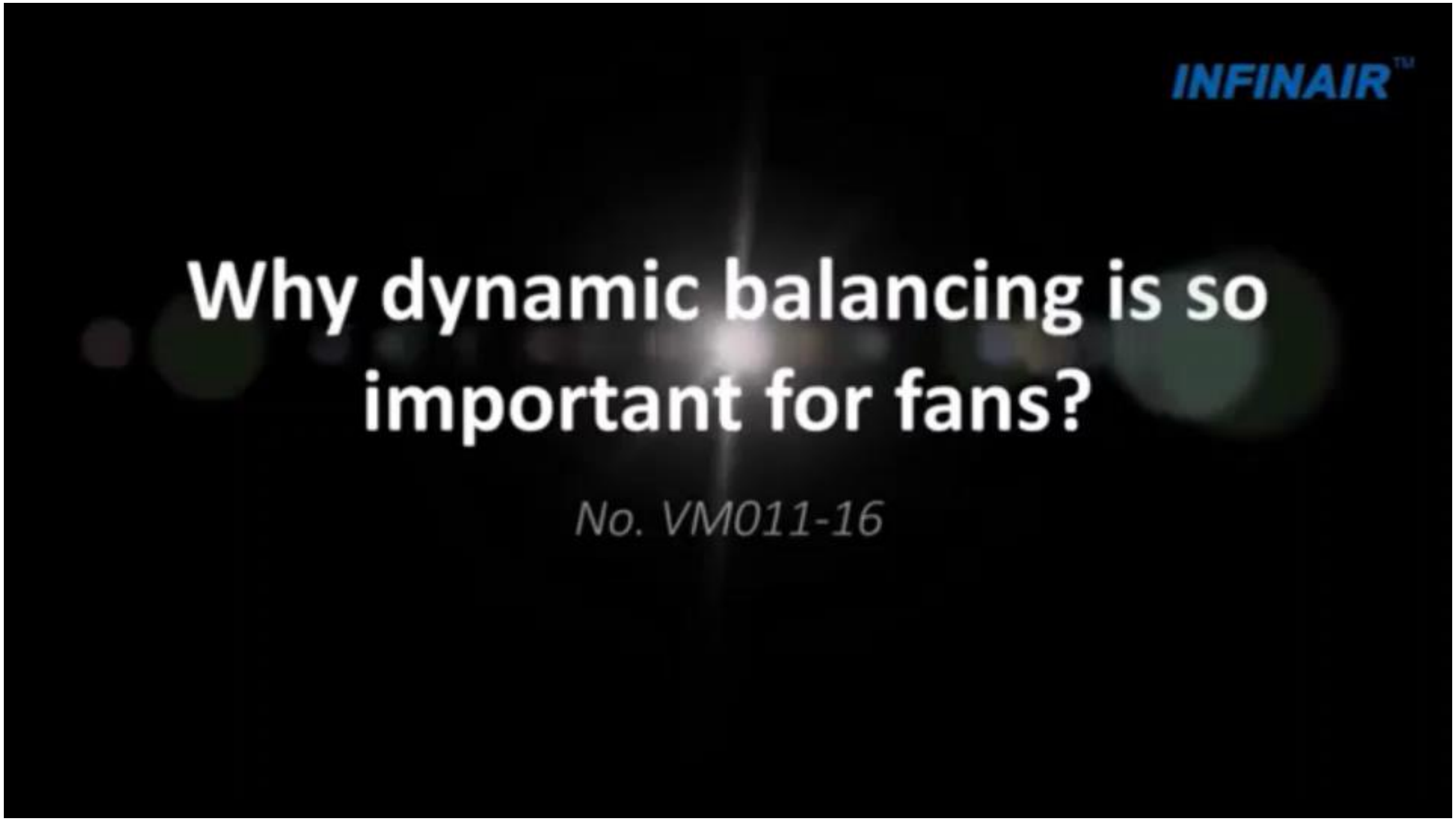
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